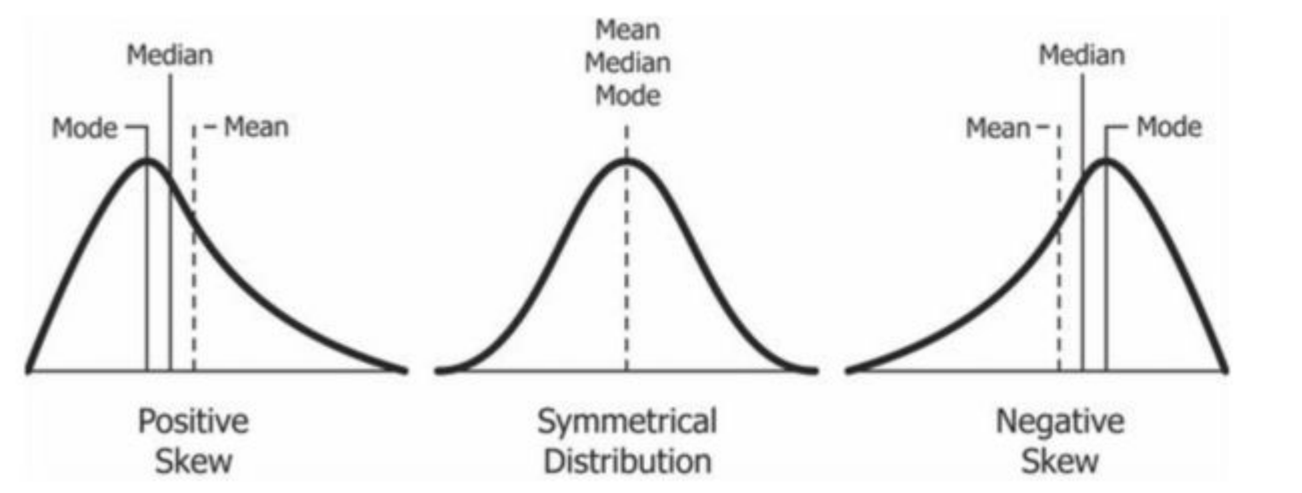


Econ 2250: Stats for Econ

Fall 2022

Source for pic stats above.

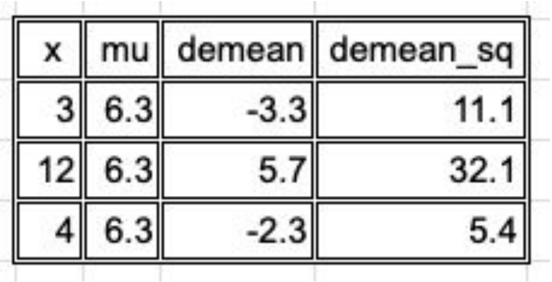
- **Today**
    - **Intro in linear regression**
        - **Variance**
        - **Covariance**
        - **Slope of line**
        - **Predicted value**
        - **Error term**
        - **Examples**

Variance

$V(X) = E((X - E(X))^2)$

From the example above

$$\sigma^2 = \frac{1}{n}\sum(x_i - \mu_x)^2$$



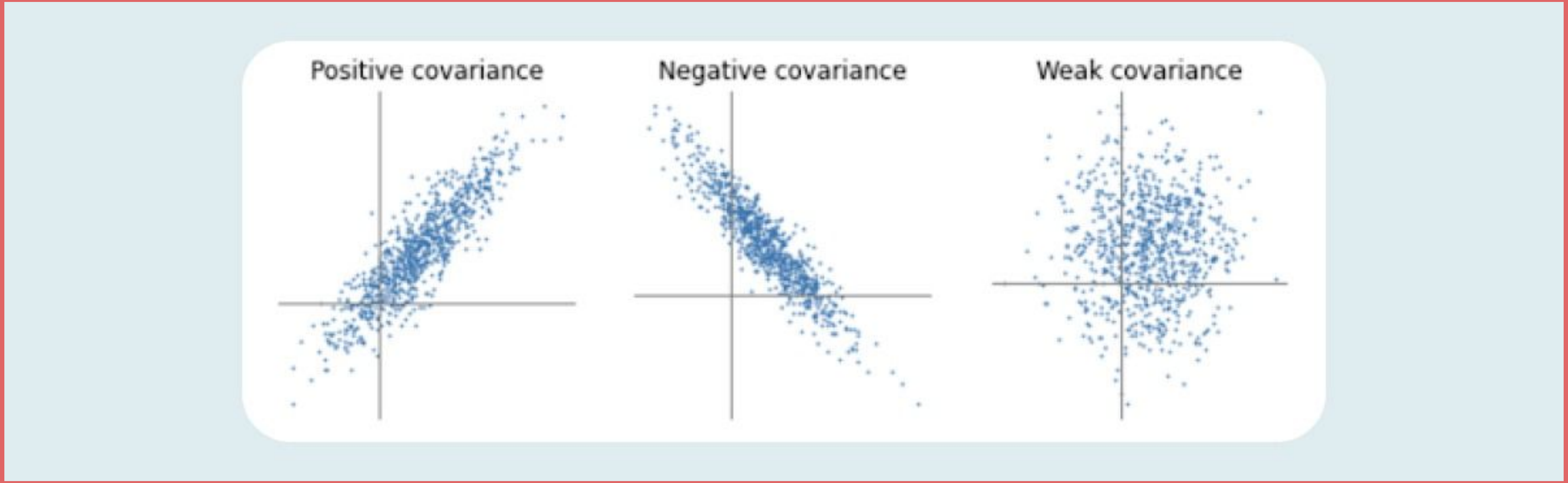
x	mu	demean	demean_sq
3	6.3	-3.3	11.1
12	6.3	5.7	32.1
4	6.3	-2.3	5.4

Sum of demean_sq = 11.1 + 32.1 + 5.4 = 48.6

48.6/3 = 16.2

But, notice our deviations (-3.3, 5.7, -2.3), 16.2 is an awful absolute value estimate. That is because we squared the errors, and x is in levels (not squared).

