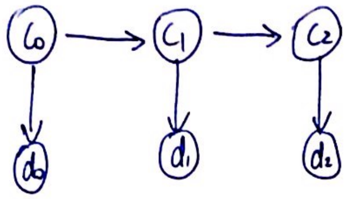
1.a

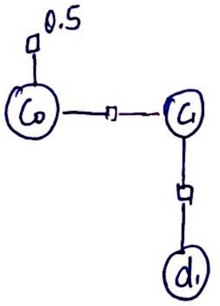
Song Han



$$P(C_t \mid C_{t-1}) = \begin{cases} \varepsilon & (C_t \neq C_{t-1}) \\ 1-\varepsilon & (C_t = C_{t-1}) \end{cases}$$

$$P(d_t \mid C_t) = \begin{cases} \eta & (d_t \neq C_t) \\ 1-\eta & (d_t = C_t) \end{cases}$$

$P(C_1 = 1 \mid d_1 = 0)$:



$$P(C_1=1 \mid d_1=0) \propto \sum_{C_0} P(C_0)\, P(C_1=1 \mid C_0)\, P(d_1=0 \mid C_1=1)$$

$$= \underline{P(C_0=0)}\, P(C_1=1 \mid C_0=0)\, P(d_1=0 \mid C_1=1) + \underline{P(C_0=1)}\, P(C_1=1 \mid C_0=1)\, P(d_1=0 \mid C_1=1)$$

$$= 0.5\,\varepsilon\eta + 0.5(1-\varepsilon)\eta = 0.5\eta$$

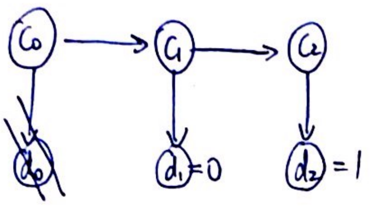
$$P(C_1=0 \mid d_1=0) \propto \sum_{C_0} P(C_0)\, P(C_1=0 \mid C_0)\, P(d_1=0 \mid C_1=0)$$

$$= P(C_0=0) \cdot P(C_1=0 \mid C_0=0) \cdot P(d_1=0 \mid C_1=0) + P(C_0=1) \cdot P(C_1=1 \mid C_0=1) \cdot P(d_1=0 \mid C_1=1)$$

$$= 0.5(1-\varepsilon)(1-\eta) + 0.5(1-\varepsilon)\cdot\eta = 0.5(1-\eta)$$

$\Rightarrow$ normalized $P(C_1=1 \mid d_1=0) = \dfrac{0.5\eta}{0.5\eta + 0.5(1-\eta)} = \eta$

1.b



$$P(C_1=1|d_1=0,d_2=1) \propto \sum_{C_0} P(C_0)\cdot P(C_1=1|C_0)\cdot P(d_1=0|C_1=1)\cdot \sum_{C_2} P(C_2|C_1=1)\cdot P(d_2=1|C_2)$$

$$= (0.5\varepsilon + 0.5(1-\varepsilon))\eta \; (P(C_2=0|C_1=1)\cdot P(d_2=1|C_2=0) + P(C_2=1|C_1=1)\cdot P(d_2=1|C_2=1))$$

$$= 0.5\eta(\varepsilon\cdot\eta + (1-\varepsilon)(1-\eta))$$

$$P(C_1=0|d_1=0,d_2=1) \propto \sum_{C_0} P(C_0)P(C_1=1|C_0)\cdot P(d_1=0|C_1=0)\cdot \sum_{C_2} P(C_2|C_1=0)\cdot P(d_2=1|C_2)$$

$$= 0.5\cdot(1-\eta)((1-\varepsilon)\eta + \varepsilon(1-\eta))$$

normalize $\Rightarrow P(C_1=1|d_1=0,d_2=1) = \dfrac{\eta(\varepsilon\eta + (1-\varepsilon)(1-\eta))}{\eta(\varepsilon\eta + (1-\varepsilon)(1-\eta)) + (1-\eta)((1-\varepsilon)\eta + \varepsilon(1-\eta))}$

1.c

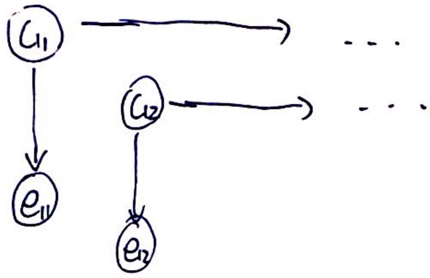
$$P(C_1=1|d_1=0) = 0.2$$

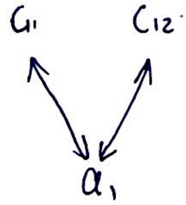
$$P(C_1=1|d_1=0,d_2=1) = 0.4157$$

Adding the second result makes $P(C_1=1)$ higher. Since at $t=2$ the observation is at 1 which makes $P(C_1=1)$ higher

5.a

$$P(c_{11}, c_{12} \mid e_1) = \frac{1}{2} P(c_{11}, c_{12}) \Big[P_N(e_{11}; \|a_1 - c_{11}\|, \sigma^2) P_N(e_{12}; \|a_1 - c_{12}\|, \sigma^2) + P_N(e_{11}; \|a_1 - c_{12}\|, \sigma^2) P_N(e_{12}; \|a_1 - c_{11}\|, \sigma^2) \Big]$$





5(b)

$$P(c_{11} \cdots c_{1K} \mid e_1) = \frac{1}{K!} P(c_{11}, c_{12} \cdots ; c_{1K}) \times \sum_{c'_{1i} \in \text{Permutation}\{c_{1i}\}} P(e_{11}; \|a_1 - c'_{1i}\|, \sigma^2) \cdot P(e_{12}; \|a_1 - c'_{12}\|, \sigma^2) \cdots P(e_{1K}; \|a_1 - c'_{1K}\|, \sigma^2)$$

Assume $P(c_{1i}^* \cdots c_{1K}^*)$ achieves the maxium value, since $P(c_{1i})$ is the same, if we permute $c_{1i}^* \cdots c_{1K}^*$, it will still achieve the maximum value. There're $K!$ such permutes, so the number of car locations that maximize $P(c_{11} \cdots c_{1K} \mid e_1)$ is $K!$

5(c)

Treewidth = K