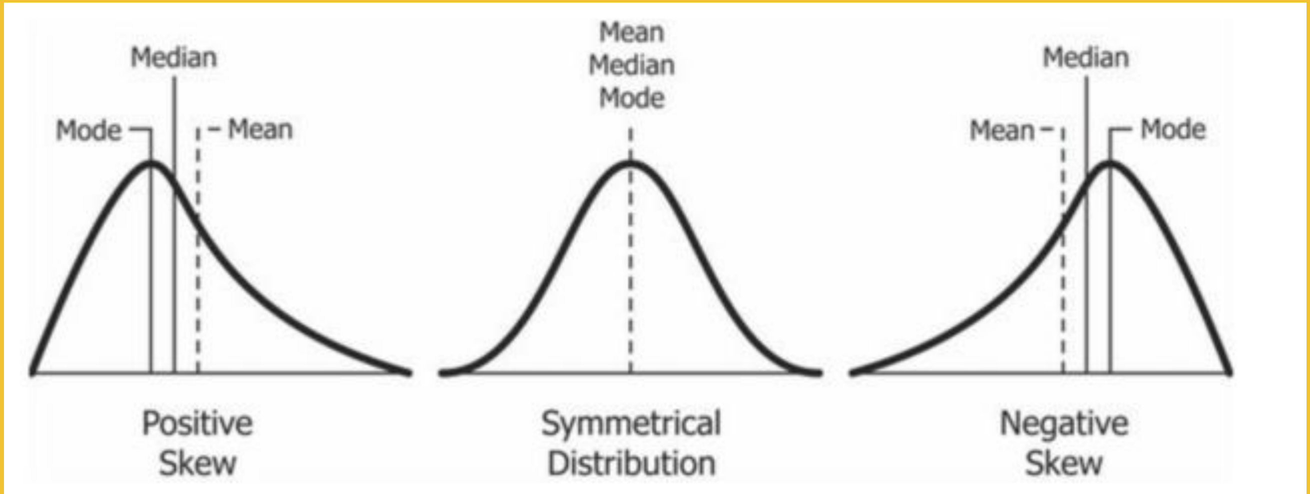


Econ 2250: Stats for Econ

Fall 2022

[Source for pic stats above.]

Announcements

- Homework 4 is due on Sunday

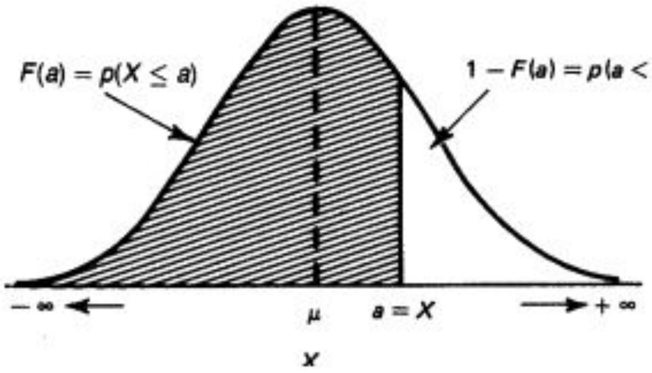
What we will do today?

- Review probability rules
- Review HW3
- Read through Hw4
- Review AND and OR rules for unconditional prob
- Discuss Conditional Probability

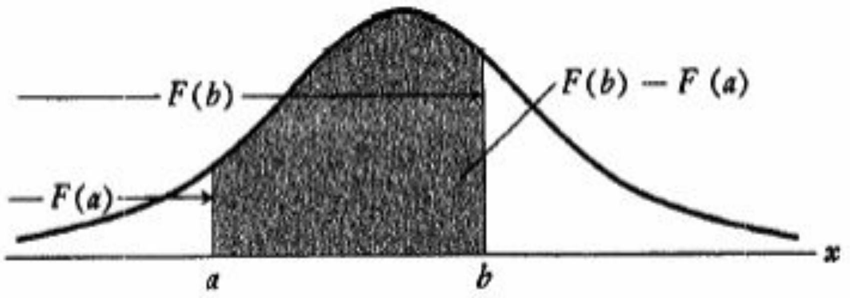
Probability Distribution Function (PDF): specification of the probability associated with each value of a random variable.

For continuous r.v.s:

$$F(a) = p(X \leq a) = \int_{-\infty}^{a} f(x)\,dx = \text{Area up to X} = \text{a}$$



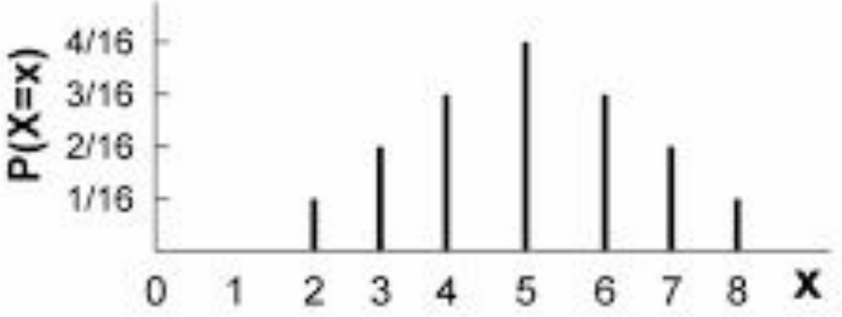
p(a ≤ X ≤ b)= F(b)- F(a)



