The surprizing complexity of generalized reachability games

GAMES’2010 workshop

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Outline

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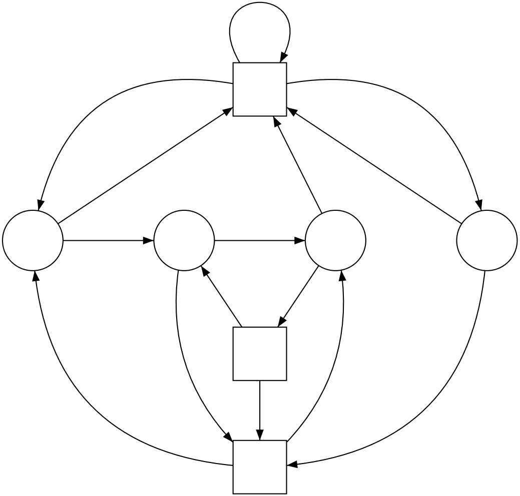
1. Generalized reachability games
   - Games
   - Reachability
   - Generalized
2. Complexity
   - PSPACE-hardness: encoding QBF
   - Memory requirements

Games

Players

Two players: **Eve** and **Adam**.

Playing



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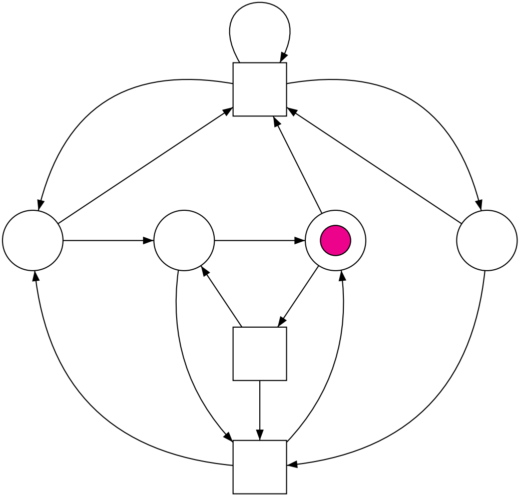

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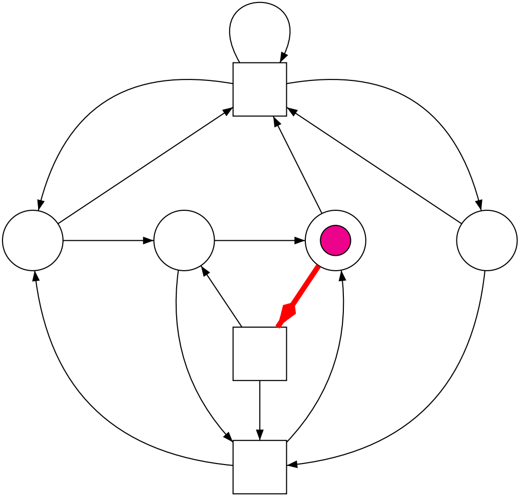

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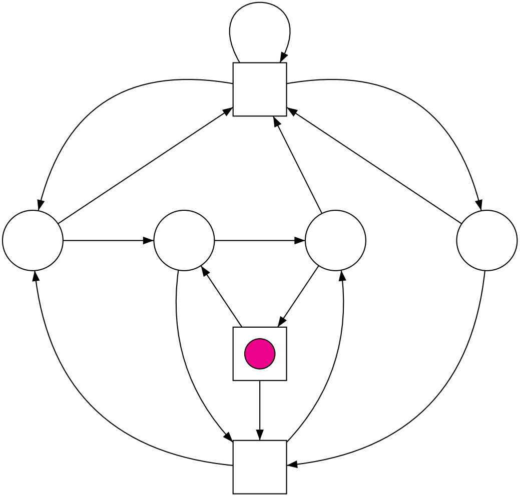
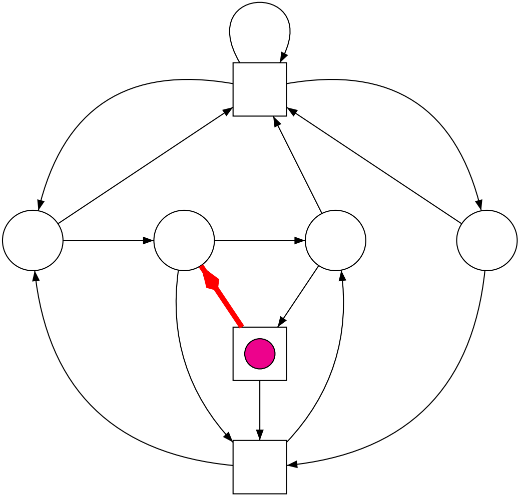


