Boundedness problems in games: a roadmap

GT Jeux'2012

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(based on a joint work with Martin Zimmermann)

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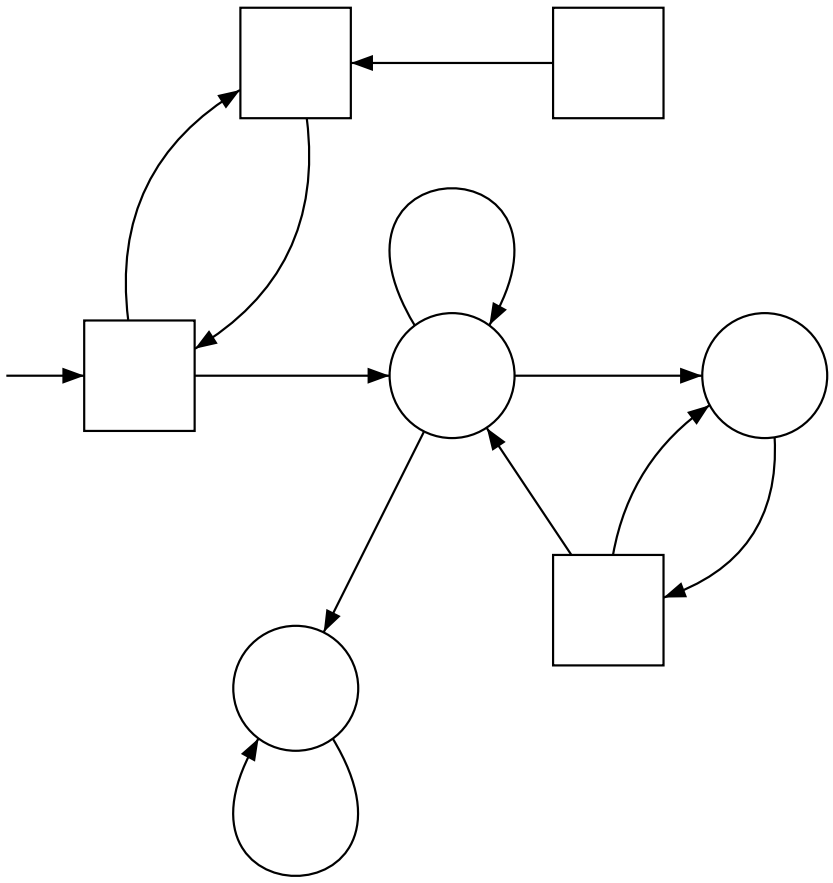
Games with counter

Two-player turn-based games over **finite** graphs



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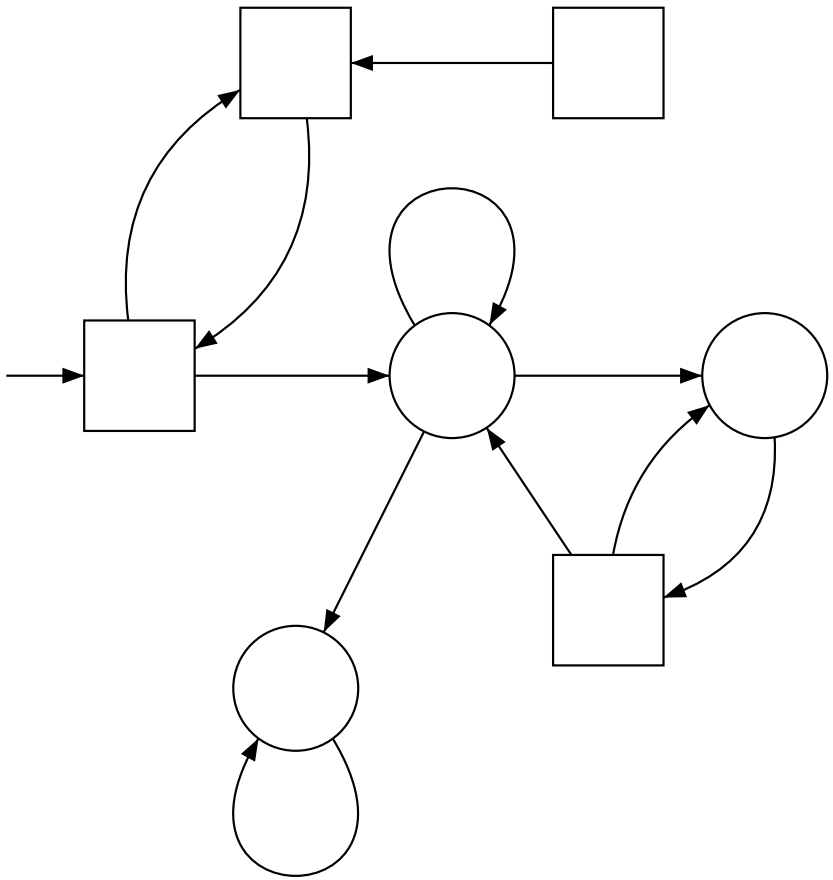

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$$c_2 \doteq 0$$

Winning condition: “there exists B, such that . . .”

