Games on graphs

Séminaire Thésards

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Outline

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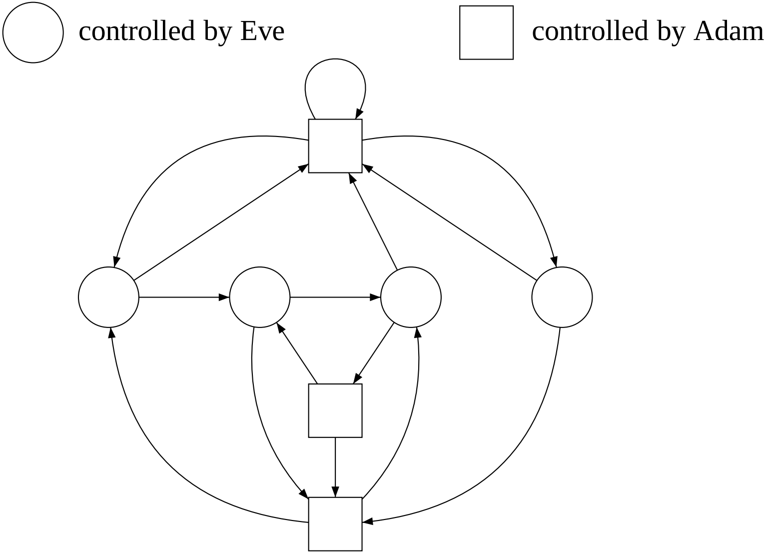
Players

Two players: **Eve** and **Adam**.

Plays



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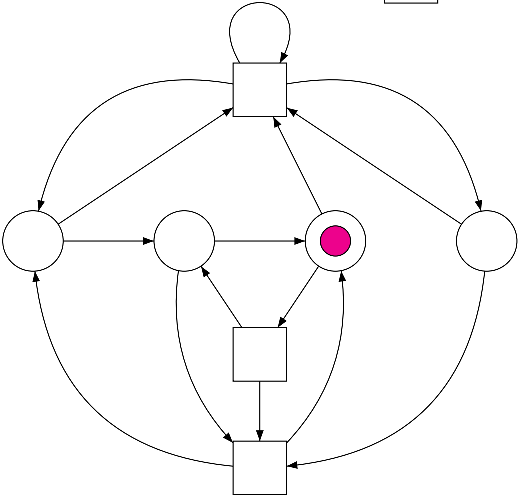
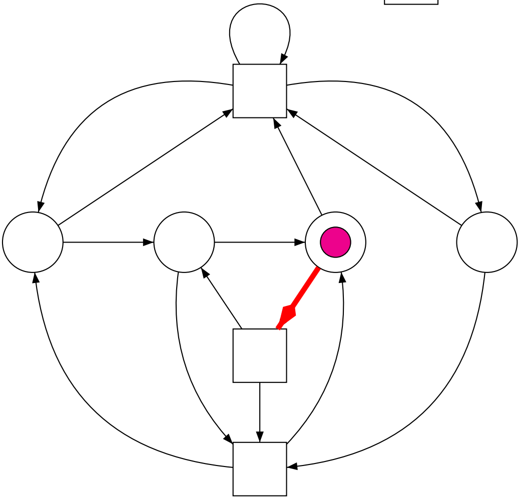


Plays

controlled by Eve

controlled by Adam



Plays

controlled by Eve

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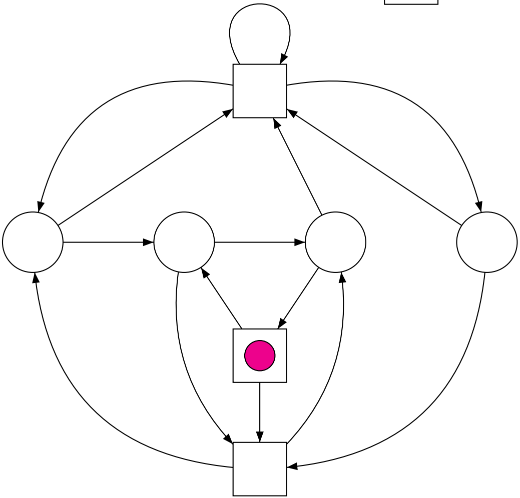

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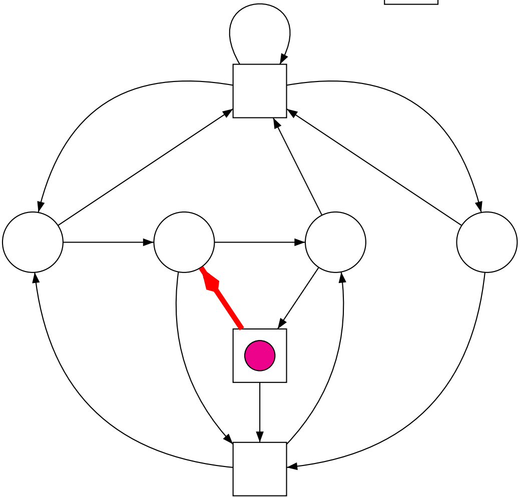

