

Boundedness games

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Highlights, September 19th, 2013

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This talk is about our *joint* effort to understand boundedness games.

Motivation: expressing boundedness properties



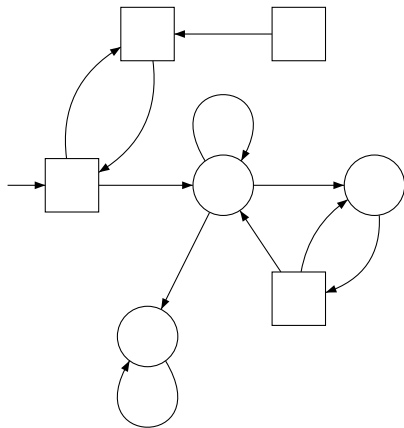
$\text{MSO} + \mathbb{U}$



cost MSO

A lot is known, and even more is not known about those two logics!

Definition of boundedness games

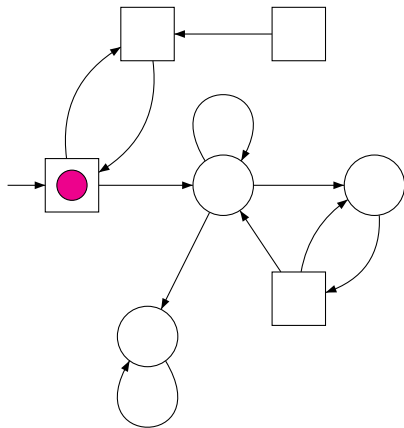


controlled by Eve

5

controlled by Adam

Definition of boundedness games

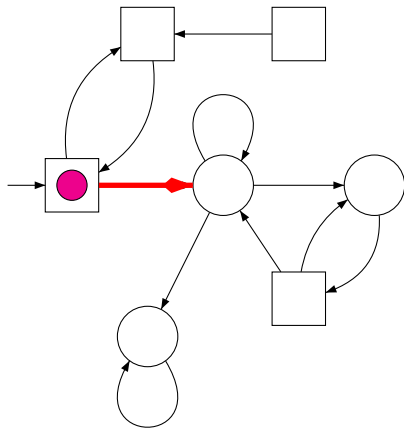


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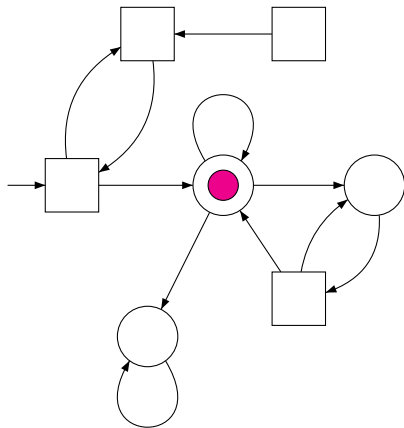


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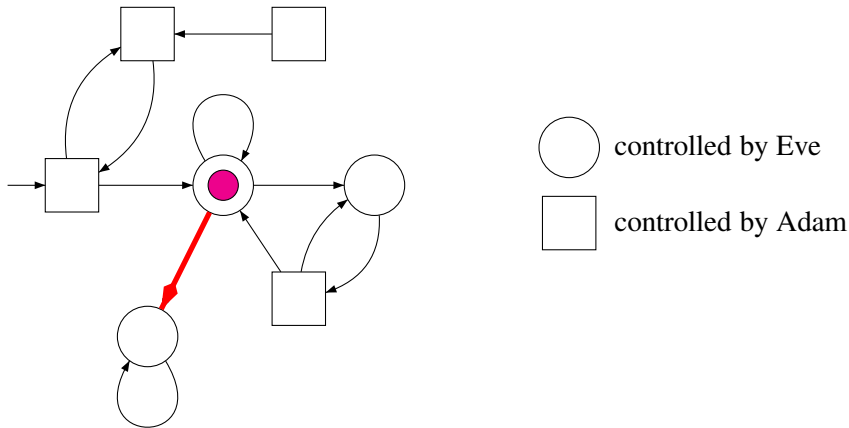


