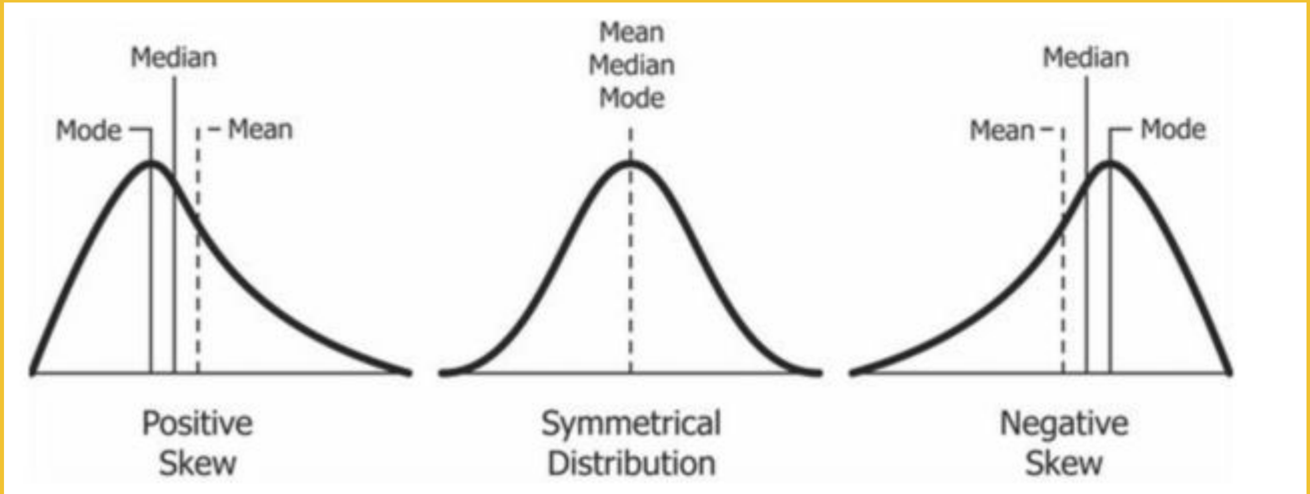

Econ 2250: Stats for Econ

Fall 2022

[Source for pic stats above.]

Announcements

- Homework 5 is due on Sunday

What we will do today?

- Expected value
- Variance
- Covariance
- Correlation

Summary

$$\sum_{i=1}^{n} x_i \equiv x_1 + x_2 + \ldots + x_n$$

Summary Operator Properties

1.) $\displaystyle\sum_{i=1}^{n} c = nc$

2.) $\displaystyle\sum_{i=1}^{n} cx_i = c\sum_{i=1}^{n} x_i$

3.) For any constant a and b: $\displaystyle\sum_{i=1}^{n}(ax_i + by_i) = a\sum_{i=1}^{n} x_i + b\sum_{j=1}^{n} y_i$

Gotchas! Be Careful

$$\sum_{i}^{n} \frac{x_i}{y_i} \neq \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} y_i}$$

$$\sum_{i=1}^{n} x_i^2 \neq \left(\sum_{i=1}^{n} x_i\right)^2$$

$$\begin{aligned} E(X) &= x_1 f(x_1) + x_2 f(x_2) + \cdots + x_k f(x_k) \\ &= \sum_{j=1}^{k} x_j f(x_j) \end{aligned}$$

Expected Value Operator Properties

$$E(c) = c$$

$$E(aX + b) = E(aX) + E(b) = aE(X) + b.$$

$$E\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i E(X_i) \longrightarrow E\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} E(X_i)$$

$$E(W + H) = E(W) + E(H), \qquad E\Big(W - E(W)\Big) = 0$$

Variance

$$V(X) \equiv \sigma^2 = E[(X - E(X))^2]$$

Population model:

$$V(X) = \sigma^2 = E[(X - E(X))^2]$$
$$= E[(X - \mu_x)^2] = \sum (x_i - \mu_x)^2 * P(x_i)$$

if $P(x_i)$ is $\frac{1}{n}$ for all $i = 1, 2, \ldots, n$

$$V(X) = \sum (x_i - \mu_x)^2 * \frac{1}{n} = \frac{1}{n} \sum (x_i - \mu_x)^2$$

bring squared values back the units of x

$$\sqrt{V(X)} = \sqrt{\frac{1}{n} \sum (x_i - \mu_x)^2}$$

Covariance

$$\operatorname{cov}(X,Y) = \operatorname{E}\left[(X - \operatorname{E}[X])\,(Y - \operatorname{E}[Y])\right]$$

$$\begin{aligned}
\operatorname{cov}(X,Y) &= \operatorname{E}\left[(X - \operatorname{E}[X])\,(Y - \operatorname{E}[Y])\right] \\
&= \operatorname{E}\left[XY - X\operatorname{E}[Y] - \operatorname{E}[X]Y + \operatorname{E}[X]\operatorname{E}[Y]\right] \\
&= \operatorname{E}[XY] - \operatorname{E}[X]\operatorname{E}[Y] - \operatorname{E}[X]\operatorname{E}[Y] + \operatorname{E}[X]\operatorname{E}[Y] \\
&= \operatorname{E}[XY] - \operatorname{E}[X]\operatorname{E}[Y],
\end{aligned}$$

Population

$$\begin{aligned}
Cov(X,Y)) &= E[(X - E(X))(Y - E(Y))] \\
&= \sum (x_i - \mu_x)(y_i - \mu_y)
\end{aligned}$$

Correlation

$$\rho_{X,Y} = \operatorname{corr}(X,Y) = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{\mathrm{E}[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}, \quad \text{if } \sigma_X \sigma_Y > 0$$

$$r_{xy} \quad \stackrel{\text{def}}{=} \quad \frac{\sum\limits_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{(n-1)s_x s_y} = \frac{\sum\limits_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum\limits_{i=1}^{n}(x_i-\bar{x})^2 \sum\limits_{i=1}^{n}(y_i-\bar{y})^2}}$$

Nice correlation app

https://shiny.rit.albany.edu/stat/rectangles/

End of class form

https://forms.gle/kgT2w9wPZo3vJcjA8