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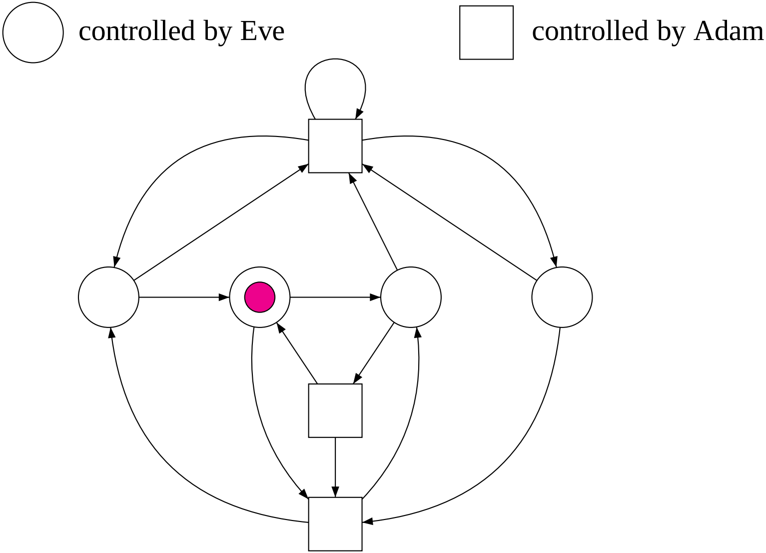

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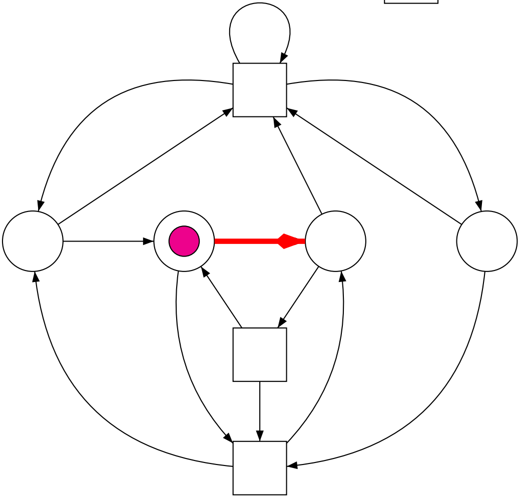

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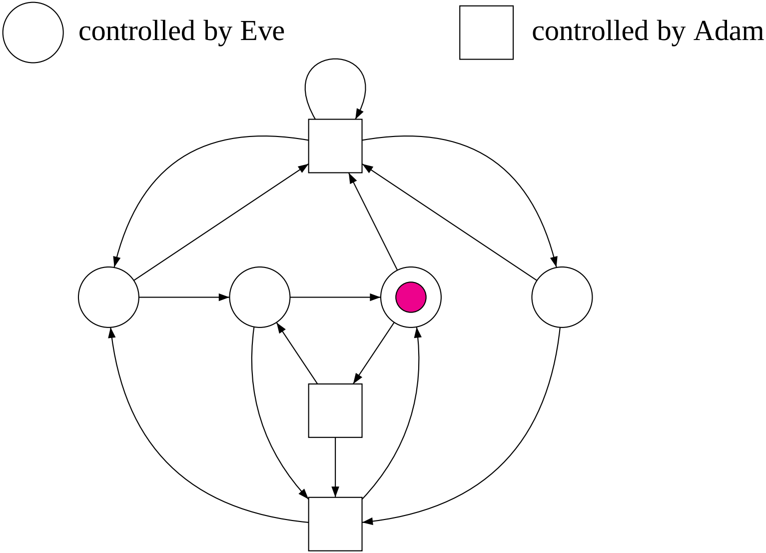