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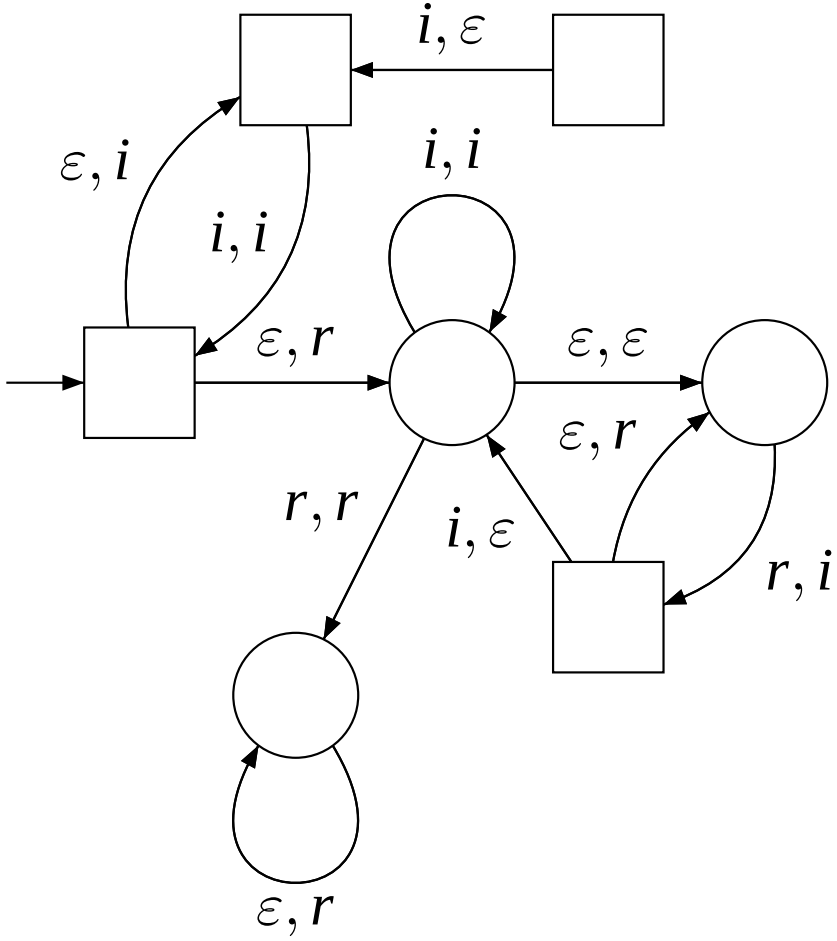

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Actions on the counters

increment: i or "$c \longleftarrow c + 1$"

no action: ε

reset: r or "$c \longleftarrow 0$"

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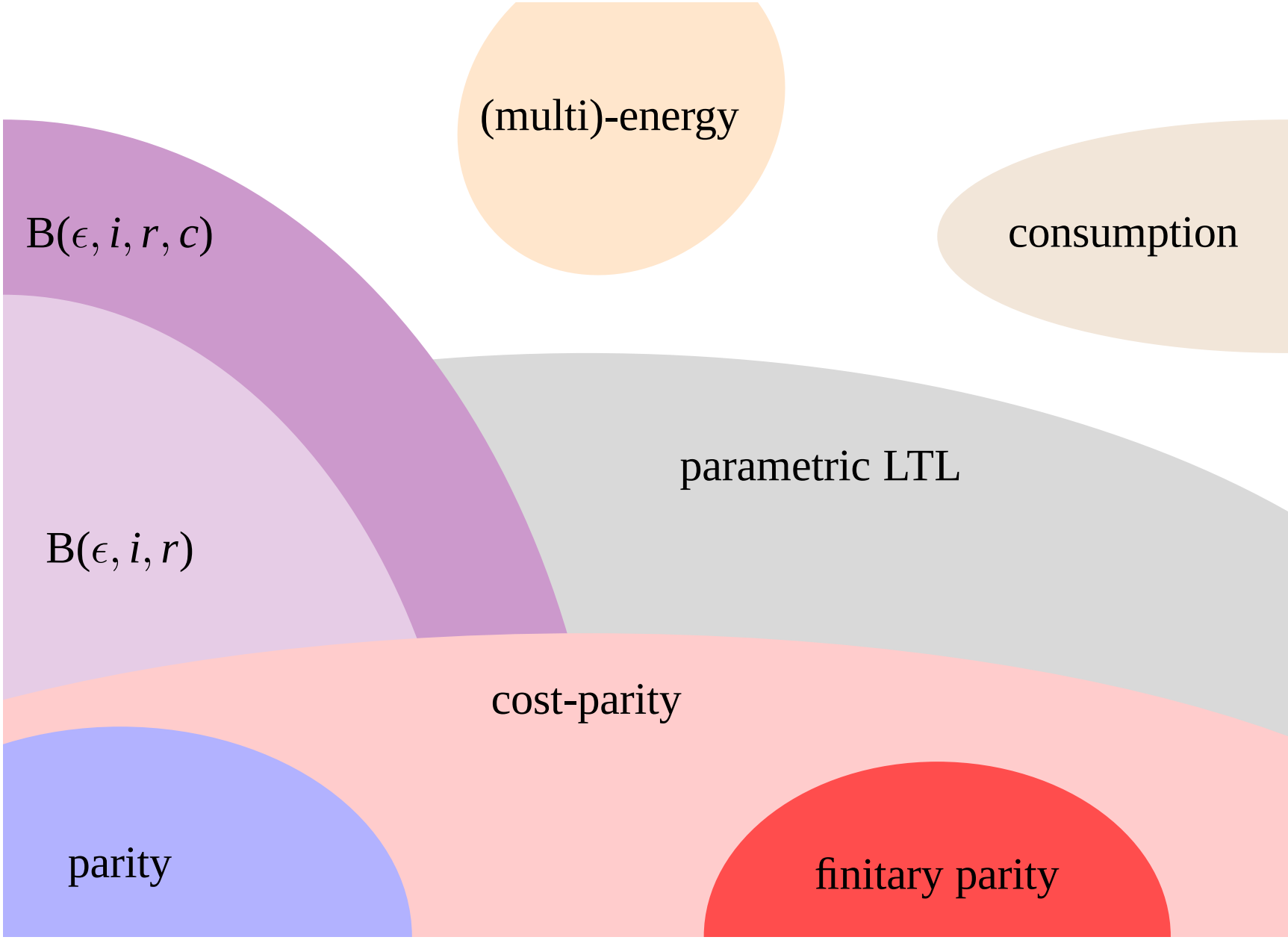
Roadmap


(multi)-energy
consumption
B(ϵ, i, r, c)
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The robust: B-conditions (Bojańczyk, Colcombet)

Actions: ε, i, r.

