Course Scheduling

Stanford CS221 Fall 2014-2015

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Note: grader.py only provides basic tests. Passing grader.py does not by any means guarantee full points.

a. **Revision 1** (12PM, Oct. 29th): bug fix to util.py, please re-download it
b. This icon means a written answer is expected in your writeup.pdf
c. This icon means a coded solution is expected in your submission.py
d. There will be a lot of reading in this assignment. Be patient. It's worth your time! :)
e. **Start early**. Ask questions. Have fun.

What courses should you take in a given quarter? Answering this question requires balancing your interests, satisfying prerequisite chains, graduation requirements, availability of courses; this can be a complex tedious process. In this assignment, you will write a program that does automatic course scheduling for you based on your preferences and constraints. The program will cast the course scheduling problem (CSP) as a constraint satisfaction problem (CSP) and then use backtracking search to solve that CSP to give you your optimal course schedule.

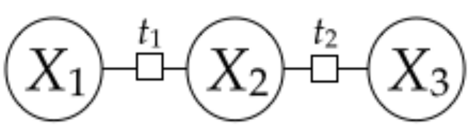
You will first get yourself familiar with CSP by doing warmup exercises in Problem 0. In Problem 1, you will implement two of the three heuristics you learned from the lectures that will make CSP solving much faster. In problem 2, you will add a helper function to reduce n-ary potentials to unary and binary potentials. Lastly, in Problem 3, you will create the course scheduling CSP and solve it using the code from previous parts.

Problem 0: Warmup

a. Let's create a CSP. Suppose you have n light bulbs, where each light bulb $i = 1, \dots, n$ is initially off. You also have m buttons which control the lights. For each button $j = 1, \dots, m$, we know the subset $T_j \subset \{1, \dots, n\}$ of light bulbs that it controls. When button j is pressed, it toggles the state of each light bulb in T_j (For example, if $3 \in T_j$ and light bulb 3 is off, then after the button is pressed, light bulb 3 will be on, and vice versa).

   Your goal is to turn on all the light bulbs by pressing a subset of the buttons. Construct a CSP to solve this problem. Your CSP should have m variables and n constraints. *For this problem only*, you can use n-ary constraints. Describe your CSP precisely and concisely.

b. Let's consider a simple CSP with 3 variables and 2 binary potentials:



   where $X_1, X_2, X_3 \in \{0, 1\}$ and t_1, t_2 are XOR functions (that is $t_1(X) = x_1 \bigoplus x_2$ and $t_2(X) = x_2 \bigoplus x_3$).

   i. How many consistent assignments are there for this CSP?
   ii. To see why variable ordering is important, let's use backtracking search to solve the CSP, *without using any heuristic*. How many times will `backtrack()` be called if we use the fixed ordering X_1, X_2, X_3?

What if the ordering is X_1, X_3, X_2? (You should use the Backtrack algorithm from the slides. The initial arguments are $x = \emptyset$, $w = 1$, and the original Domain.)

In the code, this will be `BacktrackingSearch.numOperations`.

iii. To see why lookahead can be useful, let's do it again with the ordering X_1, X_3, X_2 and AC-3. How many times will Backtrack be called?

c. Now let's consider a general case: given a factor graph with n variables $X_1, \ldots, X_n$ and $n-1$ binary potentials $t_1, \ldots, t_{n-1}$ where $X_i \in \{0, 1\}$ and $t_i(X) = x_i \bigoplus x_{i+1}$. Note that the CSP has a chain structure. Implement `create_chain_csp()` by creating a generic chain CSP with XOR as potentials.

Note: We've provided you with a CSP implementation in `util.py` which supports unary and binary potentials. For now, you don't need to understand the implementation, but please read the comments and get yourself familiar with the CSP interface. For this problem, you'll need to use `CSP.add_variable()` and `CSP.add_binary_potential()`.

