5/8/2021 OneNote

Section 6.1: Multi-step methods

Tuesday, March 30, 2021 8:56 AM

What is something Euler's Method, Taylor Methods, RK Method's have in common?

- they only rdy on previous step.

With depends only on w; (not w;,, w;-z, ... etc.)

recall Enler's Method

With = w; + st [f(t; w;)]

Goal: obtain higher accuracy by using more information. (i.e., with unit depend on with unit depend on

General form for an "m" step method is

 $\begin{array}{lll}
(\omega_0 = \alpha_0) & & & \\
(\omega_1 = \alpha_1) & & & & \\
(\omega_2 = \alpha_2) & & & & \\
(\omega_3 = \alpha_3) &$

Special cose

If b=0, we call method "explicit" (only depends on past)
If b+0, one call method "implicit" (using on both sides)

How to obtain a MS method?

Recall that we are solving y'=fct,y), yctol=d.

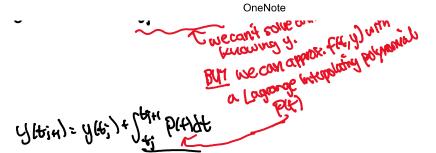
Let's integrate over [ti, tim]

Stim

July (H) et = Stiff(ti, y) et

y(bj+)-y(bj) = Stitl f(t,y)dt.

Thout.



Let's do an example.

Use 2 points to create Lagrange Interpolating polynomial Of fitig) explinitly (use ti, ti-1)

P(t) =
$$\int [t_{i-1}, y_{i-1}] \int_{b_i} (t_i) \int_{b_i} (t_i)$$

exercise for = - f(f)-1.42-1) PA + 3 f(f).(1) PA

So all together

y(t;+1) = y(t;)+ bt [3 f(t;,yi) - 2 f(t;,yi)] "Adams-Bashforth" 2-step Method (LTE OCKY))

ex what if we used L.L.P. implicitly (tri, ti,1)? PH) = f(+in, 4)+1) lo(+) + f(+i,4i) l, (+) \$ (t-ti) \frac{1}{4}(t-tin)

Win=w; + st [5 { bit, win) + 8 f(ti, wi) - f(ti, wi-)]

"Adams - Moulton" 2- step Method (LTE OCAL4)

ex Consider y'= y-t2+1, 04+2, y(0)=0.5 Use AM2 with N=10 te estimate solution.

$$W_{2} = W_{1} + \frac{\Delta t}{12} \left[5 f(t_{2}, w_{2}) + 8 f(t_{3}, w_{4}) - f(t_{3}, w_{3}) \right]$$

$$= W_{1} + \frac{\Delta t}{12} \left(5 \left[w_{2} - t_{2}^{2} + 1 \right] + 8 \left[w_{1} - t_{1}^{2} + 1 \right] - \left[w_{3} - t_{3}^{2} + 1 \right] \right)$$

$$= W_{1} + \left(\frac{5 \Delta t}{12} w_{2} - \frac{5 \Delta t}{12} t_{2}^{2} + \frac{5 \Delta t}{12} + \frac{8 \Delta t}{12} w_{1} - \frac{8 \Delta t}{12} t_{2}^{2} + \frac{8 \Delta t}{12} \right)$$

$$- \frac{\Delta t}{12} w_{3} + \frac{\Delta t}{12} t_{3}^{2} + \frac{\Delta t}{12} \right)$$

Wz= 位((+音)4)4-台w+台-5台也-3412+台台 Ugh. That was a lot of work.

@ (onsuler y'= y-t2+), 0 = t=2, y(0)=0.5. Use AB 2 method to approximate solution WN=10.

How can we find yo?

USC a 1-step method from Unt 5 (Euler, RK2, RK9) to generate our initial conditions) In particular, use a 1-step method with comparable LTE.