Standard Deviation = square root of σ^2 , sqrt(16.2) = 4.02



Covariance

$Cov(X,Y) = E[(X-E(X)(Y-E(Y)]$

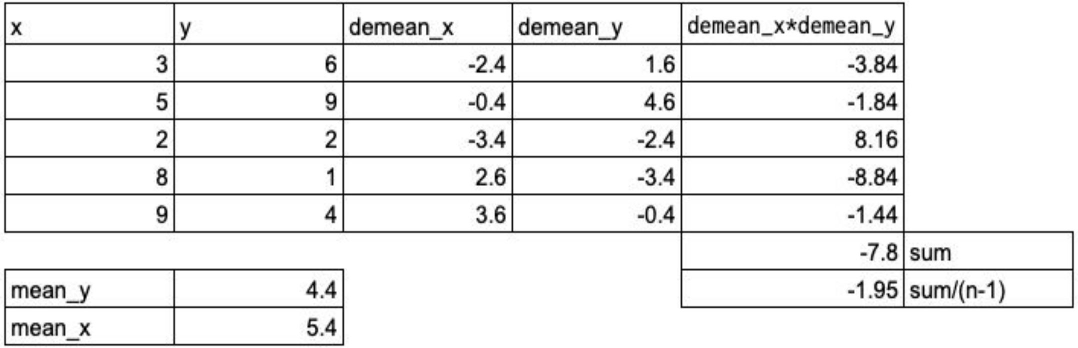
image

Covariance

$$
\begin{aligned}
Cov(x, y) &= E[(X - E(X))(Y - E(Y))] \\
&= \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n - 1}
\end{aligned}
$$

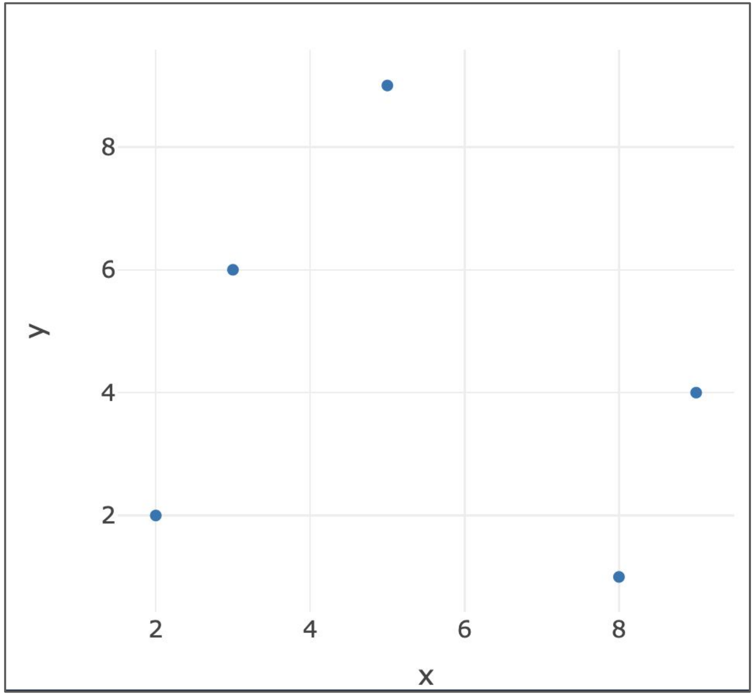
Example covariance

$$\frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n - 1}$$



x	y	demean_x	demean_y	demean_x*demean_y	
3	6	-2.4	1.6	-3.84	
5	9	-0.4	4.6	-1.84	
2	2	-3.4	-2.4	8.16	
8	1	2.6	-3.4	-8.84	
9	4	3.6	-0.4	-1.44	
				-7.8	sum
				-1.95	sum/(n-1)

