Rutgers University
CS440: Introduction to Artificial Intelligence, Spring 2017
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# Homework 4: James Carroll and Joel Carrillo

***Decision Making under Uncertainty and Learning***

1. Question 1

   (a) Starting with an arbitrary discount of 0.4 and values (0, 0, 0, 0), it took one millisecond and five iterations to find the optimal policy of: Action 2 at State 1; Action 2 at State 2; Action 3 at State 3; and Action 1 at State 4.

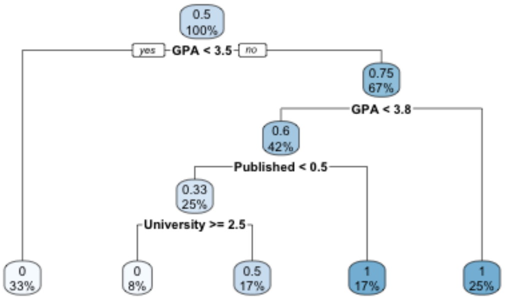
   The optimal utility at the respective states are 0.14690304000000004, 0.39942144000000007, 1.1707264, and 0.43098624000000013. The method was repeatedly applying the Bellman equation to find the policy that maximizes the utility function. Intermediate values:

   i. Policy [0, 0, 0, 0] Values [0, 0, 1, 0]
   ii. Policy [0, 0, 0, 0] Values [0, 0.32, 1, 0.36]
   iii. Policy [0, 1, 0, 0] Values [0.1152, 0.3456, 0.144, 0.3744]
   iv. Policy [1, 1, 2, 0] Values [0.129024, 0.393728, 1.14976, 0.426816]
   v. Policy [1, 1, 2, 0] Values [0.14690304, 0.39942144, 1.1707264, 0.43098624]

2. Question 2

   (a) Yes.

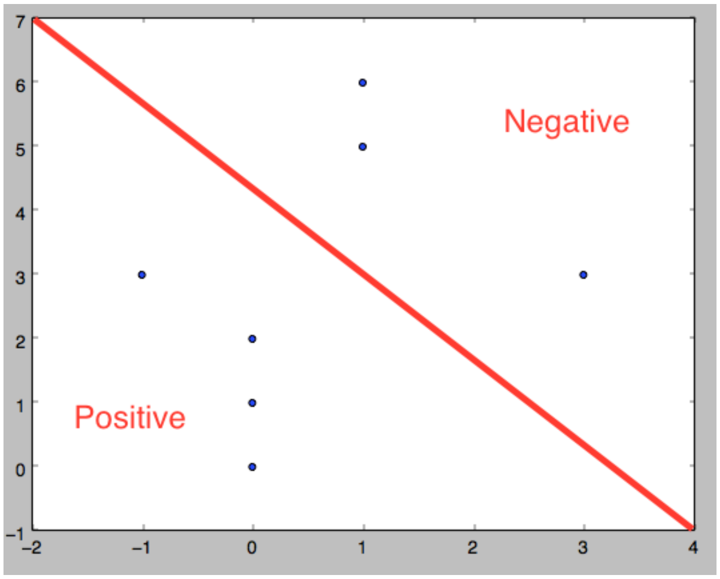
   (b) Gain(GPA) = $I(\frac{p}{p+n}, \frac{n}{p+n})$ - Remainder(GPA)
   $I(\frac{6}{6+6}, \frac{6}{6+6}) = I(\frac{1}{2}, \frac{1}{2}) = 1$
   Gain(GPA) = 1 - $\sum_{i=1}^{v} \frac{p_i+n_i}{p+n} I(\frac{p_i}{p_i+n_i}, \frac{n_i}{p_i+n_i})$
   Gain(GPA) = 1 - $[\frac{4}{12}I(0,1) + \frac{5}{12}I(\frac{3}{5}, \frac{2}{5}) + \frac{3}{12}I(1,0)]$
   Gain(GPA) = 1 - $\frac{4}{12} * 0 + \frac{5}{12} * (0.9710) + \frac{3}{12} * 0 = 0.5954$
   Gain (Pub) = 1 - $[\frac{7}{12}I(\frac{3}{7}, \frac{4}{7}) + \frac{5}{12}I(\frac{3}{5}, \frac{2}{5}))] = 0.0207$



(c) Yes, because the information gain for GPA is highest, Publications is second-highest, and so forth.