B-conditions are (positive) boolean combination of:

- "counter c is bounded"
- Büchi(F): "F appears infinitely often"
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Eve wins "counter c is bounded"
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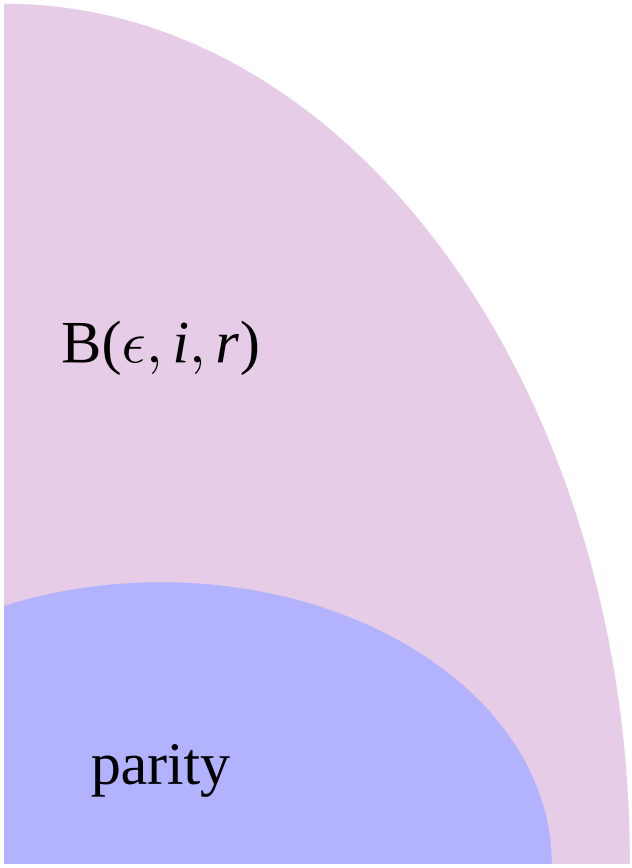
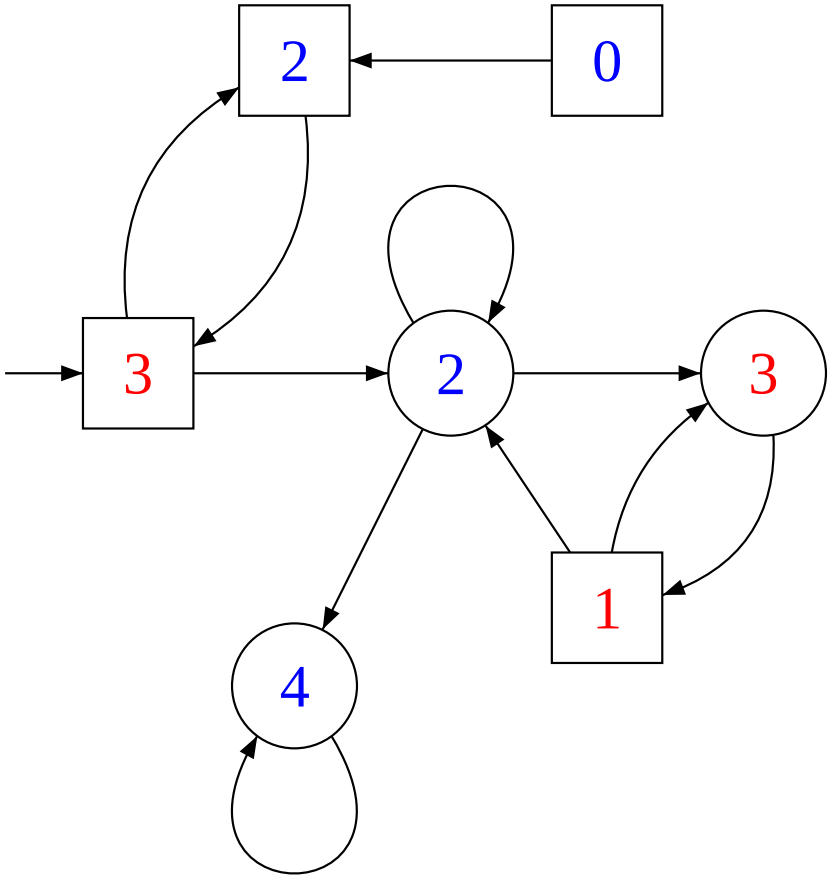
Corollary

One can decide who wins a B-game and Eve needs finite memory.

Roadmap



The surprising: finitary parity (Alur,Henzinger)



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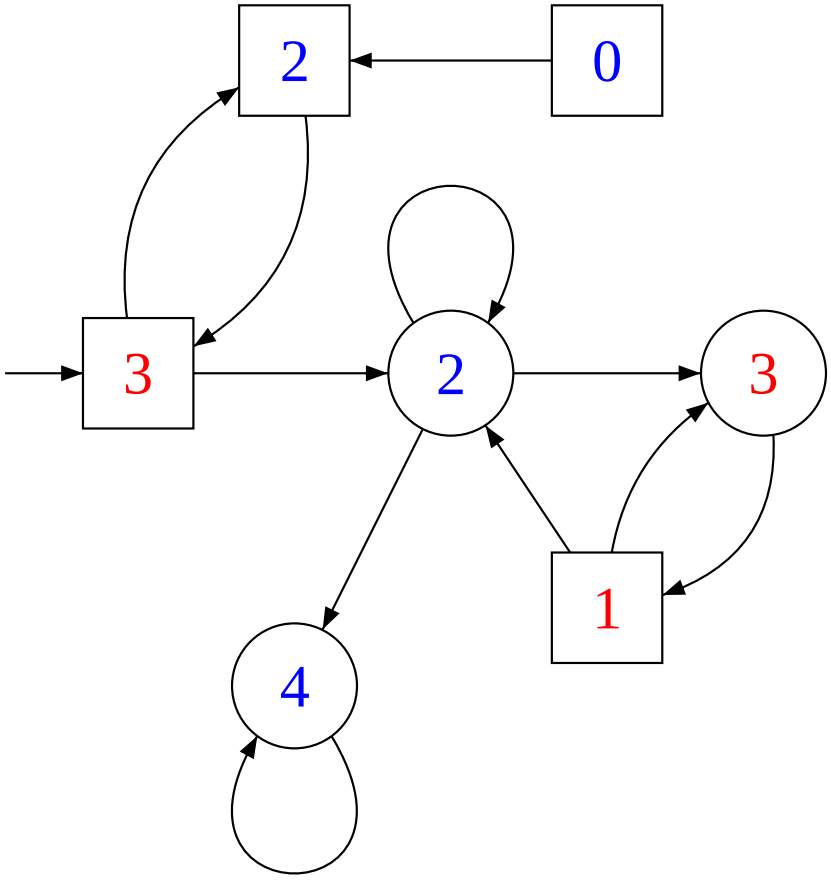