mean_y	4.4
mean_x	5.4

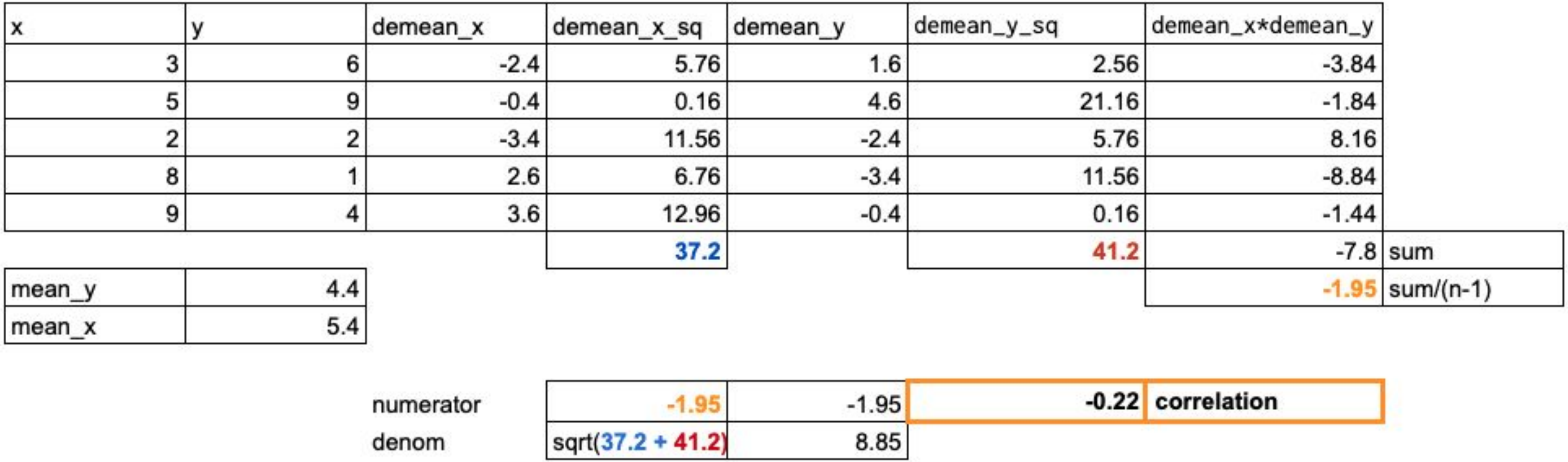


Correlation

$$\rho_{X,Y} = \operatorname{corr}(X, Y) = \frac{\operatorname{cov}(X, Y)}{\sigma_X \sigma_Y}$$

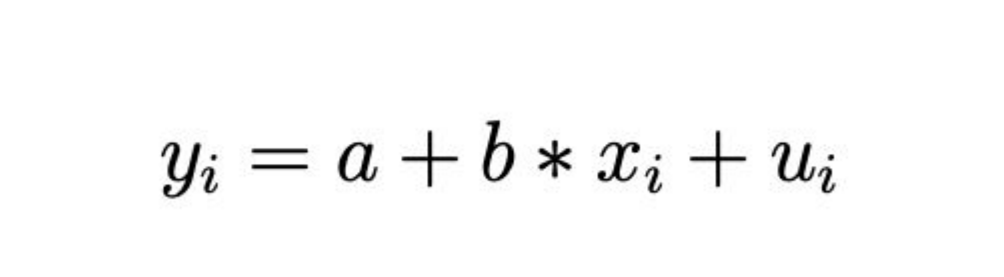
Example

$$\frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$



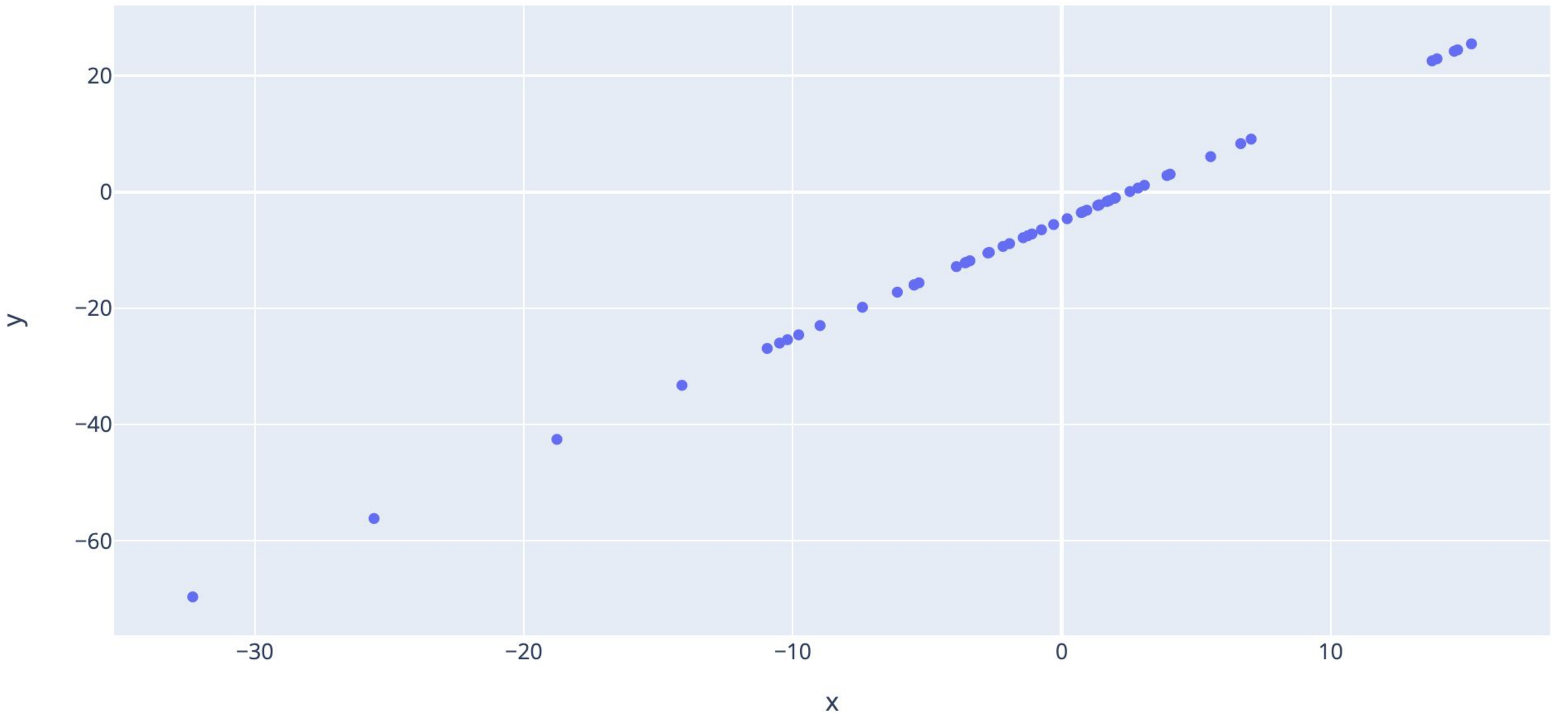
x	y	demean_x	demean_x_sq	demean_y	demean_y_sq	demean_x*demean_y	
3	6	-2.4	5.76	1.6	2.56	-3.84	
5	9	-0.4	0.16	4.6	21.16	-1.84	
2	2	-3.4	11.56	-2.4	5.76	8.16	
8	1	2.6	6.76	-3.4	11.56	-8.84	
9	4	3.6	12.96	-0.4	0.16	-1.44	
			37.2		**41.2**	-7.8	sum
mean_y	4.4					**-1.95**	sum/(n-1)
mean_x	5.4						
		numerator	**-1.95**	-1.95	**-0.22**	**correlation**	
		denom	sqrt(**37.2** + **41.2**)	8.85			

