# The Flea Collar Dilemma

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# 1 Abstract

When owning a retail business, it's important to have an efficient method of stocking products. If an owner doesn't take care to make sure they have a product in stock, they face the consequences of losing current sales, possibly customers which will in turn effect future sales, and profit. I've taken data from a local veterinary clinic that sells flea collars. I've found that this clinic sells an average of 5 collars a week, and each box that they order contains 7 collars. Their current method of stocking the flea collars is to wait until they are completely out of the collar to place a new order. In this paper I will analyze their current stocking methods, and attempt to find a different stocking method to reduce their potential loss of profit.

## 2 Background

The veterinary clinic I chose is in a somewhat small town and they are the only veterinary clinic servicing the area. The fact that they are regularly running out of flea collars is a big problem not only for the company that is losing potential sales, but for the community they serve as well. Fleas are a major problem for animals, causing numerous health conditions. Animals can contract tapeworms if they happen to swallow one of their fleas. Tapeworms are parasites that live in your pet's intestinal tract, and can rob them from vital nutrients. If an animal has enough fleas, they can become anemic from the loss of blood that results from the fleas biting them. The most dangerous possibility is an animal developing a Bartonella infection, which can quickly lead to death. Fleas can also follow your pet into your house and become a nuisance to your entire family. It's for these reasons that flea control is important for every community.

## 3 Analysis

I decided the best method of approach was implementing a Markov Chain. To begin with, I used a Poisson distribution to find the initial probabilities of the demand of the collars. This formula is

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

Where $\lambda$ is the average number of collar sales in a week, and x is the actual number of collars. We know the clinic sells 5 collars a week on average, and that there are 7 collars in each box. So for our analysis, we will use this distribution to find the probability of the demand of 0-7 collars, and the probability that the demand will exceed the 7 collars.

$$P(D_n = 0) = 0.0067$$

$$P(D_n = 1) = 0.0337$$

$$P(D_n = 2) = 0.0842$$

$$P(D_n = 3) = 0.1404$$

$$P(D_n = 4) = 0.1755$$

$$P(D_n = 5) = 0.1755$$

$$P(D_n = 6) = 0.1462$$

$$P(D_n = 7) = 0.1044$$

$$P(D_n > 7) = 0.1334$$

Now we need to consider the possible outcomes for each stock value at the beginning of the week. We know the vet only places an order for more collars if he completely runs out. Therefore when a new week begins, the options for the inventory range from 1-7. We will denote the current week as $S_n$, and we want to consider all of the stock options for the next week.

Given $S_n = 1$, we know that we either sell that one, or don't sell any at all. In the first case we will start the next week with a new box of 7 collars. Otherwise we begin $S_{n+1}$ with only one collar. There is not a way for us to end up with 2, 3, 4, 5, or 6 collars. Continuing this process for each of the remaining stock options we get the following values.

If $S_n = 2$, then $S_{n+1} = 1, 2, 7$

If $S_n = 3$, then $S_{n+1} = 1, 2, 3, 7$

If $S_n = 4$, then $S_{n+1} = 1, 2, 3, 4, 7$

If $S_n = 5$, then $S_{n+1} = 1, 2, 3, 4, 5, 7$

If $S_n = 6$, then $S_{n+1} = 1, 2, 3, 4, 5, 6, 7$

If $S_n = 7$, then $S_{n+1} = 1, 2, 3, 4, 5, 6, 7$

We now want to consider each of these situations, and we will do so by creating a matrix of each of the probabilities. For these values the probability matrix is

$$\begin{bmatrix} 0.0067 & 0 & 0 & 0 & 0 & 0 & 0.9933 \\ 0.0337 & 0.0067 & 0 & 0 & 0 & 0 & 0.9596 \\ 0.0842 & 0.0337 & 0.0067 & 0 & 0 & 0 & 0.8754 \\ 0.1404 & 0.0842 & 0.0337 & 0.0067 & 0 & 0 & 0.7350 \\ 0.1755 & 0.1404 & 0.0842 & 0.0337 & 0.0067 & 0 & 0.5595 \\ 0.1755 & 0.1755 & 0.1404 & 0.0842 & 0.0337 & 0.0067 & 0.3840 \\ 0.1462 & 0.1755 & 0.1755 & 0.1404 & 0.0842 & 0.0337 & 0.2445 \end{bmatrix}$$

Using this matrix, we can find what's called the steady state vector, which can help us predict what will happen over a long period of time. I used an online calculator for this matrix and found the steady state vector to be

$$\begin{bmatrix} 0.115 & 0.114 & 0.1 & 0.073 & 0.041 & 0.119 & 0.437 \end{bmatrix}$$

We can now use this vector to predict the future of the veterinarian's stocking procedures. The formula to see the probability that the demand will exceed the supply is

$$Pr(D_n > S_n) = \sum_{i=1}^{7} (Pr(D_n > S_n | X_n = i))(Pr(X_n = i))$$

Filling in our data, we get

$$Pr(D_n > S_n) = 0.9596(0.115)+0.8754(0.114)+0.735(0.1)+0.5595(0.073)+0.384(0.041)+$$

$$0.2378(0.119) + 0.1334(0.437) = 0.4268311$$

This means that the demand for the flea collars will exceed the supply 42.68% of the time, and I would highly recommend the clinic change their current stocking method. I also considered the situation where he changes his ordering methods to ordering every time his stock is 5 or less. The initial probabilities are the same, but we have a new probability matrix and steady state vector.

The new probability matrix is

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0.0067 & .9933 \\ 0 & 0 & 0 & 0 & 0 & 0.0337 & 0.9963 \end{bmatrix}$$

Again using an online calculator, I found the new steady state vector to be

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.033 & 0.967 \end{bmatrix}$$

Substituting these new values into our formula to see when the demand exceeds the supply, we get

$$Pr(D_n > S_n) = 0.2378(0.033) + 0.1334(0.967) = 0.1368$$

This new method reduces the probability the supply will be less than the demand to 13.68%. While this is a vast improvement from the 42.68%, I would encourage the veterinarian to consider ordering two boxes at a time to further reduce his supply problems.

# 4 Conclusion

We have analyzed the data given to us by the veterinary clinic and have decided that their current procedure of waiting until they run out of flea collars to reorder them is not beneficial for them, nor the community they serve. We found the probability that the demand for the collars would exceed the supply to be 42.68%. Knowing that he profits $25 for each collar leads us to conclude that he could possibly maximize his profit an additional $4,858 by selling an additional 194 collars in a year. We also found that changing up the method of stocking can help reduce the amount of potential lost sales. By ordering a new box of collars every time the inventory is 5 or less, we reduce the probability that the demand exceeds the supply to 13.68%. I personally do not believe that this is a small enough probability so I would advise the owner to consider ordering more than one case of collars at a time.