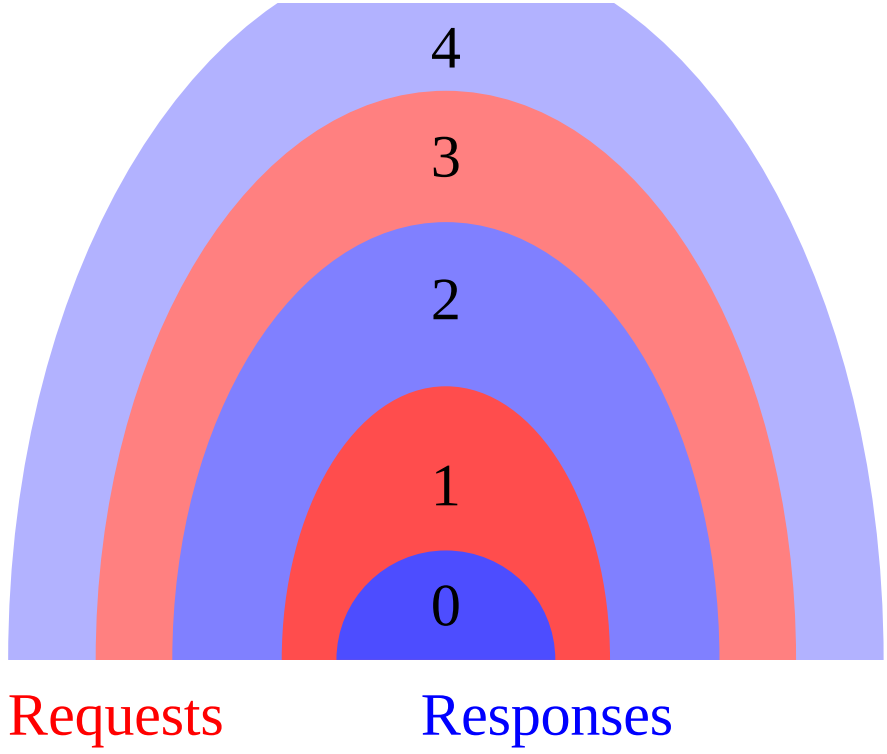

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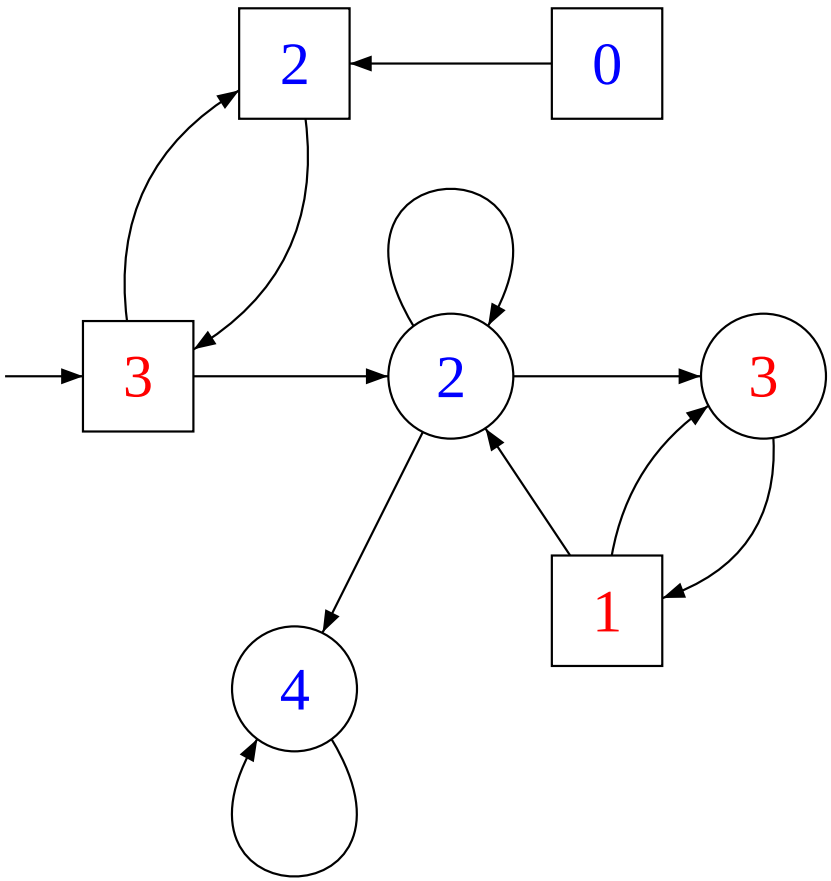

Parity:
Almost all requests are answered.

Finitary parity:
There exists a bound *b*, s.t. almost all requests are answered *within b steps*.

Parity games and finitary parity games are different

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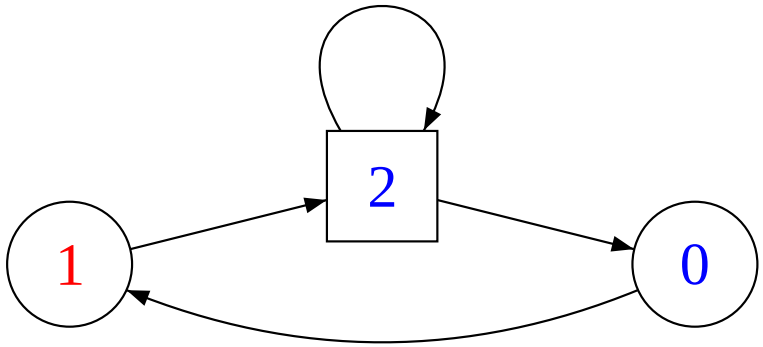

Eve wins for the parity condition,

but loses for the finitary parity condition!