Linear Regression

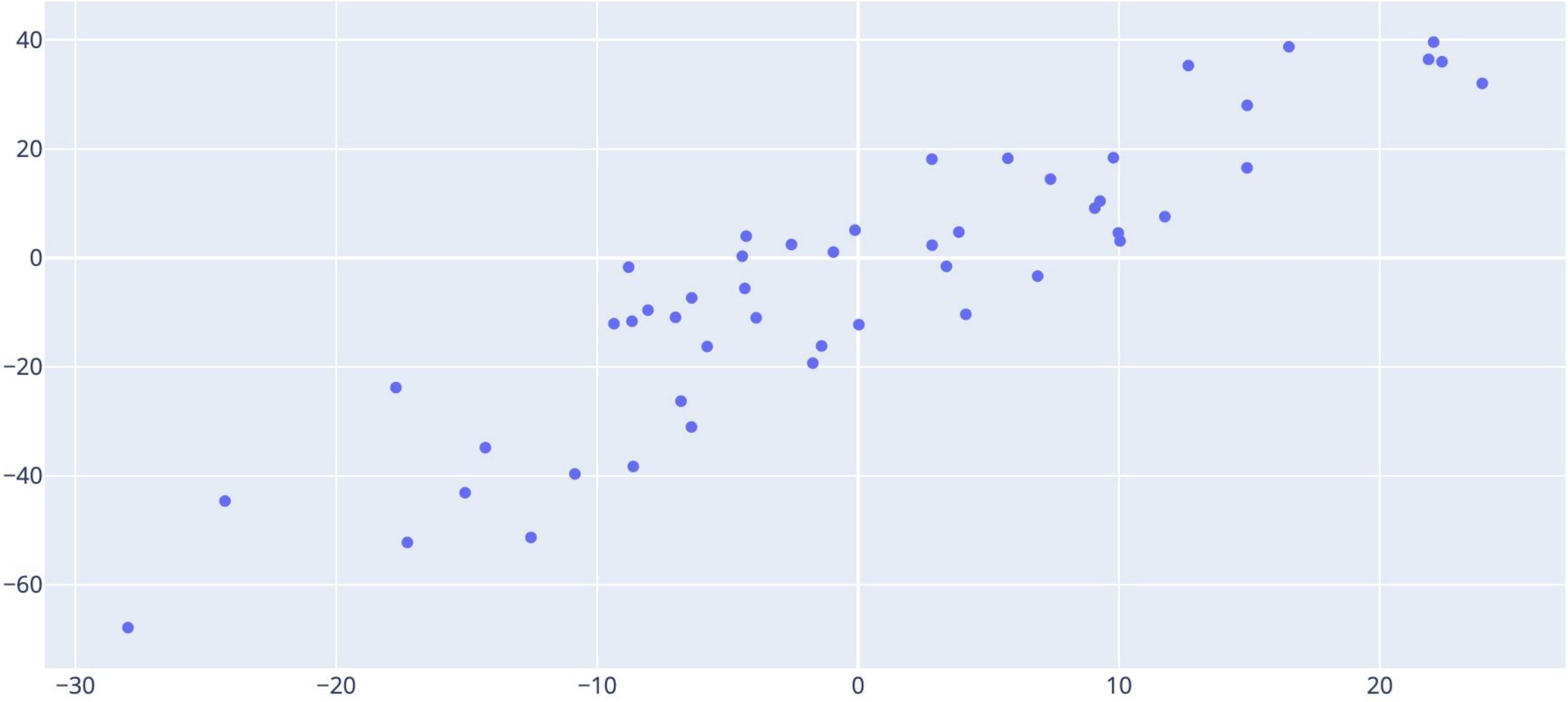


$$y_i = a + b * x_i + u_i$$

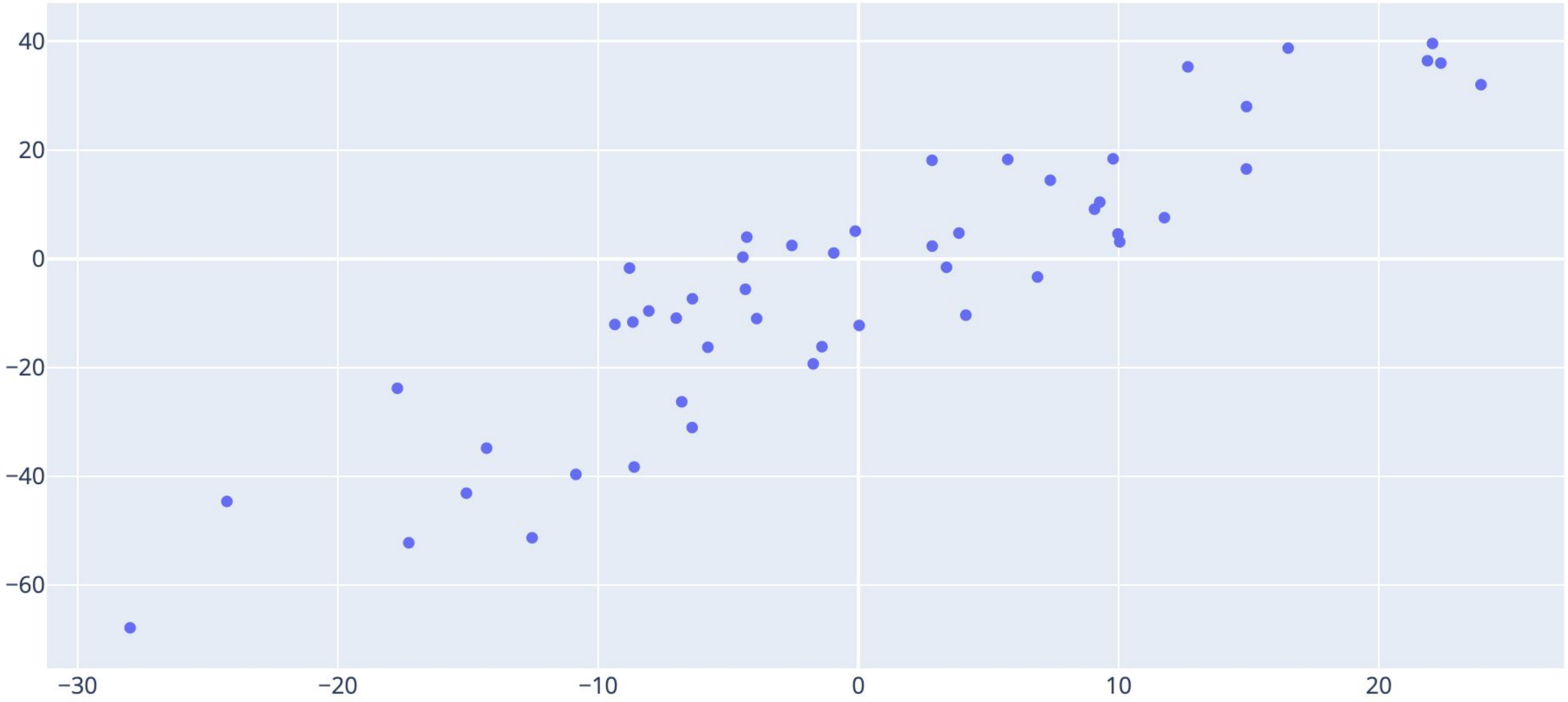
$$y = mx + b$$



$$y_i = a + b * x_i + ??$$



