Aufgabe 1

$S_i(x) = a_i + b_i(x-x_i) + c_i(x-x_i)^2 + d_i(x-x_i)^3$

$h_0 = x_1 - x_0 = 6-4 = 2$

$h_1 = x_2 - x_1 = 8-6 = 2$

$h_2 = x_3 - x_2 = 10-8 = 2$

$a_i = y_i \Rightarrow a_0 = x_0 = 6$
$a_1 = y_1 = 3$
$a_2 = y_2 = 9$

$m_i = S''(x_i)$, weil natürliche spline $m_0 = m_3 = 0$

$\frac{h_{i-1}}{6} m_{i-1} + \frac{h_{i-1}+h_i}{3} m_i + \frac{h_i}{6} m_{i+1} = \frac{y_{i+1}-y_i}{h_i} - \frac{y_i - y_{i-1}}{h_{i-1}}$

für $i=1$ $\frac{2}{6}\cdot 0 + \frac{4}{3}m_1 + \frac{2}{6}m_2 = \frac{9-3}{2} - \frac{3-6}{2} = 3+1.5 = 4.5 \Rightarrow \frac{4}{3}m_1 + \frac{1}{3}m_2 = 4.5$

für $i=2$ $\frac{2}{6}m_1 + \frac{4}{3}m_2 + \frac{2}{6}\cdot 0 = \frac{0-9}{2} - \frac{9-3}{2} = -4.5-3 = -7.5 \Rightarrow \frac{1}{3}m_1 + \frac{4}{3}m_2 = -7.5$

$\left(\frac{4}{3}m_1 + \frac{1}{3}m_2 = 4.5\right) \cdot 3 \Rightarrow 4m_1 + m_2 = 13.5$

$\left(\frac{1}{3}m_1 + \frac{4}{3}m_2 = -7.5\right) \cdot 3 \Rightarrow m_1 + 4m_2 = -22.5$

$m_2 = 13.5 - 4m_1 \Rightarrow m_2 = 13.5 - 4(5.1) \Rightarrow m_2 = -6.9$

$m_1 + 4(13.5-4m_1) = -22.5 \Rightarrow -15m_1 + 54 = -22.5 \Rightarrow -15m_1 = -76.5 \Rightarrow m_1 = 5.1$

$m_0 = 0$, $m_1 = 5.1$, $m_2 = -6.9$, $m_3 = 0$

$b_i = \frac{y_{i+1}-y_i}{h_i} - \frac{h_i}{6}(2m_i + m_{i+1})$

für $i=0$ $b_0 = \frac{3-6}{2} - \frac{2}{6}(2\cdot 0 + 5.1) = -3.2$

für $i=1$ $b_1 = \frac{9-3}{2} - \frac{2}{6}(2\cdot 5.1 - 6.9) = 1.9$

für $i=2$ $b_2 = \frac{0-9}{2} - \frac{2}{6}(2\cdot(-6.9)+0) = 0.1$

$c_i = \frac{m_i}{2}$

$c_0 = 0$

$c_1 = \frac{5.1}{2} = 2.55$

$c_2 = \frac{-6.9}{2} = -3.45$

$d_i = \frac{m_{i+1}-m_i}{6h_i}$

$d_0 = \frac{5.1-0}{12} = 0.425$

$d_1 = \frac{-6.9-5.1}{12} = -1$

$d_2 = \frac{0+6.9}{12} = 0.575$

$S_0(x),\ x \in [4,6]$ $\quad S_0(x) = 6 - 3.2(x-4) + 0.425(x-4)^3$

$S_1(x),\ x \in [6,8]$ $\quad S_1(x) = 3 + 1.9(x-6) + 2.55(x-6)^2 - (x-6)^3$

$S_2(x),\ x \in [8,10]$ $\quad S_2(x) = 9 + 0.1(x-8) - 3.45(x-8)^2 + 0.575(x-8)^3$