Boundedness games

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Nathanaël Fijalkow Florian Horn Denis Kuperberg
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Institute of Informatics, Warsaw University – Poland

LIAFA, Université Paris 7 Denis Diderot – France

Highlights, September 19th, 2013

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This talk is about our *joint* effort to understand boundedness games.

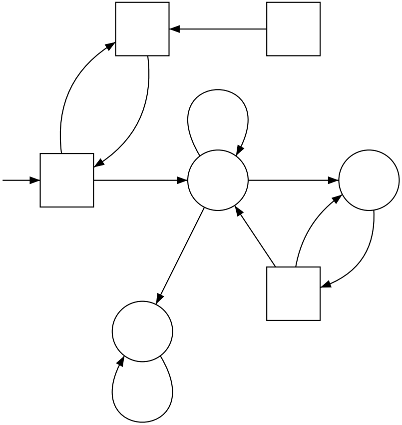
Motivation: expressing boundedness properties

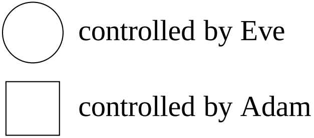
$\mathrm{MSO} + \mathbb{U}$

cost MSO

A lot is known, and even more is not known about those two logics!

Definition of boundedness games





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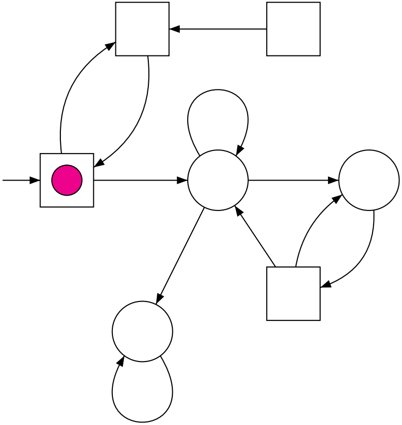

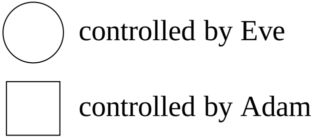

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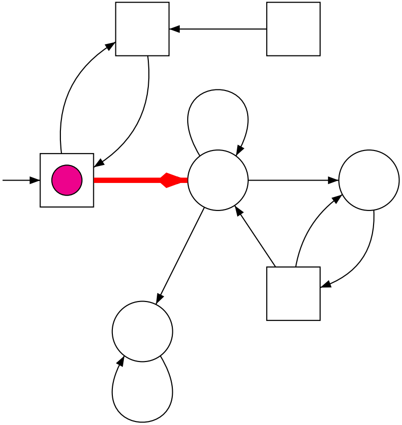

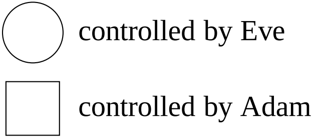

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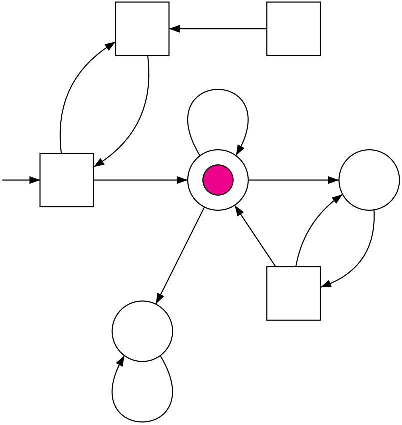

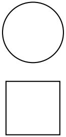

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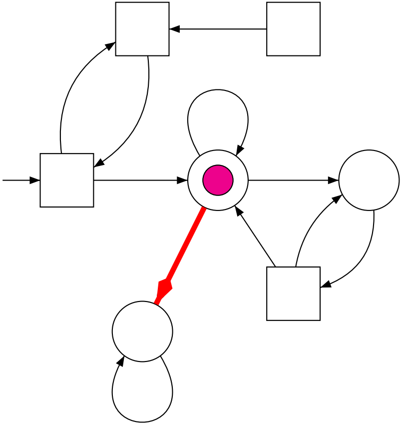

