Cost-parity games and Cost-Streett games

FSTTCS’2012

Nathanaël Fijalkow, Martin Zimmermann

Institute of Informatics, Warsaw University – Poland

LIAFA, Université Paris 7 Denis Diderot – France

December 15th, 2012

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Framework

This paper is about two-player games over finite graphs,
as used in automata theory and verification (synthesis problem).

We are after efficient algorithms to:

- decide the winner, *and*
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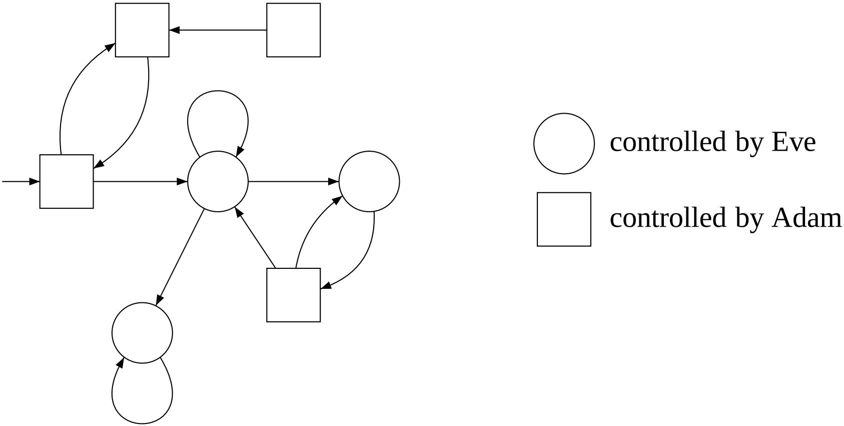
Starting point: good understanding of ω-regular specifications.

Objective: add boundedness specifications.

Games



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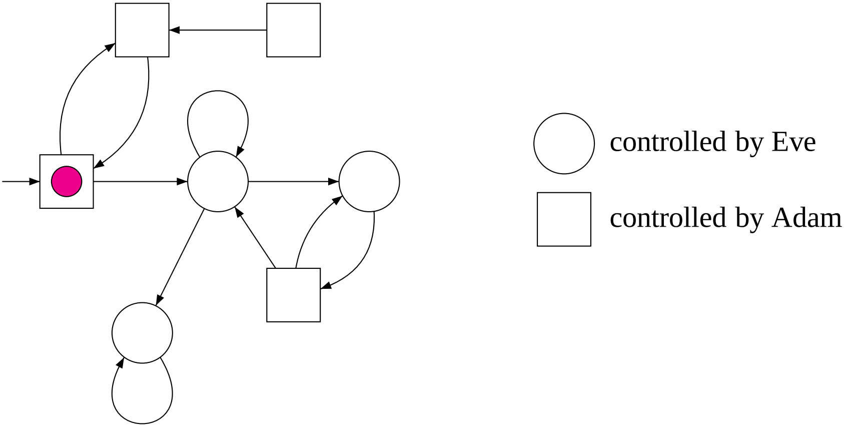

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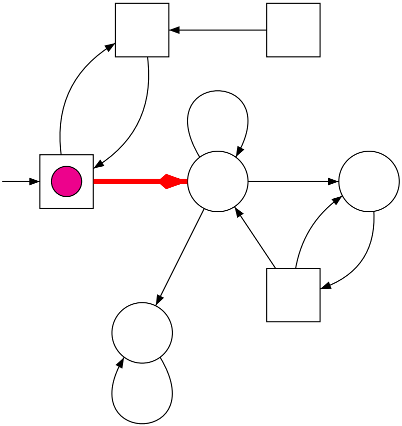

