ACME: Automata with Counters, Monoids and Equivalence

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An algorithmic back-end: Stabilization Monoids

What?

- An algebraic structure with two operations: a binary composition and a unary operator $\sharp$,
- Generalizes the transition monoid of a non-deterministic automaton to two weighted settings.

Where? When?

- First appeared in the Theory of Regular Cost Functions (Colcombet 2009),
- Later used for Probabilistic Automata (F., Gimbert, Oualhadj 2012).

Non-deterministic automata

a a

a

a, b $\perp$ b 0 1 b F a, b

b

$$\langle a \rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \langle b \rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$I \cdot \langle u \rangle \cdot F = 1$ if and only if u is accepted.

Probabilistic automata

$a : 0.5$ a

$a : 0.5$

a, b $\perp$ b 0 1 $b : 0.3$ F a, b

$b : 0.7$

$$\langle a \rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \langle b \rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0.3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$I \cdot \langle u \rangle \cdot F = \mathbb{P}_{\mathcal{A}}(u)$$

B-automata

$a : +1$ a

$a : r$

a, b $\perp$ $b : +1$ 0 1 b F a, b

$b : r$

$$\langle a \rangle = \begin{pmatrix} 0 & \perp & \perp & \perp \\ \perp & 1 & r & \perp \\ \perp & \perp & 0 & \perp \\ \perp & \perp & \perp & 0 \end{pmatrix} \qquad \langle b \rangle = \begin{pmatrix} 0 & \perp & \perp & \perp \\ 1 & \perp & \perp & \perp \\ \perp & r & \perp & 0 \\ \perp & \perp & \perp & 0 \end{pmatrix}$$

$$I \cdot \langle u \rangle \cdot F = \mathcal{A}(u)$$

Computing in Infinite Semirings

Consider either the rational semiring $(\mathbb{Q}, +, \times)$ or the tropical semiring $(\mathbb{N} \cup \{\infty\}, \min, +)$:

- An automaton $\mathcal{A}$ is given by a matrix $\langle a \rangle$ for each letter $a \in A$,
- We would like to finitely represent $\{\langle u \rangle \mid u \in A^*\}$.

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- a binary composition law: matrix multiplication,
- a stabilization unary operator $\sharp$.

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Intuitively, $\langle u \rangle^\sharp$ represents $\lim_n \langle u^n \rangle$.

Stabilization Monoid of an Automaton

Definition

The Stabilization Monoid of $\mathcal{A}$ is the closure of $\{\langle a \rangle \mid a \in A\}$ under both operators.

The Stabilization Monoid of $\mathcal{A}$ contains a lot of informations about $\mathcal{A}$!

Using the Stabilization Monoid

B-Automata

- Decide whether a B-automaton is bounded,
- Decide whether two B-automata are equivalent.

Probabilistic Automata

- Decide whether a probabilistic automaton has (probably) value 1,
- Decide whether a probabilistic automaton is leaktight.

The end.

Thank you for your attention!