3. Question 3

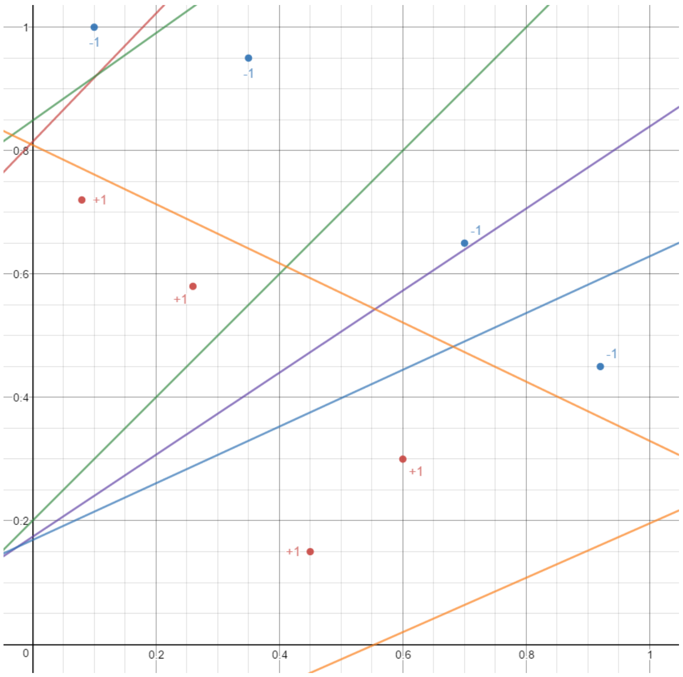
(a)



(b) For this line, w = −1.3164 and b = 4.2658.

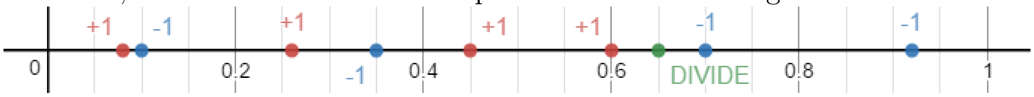
4. Question 4

(a) For the purposes of keeping the rest of the document clean, the coded printout is attached to the end of the document, after Question 5. This particular sample found the division in 14 total steps, but it can vary.
The six lines prior depict the first six steps, including the initial line (step 0, in this case.)



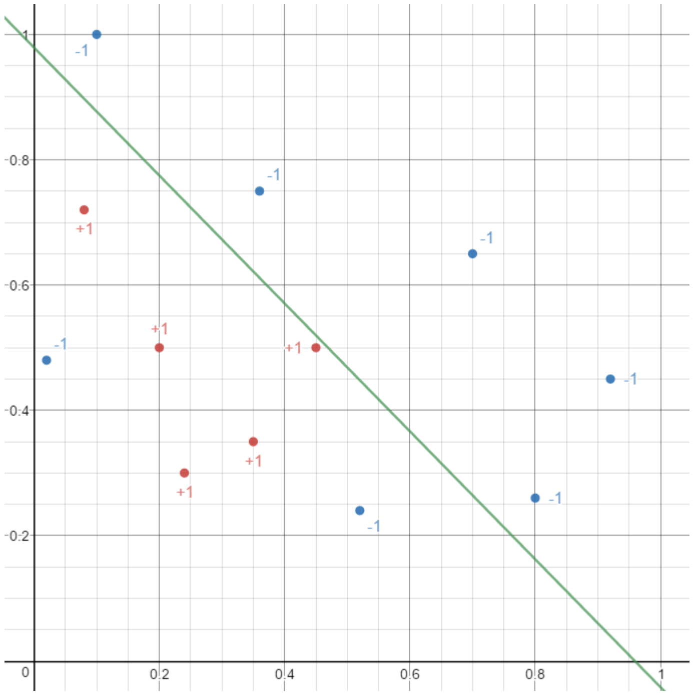
(b) This work is shown in (a), but the weights derived are the following:
w0: 0.7 w1: -0.415 w2: -0.865

(c) Because the graph can only be depicted one-dimensionally, the best way to visualize this is a straight line along which all the samples' $i_1$ locations are placed. The minimum error is 2, with the two 'earliest' samples from Class -1 being cut off from the others.



5. Question 5

(a) The minimum error for this chart is 2, as shown below. It is impossible for a straight line to perfectly separate all the samples by the nature of the plotted samples.



(b) In order to tackle this problem, we need to encapsulate the Class +1 samples in an area surrounded by Class -1 samples, effectively boxing it in. A multi-layer perceptron is required that should operate with the following steps:

i. Given input weights and a bias are inserted three times into three different 'hidden' perceptrons.

