Section 1.1: Errors

Recall from previous courses

- continuity
- differentiation / tangent lines
- Integration / Riemann Sums
- Mean Value Thm
- Intermediate Value Thm
- ODEs
- $\boxed{\text{Taylor Series}}$

ex/ π cannot be exactly stored in a computer. If we have 4 total digits

rounding: 3.142 (round to next number)

Chopping: 3.141 (chop at the desired # of digits)

Def For an approximation p^* to a number p, we measure

(a) Absolute error: $|p-p^*|$

(b) Relative error: $\dfrac{|p-p^*|}{|p|}$

Def The approximation p^* is said to approximate p to t significant digits if t is the largest integer for which

$t=1$ $t=2$ $t=3$

$$\frac{|p-p^*|}{|p|} \le 5 \times 10^{-t} \quad (0.5, 0.05, 0.005, \ldots)$$

relative error!

ex

$p = 0.54617 \quad q = 0.54601$

a) Find $r = p - q$

$r = p - q = 0.00016$

b) Find p^*, q^*

$p^* = 0.5462$ (rounding)

$q^* = 0.5460$ (rounding)

"chopping"

$p^* = 0.5461$

$q^* = 0.5460$

c) Find $r^* = p^* - q^*$

$r^* = 0.0002$ (rounding)

d) Find absolute & relative errs

$\text{abs} = |r - r^*| = |0.00016 - 0.0002|$

$= 0.00004$

$\text{rel} : \frac{|r - r^*|}{|r|} = \frac{0.00004}{0.00016} = 0.25$

e) How many sig digits p^*, q^*, r^*

r^* is accurate to 1 sig digit

whereas p^*, q^* accurate to 4 sig. digits

$0.25 < 0.5$ but not ~~not~~ less than 0.05

$< 5 \times 10^{-1}$ $t = 1$