a) Euler Verfahren

$y_{i+1} = y_i + h \cdot f(x_i, y_i)$

Iteration von $x_0 = 0$ zu $x_1 = 0.7$

$y_1 = 2 + 0.7 \cdot f(0,2) = 2$

Iteration von $x_1 = 0.7$ zu $x_2 = 1.4$

$y_2 = 2 + 0.7 \cdot f(0.7, 2) = 2.1715$

b) Mittelpunkt verfahren

$k_1 = f(x_i, y_i)$
$y_{i+x_2} = y_i + \frac{h}{2} \cdot k_1$
$x_{i+x_2} = x_i + \frac{h}{2}$
$k_2 = f(x_{i+x_1}, y_{i+x_1})$
$y_{i+1} = y_i + h \cdot k$

Iteration von $x_0 = 0$ zu $x_1 = 0.7$
$k_1 = f(0,2) = 0$
$y_{0.5} = 2 + 0.35 \cdot 0 = 2$
$x_{0.5} = 0 + 0.35 = 0.35$
$k_2 = f(0.35, 2) = 0.06125$
$y_1 = 2 + 0.7 \cdot 0.06125 = 2.0429$

Iteration von $x_1 = 0.7$ zu $x_2 = 1.4$

$k_1 = f(0.7, 2.0429) = 0.24$
$y_{1.5} = 2.0429 + 0.35 + 0.24 = 2.1269$
$x_{1.5} = 0.7 + 0.35 = 1.05$
$k_2 = f(1.05, 2.1269) = 0.518$
$y_2 = 2.0429 + 0.7 \cdot 0.518 = 2.4055$

c) Modifiziertes Euler verfahren

$k_1 = f(x_i, y_i)$
$k_2 = f(x_i + h, y_i + h \cdot k_1)$
$y_{i+1} = y_i + \frac{h}{2}(k_1 + k_2)$

Iteration von $x_0 = 0$ zu $x_1 = 0.7$

$k_1 = f(0,2) = 0$
$k_2 = f(0.7, 2 + 0.7 \cdot 0) = 2.45$
$y_1 = 2 + \frac{0.7}{2}(0 + 0.245) = 2.0858$

Iteration von $x_1 = 0.7$ zu $x_2 = 1.4$
$k_1 = f(0.7, 2.0858) = 0.235$
$k_2 = f(1.4, 2.0858 + 0.7 \cdot 0.235) = 0.891$
$y_2 = 2.0858 + \frac{0.7}{2}(0.235 + 0.891) = 2.4729$

Tehleranalyse

exakte lösung

$y(x) = \sqrt{\frac{2x^3}{3} + 4}$

An den Stellen
$y(0) = \sqrt{0+4} = 2$
$y(0.7) = \sqrt{0.2287 + 4} = 2.056$
$y(1.4) = \sqrt{1.829 + 4} = 2.414$

Methode	Abs Fehler bei $x=0.7$	Abs Fehler bei $x=1.4$
Euler	$\lvert 2 - 2.056 \rvert = 0.056$	$\lvert 2.1715 - 2.414 \rvert = 0.2425$
Mittelpunkt	$\lvert 2.0429 - 2.056 \rvert = 0.0131$	$\lvert 2.4055 - 2.414 \rvert = 0.0085$
Mod. Euler	$\lvert 2.0858 - 2.056 \rvert = 0.0298$	$\lvert 2.4729 - 2.414 \rvert = 0.0589$