

ACME: Automata with Counters, Monoids and Equivalence

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What?

- An algebraic structure with two operations: a binary composition and a unary operator \sharp ,
- Generalizes the transition monoid of a non-deterministic automaton to two weighted settings.

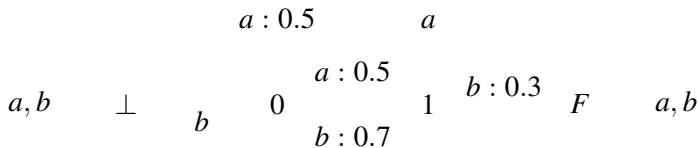
Where? When?

- First appeared in the Theory of Regular Cost Functions (Colcombet 2009),
- Later used for Probabilistic Automata (F., Gimbert, Oualhadj 2012).

$$\begin{array}{ccccccc}
 & & a & & a & & \\
 a, b & \perp & b & 0 & a & 1 & b & F & a, b \\
 & & & & b & & & &
 \end{array}$$

$$\langle a \rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \langle b \rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$I \cdot \langle u \rangle \cdot F = 1 \quad \text{if and only if} \quad u \text{ is accepted.}$$



$$\langle a \rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \langle b \rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0.3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$I \cdot \langle u \rangle \cdot F = \mathbb{P}_{\mathcal{A}}(u)$$

$$\begin{array}{ccccccc}
 & & a : +1 & & a & & \\
 a, b & \perp & & a : r & & b & F \quad a, b \\
 & & b : +1 & 0 & 1 & & \\
 & & & b : r & & &
 \end{array}$$

$$\langle a \rangle = \begin{pmatrix} 0 & \perp & \perp & \perp \\ \perp & 1 & r & \perp \\ \perp & \perp & 0 & \perp \\ \perp & \perp & \perp & 0 \end{pmatrix} \quad \langle b \rangle = \begin{pmatrix} 0 & \perp & \perp & \perp \\ 1 & \perp & \perp & \perp \\ \perp & r & \perp & 0 \\ \perp & \perp & \perp & 0 \end{pmatrix}$$

$$I \cdot \langle u \rangle \cdot F = \mathcal{A}(u)$$

Consider either the rational semiring $(\mathbb{Q}, +, \times)$ or the tropical semiring $(\mathbb{N} \cup \{\infty\}, \min, +)$:

- An automaton \mathcal{A} is given by a matrix $\langle a \rangle$ for each letter $a \in A$,
- We would like to finitely represent $\{\langle u \rangle \mid u \in A^*\}$.

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Intuitively, $\langle u \rangle^\sharp$ represents $\lim_n \langle u^n \rangle$.

Definition

The Stabilization Monoid of \mathcal{A} is the closure of $\{\langle a \rangle \mid a \in A\}$ under both operators.

The Stabilization Monoid of \mathcal{A} contains a lot of informations about \mathcal{A} !

***B*-Automata**

- Decide whether a *B*-automaton is bounded,
- Decide whether two *B*-automata are equivalent.

Probabilistic Automata

- Decide whether a probabilistic automaton has (probably) value 1,
- Decide whether a probabilistic automaton is leaktight.

The end.



Thank you for your attention!