Parity games and finitary parity games are different

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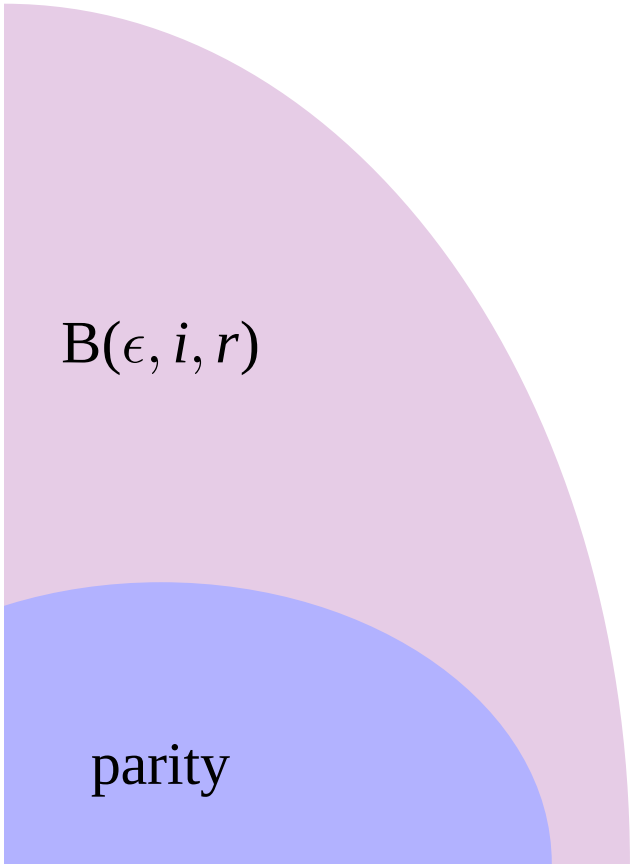
- Both players have positional winning strategies.
- Deciding the winner is in NP ∩ coNP.

- Eve has positional winning strategies.
- Adam needs infinite memory.
- Deciding the winner is in PTIME.

Roadmap



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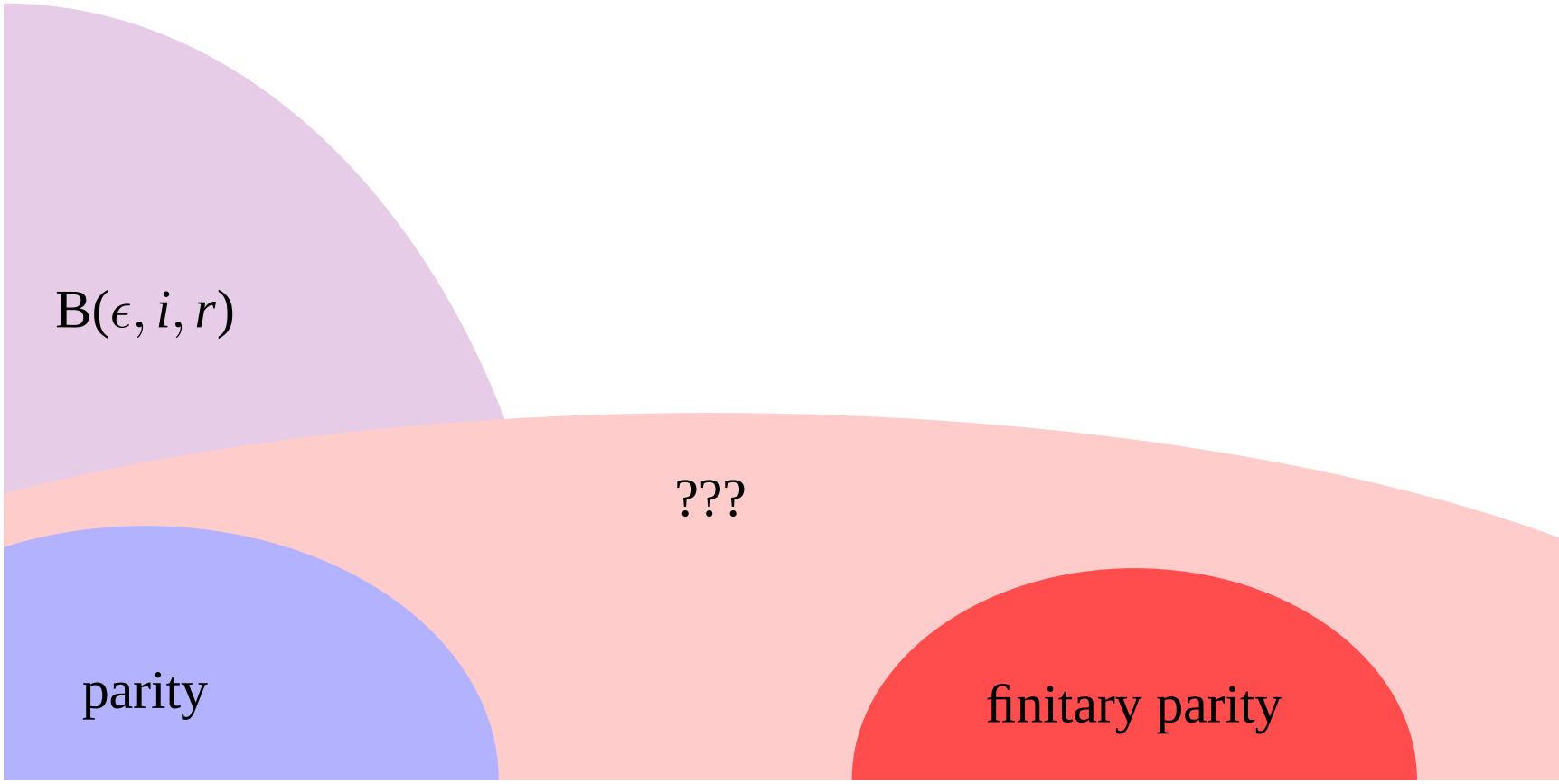

Cost-parity conditions

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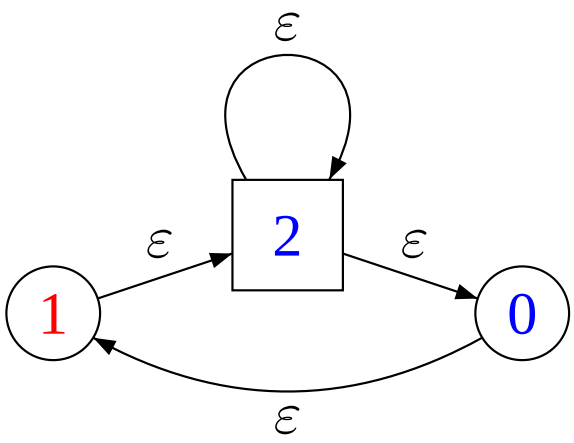
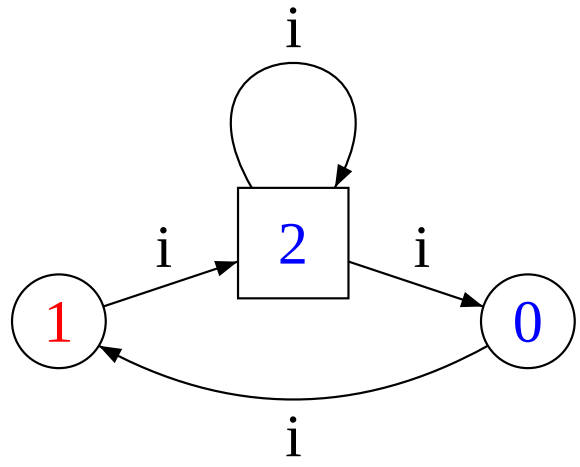
Cost-parity:
There exists a bound *b*, s.t.
almost all requests are answered *with cost at most b*.