Probability Mass Function (PMF)

A probability distribution involving only discrete values of X. Aggregates different possible values of X, and the different possible values of P(x).

x	P(x)
2	1/16
3	2/16
4	3/16
5	4/16
6	3/16
7	2/16
8	1/16



Properties:

$0 \leq P(X = x) \leq 1$

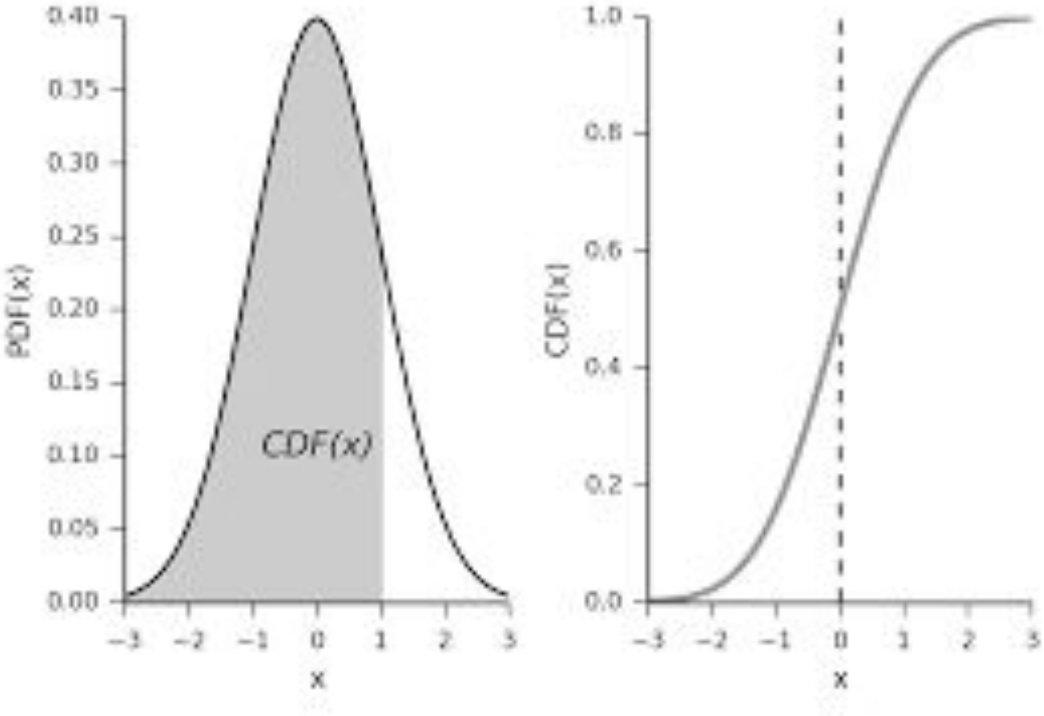
$\Sigma P(X = x) = 1.$

Cumulative Density Function (CDF)

The probability that a random variable X takes on a value less than or equal to some particular value a is often written as

$$F(a) = p(X \le a) = \sum_{X \le a} p(x)$$

(for discrete variables, integral for continuous)



Basic Rules of Probability

1. For any event $P(E)$ $[0,1]$
2. If an event cannot occur $P(E) = 0$
3. If an event is certain to occur $P(E) =1$
4. The sum of the probability of all outcomes must equal 1.

Likelihood of event

$$P(\text{event}) = \frac{\#\text{ of outcomes of event}}{\#\text{ of outcomes in }\Omega}$$

Probability Jargon

Marginal Probability: P(A)

Joint Probability: P(A and B) = P(A,B)

Conditional Probability: P(A given B) = P(A|B)

P(A|B) = P(A,B)/P(B)

NOTICE: P(A|B) NOT EQUAL P(B|A)

Here are some examples of bayes rule

https://www.mathsisfun.com/data/bayes-theorem.html

Make sure to check out the test questions at the bottom. You should be able to identify