Problem 1: CSP solving

So far, we've only worked with unweighted CSPs, where $f_j(x) \in \{0, 1\}$. In this problem, we will work with weighted CSPs, which associates a weight for each assignment x based on the product of m potential functions $f_1, \ldots, f_m$:

$$\text{Weight}(x) = \prod_{j=1}^{m} f_j(x)$$

where each potential $f_j(x) \geq 0$. Our goal is to find the assignment(s) x with the *highest* weight. As in problem 0, we will assume that each potential is either a unary potential (depends on exactly one variable) or a binary potential (depends on exactly two variables).

For weighted CSP construction, you can refer to the CSP examples we provided in `util.py` for guidance (`create_map_coloring_csp()` and `create_weighted_csp()`). You can try these examples out by running

```
python run_p1.py
```

Notice we are already able to solve the CSPs, because in `submission.py`, a basic backtracking search is already implemented. Recall that backtracking search operates over partial assignments and associates each partial assignment with a weight, which is the product of all the potentials that depend only on the assigned variables. When we assign a value to a new variable X_i, we multiply in all the potentials that depend only on X_i and the previously assigned variables. The function `get_delta_weight()` returns the contribution of these new potentials based on the `unaryPotentials` and `binaryPotentials`. An important case is when `get_delta_weight()` returns 0. In this case, any full assignment that extends the new partial assignment will also be zero, so *there is no need to search further with that new partial assignment*.

Take a look at `BacktrackingSearch.reset_results()` to see the other fields which are set as a result of solving the weighted CSP. You should read `submission.BacktrackingSearch` carefully to make sure that you understand how the backtracking search is working on the CSP.

a. Let's create a CSP to solve the n-queens problem: Given an $n \times n$ board, we'd like to place n queens on this board such that no two queens are on the same row, column, or diagonal. Implement `create_nqueens_csp()` by **adding n variables** and some number of binary potentials. Note that the solver collects some basic statistics on the performance of the algorithm. You should take advantage of these statistics for debugging and analysis. You should get 92 (optimal) assignments for $n = 8$ with exactly 2057 operations (number of calls to `backtrack()`).

Hint: If you get a larger number of operations, make sure your CSP is minimal.

b. You might notice that our search algorithm explores quite a large number of states even for the 8×8 board. Let's see if we can do better. One heuristic we discussed in class is using most constrained variable (MCV): To choose an unassigned variable, pick the X_j that has the fewest number of values a which are consistent with the current partial assignment (a for which `get_delta_weight()` on $X_j = a$ returns a non-zero value). Implement this heuristic in `get_unassigned_variable()` under the condition `self.mcv = True`. It should take you exactly 1361 operations to find all optimal assignments for 8 queens CSP. — that's 30% fewer!

Note: in `CSP` and `BacktrackingSearch` we always use indices to represent variables and their values. While the variable name can be any hashable object (for example `int`, `str` or tuple of hashable elements), there's a unique index associated with it in a given CSP. For example, if we start with an empty CSP and add a variable `'Victoria'`, its index will be `0` as it's the first variable added. Given the index (`0` for this case), you can get the actual variable name by accessing `csp.varNames[0]` and get its domain by accessing `csp.valNames[0]`. Similarly, a variable's value can be *any* object (not necessarily hashable) and there's also a unique index associated with each value given a variable. Suppose the previous variable `'Victoria'` has three possible values `['red', 'green', 'blue']`, the index for `'red'` would be `0`, the index for `'green'` would be `1` and so on. **Whenever you see `var` (for a variable) and `val` (for a value) in the code, they almost always are the indices, not names.**

Why do we do this? This is done for efficiency as accessing arrays is faster than accessing hash tables. For example:

- `csp.varNames[var]` gives you the variable name
- `csp.valNames[var][val]` gives you the val-th variable value
  valNames[var] gives the whole domain, [var][val] gives a particular value
- `csp.unaryPotentials[var][val]` gives the unary potential value
- `csp.binaryPotentials[var1][var2][val1][val2]` gives the binary potential value
- In `BacktrackingSearch`, `assignment[var]` gives the index of assigned value

where `var1` and `var2` are indices of variables and `val1` and `val2` are indices of their corresponding values.

c. The previous heuristics looked only at the local effects of a variable or value. Let's now implement arc consistency (AC-3) that we discussed in lecture. After we set variable X_j to value a, we remove the values b of all neighboring variables X_k that could cause arc-inconsistencies. If X_k's domain has changed, we use X_k's domain to remove values from the domains of its neighboring variables. This is repeated until no domains have changed. Note that this may significantly reduce your branching factor, although at some cost. In `backtrack()` we've implemented code which copies and restores domains for you. Your job is to fill in `arc_consistency_check()`.