controlled by Eve

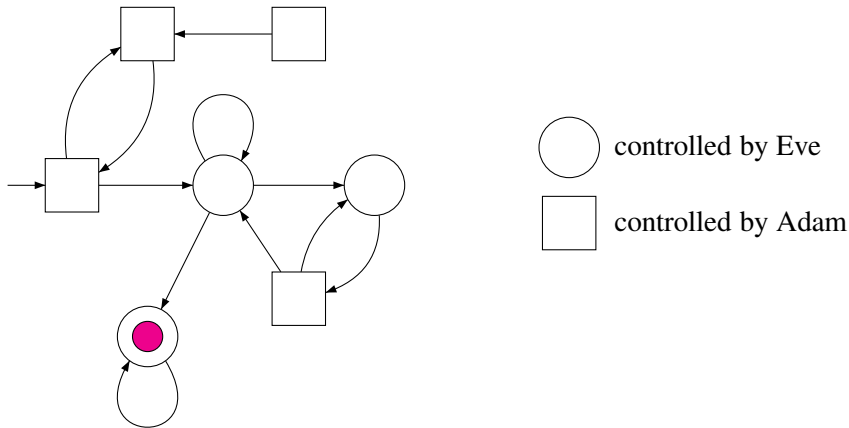


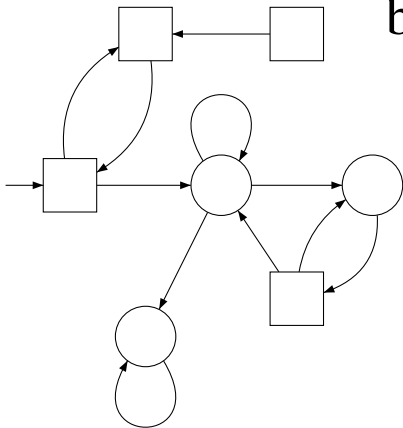
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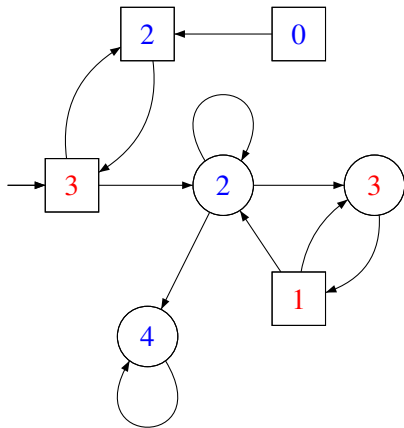
Definition of boundedness games





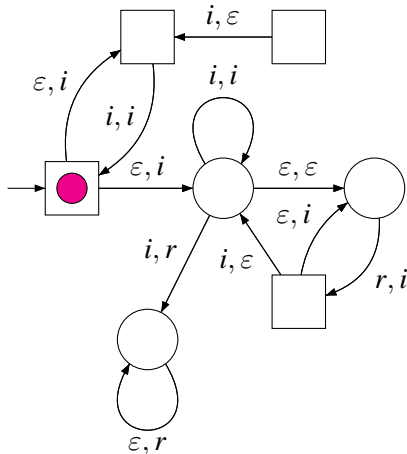
boundedness condition:

parity
and
all counters
are bounded



parity condition:
the minimal priority
seen infinitely often
is even

Definition of boundedness games



$$c_1 = 0$$

$$c_2 = 0$$

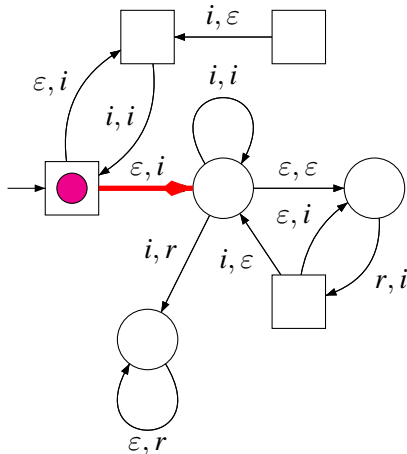
ϵ : nothing

i : increment

r : reset

Definition of boundedness games

2



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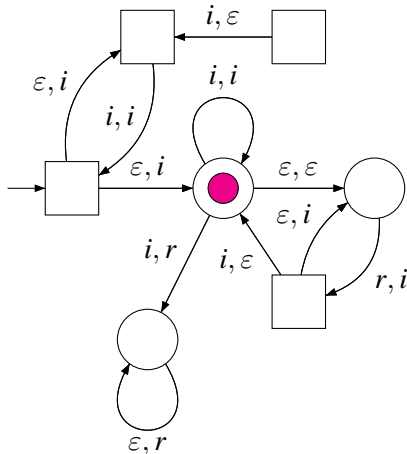
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Definition of boundedness games

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$$c_1 = 0$$

$$c_2 = 1$$

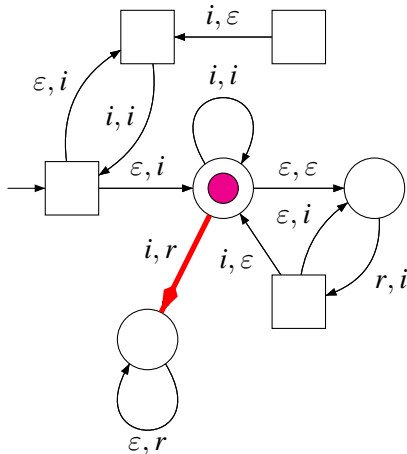
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Definition of boundedness games