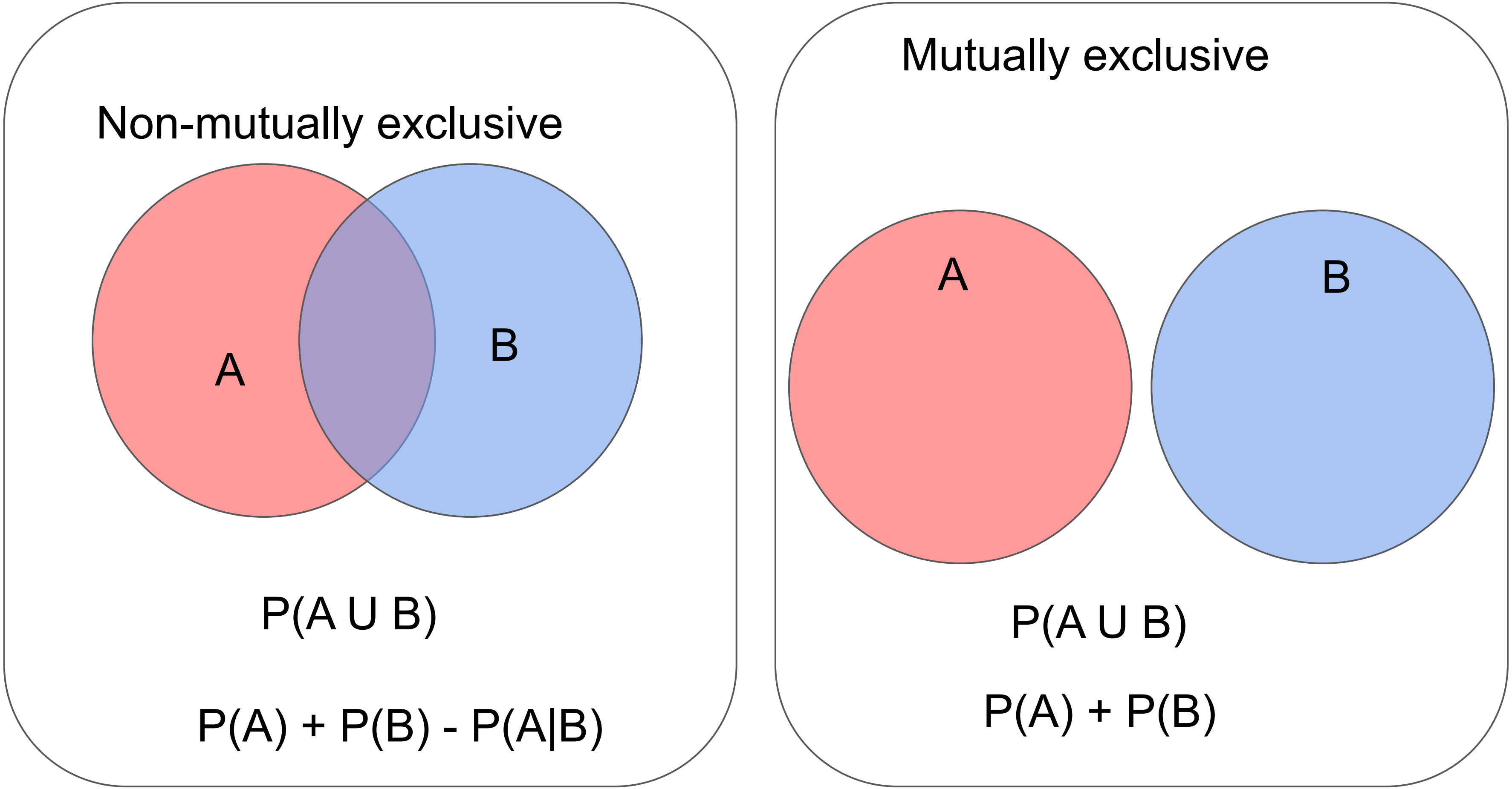
P(A|B) (what you're looking for),

P(B|A) the prior,

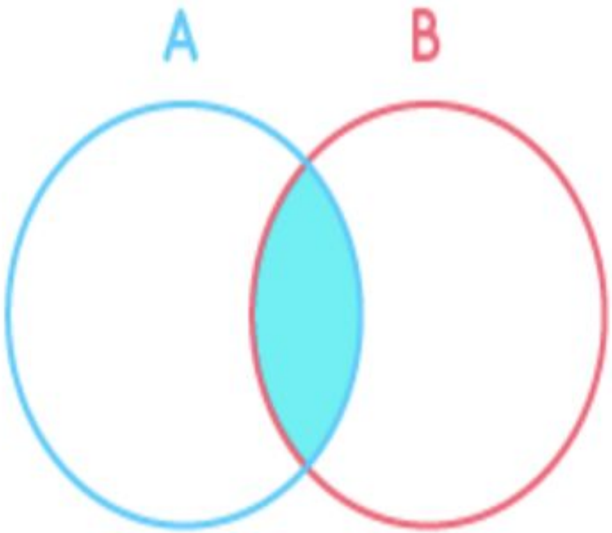
P(A) the marginal of the conditional that you are looking for, and

P(B) marginal of the condition (or how to find it)

If no P(B), define P(A)P(B|A) + P(not A)P(not A|not B)


Non-mutually exclusive
A
B
P(A U B)
P(A) + P(B) - P(A|B)
Mutually exclusive
A
B
P(A U B)
P(A) + P(B)

Independent



$$P(A \cap B)$$

P(A) * P(B)

