

Aufgabe 2:

a) manuelles partielles Ableiten, um zu zeigen, dass die Funktionen die Wellengleichung erfüllen.

Wellengleichung: $\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2} \rightarrow$ es muss also gelten dass: $x^2 = c^2 \cdot t^2$

I) $w(x, t) = \sin(x + ct)$ 2. part. Abl.

$$\left. \begin{aligned} \frac{\partial w}{\partial x} &= \cos(x + ct) \\ \frac{\partial w}{\partial t} &= \cos(x + ct) \cdot c \end{aligned} \right\} \begin{aligned} \frac{\partial^2 w}{\partial x^2} &= -\sin(x + ct) \\ \frac{\partial^2 w}{\partial t^2} &= c^2 \cdot (-\sin(x + ct)) \end{aligned} \Rightarrow \text{erfüllt Wellengleichung}$$

II) $v(x, t) = \sin(x + ct) + \cos(2x + 2ct)$

$$\left. \begin{aligned} \frac{\partial v}{\partial x} &= \cos(x + ct) - 2\sin(2x + 2ct) \\ \frac{\partial v}{\partial t} &= c \cdot \cos(x + ct) - 2c \cdot \sin(2x + 2ct) \end{aligned} \right\} \begin{aligned} \frac{\partial^2 v}{\partial x^2} &= -\sin(x + ct) - 4\cos(2x + 2ct) \\ \frac{\partial^2 v}{\partial t^2} &= c^2 \cdot (-\sin(x + ct) - 4\cos(2x + 2ct)) \end{aligned} \Rightarrow \text{erfüllt Wellengleichung}$$