Games

Playing



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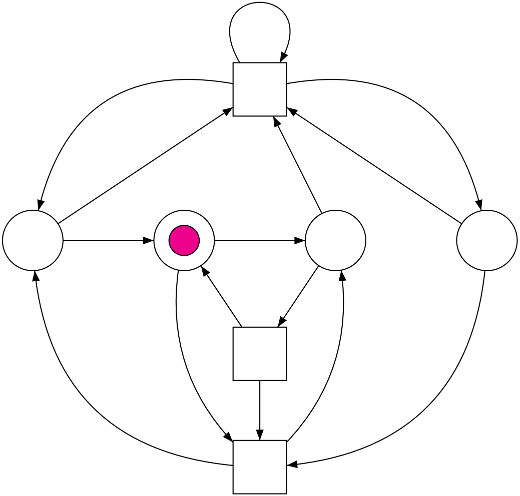
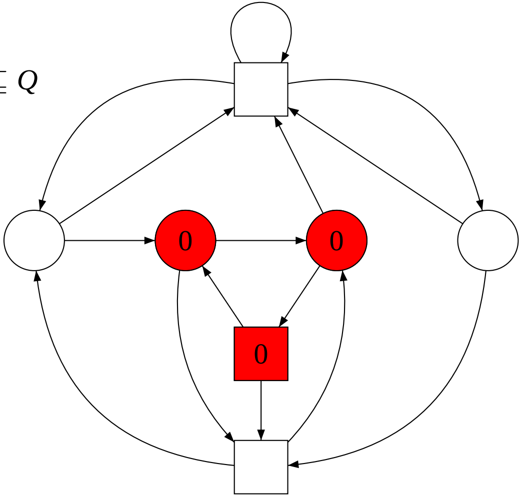


Outline

Solving reachability objectives

Given $F \subseteq Q$



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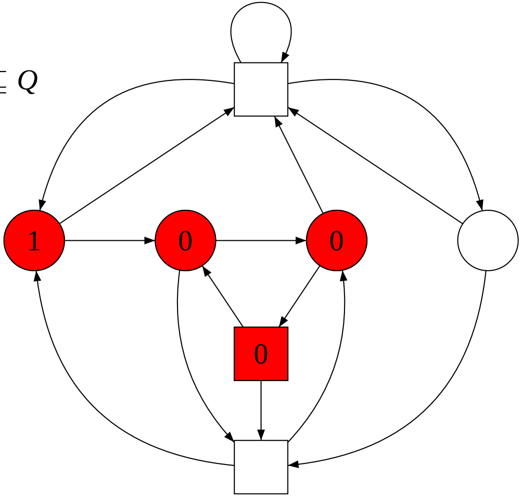
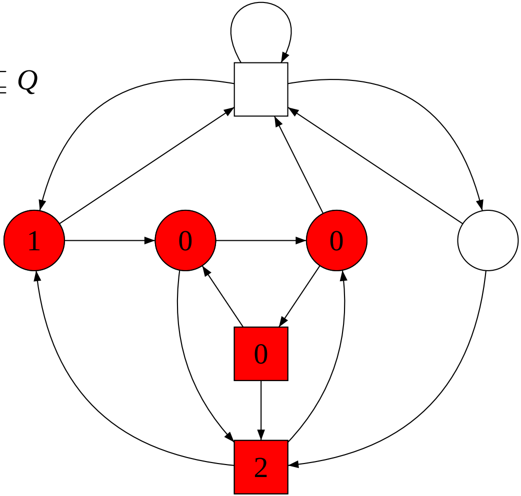


