Emptiness Of Alternating Tree Automata Using Games With Imperfect Information

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Objectives

This talk is about *one* approach to deal with alternating tree automata.

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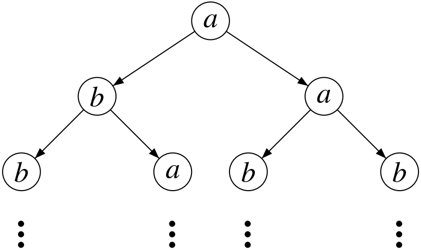
We consider two settings: the first is “classical”, the second is “qualitative”.

Alternating tree automata: a powerful tool

- *What is it?* A finite-state machine that defines a property, given as a set of infinite (binary) trees.



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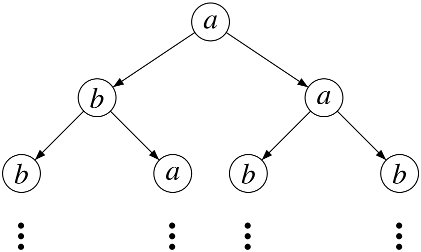
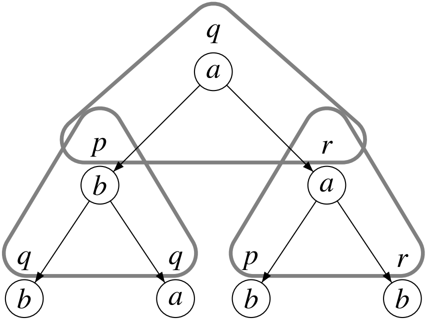


- *What for?* Define **regular** properties, used for instance for program verification.

Definition of alternating tree automata (1/2)

$\mathcal{A} = (Q, Q_\exists, Q_\forall, q_0, \delta, \text{Parity})$

The transition relation $\delta \subseteq Q \times A \times Q \times Q$ gives local constraints:



$(q, a, p, r) \in \delta$

$(p, b, q, q) \in \delta$
$(r, a, p, r) \in \delta$

Definition of alternating tree automata (2/2)

Let $\mathcal{A}$ be an alternating automaton and t a tree.
This induces a two-player zero-sum game, where Eve tries to show that the tree is accepted:

- Eve picks the transitions from $Q_\exists$,
- Adam picks the transitions from $Q_\forall$,
- Adam also chooses directions.

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Classical versus *Qualitative* semantics:

- $\mathcal{A}$ accepts t if Eve has a strategy such that **all plays** consistent with the strategy are winning.
- $\mathcal{A}$ qualitatively accepts t if Eve has a strategy such that **almost all plays** consistent with the strategy are winning.

Classical versus Qualitative semantics

- $L(\mathcal{A})$ is the set of trees accepted by $\mathcal{A}$.
- $L^{=1}(\mathcal{A})$ is the set of trees qualitatively accepted by $\mathcal{A}$.

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- $\{L(\mathcal{A}) \mid \mathcal{A}\}$ describes the class of regular languages.
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Input: An alternating automaton $\mathcal{A}$

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Theorem (Müller and Schupp)

The emptiness problem for alternating parity automata with the classical semantics is EXPTIME*-complete.*

Theorem (Our paper)

- *The emptiness problem for alternating Büchi automata with the qualitative semantics is* EXPTIME*-complete.*
- *The emptiness problem for alternating CoBüchi automata with the qualitative semantics is undecidable.*

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We use a different technique, through games of imperfect information.

Outline

A sketch of the solution

Let $\mathcal{A}$ be an alternating automaton.

1. simulation: construct an equivalent non-deterministic automaton $\mathcal{B}$ (*i.e.* where $Q_\forall = \emptyset$),
2. solve the emptiness problem for $\mathcal{B}$.

A sketch of the solution

Let $\mathcal{A}$ be an alternating automaton.

1. simulation: construct an equivalent non-deterministic automaton $\mathcal{B}$ (*i.e.* where $Q_\forall = \emptyset$),
2. solve the emptiness problem for $\mathcal{B}$.

The emptiness problem for non-deterministic automata reduces to solving a two-player game, where Eve tries to show that there exists an accepted tree:

- Eve picks the tree t *and* the transitions,
- Adam only chooses directions.

Questions

- Can we directly reduce the emptiness problem for alternating automata to solving a two-player game?
- How to handle the qualitative semantics, for which no simulation is known?

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Consider a universal word automaton where Adam can:

1. check that the word contains a b,
2. or check that the word does not contain any b

The language accepted is empty but Eve can wait to see whether Adam chooses option 1 or 2 and pick the next letters to contradict Adam's choice.

Outline

Games of imperfect information

We directly reduce the emptiness problem for alternating automata to solving a two-player imperfect-information game, where Eve tries to show that there exists an accepted tree:

- Eve picks the tree t *and* a positional strategy for her transitions,
- Adam chooses transitions and directions.
- Eve does not see what Adam does!

Games of imperfect information

We directly reduce the emptiness problem for alternating automata to solving a two-player imperfect-information game, where Eve tries to show that there exists an accepted tree:

- Eve picks the tree *t and* a positional strategy for her transitions,
- Adam chooses transitions and directions.
- Eve does not see what Adam does!

The new key technical ingredient is a positional determinacy theorem for infinite (chronological) arenas:

Theorem

If Eve has an almost surely winning strategy for Büchi, then she has a positional one.

A generic result

To solve the emptiness problem of alternating automata with a semantics *Acc*, one can reduce it to a two-player imperfect-information game, and:

1. to prove the construction correctness, show a positional determinacy result for *Acc*,
2. solve the obtained imperfect-information game with winning condition *Acc*.

A generic result

To solve the emptiness problem of alternating automata with a semantics Acc, one can reduce it to a two-player imperfect-information game, and:

1. to prove the construction correctness, show a positional determinacy result for Acc,
2. solve the obtained imperfect-information game with winning condition Acc.

Thank you!