Finitary Languages

Presentation for LATA 2011

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Introduction: system specification

- non-terminating (*e.g* web server);
- discrete time;
- non-deterministic.

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- non-terminating;
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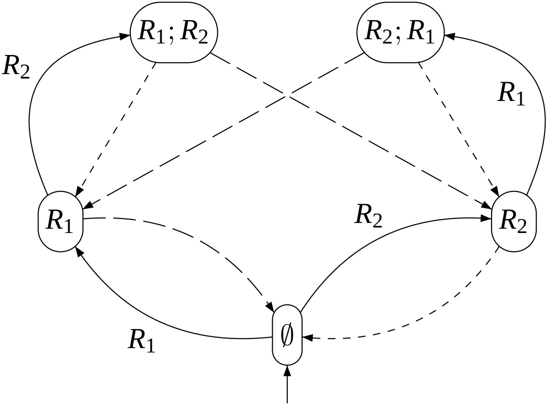
- non-deterministic.

- a finite alphabet Σ represent propositions;
  (*e.g* “available”, “waiting”, “critical error”)
- runs are infinite words $w = w_0 \cdot w_1 \ldots w_n \ldots \in \Sigma^\omega$;
- specification given as a language $L \subseteq \Sigma^\omega$.

Introduction: system specification



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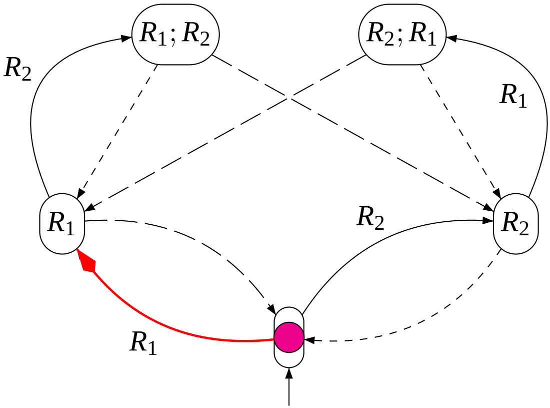

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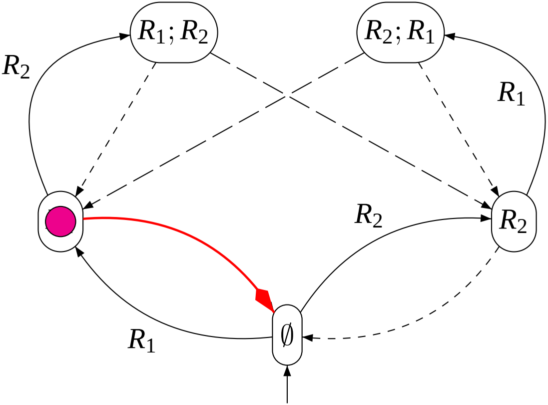

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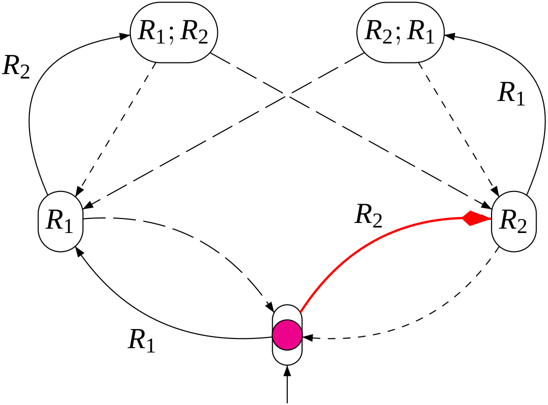

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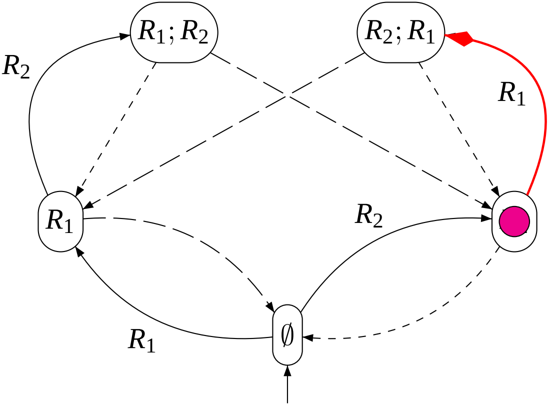