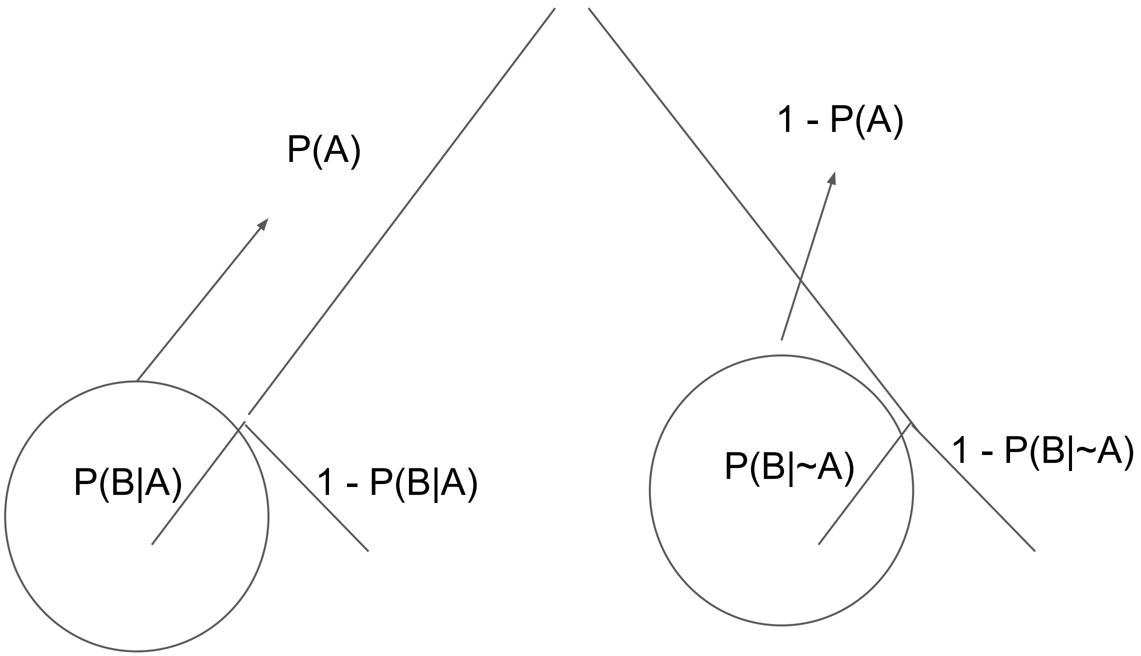
Summary of probabilities

Event	Probability
A	$P(A) \in [0,1]$
not A	$P(A^{\complement}) = 1 - P(A)$
A or B	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cup B) = P(A) + P(B) \qquad \text{if A and B are mutually exclusive}$
A and B	$P(A \cap B) = P(A\|B)P(B) = P(B\|A)P(A)$ $P(A \cap B) = P(A)P(B) \qquad \text{if A and B are independent}$
A given B	$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B\|A)P(A)}{P(B)}$

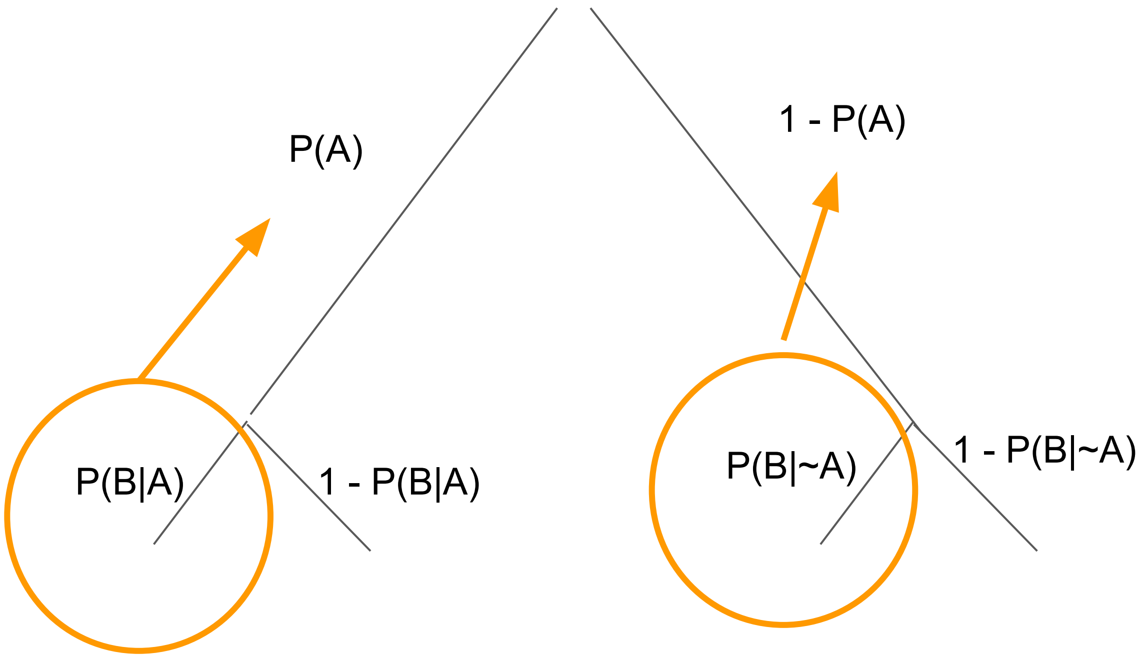
Bayes Rule

- $P(A|B) = P(B|A) * P(A) / P(B)$
- NOTE: we often do not have access to P(B) and have to calculate by looking at all possible cases:
- $P(B) = P(B|A) * P(A) + P(B|\text{not } A) * P(\text{not } A)$

Two Possibilities Graph



Two Possibilities Graph: $P(B) = P(B|A) * P(A) + P(B|\text{not } A) * P(\text{not } A)$



Bayes example 1

At a School, 60% of the boys play football and 36% of the boys play ice hockey.