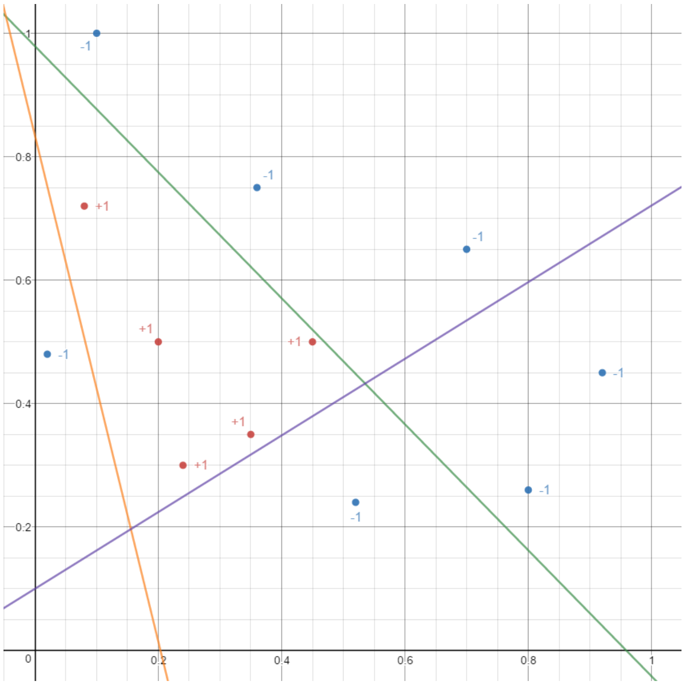
ii. The hidden perceptrons calculate as normal, each with their own lines and their own means of determining missed specimens.

iii. The perceptrons place their output in the 'output' perceptron, which uses these outputs as their own lines.

iv. The output will check if the lines function as an 'area' and successfully encapsulate the Class +1 specimens.

v. If it does, then stop. If not, then modify the weights inserted into each hidden perceptron until it is successful.

We can actually use the unsuccessful line used in (a) towards this end.



(c) Line 1: -1.02098x + 0.97902
Line 2: -4.09790x + .834
.62098x + 0.08379

Question 4, Part (a) Code Printout:

```
====================
step: 0 info: w0: 0.2 w1: 1.0 w2: -1.0 misses: 4
slope: 1.0 bias/w2: -0.2
visual line: 1.0x - -0.2
====================
step: 1 info: w0: -0.3 w1: 0.54 w2: -1.225 misses: 4
slope: 0.44081632653061226 bias/w2: 0.24489795918367344
visual line: 0.44081632653061226x - 0.24489795918367344
====================
step: 2 info: w0: 0.2 w1: 0.765 w2: -1.1500000000000001 misses: 3
slope: 0.6652173913043478 bias/w2: -0.17391304347826086
visual line: 0.6652173913043478x - -0.17391304347826086
====================
step: 3 info: w0: 0.7 w1: 0.895 w2: -0.8600000000000001 misses: 3
slope: 1.0406976744186045 bias/w2: -0.8139534883720929
visual line: 1.0406976744186045x - -0.8139534883720929
====================
step: 4 info: w0: 0.19999999999999996 w1: 0.545 w2: -1.185 misses: 3
slope: 0.459915611814346 bias/w2: -0.16877637130801684
visual line: 0.459915611814346x - -0.16877637130801684
====================
step: 5 info: w0: 0.7 w1: 0.5850000000000001 w2: -0.8250000000000001 misses: 3
slope: 0.7090909090909091 bias/w2: -0.8484848484848484
visual line: 0.7090909090909091x - -0.8484848484848484
====================
step: 6 info: w0: 0.19999999999999996 w1: 0.12500000000000006 w2: -1.05 misses: 3
slope: 0.1190476190476191 bias/w2: -0.19047619047619044
visual line: 0.1190476190476191x - -0.19047619047619044
====================
step: 7 info: w0: 0.7 w1: 0.25500000000000006 w2: -0.76 misses: 3
slope: 0.3355263157894738 bias/w2: -0.9210526315789473
visual line: 0.3355263157894738x - -0.9210526315789473
====================
step: 8 info: w0: 0.19999999999999996 w1: -0.09499999999999992 w2: -1.085 misses: 4
slope: -0.08755760368663587 bias/w2: -0.184331797235023
visual line: -0.08755760368663587x - -0.184331797235023
====================
step: 9 info: w0: 0.7 w1: -0.05499999999999992 w2: -0.725 misses: 2
slope: -0.07586206896551713 bias/w2: -0.9655172413793103
visual line: -0.07586206896551713x - -0.9655172413793103
====================
step: 10 info: w0: 0.19999999999999996 w1: -0.4049999999999999 w2: -1.05 misses: 4
slope: -0.3857142857142856 bias/w2: -0.19047619047619044
visual line: -0.3857142857142856x - -0.19047619047619044
====================
step: 11 info: w0: 0.7 w1: -0.10499999999999993 w2: -0.9 misses: 2
slope: -0.11666666666666659 bias/w2: -0.7777777777777777
visual line: -0.11666666666666659x - -0.7777777777777777
====================
step: 12 info: w0: 0.19999999999999996 w1: -0.4549999999999999 w2: -1.225 misses: 4
slope: -0.37142857142857133 bias/w2: -0.16326530612244894
visual line: -0.37142857142857133x - -0.16326530612244894
====================
step: 13 info: w0: 0.7 w1: -0.4149999999999999 w2: -0.8650000000000001 misses: 0
slope: -0.4797687861271675 bias/w2: -0.8092485549132946
visual line: -0.4797687861271675x - -0.8092485549132946
```