Solving reachability objectives

Given $\color{red}{F} \subseteq Q$



Solving reachability objectives

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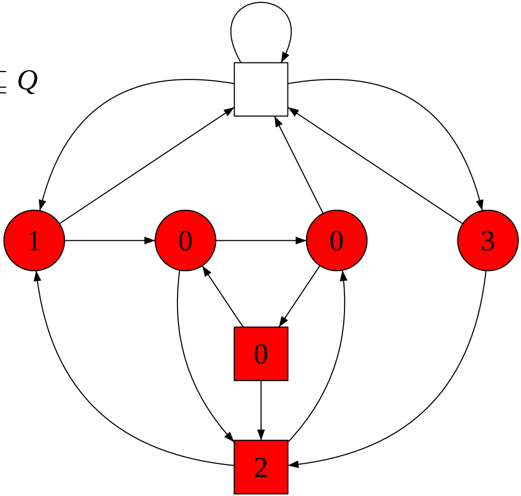


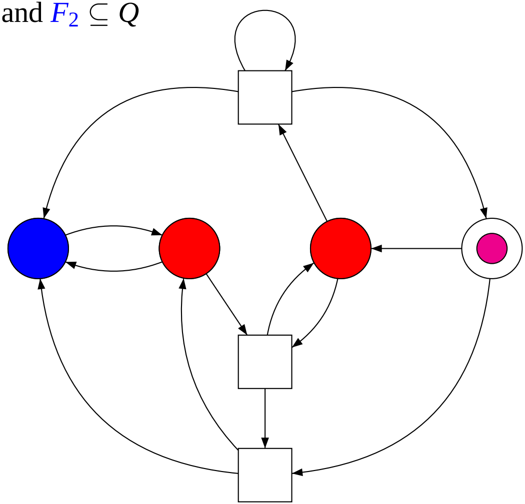
Outline

Generalized reachability objectives

- Reachability objectives: given $F \subseteq Q$, reach at least one vertex in F;
- Generalized reachability objectives: given $F_1, F_2, \ldots, F_p \subseteq Q$, reach at least one vertex in each F_i.

Example

Given F_1 and $F_2 \subseteq Q$



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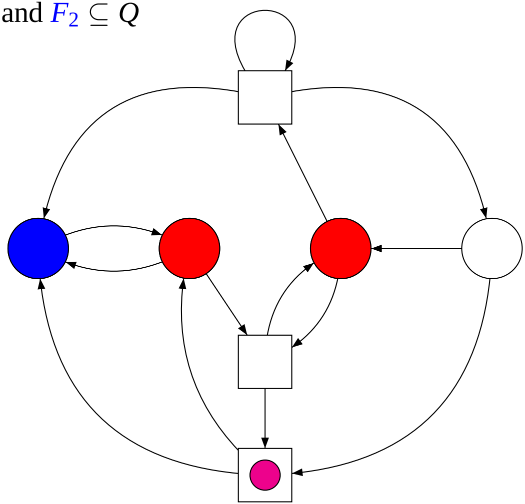

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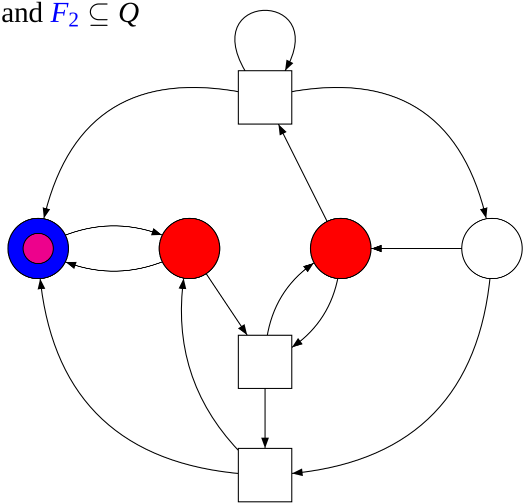

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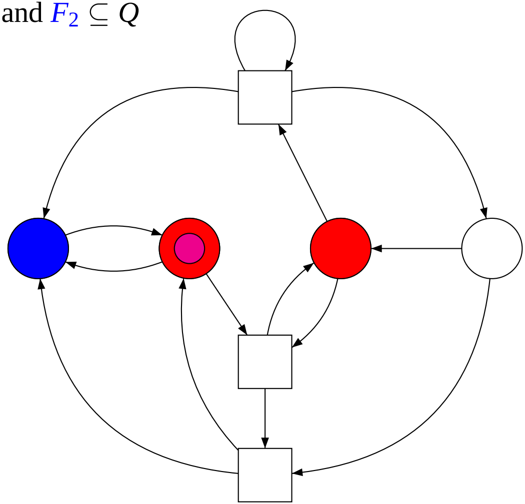

