The Value 1 Problem for Probabilistic Automata

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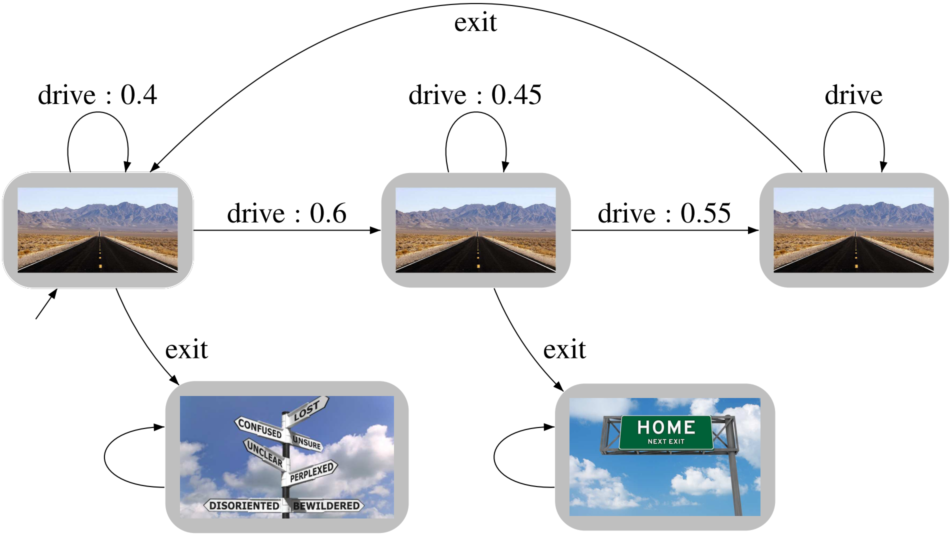
A Real-life Situation



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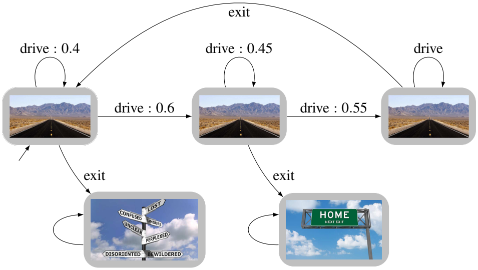
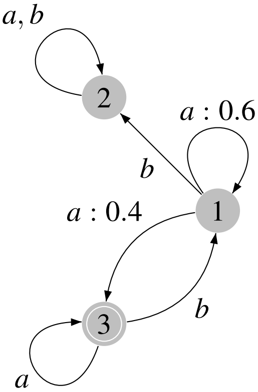


- No sequence of actions ensure to reach home *almost surely*.
- For every $\varepsilon > 0$, there exists a sequence of actions ensuring to reach home with probability at least $1 - \varepsilon$!
- This is not true anymore if the probabilities change!

The Value 1 Problem



$$\mathbb{P}_{\mathcal{A}} : A^* \to [0,1]$$

$\mathbb{P}_{\mathcal{A}}(w)$ is the probability that a run for w is successful.

INPUT: $\mathcal{A}$ a probabilistic automaton OUTPUT: for all $\varepsilon > 0$, there exists $w \in A^*$, $\mathbb{P}_{\mathcal{A}}(w) \geq 1 - \varepsilon$.

In other words, define $\mathrm{val}(\mathcal{A}) = \sup_{w \in A^*} \mathbb{P}_{\mathcal{A}}(w)$, is $\mathrm{val}(\mathcal{A}) = 1$?

Starting point:

Theorem (Gimbert and Oualhadj, 2010)

The value 1 *problem is undecidable.*

But *to what extent*?