Given that 40% of those that play football also play ice hockey, what percent of those that play ice hockey also play football?

P(A|B)?

P(A) = 60% = 0.6

P(B) = 36% = 0.36

P(B|A) = 40% = 0.4

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)} = \frac{0.6 \times 0.4}{0.36} = \frac{0.24}{0.36} = 66\tfrac{2}{3}\%$$

Example 1b

Now, for the problem above, what is the percentage of those that do not play football that play hockey?

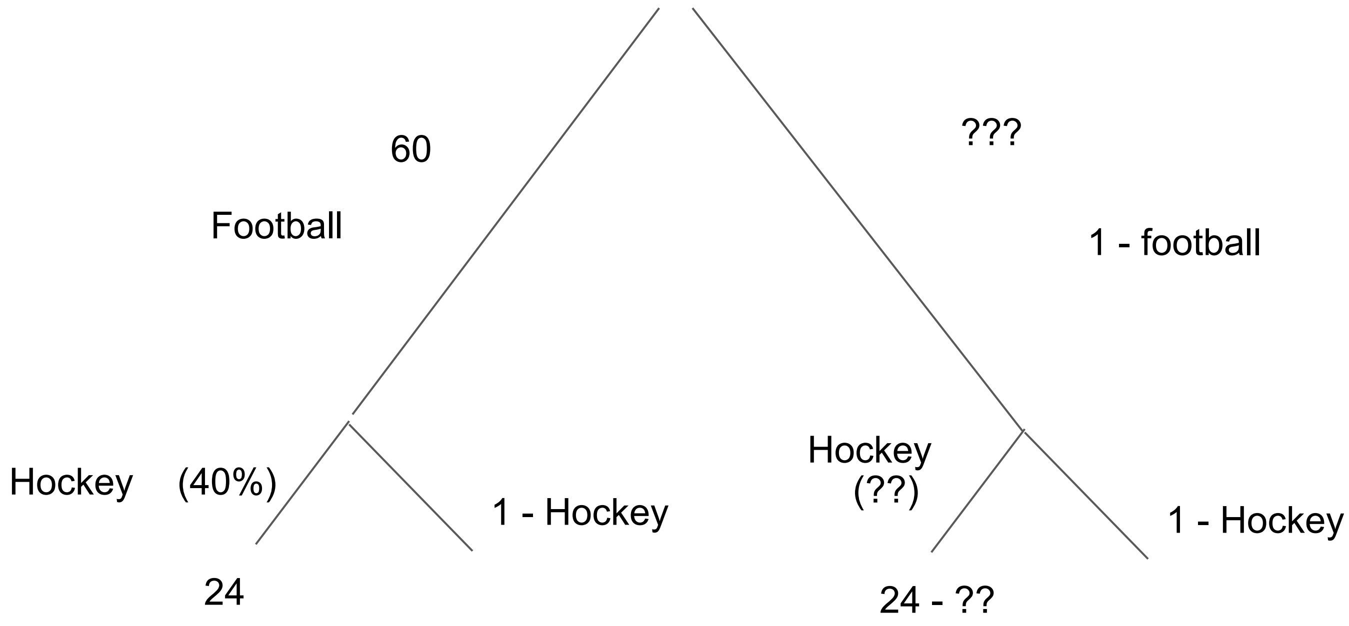
What is the $P(H|\sim F)$?