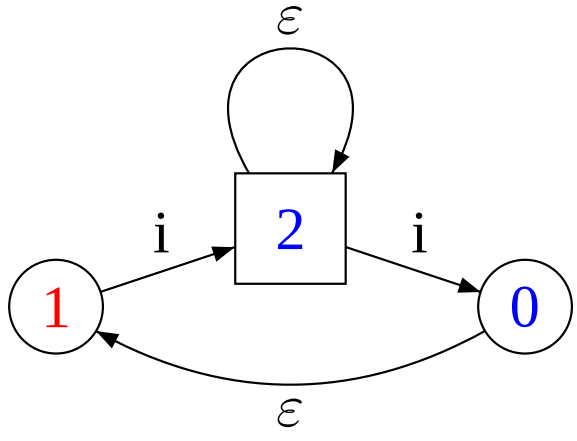
Costs



Parity game



Finitary parity game



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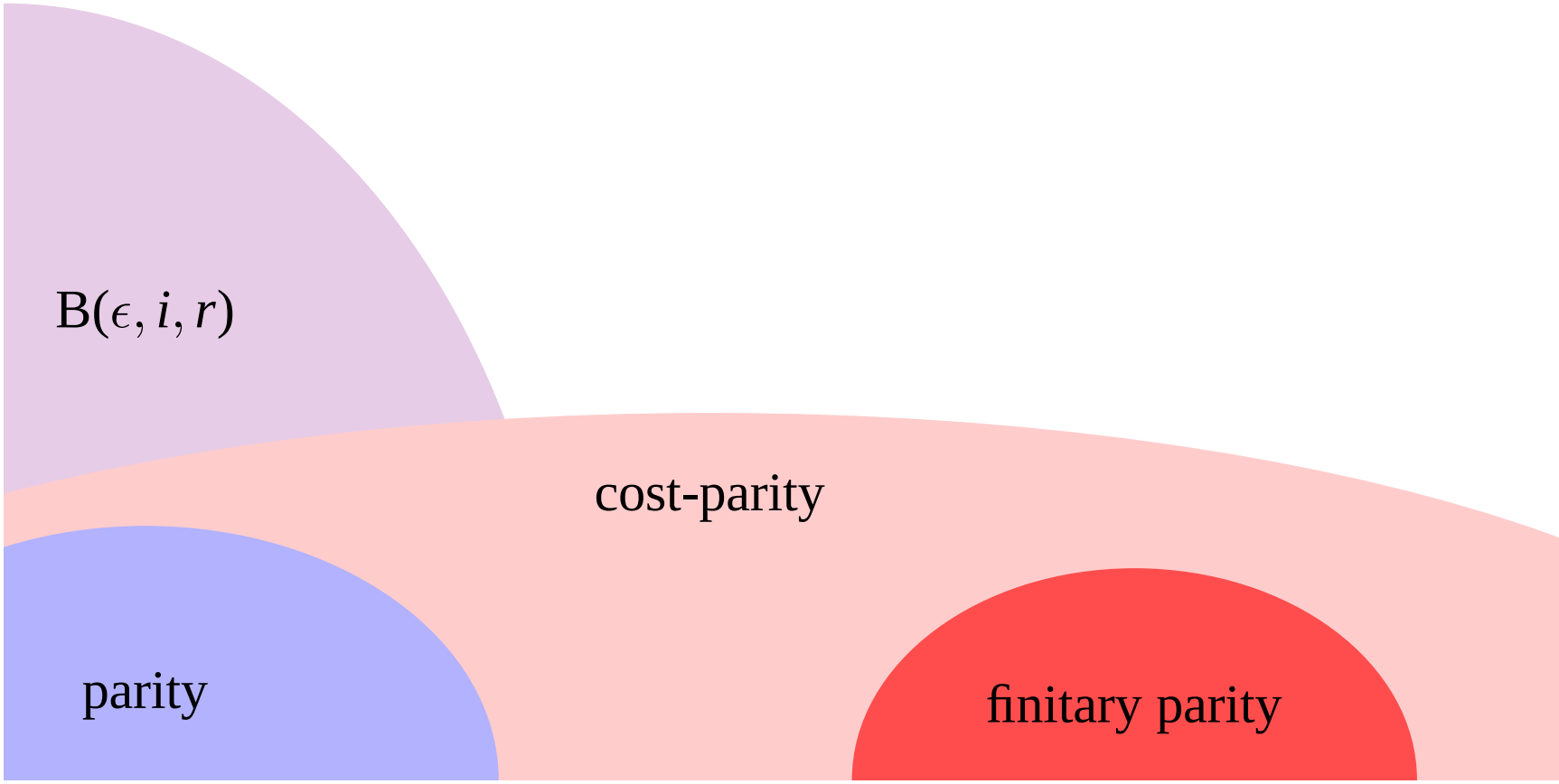

Deciding the winner in cost-parity games

n: number of vertices
m: number of edges
d: number of colors

Theorem (F., Zimmermann 2012)

Given a parity games solver of complexity $T(n, m, d)$,
we construct a cost-parity games solver of complexity

$$O(n \cdot T(n \cdot d, m \cdot d, d + 2)) \ .$$

Positional determinacy for Eve

Objective: **strategy optimization**

Assume σ is a winning strategy.
How to construct a memoryless winning strategy σ' from σ?