You should make sure that your existing MCV implementation is compatible with your AC-3 algorithm as we will be using all three heuristics together during grading. With AC-3 enabled, it should take you 769 operations only to find all optimal assignments to 8 queens CSP — That is almost 45% fewer even compared with MAC!

Take a deep breath! This part requires time and effort to implement — be patient.

Hint 1: documentation for `CSP.add_unary_potential()` and `CSP.add_binary_potential()` can be helpful.
Hint 2: although AC-3 works recursively, you may implement it iteratively. Using a queue might be a good idea.

Problem 2: Handling n-ary potentials

So far, our CSP solver only handles unary and binary potentials, but for course scheduling (and really any non-trivial application), we would like to define potentials that involve more than two variables. It would be nice if we could have a general way of reducing n-ary constraint to unary and binary constraints. In this problem, we will do exactly that for two types of n-ary constraints.

Suppose we have boolean variables X_1, X_2, X_3, where X_i represents whether the i-th course is taken. Suppose we want to enforce the constraint that $Y = X_1 \vee X_2 \vee X_3$, that is, Y is a boolean representing whether at least one course has been taken. In `submission.py`, the function `get_or_variable()` does such a reduction. It takes in a list of variables and a target value, and returns a boolean variable with domain `[True, False]` whose value is constrained to the condition of having at least one of the variables assigned to the target value. For example, we would call `get_or_variable()` with arguments $(X_1, X_2, X_3, \text{True})$, which would return a new (auxiliary) variable X_4, and then add another constraint $[X_4 = \text{True}]$.

The second type of n-ary potentials is constraints on the sum over n variables. You are going to implement reduction of this type but let's first look at a simpler problem to get started:

a. Suppose we have a CSP with three variables X_1, X_2, X_3 with the same domain $\{0, 1, 2\}$ and a ternary constraint $[X_1 + X_2 + X_3 \leq 6]$. How can we reduce this CSP to one with only unary and/or binary constraints? Explain what auxiliary variables we need to introduce, what their domains are, what unary/binary potentials you'll add, and why your scheme works. Add a graph if you think that'll better explain your scheme.

Hint: draw inspiration from the example of enforcing $[X_i = 1 \text{ for exactly one } i]$ *which Percy did in the lecture.*

b. Now let's do the general case in code: implement `get_sum_variable()`, which takes in a sequence of non-negative integer-valued variables and returns a variable whose value is constrained to equal the sum of the variables. You will need to access the domains of the variables passed in, which you can assume contain only non-negative integers. The parameter `maxSum` is the maximum sum possible of all the variables. You can use this information to decide the proper domains for your auxiliary variables.

How do you use this? Suppose we wanted to enforce the constraint $[X_1 + X_2 + X_3 \leq K]$. We would call `get_sum_variable()` on (X_1, X_2, X_3) to get some auxiliary variable Y, and then add the constraint $[Y \leq K]$.

Problem 3: Course Scheduling

In this problem, we will apply your weighted CSP solver to the problem of course scheduling. We have scraped a subset of courses that are offered this year from Stanford's Bulletin. For each course in this dataset, we have information on which quarters it is offered, the prerequisites (which may not be fully accurate due to ambiguity in the listing), and the range of units allowed. You can take a look at all the courses in `courses.json`. Please refer to `util.Course` and `util.CourseBulletin` for more information.