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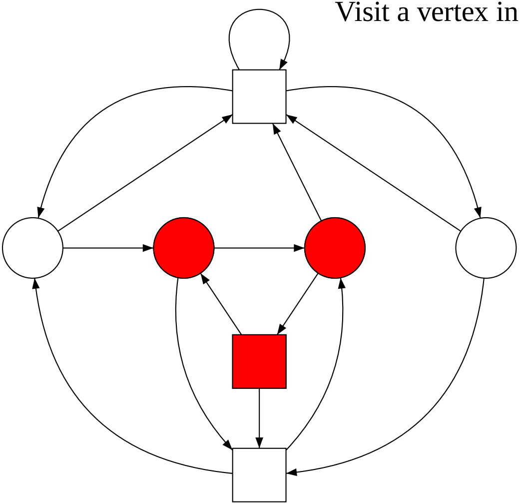
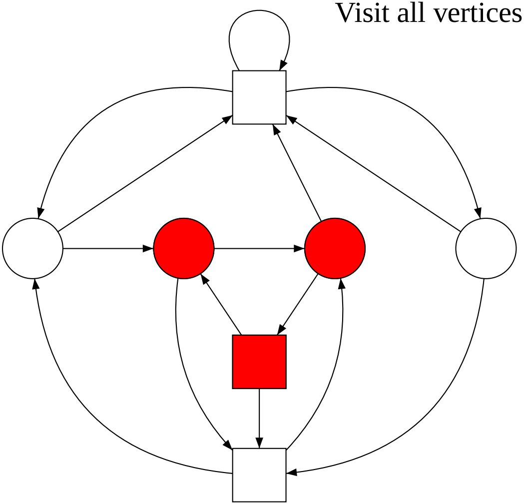
Reachability and Büchi objectives

Visit a vertex in F



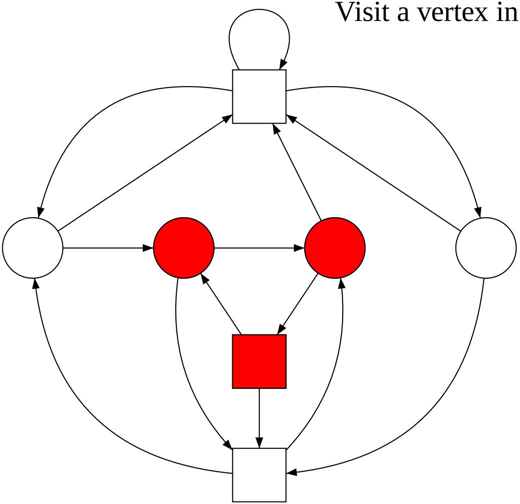
Reachability and Büchi objectives

Visit all vertices in F



Reachability and Büchi objectives

Visit a vertex in F infinitely often



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1. Games
   - Playing
   - Winning
2. Motivations
   - Deciding tree automata membership
   - Deciding logics
3. Problems and tools
   - Deciding a winner
   - Strategy complexity
   - Stochastic games
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Assume we can construct finitely representable winning strategies for Adam. Then we can use them as witnesses to complement automata.

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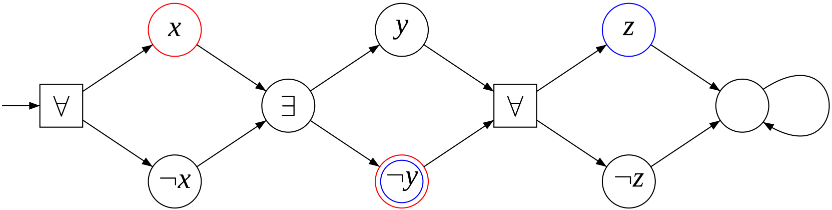
$$F_1 = \{x, \neg y\} \qquad F_2 = \{\neg y, z\}$$



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Hence we focus on ω-regular winning conditions, that is given by a Büchi automaton.

For these games, the winner problem is decidable.

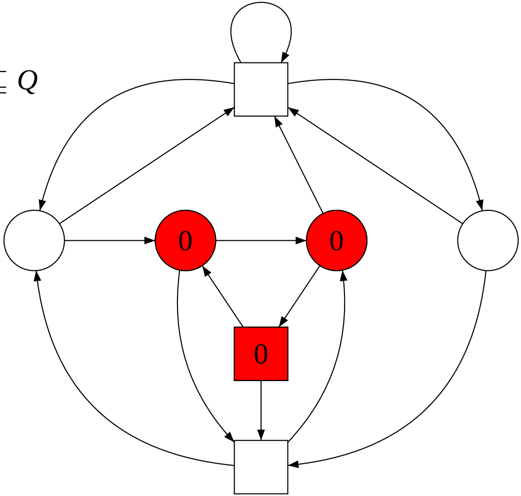
A sketchy sketch of a proof

Let $\mathbb{G}$ an arena and $\mathcal{A}$ a deterministic Büchi automaton.