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Tool: **scoring functions**

“à la Müller and Schupp” past-oriented proof

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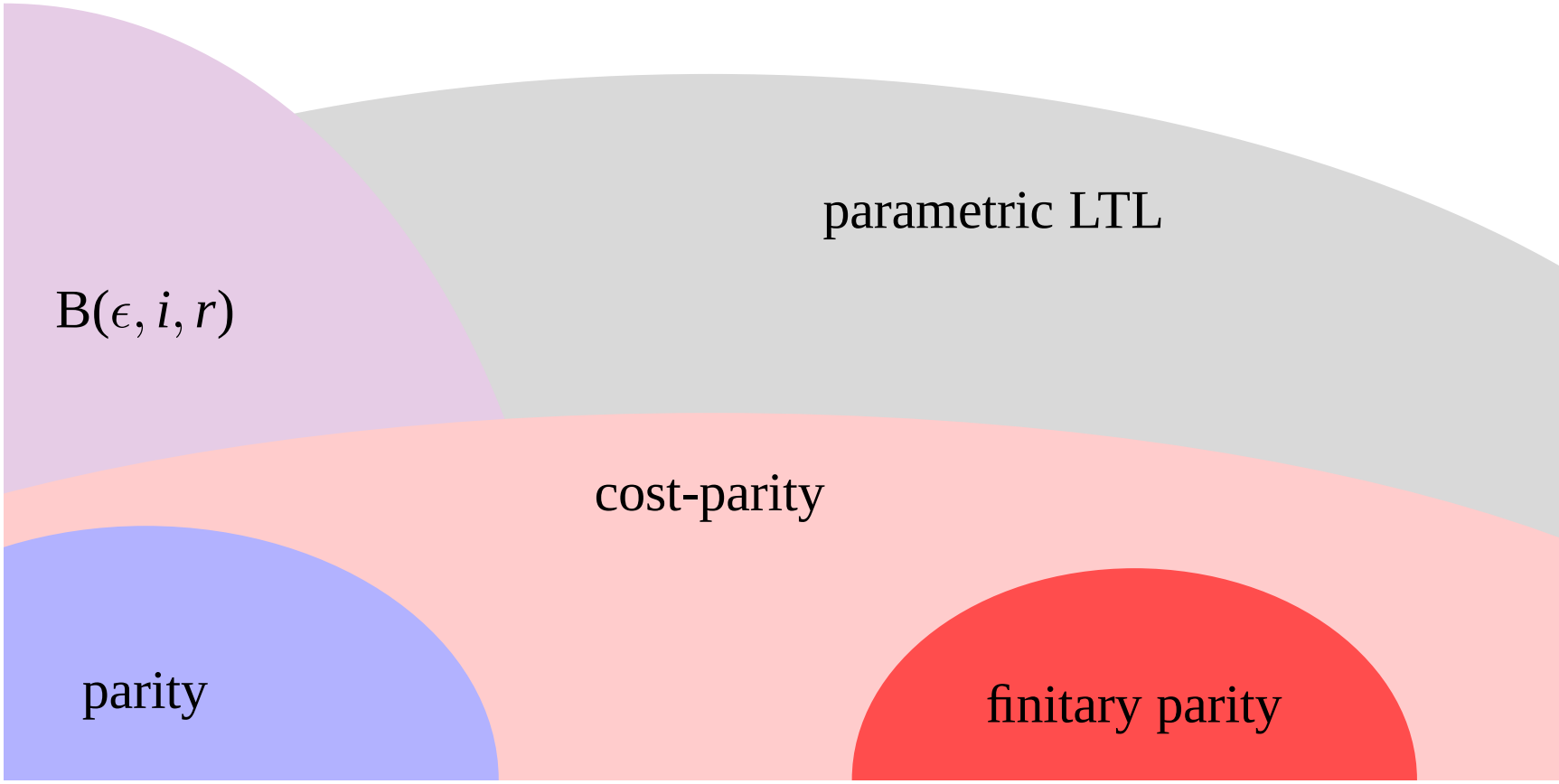

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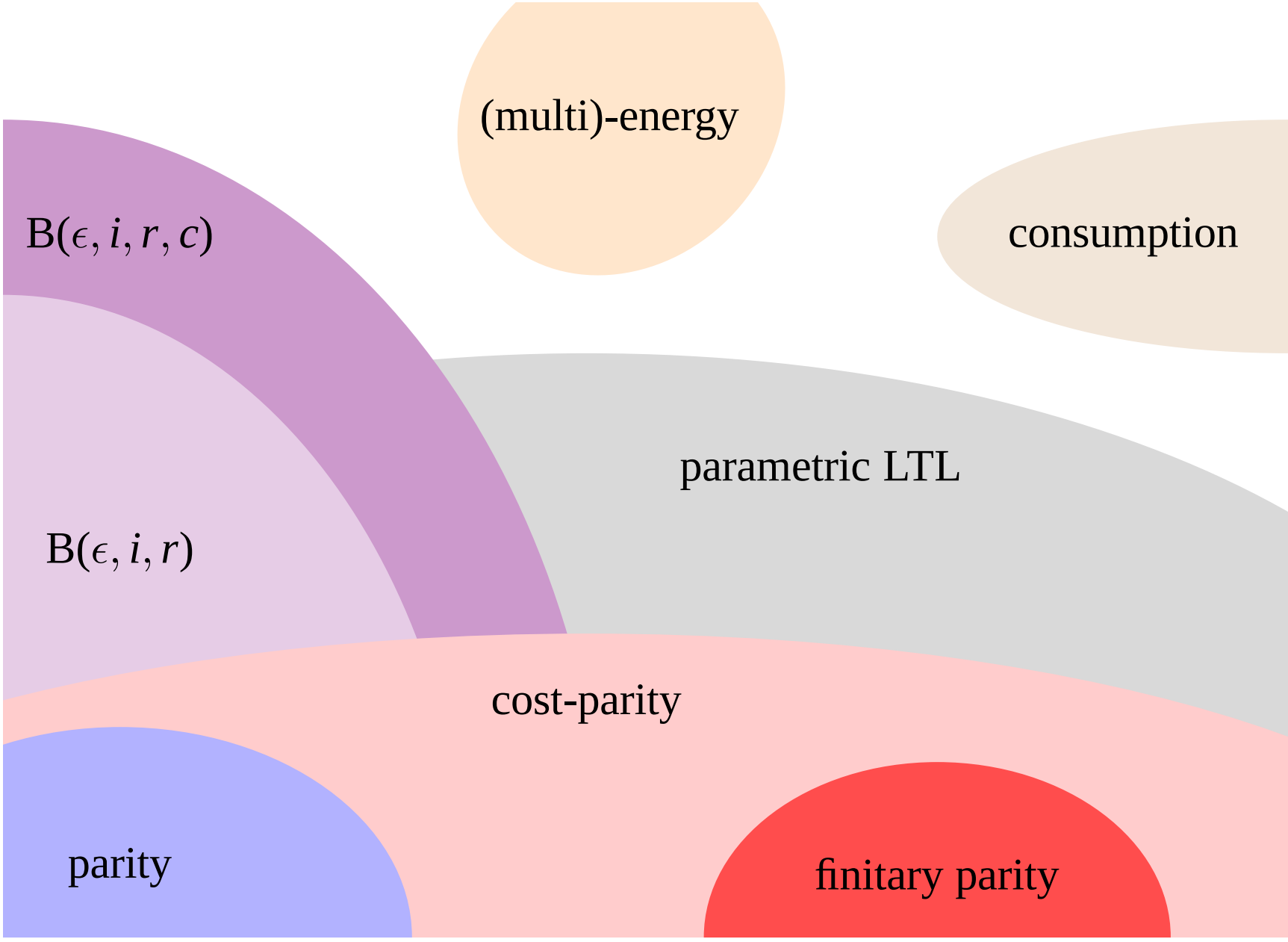
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Multi-energy