To specify a desired course plan, you would need to provide a *profile* which specifies your constraints and preferences for courses. A profile is specified in a text file (see `profile*.txt` for examples). The profile file has four sections. The first section specifies a fixed minimum and maximum (inclusive) number of units you need to take for each quarter. In the second section, you `register` for the quarters that you want to take your courses in. For example, `register Aut2013` would sign you up for this quarter. The quarters need not to be contiguous, but they must follow the exact format `XxxYYYY` where `Xxx` is one of `Spr, Sum, Aut, Win` and `YYYY` is the year. The third section specifies the list of courses that you've taken in the past and elsewhere using the `taken` keyword. The the last section is a list of courses that you would like to take during the registered quarters, specified using `request`. Not every course listed in `request` must appear in the generated schedule. Conversely, a list of requests could potentially result in an infeasible schedule due to the additional constraints we will discuss next.

To allow for more flexibility in your preferences, we allow some freedom to customize the requests. For instance, if you only want to take exclusively one of several courses but not sure which one, then specify:

```
request CS229 or CS229A or CS229T
```

Note that these courses do not necessarily have to be offered in the same quarter. The final schedule can have at most one of these three courses. **Each course can only be requested at most once.**

If you want to take a course in one of a specified set of quarters, use the `in` modifier. For example, if you want to take one of CS221 or CS229 in either Aut2013 **or** Sum2016, do:

```
request CS221 or CS229 in Aut2013,Sum2016
```

Another operator you can apply is `after`, which specifies that a course must be taken after another one. For example, if you want to choose one of CS221 or CS229 and take it after both CS109 **and** CS161, add:

```
request CS221 or CS229 after CS109,CS161
```

Note that this implies that if you take CS221 or CS229, then you must take both CS109 and CS161. In this case, we say that CS109 and CS161 are `prereqs` of this request. (Note that there's **no space** after the comma.) If you request course A and B (separately), and A is an official prerequisite of B based on the `CourseBulletin`, we will automatically add A as a prerequisite for B; that is, typing `request B` is equivalent to `request B after A`. Note that if B is a prerequisite of A, to request A, you must either request B or declare you've taken B before.

Finally, the last operator you can add is `weight`, which adds non-negative weight to each request. All requests have a default weight value of 1. Requests with higher weight should be preferred by your CSP solver. Note that you can combine all of the aforementioned operators into one as follows (again, no space after comma):

```
request CS221 or CS229 in Win2014,Win2015 after CS131 weight 5
```

In the code, we use the `Request` class to represent the requests. For example, the request above will be parsed to a `Request` object with the following properties:

- `cids` of value `['CS221', 'CS229']`
- `quarters` of value `['Win2014', 'Win2015']`
- `prereqs` of value `['CS131']`
- `weight` of value `5.0`

It's important to note that a request does not have to be fulfilled, *but if it is*, the constraints specified by the various operators `after,in` must also be satisfied.

You shall not worry about parsing the profiles because we have done all the parsing of the bulletin and profile for you, so all you need to work with is the collection of `Request` objects in `Profile` and `CourseBulletin` to know when courses are offered and the number of units of courses.

Well, that's a lot of information! Let's open a python shell and see them in action:

```
import util
# load bulletin
bulletin = util.CourseBulletin('courses.json')
# retrieve information of CS221
cs221 = bulletin.courses['CS221']
print cs221
# look at various properties of the course
print cs221.cid
print cs221.minUnits
print cs221.maxUnits
print cs221.prereqs  # the prerequisites
print cs221.is_offered_in('Aut2014')
print cs221.is_offered_in('Win2015')

# load profile from profile_example.txt
profile = util.Profile(bulletin, 'profile_example.txt')
# see what it's about
profile.print_info()
# iterate over the requests and print out the properties
for req in profile.requests:
    print req.cids, req.quarters, req.prereqs, req.weight
```

Your task is to take a profile and bulletin and construct a CSP. We have started you off with code in `SchedulingCSPConstructor` that constructs the core variables of the CSP as well as some basic constraints. The variables are all pairs of requests and registered quarters `(request, quarter)`, and the value of such a variable is one of the course IDs in that Request or `None`, which indicates none of the courses should be taken in that quarter. We will add auxiliary variables later. We have also implemented some basic constraints: `add_bulletin_constraints()`, which enforces that a course can only be taken if it's offered in that quarter (according to the bulletin), and `add_norepeating_contstraints()`, which constrains that no course can be taken more than once.