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Reduction from QBF to generalized reachability games

$$\phi = \forall x \, \exists y \, \forall z \;\; (x \vee \neg y) \wedge (\neg y \vee z)$$

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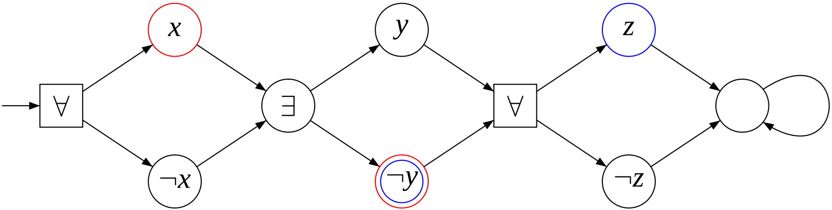

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$$F_1 = \{x, \neg y\} \qquad\qquad F_2 = \{\neg y, z\}$$

Note that the number of literals in a clause is the size of the corresponding reachability set.

Results

Theorem (Lower bounds for generalized reachability games)

- *Solving two players generalized reachability games is* PSPACE-*hard;*
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Special cases where reachability sets have size less than 3 *might* be easier…

An easier case

Lemma (A polynomial special case)

Solving two players generalized reachability games where reachability sets are singletons is in P.

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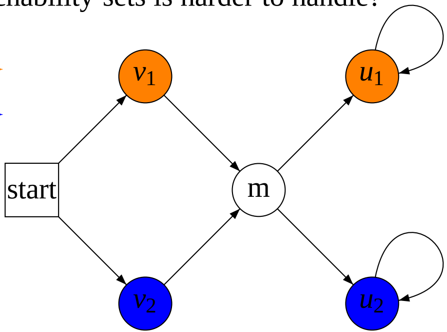
Why bigger reachability sets is harder to handle?

$F_1 = \{v_1, u_1\}$

$F_2 = \{v_2, u_2\}$



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Lemma (A polynomial special case)

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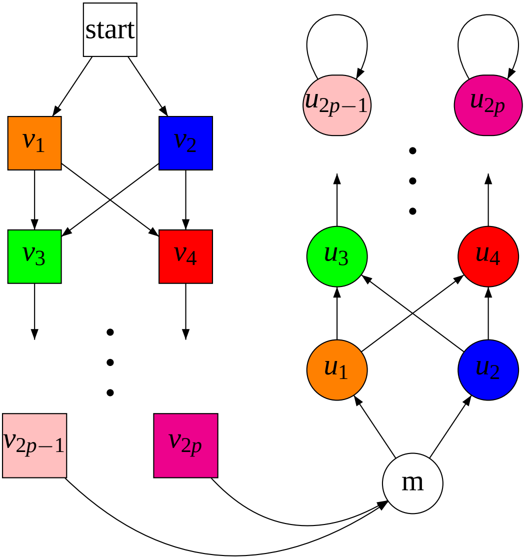
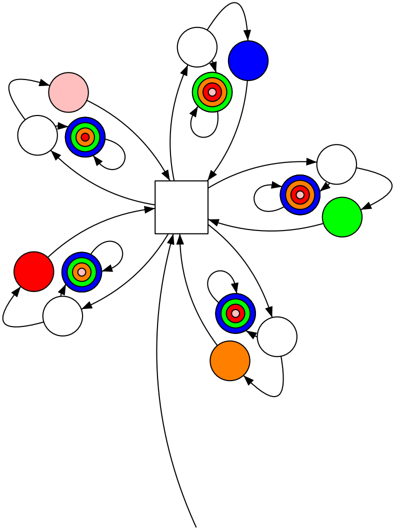
If reachability sets are singletons, then Eve can predict the objectives vertices appearance order.

Outline

Exponential lower bound for Eve, reachability sets of size 2



Florian's piece of art; exponential lower bound for Eve



Conclusion and further work

- Conjuntions of easy objectives may be way harder to solve;
- Open case: reachability sets of size 2.

The end.

Thank for your attention!