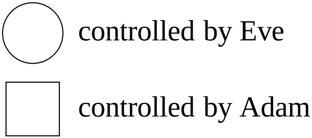

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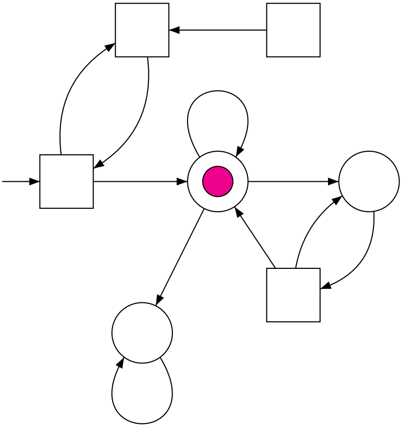

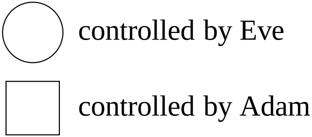

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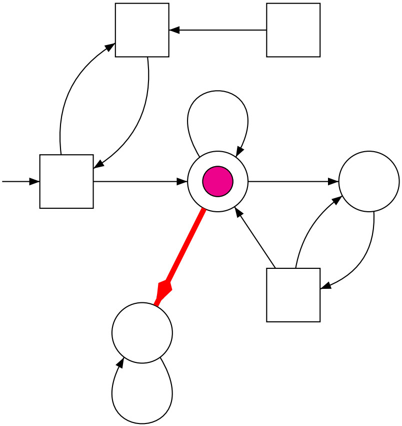

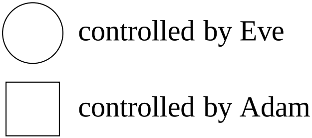

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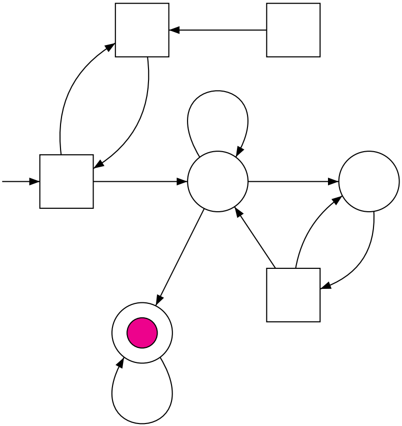
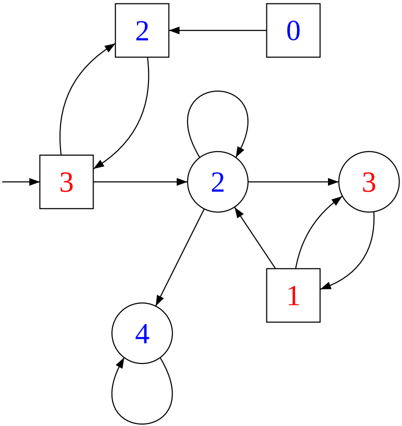
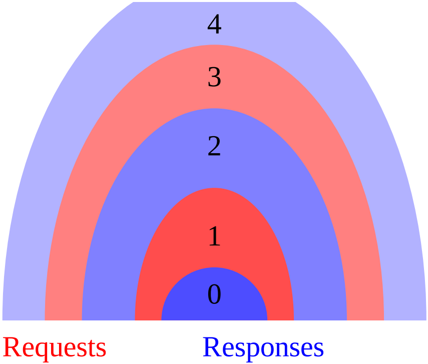


controlled by Eve

controlled by Adam

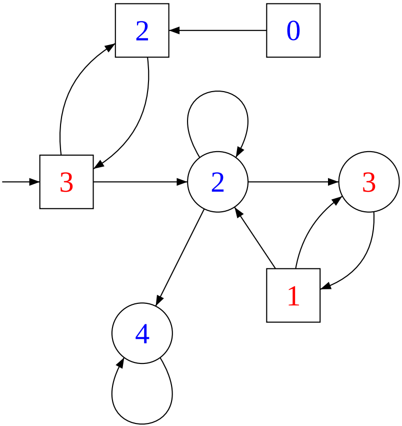
Parity conditions





Almost all requests are answered.