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(Tomáš Brázdil, Jakub Chaloupka, Krishnendu Chatterjee, Laurent Doyen, Uli Fahrenberg, Petr Jančar, Line Juhl, Antonín Kučera, Kim G. Larsen, Jean-François Raskin, Mickael Randour, Jiři Srba,. . .)

Consumption

~~**increment:** i or "$c \longleftarrow c + 1$"~~

no action: ε

decrement: "$c \longleftarrow c - 1$"

refill: "$c \longleftarrow c + \omega$"

(Tomáš Brázdil, Krishnendu Chatterjee, Antonín Kučera, Petr Novotný, CAV'2012)

B-conditions with checks

check: "counter c is bounded on **checked** values"

increment: i or "$c \longleftarrow c + 1$"

no action: ε

reset: "$c \longleftarrow 0$"

max: "$c \longleftarrow \max(c', c'')$"

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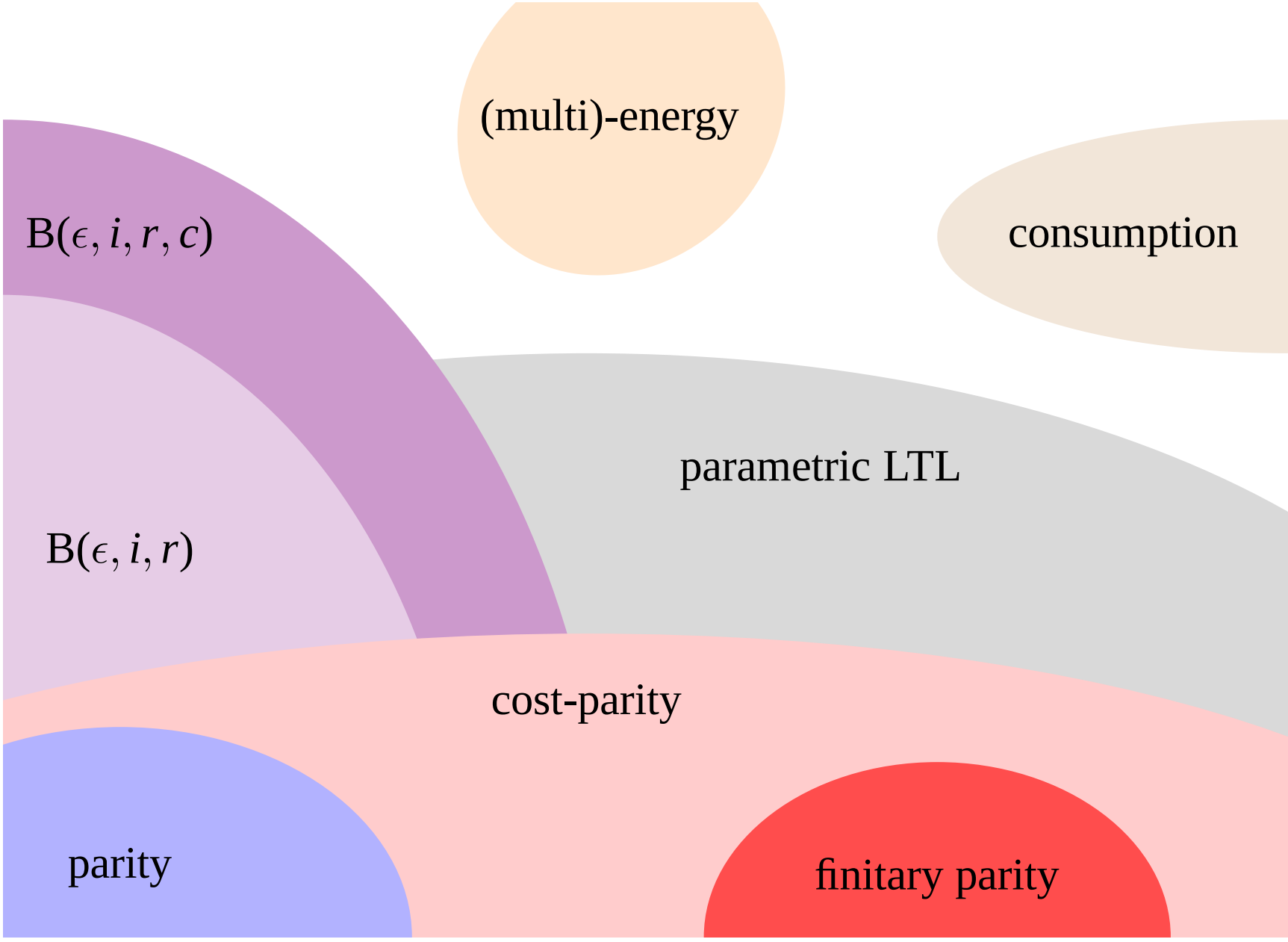
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