$$y_i = a + b * x_i + error_i$$



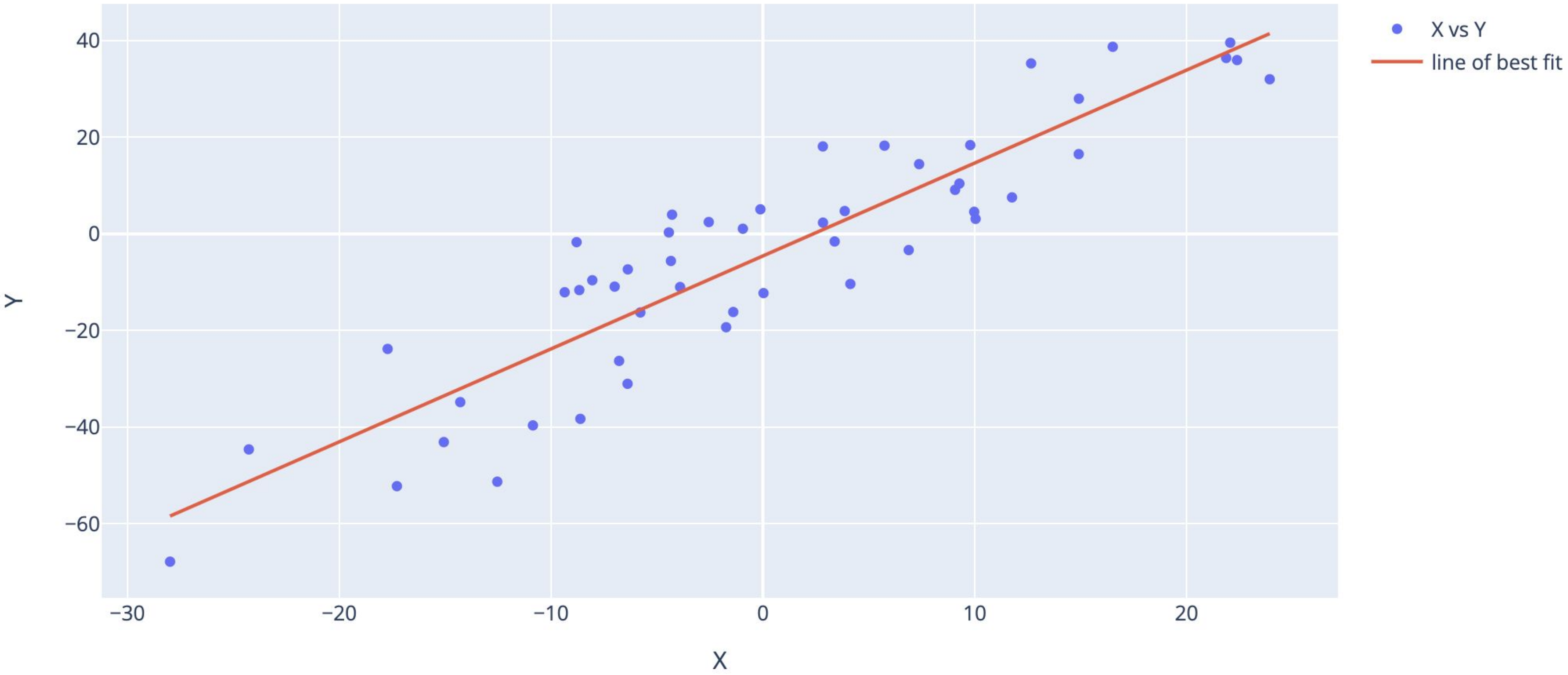
$$\hat{y}_i = \text{best guess intercept} + \text{best guess slope} * x_i$$