Parity conditions



- Parity conditions allow to express all ω-regular specifications.
- Both players have positional winning strategies.
- Deciding the winner is in NP $\cap$ coNP.

Parity and finitary parity

Finitary specifications: proposed by Alur and Henzinger, games studied by Chatterjee, Henzinger and Horn.

Parity:
Almost all requests are answered.

Finitary parity:
There exists a bound *b*, s.t. almost all requests are answered *within b steps*.

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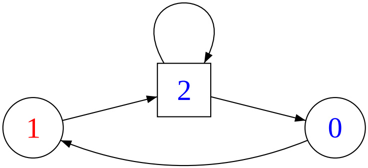


Eve wins for the parity condition,

but loses for the finitary parity condition!

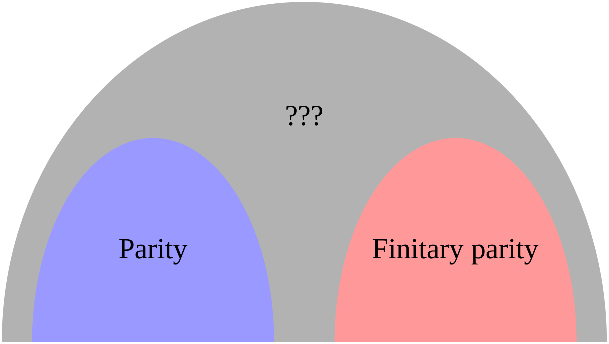
Parity and finitary parity

Parity: Almost all requests are answered.	**Finitary parity:** There exists a bound *b*, s.t. almost all requests are answered *within b steps*.

- Both players have positional winning strategies.
- Deciding the winner is in NP $\cap$ coNP.

- Eve has positional winning strategies.
- Adam needs infinite memory.
- Deciding the winner is in PTIME.

Cost-parity games



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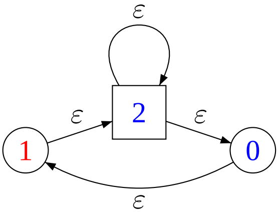
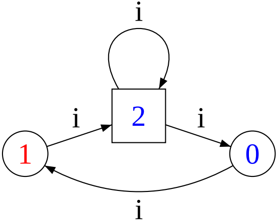
Cost-parity:
There exists a bound b, s.t.
almost all requests are answered *with cost at most b*.

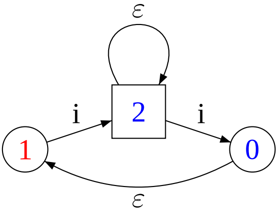
Costs



Parity game



Finitary parity game



Positional determinacy for Eve

Objective: **strategy optimization**

Assume σ is a winning strategy.
How to construct a memoryless winning strategy σ' from σ?

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Tool: **scoring functions**

“à la Müller and Schupp” past-oriented proof

A general framework

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Define a scoring function $\mathrm{Sc} : V^* \to (S, \leq)$ satisfying:

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Construct a memoryless strategy σ':

“play according to σ assuming the worst play prefix”.

Deciding the winner in cost-parity games

n: number of vertices
m: number of edges
d: number of colors

Theorem

Given a parity games solver of complexity $T(n, m, d)$, there exists a cost-parity games solver of complexity

$$O(n \cdot T(n \cdot d, m \cdot d, d + 2)) \ .$$

Results

winning condition	complexity	Eve	Adam
parity	$\mathrm{NP} \cap \mathrm{coNP}$	memoryless	memoryless
finitary parity	PTIME	memoryless	infinite
cost-parity	$\mathrm{NP} \cap \mathrm{coNP}$	memoryless	infinite
Streett	coNP-complete	finite	memoryless
finitary Streett	EXPTIME-complete	finite	infinite
cost-Streett	EXPTIME-complete	finite	infinite

Towards ωB-conditions