Section 1.3: Speed and Convergence

Def An algorithm is a procedure that describes a finite sequence of steps to be performed in a specific order.

Def Pseudocode describes algorithms, but without specific syntax.

ex Calculate the sum of the first n integers
$$\sum_{i=1}^{n} i.$$

For Loop

Pseudocode

```
inputs: n
output: summy (sum_{i=1}^{n} i)
    summy = 0.
    for i = 1, 2, ..., n, do
        sumy = summy + i
    end % for
```

While loop

```
inputs: n
output: summy (sum_{i=1}^{n} i)
    summy = 0.
    i = 1
    while i ≤ n, do
        summy = summy + i
        i = i + 1
    end % while
```

Two different algorithms do the same thing!

Which is better?

Def The sequence $\{p_n\}$ converges to p with a rate of convergence $O(b_n)$ if $\{b_n\}$ converges to 0 and

approx, what trying to approx, "big Oh of bn"

$$|p_n - p| \le \lambda |b_n|$$

for some constant λ and sufficently large n.

Anothe def.

Def Suppose $G(h) \to 0$ as $h \to 0$ and $F(h) \to L$ as $h \to 0$. Then if

$$|F(h) - L| \le C\,|G(h)|$$

we say that $F(h)$ coneges to L w/ a rate of convergence $O(G(h))$

ex Find the rate of convergence of f as $h \to 0$, $f(h) = \cos(h)$ using the 1st order Taylor Polynomial.

$$\cos(h) = \underbrace{f(x_0) + \frac{f'(x_0)}{1}\overset{h-0}{(x - x_0)}}_{\text{Taylor Poly of order 1}} + \underbrace{\frac{f''(\xi)}{2!}\overset{(h-0)^2}{(x-x_0)^2}}_{\text{Remainder "error"}}$$

note: $x_0 = 0$, $f(x) = \cos(x)$

$$\cos(h) = \cos(0) - \sin(0)(h) - \frac{\cos(\xi(h))}{2} h^2$$

$$\cos(h) = 1 - \frac{\cos(\xi(h))}{2} h^2$$

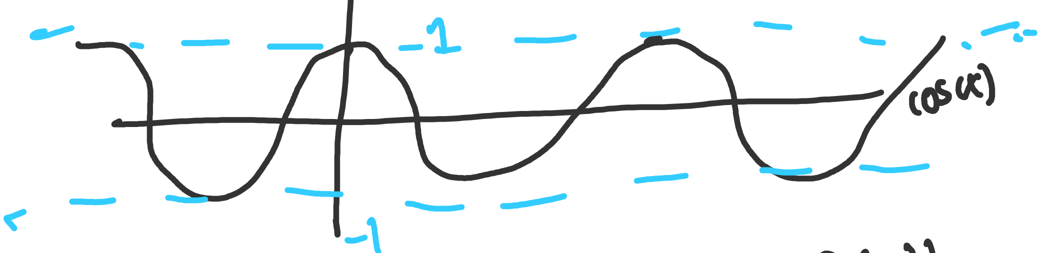
exact so   approx   remainder

$$|\underbrace{\cos(h)}_{\text{exact}} - \underbrace{1}_{\text{approx TP order 1}}| \le \left| \underbrace{\frac{-\cos(\xi(h))}{2}}_{\text{remainder term}} h^2 \right| \le \underbrace{\frac{1}{2}}_{C} h^2$$

abs error.

(since $|\cos(\xi(h))| \le 1$)

worst case scenario



rate of convergence is $O(h^2)$

"order h^2"