- First compute the synchronous product $\mathbb{G} \times \mathcal{A}$. A play is winning if its second component is accepted by $\mathcal{A}$;
- The resulting game is a Büchi game.
- We are now left to solve Büchi games. This can be done efficiently (quadratic time in the size of the arena), using the following ideas:

Solving reachability games: attractor computation

Given $F \subseteq Q$



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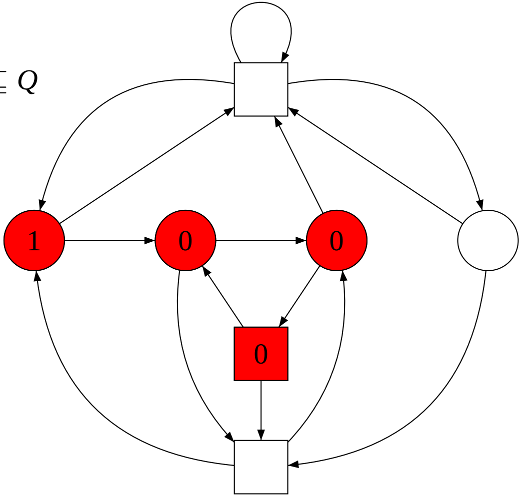

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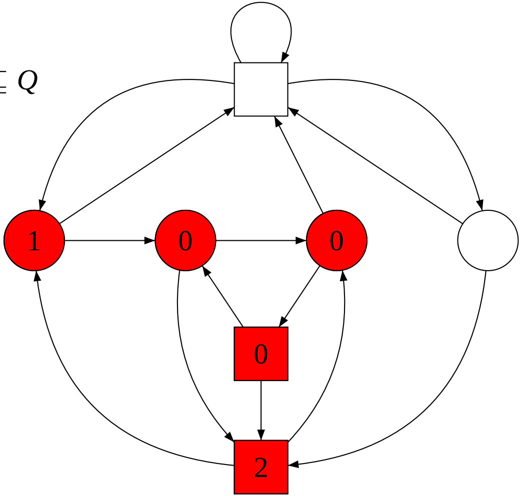

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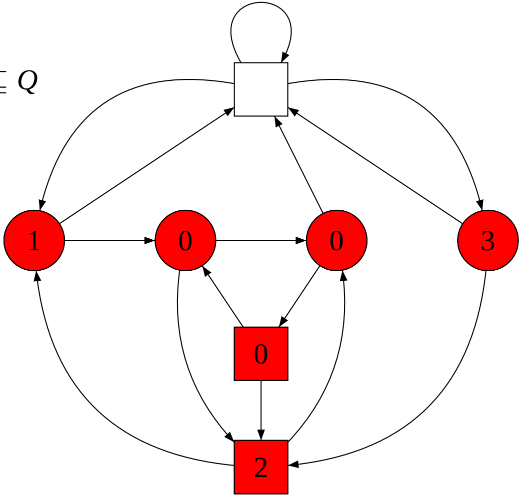

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Informally, to ensure to reach F at least $k+1$ times, you have to ensure to reach it k times and then to ensure at least one visit.
We can show that for $k \geq n+1$, the k-repeated reachability games and Büchi games are equivalent.

Questions

- What about non-deterministic automata?
- Do these proofs allow to construct winning strategies?

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Different models of strategy

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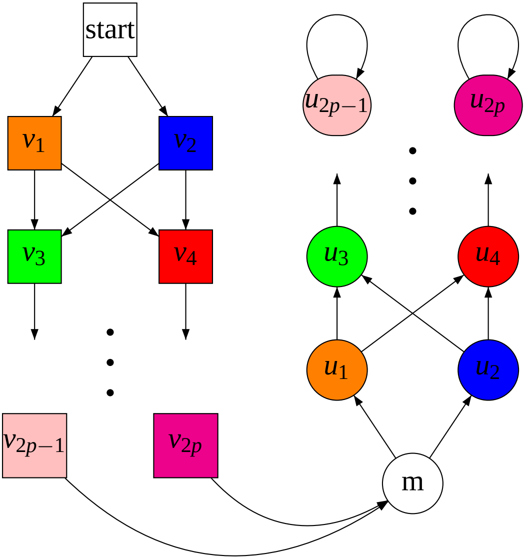
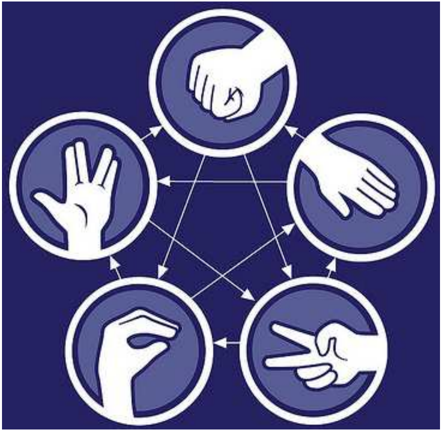
Strategies can use memory or randomization.

Why memory?



Why randomization?



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The end.

Thank for your attention!