$$\hat{a} = \text{best guess intercept}$$

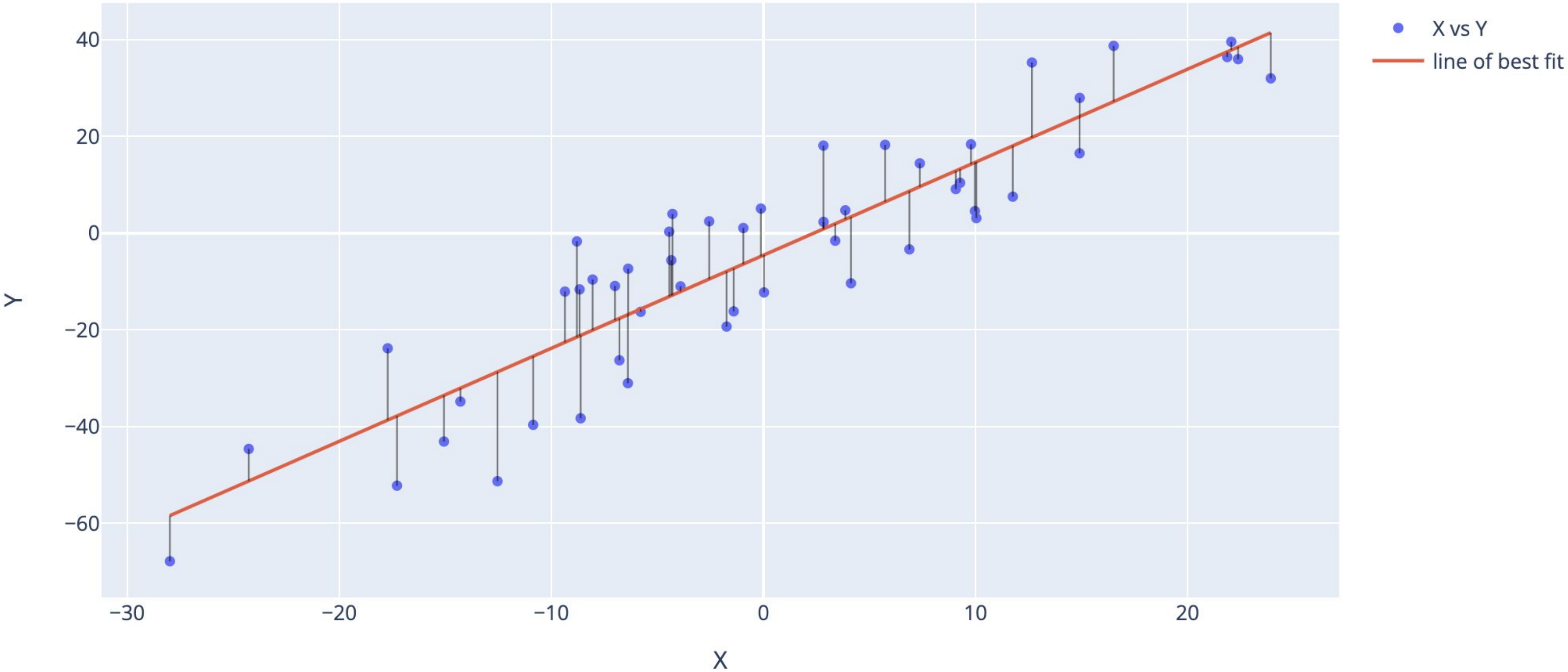
$$\hat{b} = \text{best guess slope}$$

$$\hat{y}_i = \hat{a} + \hat{b} * x_i$$

$$\hat{y}_i = \hat{a} + \hat{b} * x_i$$



$$y_i = a + b * x_i + error_i$$



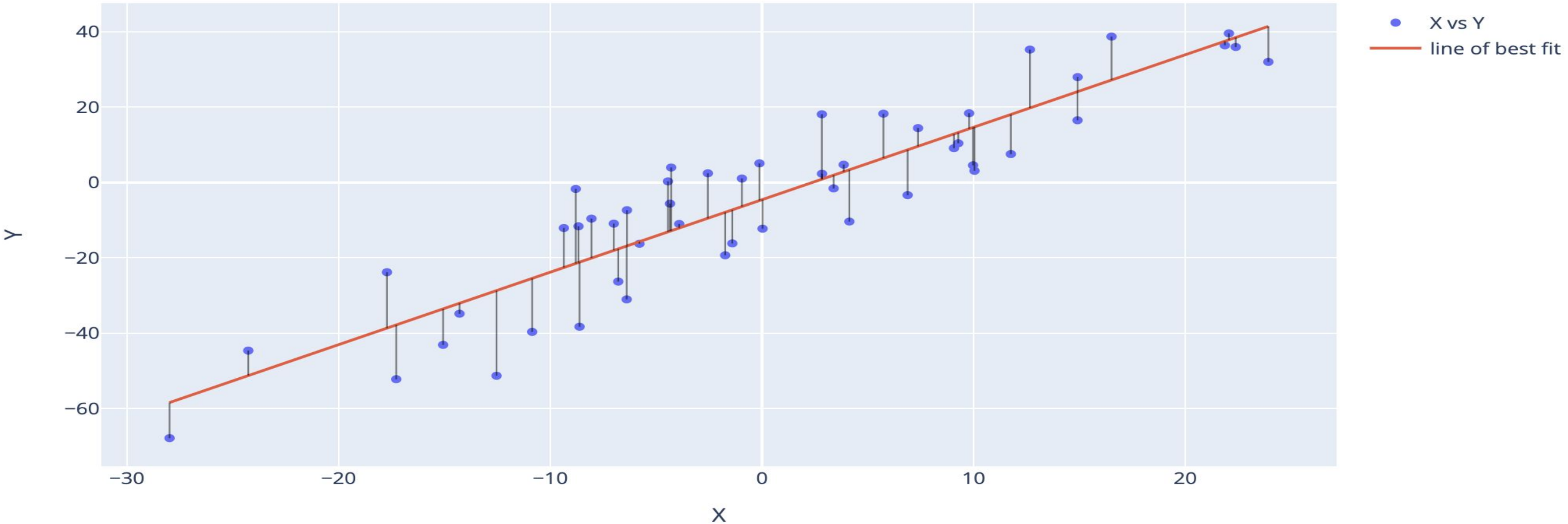
$$\hat{y}_i = \hat{a} + \hat{b} * x_i$$

$$\hat{b} = \frac{\mathrm{cov}(x, y)}{\mathrm{var}(x)} = \frac{\sum(x_i - \bar{x}) \sum(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