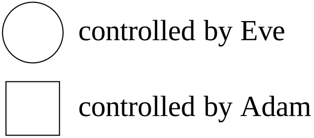

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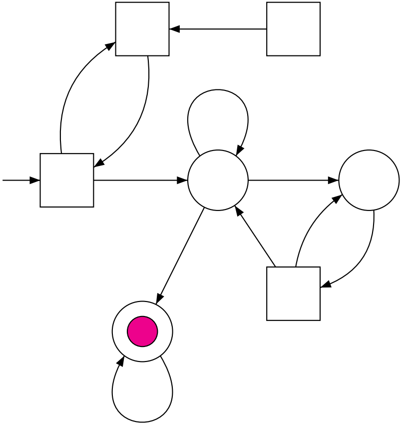
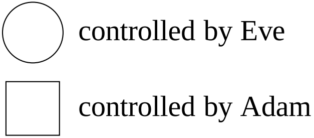
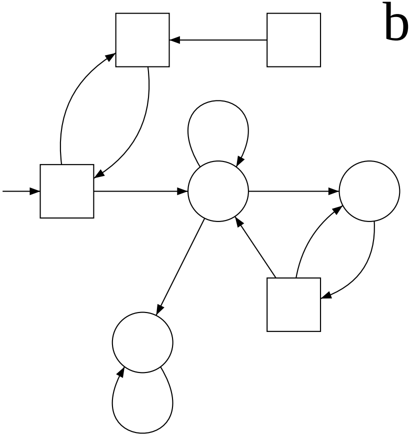
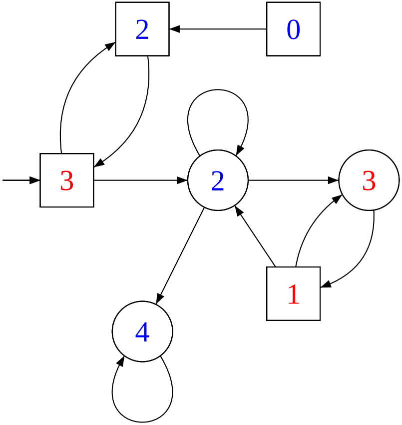






boundedness condition:

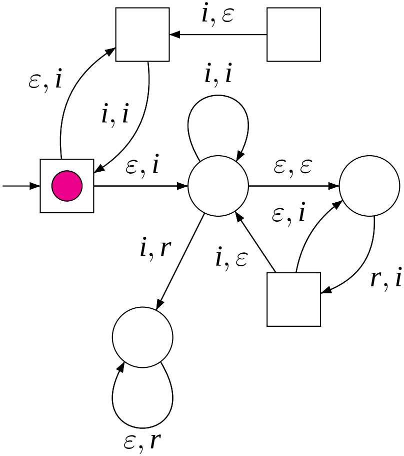
parity
and
all counters
are bounded



parity condition:

the minimal priority seen infinitely often is even

Definition of boundedness games



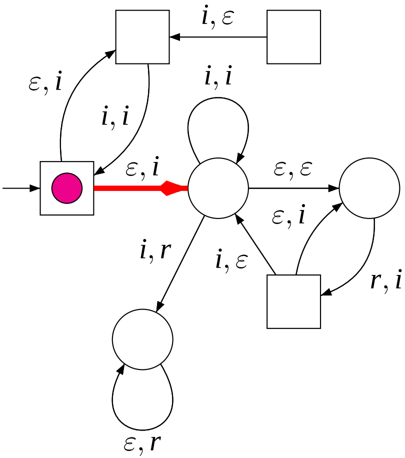
$$c_1 = 0$$
$$c_2 = 0$$

ε : nothing

i : increment

r : reset

Definition of boundedness games



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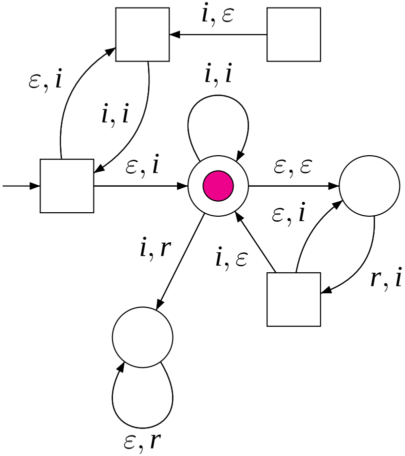

$$c_1 = 0$$
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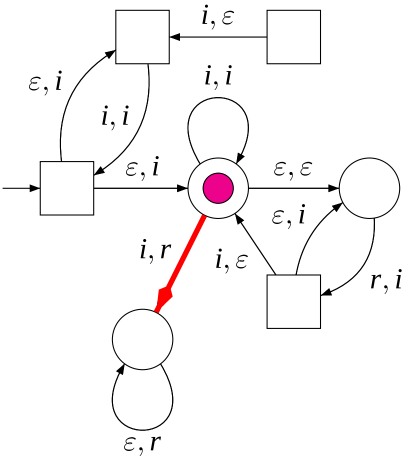
Definition of boundedness games