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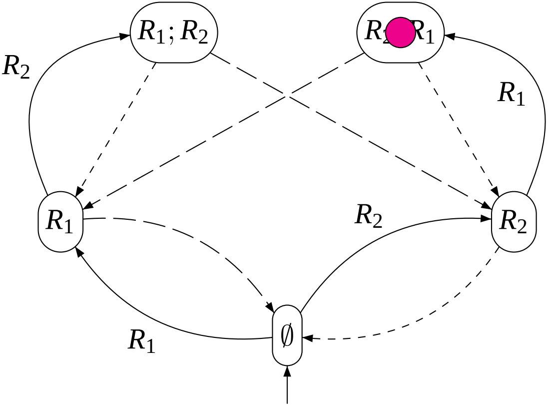

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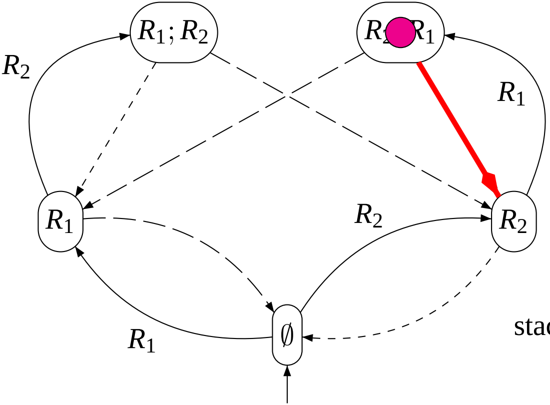

stack strategy

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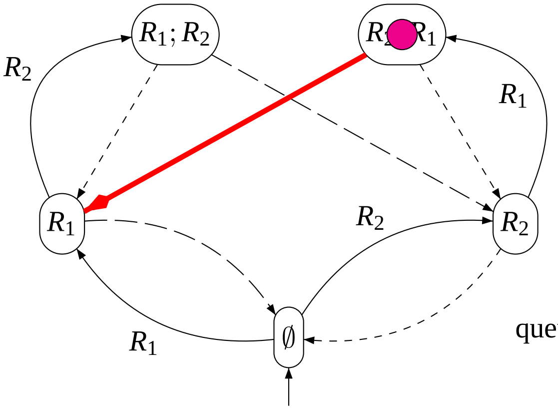

queue strategy

Introduction: system specification

- ω-regular language: safety + liveness;
- liveness properties: “something good happens eventually”.

Classical liveness properties

A first example, Büchi:

- a given set of propositions appears infinitely often;
  (e.g “job done”)

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- if R_i is requested infinitely often, then it is serviced (G_i) infinitely often.

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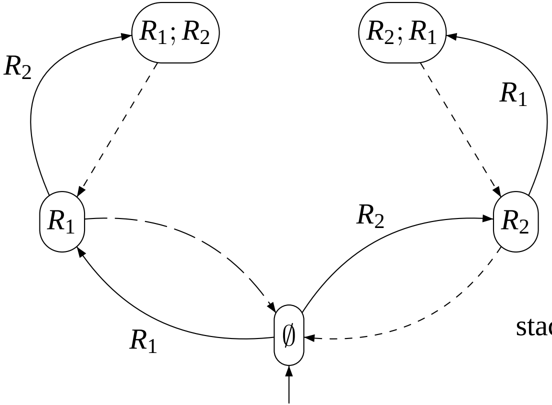
(special case: parity)

Outline

A drawback of classical ω-regular specifications



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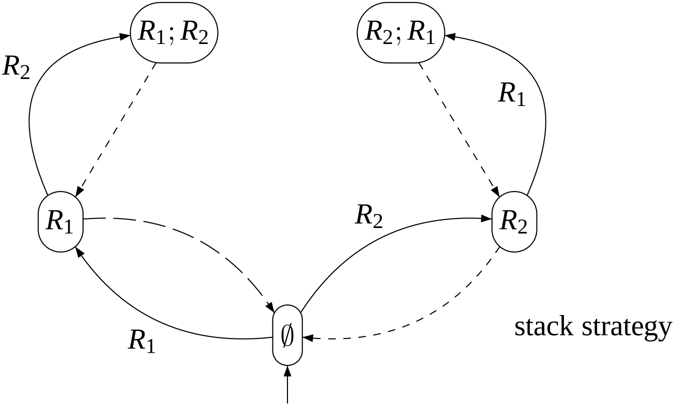

Streett specification: for $i \in \{1,2\}$, if R_i is requested infinitely often, then it is serviced infinitely often.