$P(F) = 60\% = 0.6$

$P(H|F) = 40\% = 0.4$

$P(H) = 36\% = P(F)P(H|F) + P(\sim F)P(H|\sim F) =0.36$

Let’s imagine that there are 100 students…



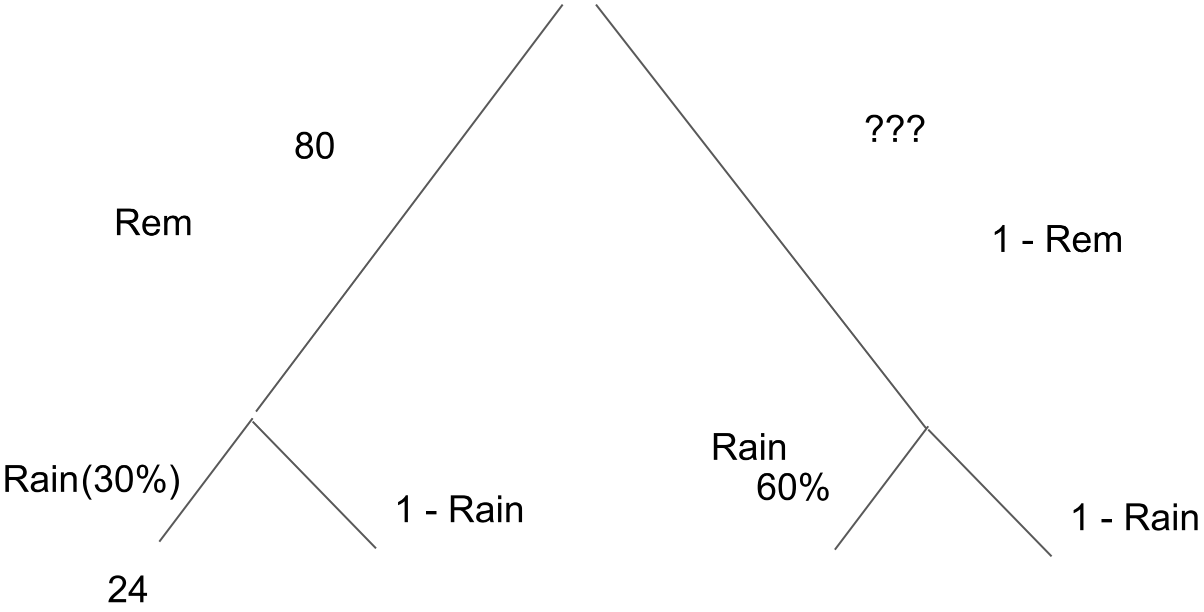
Bayes Example 3

Dr. Foster remembers to take his umbrella with him 80% of the days.

It rains on 30% of the days when he remembers to take his umbrella, and it rains on 60% of the days when he forgets to take his umbrella.

What is the probability that he remembers his umbrella when it rains?

Let's imagine that there are 100 students…



P(Remembers | Rains)

P(Remembers) = 80%

P(Rains | Remembers) = 30%

R(Rains) = 30% × 80% + 60% × 20% = 0.24 + 0.12 = 0.36

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)} = \frac{0.8 \times 0.3}{0.36} = \frac{0.24}{0.36} = \frac{2}{3}$$

Bayes Example 4

In a factory, machine X produces 60% of the daily output and machine Y produces 40% of the daily output.

2% of machine X's output is defective, and 1.5% of machine Y's output is defective.

One day, an item was inspected at random and found to be defective. What is the probability that it was produced by machine X?

P(X|defective)

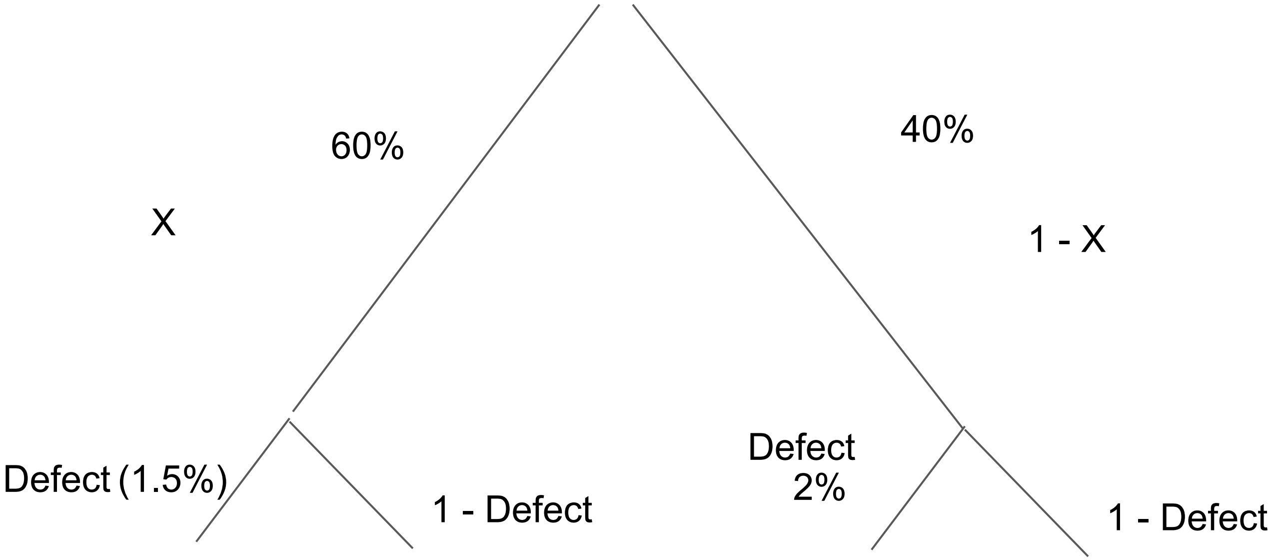
$P(X) = 60\% = 0.6$

$P(\text{defective}) = 2\% \times 60\% + 1.5\% \times 40\% = 0.012 + 0.006$
$= 0.018$