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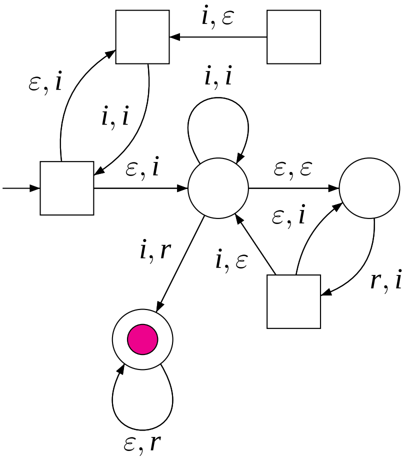

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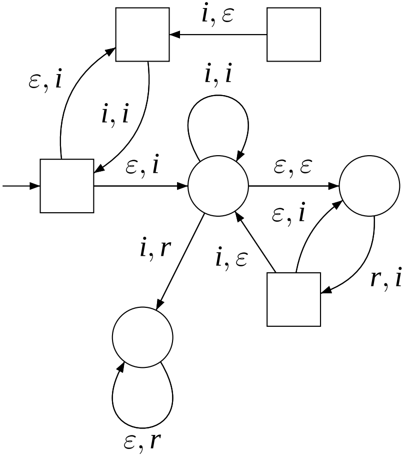

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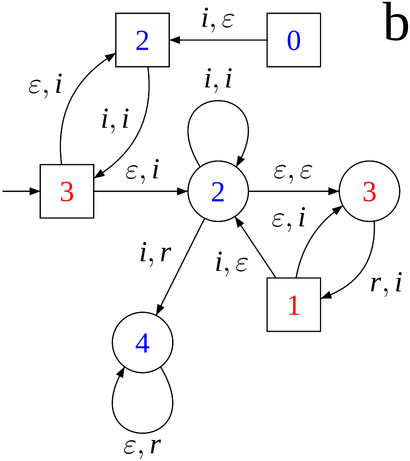

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Quantification

Eve wins means:

$\exists \sigma$ (strategy for Eve),
$\forall \pi$ (paths),
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non-uniform
$(\mathrm{MSO} + \mathbb{U})$

uniform
(cost MSO)

Research questions and some answers

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- When does Eve has finite-memory winning strategies?
  ↬ Uniform quantifications, the Büchi case over infinite chronological arenas [Vanden Boom, 2011].
  ↬ Uniform quantifications, the parity case over thin tree arenas [F., Horn, Kuperberg, Skrzypczak, unpublished].

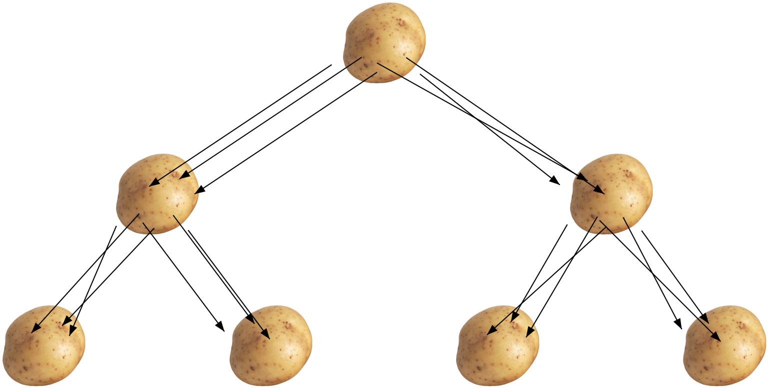
Why finite-memory strategies?

Thomas Colcombet's habilitation:

Conjecture 9.3. *Les objectifs* $\mathtt{hB} \wedge \mathtt{parité}$ *et* $\neg\mathtt{B} \wedge \mathtt{parité}$ *sont à* $\approx$*-mémoire finie, sur toutes les arènes/sur les arènes chronologiques/sur les arènes «arborescentes».*

Existence of finite-memory strategies in (some) boundedness games
$\Longrightarrow$ Decidability of cost MSO over infinite trees
$\Longrightarrow$ Decidability of the index of the non-deterministic Mostowski's hierarchy (open for 40 years)!

Working with potato trees



Theorem (F., Horn, Kuperberg, Skrzypczak)

The B-part of Colcombet's conjecture holds for thin tree arenas!