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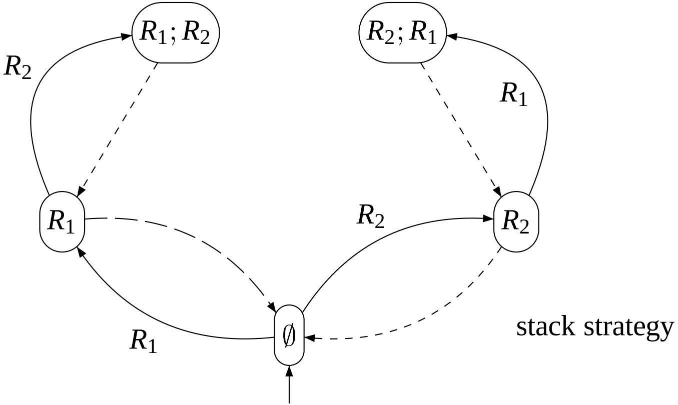

Streett specification: for $i \in \{1, 2\}$, if R_i is requested infinitely often, then it is serviced infinitely often.

Satisfied, but the "service time" may grow unbounded!

A stronger formulation of liveness: finitary liveness [AH94]

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unknown: retain independence from granularity.

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It can be expressed as a finitary operator on languages:

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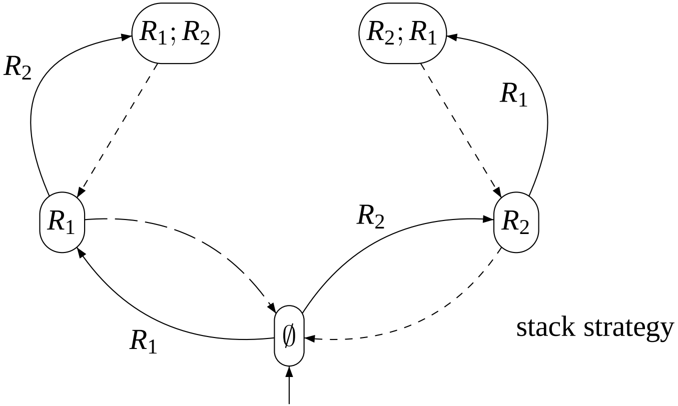
- closed: involves Cantor topology;
- ω-regular: involves ω-regularity;
- restriction operator: $\text{fin}(L) \subseteq L$.

Back to the example



Finitary Streett specification: there exists a bound b, such that *in the limit*, for $i \in \{1, 2\}$, if R_i is requested, then it is serviced within b transitions.

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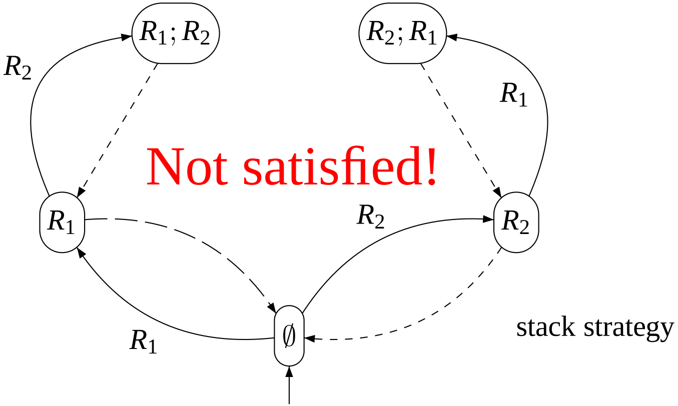

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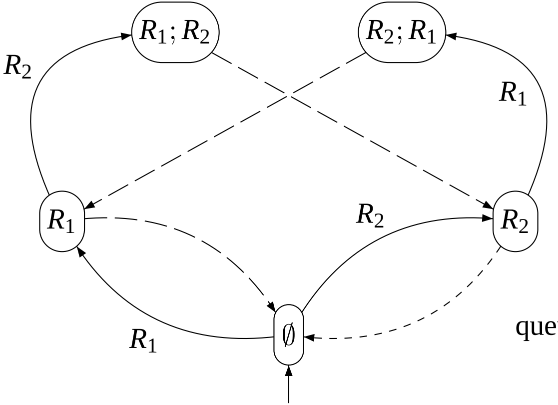

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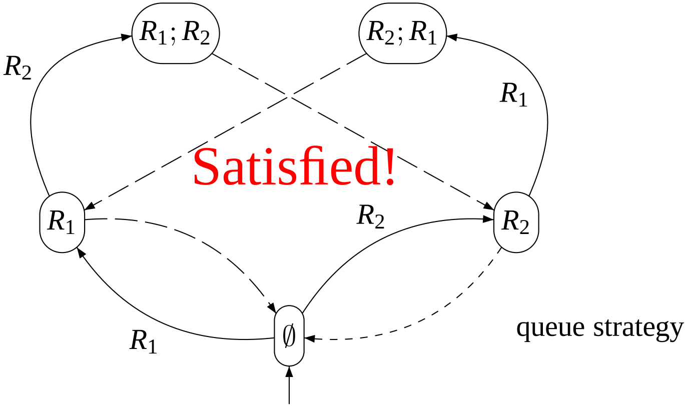

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Outline

Describing classical finitary objectives: Büchi

Let $F \subseteq \Sigma$,

$$\text{Büchi}(F) = \{w \mid \text{Inf}(w) \cap F \neq \emptyset\}$$

$\text{Inf}(w)$ is the set of propositions that appear infinitely often in w.