$$\hat{a} = \bar{y} - \hat{b} * \bar{x}$$

$$\hat{y}_i = \hat{a} + \hat{b} * x_i$$

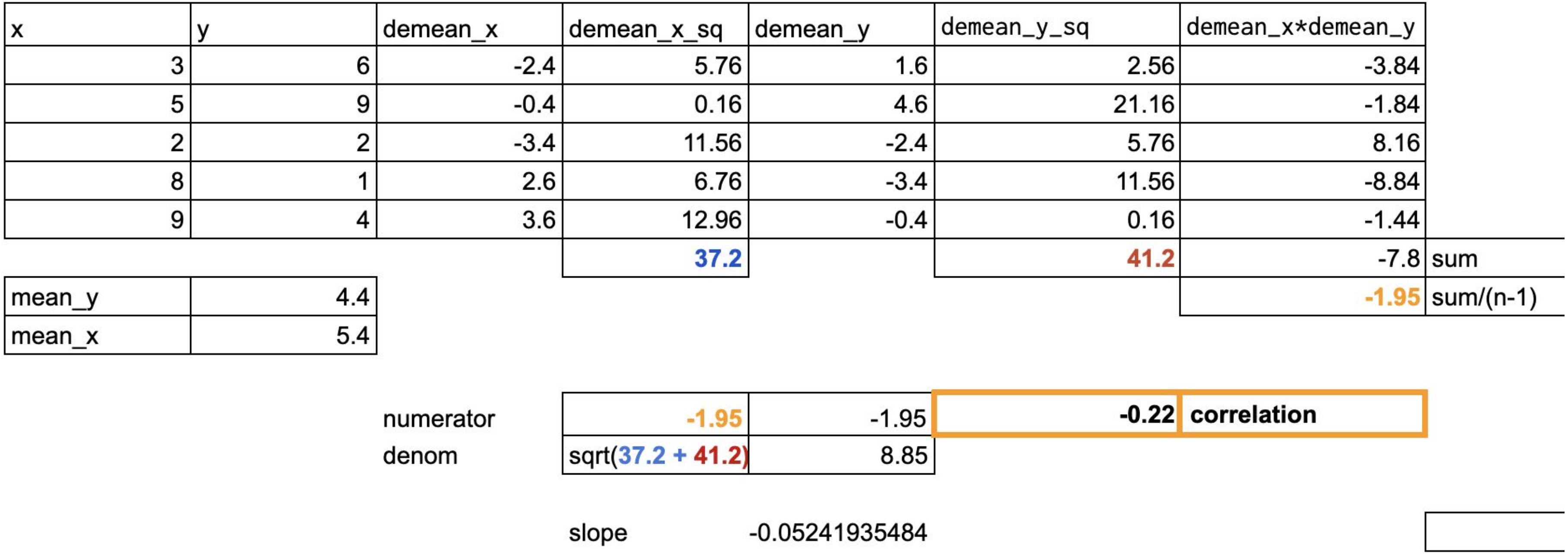
$$\hat{u}_i = y_i - \hat{y}_i$$



Sum of Squared Residuals

$$SSR = \sum (y_i - \hat{y}_i)^2 = \sum u_i^2$$

Look at excel



x	y	demean_x	demean_x_sq	demean_y	demean_y_sq	demean_x*demean_y	
3	6	-2.4	5.76	1.6	2.56	-3.84	
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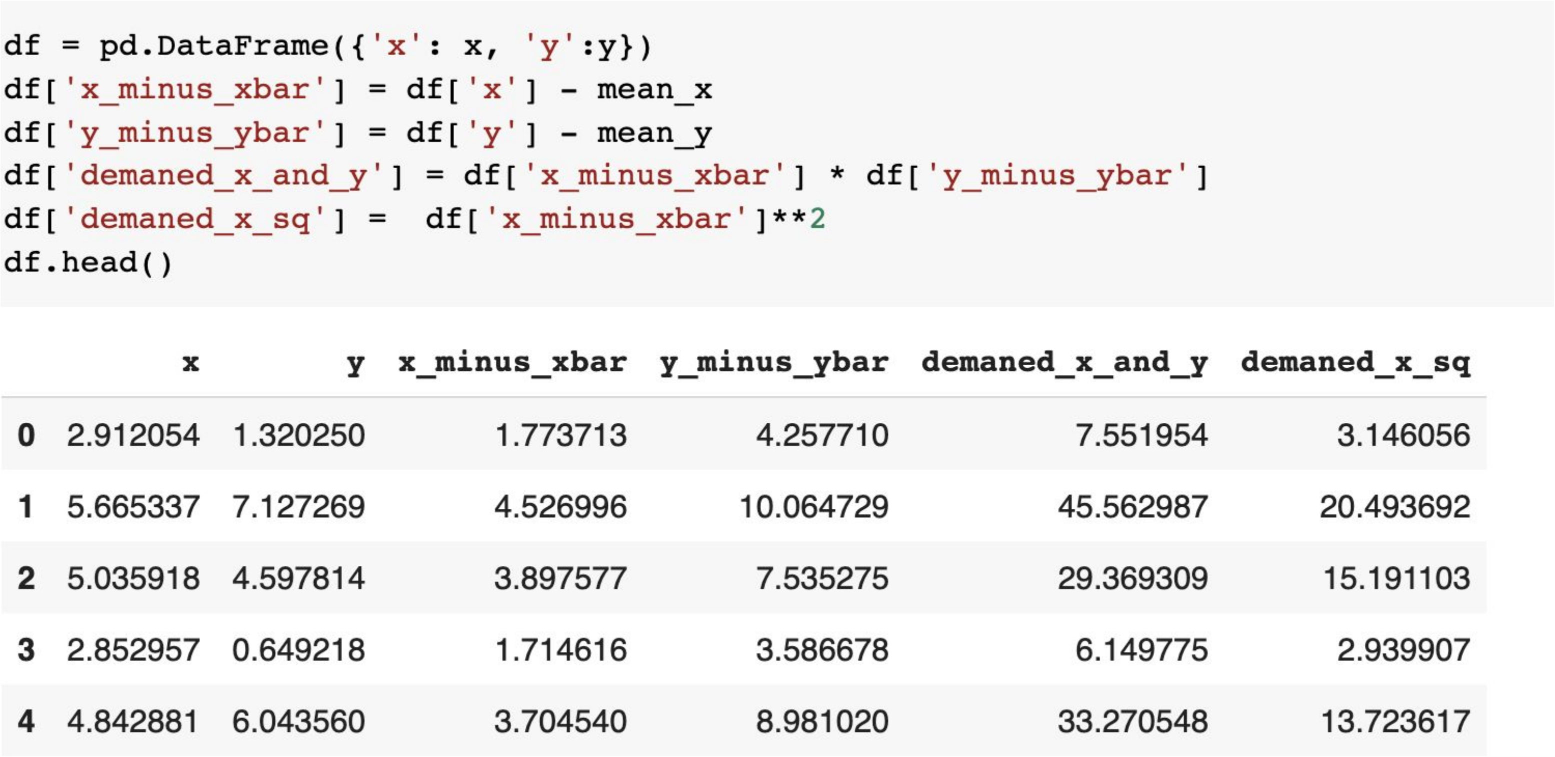
9	4	3.6	12.96	-0.4	0.16	-1.44	
			37.2		41.2	-7.8	sum
						-1.95	sum/(n-1)

mean_y	4.4
mean_x	5.4

numerator	-1.95	-1.95	**-0.22**	**correlation**
denom	sqrt(37.2 + 41.2)	8.85		

slope	-0.05241935484

Look at Colab



```
df = pd.DataFrame({'x': x, 'y':y})
df['x_minus_xbar'] = df['x'] - mean_x
df['y_minus_ybar'] = df['y'] - mean_y
df['demaned_x_and_y'] = df['x_minus_xbar'] * df['y_minus_ybar']
df['demaned_x_sq'] =  df['x_minus_xbar']**2
df.head()
```

	x	y	x_minus_xbar	y_minus_ybar	demaned_x_and_y	demaned_x_sq
0	2.912054	1.320250	1.773713	4.257710	7.551954	3.146056
1	5.665337	7.127269	4.526996	10.064729	45.562987	20.493692
2	5.035918	4.597814	3.897577	7.535275	29.369309	15.191103
3	2.852957	0.649218	1.714616	3.586678	6.149775	2.939907
4	4.842881	6.043560	3.704540	8.981020	33.270548	13.723617

End of class form

https://forms.gle/My9wHi2QFKNLedGC7