You should take a look at `add_bulletin_constraints()` and `add_norepeating_contstraints()` to get a basic understanding how the CSP for scheduling is represented. Nevertheless, we'll highlight some important details to make it easier for you to implement:

- The existing variables are tuples of `(req, quarter)` where `req` is a `Request` object (like the one shown above) and `quarter` is a `str` representing a quarter (e.g. `'Aut2013'`). For detail please look at `SchedulingCSPConstructor.add_variables()`.
- The domain for `req` is the course IDs of the request **plus** `None` (e.g. `['CS221', 'CS229', None]`). When `req` is `None`, this means no course is scheduled. **Always remember to check if `req` is `None`.**
- The domain for `quarter` is all possible quarters (`self.profile.quarters`, e.g. `['Win2014', 'Win2015']`).
- Given a course ID `cid`, you can get the corresponding `Course` object by `self.bulletin.courses[cid]`.

a. Implement the `add_quarter_constraints()`. This is when your profile specifies which quarter(s) you want your requested courses to be taken in. This does not saying that one of the courses must be taken, *but if it is*, then it must be taken in any one of the specified quarters. Also note that this constraint will apply to all courses in that request. We have written a `verify_schedule()` function in `grader.py` that determines if your schedule satisfies all of the given constraints. Note that since we are not dealing with units yet, it will print `None` for the number of units of each course.

b. Let's add the weight potential in `add_request_weights()`. By default, all requests have a weight of 1 regardless whether it's satisfied or not. When a weight is explicitly specified, it should only contribute to the final weight if one of the requested courses is in the solution. **NOTE:** Each grader test only tests the function you are asked to implement. To test your CSP with multiple constraints you can use `run_p3.py` and changing the constraints that you want to add.

c. Let's now add the unit constraints in `add_unit_constraints()`. You must ensure that the sum of units per quarter for your schedule are within the min and max threshold inclusive. You should use `get_sum_variable()`. In order for our solution extractor to obtain the number of units, for every course, you must add a variable `(courseId, quarter)` to the CSP taking on a value equal to the number of units being taken for that course during that quarter. When the course is not taken during that quarter, the unit should be 0.

Hint: If your code times out, your `maxSum` passed to `get_sum_variable()` might be too large.

d. Now try to use the course scheduler for the winter and spring (and next year if applicable). Create your own `profile.txt` and then run the course scheduler:

```
python run_p3.py profile.txt
```

You might want to turn on the appropriate heuristic flags to speed up the computation. Does it produce a reasonable course schedule? Please submit your `profile.txt`; we're curious how it worked out for you!

Extra Credit: weighted CSPs with notable patterns (3 points)

Want more challenges about CSP? Here we go. :D

Suppose we have a weighted CSP with variables $X_1, \dots, X_n$ with domains $\mathrm{Domain}_i = \{1, \dots, K\}$. We have a set of basic potentials which depend only on adjacent pairs of variables in the same way: there is some function g such that $f_i(x) = g(x_i, x_{i+1})$ for $i = 1, \dots, n-1$. In addition, we have a small set of *notable patterns* P, where each $p \in P$ is a sequence of elements from the domain.

Let n_p be the number of times that p occurs in an assignment $x = (x_1, \dots, x_n)$ as a consecutive sequence. Define the weight of an assignment x to be $\prod_{i=1}^{n-1} f_i(x) \prod_{p \in P} \gamma^{n_p}$. Intuitively, we multiply the weight by γ every time a notable pattern appears.

For example, suppose $n = 4$, $\gamma = 3$, $g(a,b) = 2[a = b] + 1[a \neq b]$ and $P = \{[1,3,3],[1,2,3]\}$. Then the assignment $x = [1,3,3,2]$ has weight $2 \cdot 3 = 6$.

a. If we were to include the notable patterns as potentials into the CSP, what would be the worst case treewidth? (You can assume each p has a maximum length of n.)

b. The treewidth doesn't really tell us the true complexity of the problem. Devise an efficient algorithm to compute the maximum weight assignment. You need to describe your algorithm in enough detail but don't need to implement it. Analyze your algorithm's time and space complexities. You'll get points only if your algorithm is much better than the naive solution.