Describing classical finitary objectives: Büchi

Let $F \subseteq \Sigma$,

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$$w = v_0 \ldots v_k \underbrace{v_{k+1} \ldots v_{k'-1}}_{\notin F} \underbrace{v_{k'}}_{\in F}$$

waiting time from the k^{th} position.

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Lemma

$$\text{fin}(\text{Büchi}(F)) = \{w \mid \limsup_k \text{next}_k(w, F) < \infty\}$$

Topological classification in Borel hierarchy

Theorem

$\text{fin}(\text{Büchi}(F))$, $\text{fin}(\text{Parity}(p))$ *and* $\text{fin}(\text{Streett}(R, G))$ *are* Σ_2*-complete.*

Automata-theoretic expressive power

We consider automata over infinite words, whose acceptance conditions are finitary Büchi, finitary parity or finitary Streett.

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$$\left\{ \begin{array}{c} D \\ N \end{array} \right\} \cdot \left\{ \begin{array}{c} \varepsilon\ (\textit{classical}) \\ F(\textit{finitary}) \end{array} \right\} \cdot \left\{ \begin{array}{c} B\ (\textit{Büchi}) \\ P\ (\textit{parity}) \\ S\ (\textit{Streett}) \end{array} \right\}$$

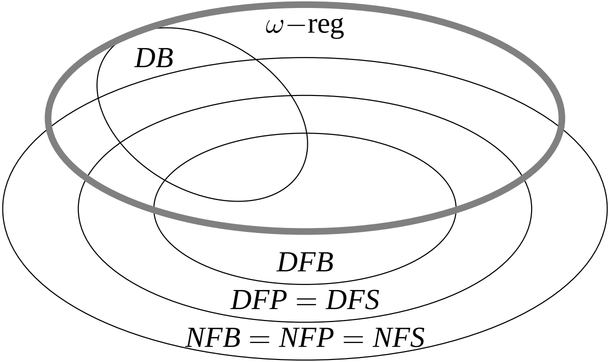

Figure: Expressive power classification

Outline

Regular and ω-regular expressions

Regular expressions defines regular languages over finite words:

$$L := \emptyset \mid \varepsilon \mid \sigma \mid \underbrace{L \cdot L}_{\text{concatenation}} \mid \underbrace{L^*}_{\text{star}} \mid \underbrace{L + L}_{\text{union}}; \quad \sigma \in \Sigma$$

ω-regular languages are finite union of $L_1 \cdot L_2^\omega$, where L_1 and L_2 are regular languages over finite words.

The bound operator B [BC06]

$$L^\omega = \{u_0 \cdot u_1 \cdot \ldots \cdot u_k \ldots \mid u_0, u_1, \ldots, u_k, \ldots \in L\}$$

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`Example:` $(a^* \cdot b)^\omega$ expresses "infinitely many b's".
`Example:` $(a^B \cdot b)^\omega$ expresses "infinitely many b's with an upper bound on the length of a's blocks".

Star-free ωB-regular expressions

B-regular languages are described by the grammar:

$$M := \emptyset \mid \varepsilon \mid \sigma \mid M \cdot M \mid M^* \mid M^B \mid M + M; \quad \sigma \in \Sigma$$

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- L is a regular language over finite words;
- M is a B-regular language over infinite words.

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Star-free ωB-regular languages are finite union of $L \cdot M^\omega$, where

- L is a regular language over finite words;
- M is a **star-free** B-regular language over infinite words.

“no star operator under the ω-operator”.

Equivalence

Theorem

NFB (non-deterministic finitary Büchi automata) has exactly the same expressive power as star-free ωB-regular expressions.

Examples

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`Second example:` $(a^B \cdot b \cdot (a^* \cdot b)^*)^\omega$ is **not** a star-free ωB-regular expression, it expresses "words of the form $a^{n_0} \cdot b \cdot a^{n_1} \cdot b \ldots$ such that $\liminf_i n_i < \infty$".

Conclusion

- finitary objectives is a refinment for specification purposes;
- for ω-regular languages, topological, logical and automata-theoretic studies are well-known;
- for finitary languages, all were missing; we established:
  - topological classification;
  - automata-theoretic characterization, comparison to ω-regular languages, closure properties;
  - characterization using by ωB-regular expressions.

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Future work:

- games (work in progress);
- a finitary logic, Myhill-Nerode equivalence relations, ...

Bibliography

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The end

Thank you for your attention!