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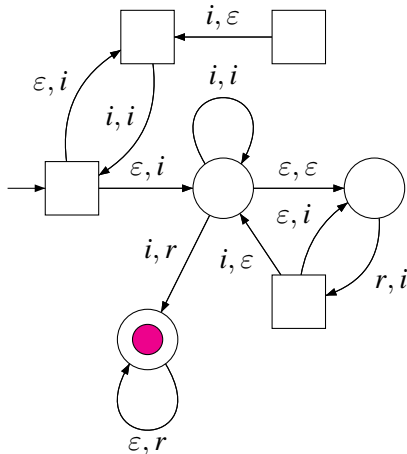
$$c_2 = 1$$

ϵ : nothing

i : increment

r : reset

Definition of boundedness games



$$c_1 = 1$$

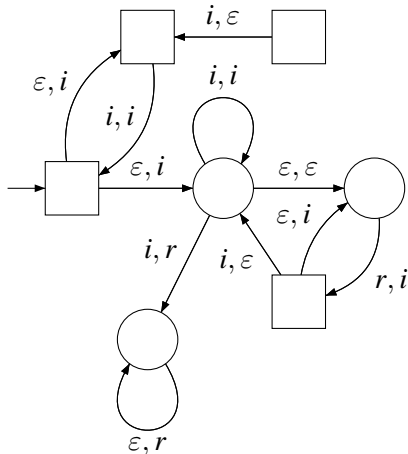
$$c_2 = 0$$

ϵ : nothing

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r : reset

Definition of boundedness games



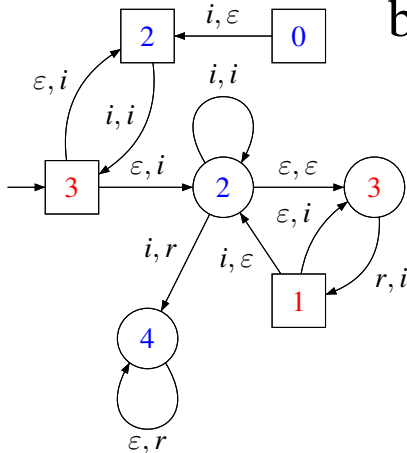
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boundedness condition:

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Quantification

Eve wins means:



$\exists \sigma$ (strategy for Eve),
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non-uniform
(MSO + \mathbb{U})



$\exists \sigma$ (strategy for Eve),
 $\exists N \in \mathbb{N}$,
 $\forall \pi$ (paths),

uniform
(cost MSO)

Research questions and some answers



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 - ↗ Non-uniform quantifications, parity games with cost [F. and Zimmermann, 2012].
- When does Eve has finite-memory winning strategies?
 - ↗ Uniform quantifications, the Büchi case over infinite chronological arenas [Vanden Boom, 2011].
 - ↗ Uniform quantifications, the parity case over thin tree arenas [F., Horn, Kuperberg, Skrzypczak, unpublished].

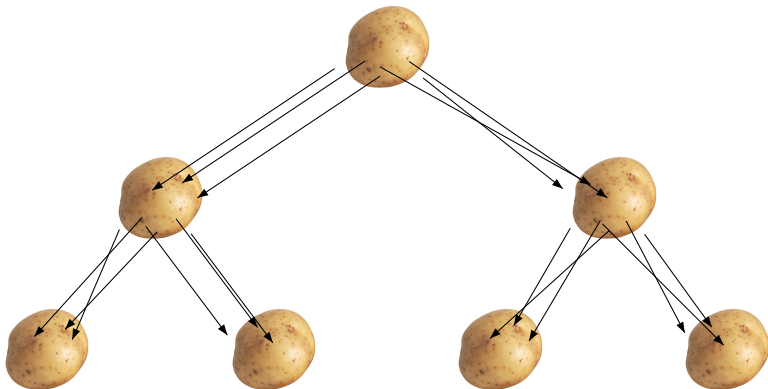
Why finite-memory strategies?



Thomas Colcombet's habilitation:

Conjecture 9.3. *Les objectifs $\mathsf{hB} \wedge \mathsf{parité}$ et $\neg \mathsf{B} \wedge \mathsf{parité}$ sont à \approx -mémoire finie, sur toutes les arènes/sur les arènes chronologiques/sur les arènes «arborescentes».*

Existence of finite-memory strategies in (some) boundedness games
 \implies Decidability of cost MSO over infinite trees
 \implies Decidability of the index of the non-deterministic Mostowski's hierarchy (open for 40 years)!



Theorem (F., Horn, Kuperberg, Skrzypczak)

The B-part of Colcombet's conjecture holds for thin tree arenas!