

Exakter wert berechnen

$$t = \int_{20}^5 \frac{10}{v^{3/2}} dv = -10 \int_{20}^5 v^{-3/2} dv$$

$$t = 10 \int_5^{20} v^{-3/2} dv = 10 \cdot [-2v^{-1/2}]_5^{20}$$

$$t = 10 \cdot (-2 \cdot \frac{1}{\sqrt{20}} + 2 \cdot \frac{1}{\sqrt{5}}) = 20 (\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{20}})$$

$$t = 20 (\frac{1}{2.2361} - \frac{1}{4.4721}) = 4.472s$$

Approximation

$$h = \frac{20-5}{5} = 3$$

Stützstellen

$$v_0 = 5, v_1 = 8, v_2 = 11, v_3 = 14, v_4 = 17, v_5 = 20$$

Funktionswerte

$$f(v) = \frac{10}{v^{3/2}} = -10 \cdot v^{-3/2}$$

$v_i$	$f(v_i)$
5	$-10/11.18 = -0.8944$
8	$-10/22.623 = -0.4420$
11	$-10/36.48 = -0.2742$
14	$-10/52.38 = -0.1909$
17	$-10/70.09 = -0.1423$
20	$-10/89.44 = -0.1118$

Rechteckes regel

$$T_{\text{rect}} = h \cdot \sum_{i=0}^4 f(v_i) = 3 \cdot (-0.8944 - 0.4420 - 0.2742 - 0.1909 - 0.1423)$$

$$T_{\text{rect}} = 3 \cdot (-1.9442) = -5.8326 \Rightarrow 5.8326s$$

$$|\text{Fehler}| = |5.833 - 4.472| = 1.361s$$

Trapezregel

$$T_{\text{trap}} = \frac{h}{2} (f(v_0) + 2f(v_1) + 2f(v_2) + 2f(v_3) + 2f(v_4) + f(v_5)) = \frac{3}{2} (-0.8944 + 2(-0.4420 - 0.2742 - 0.1909 - 0.1423) - 0.1118) = -4.6587 \Rightarrow 4.6587s$$

$$|\text{Fehler}| = |4.6587 - 4.472| = 0.187s$$

Simpson regel

$$T_{\text{simp}} = \frac{h}{3} (f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + f_5) = \frac{3}{3} (-0.8944 + 4(-0.4420) + 2(-0.2742) + 4(-0.1909) + 2(-0.1423) - 0.1118) = -4.3716 \Rightarrow 4.3716s$$

$$|\text{Fehler}| = |4.372 - 4.472| = 0.100$$