$P(\text{defective}|X) = 2\% = 0.02$

$$\frac{P(A)P(B|A)}{P(B)} = \frac{0.6 \times 0.02}{0.018} = \frac{0.012}{0.018} = \frac{2}{3}$$

Defect



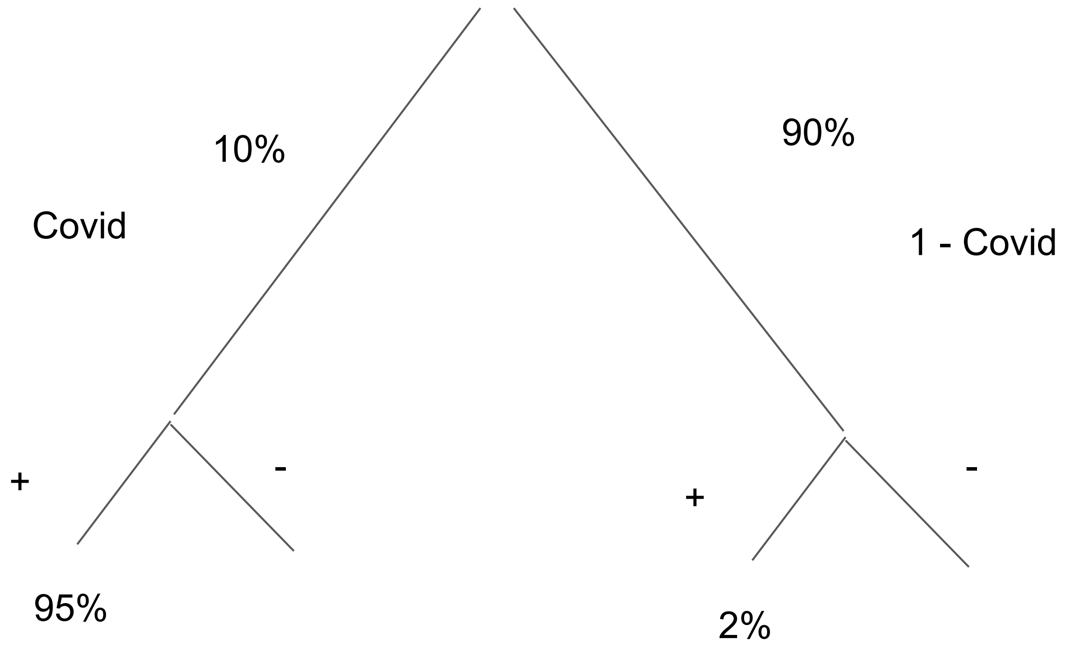
Covid

Let’s 5% of the population has COVID, and test has true positive of 85% (says you have it when you do have it), and false positive of 10% (says you have it when you don’t).

You take a test and it says positive, what is the actual chance that you have it?

Have Covid given a positive test result

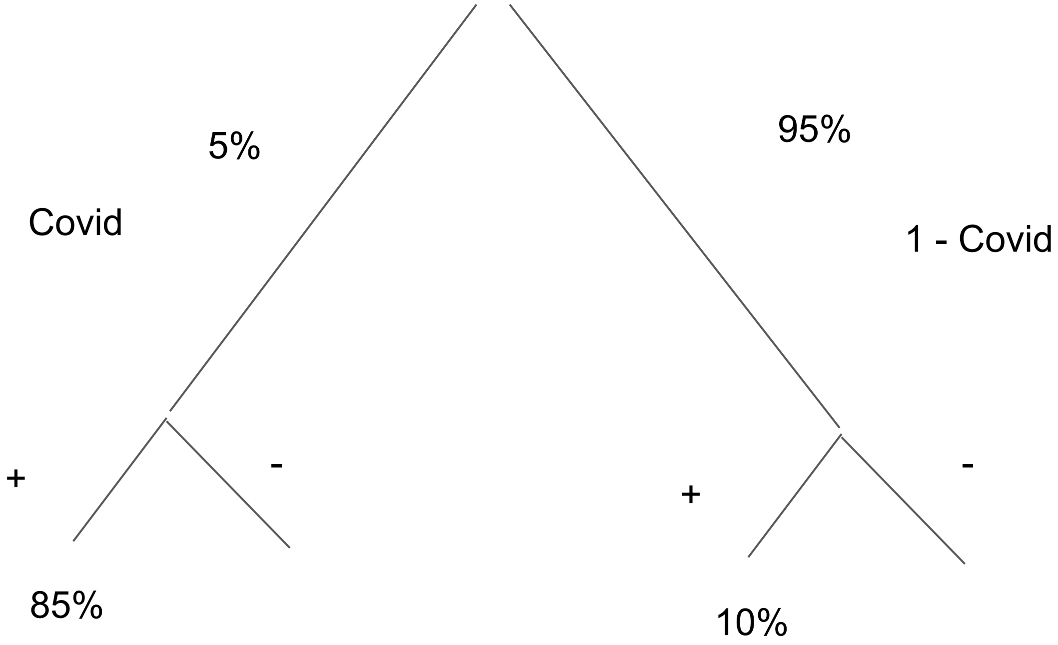
$$\frac{10\% * 95\%}{(10\% * 95\% + 90\% * 2\%)} = 84\%$$