A Research Program

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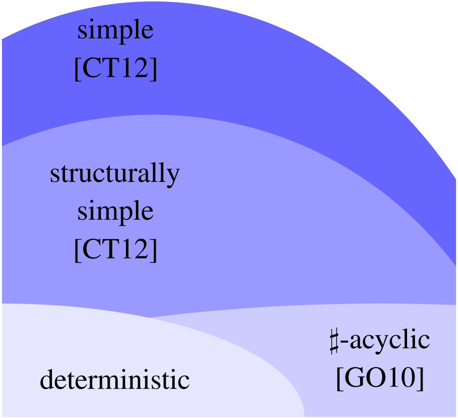
Argue that you cannot do *more often* than that.

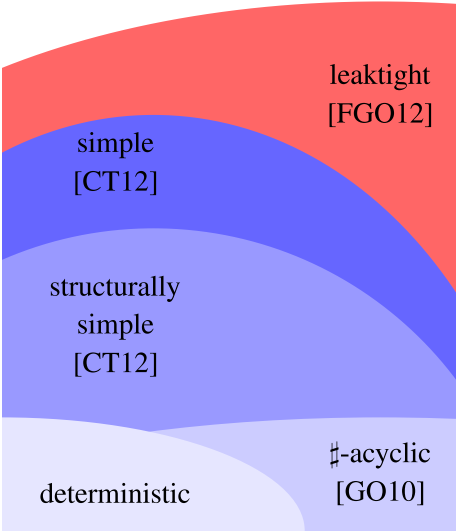
What was known?



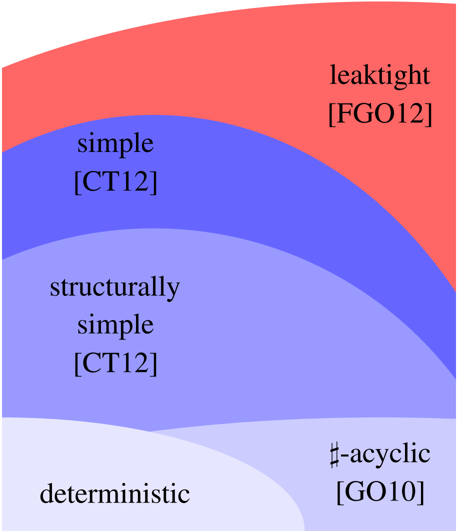
Theorem ([BBG12, CSV13])

The value 1 *problem is* Σ_2^0*-complete.*

Our Contributions



Our Contributions



In [FGO12], we introduced the Markov Monoid, generalizing the transition monoid.

Theorem ([FGO12])

The value 1 problem is decidable for leaktight automata.

Theorem ([FGKO14])

Leaktight automata strictly contain the simple automata.

Theorem ([Fij14])

The Markov Monoid algorithm is optimal.

Drawing the Decidability Frontier

The following are equivalent:

- The value 1 problem over finite words,
- The emptiness problem over prostochastic words.

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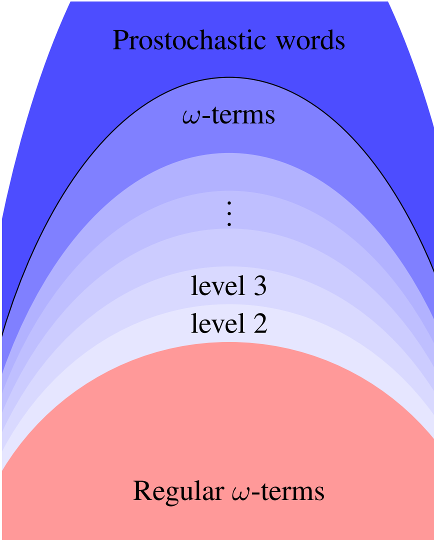
Theorem ([Fij14])

1. *The Markov Monoid Algorithm answers "YES" if and only if there exists a regular ω-term accepted by $\mathcal{A}$,*
2. *The following problem is undecidable: determine whether there exists an ω-term on the level 2 accepted by $\mathcal{A}$.*



Conclusion

We introduced the Markov Monoid Algorithm
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Thank you!

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