P(B) = P(A)P(B|A) + P(~A)P(B|~A) = 10% * 95% + 90% * 2%

Have Covid given a positive test result

$$\frac{5\% * 85\%}{(5\% * 85\% + 95\% * 10\%)} = 31\%$$



$$P(B) = P(A)P(B|A) + P(\sim A)P(B|\sim A) = 5\% * 85\% + 95\% * 10\%$$

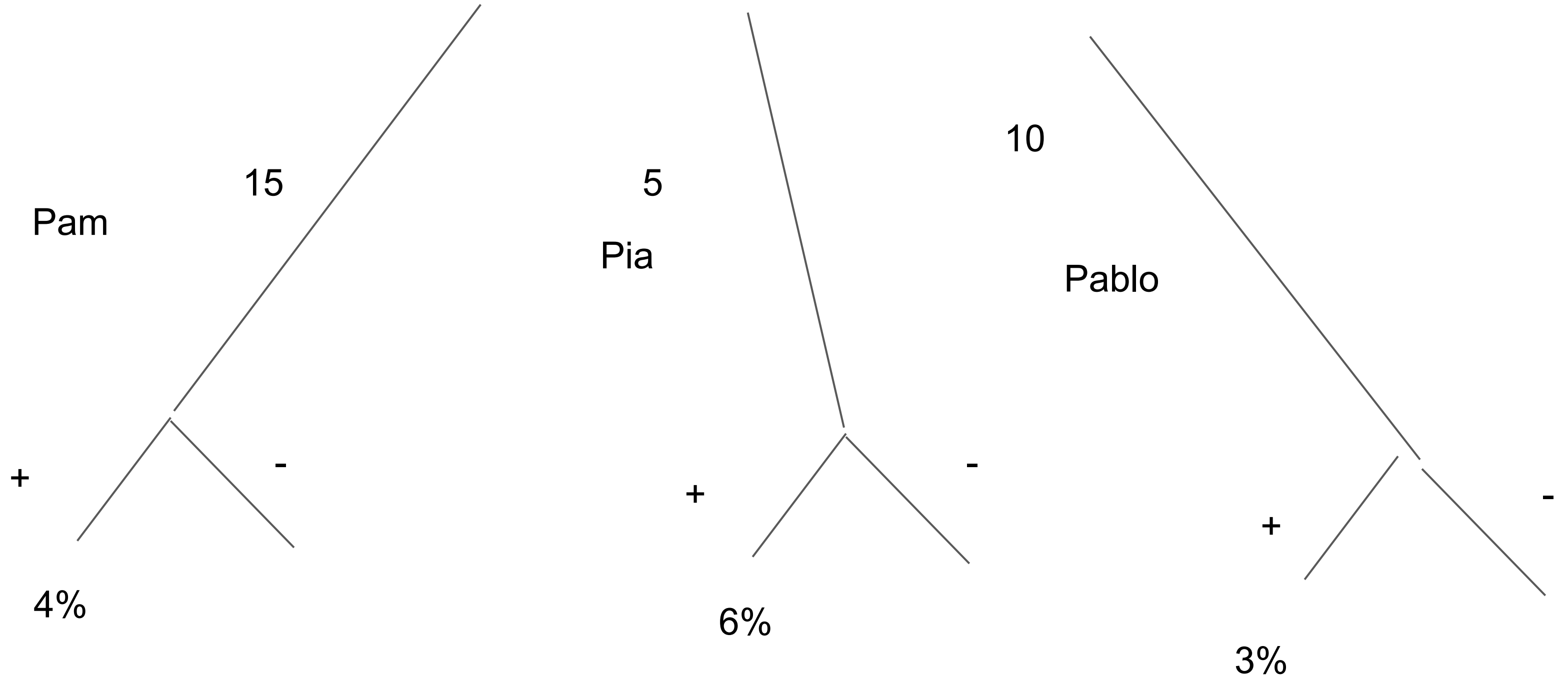
$$P(A1|B) = \frac{P(A1)P(B|A1)}{P(A1)P(B|A1) + P(A2)P(B|A2) + P(A3)P(B|A3) + \ldots etc}$$

- Pam put in 15 paintings, 4% of her works have won First Prize.
- Pia put in 5 paintings, 6% of her works have won First Prize.
- Pablo put in 10 paintings, 3% of his works have won First Prize.

What is the chance that Pam will win First Prize?

Pam wins?

$$\frac{4\%*15/30}{(4\%*15/30 + 6\%*5/30 + 4\%*10/30)} = 50\%$$



$$P(B) = P(A)P(B|A) + P(\sim A)P(B|\sim A) = 4\%*15/30 + 6\%*5/30 + 4\%*10/30$$

Review Homework 3

End of class form

https://forms.gle/8dWS1xogt49a6NUu7