Disastrous disappointments: Asset-pricing with disaster risk and disappointment aversion

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Jim Dolmas

Federal Reserve Bank of Dallas
jim@jimdolmas.net

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What does this paper do?

- Look at asset-pricing implications of a rare disasters (RD) model where representative agent is endowed with plausible degree of disappointment aversion (DA).
- Two environments in paper:
  - Similar to Rietz (1988 *JME*), 'normal times' given by Mehra-Prescott Markov chain, single disaster state, preferences collapse to EU-CRRA if no DA. Just to illustrate mechanisms at work.
  - Richer model in the spirit of Barro (2006 *QJE*) and others, distribution of disaster sizes, partial default on 'risk-free' asset in disaster states, levered equity, fully utilize EZ aggregator.
  - Focus on second in this talk.
- Disappointment aversion as defined by Gul (1991 *Ecta*), used in asset-pricing literature by Routledge and Zin (2010 *JF*) or Campanale, Castro and Clementi (2010 *RED*).

Why would we want to do this?

- Disaster risk models have recently had considerable success in rationalizing data on consumption and asset returns.
- Literature was dormant after Rietz, jump-started by Barro (2006, QJE).
- Disaster risk is hard to quantify with any precision given limited data.
  - Does a disaster happen once every couple hundred years or a couple times every hundred years?
  - Hard to say with 100 years of data.
- Barro: used cross-country evidence.

Why would we want to do this?

- Adding more countries' experiences gives more observations to estimate disaster risk with, but only works if we assume all countries draw disasters from a common distribution.
- Debatable whether the sizable risks Barro finds—*e.g.*, 1.7 percent chance of average 29% decline—are a good characterization of risk faced by, say, US investors.
- If risks are a lot smaller for the US than indicated by multi-country estimates, does disaster risk no longer explain US asset returns data?
- That's where DA comes in.

Rationale for adding DA to RD

- We have experimental and field evidence that people over-weight bad or disappointing outcomes.
- If we endow agent in RD model with plausible degree of DA, maybe we can get good results with much smaller disaster risks?
- Paper shows that is in fact the case.
- Empirical discipline imposed by keeping DA in the range of estimates from experimental literature—Choi *et al.* (2007, *AER*) and Camerer and Ho (1994, *JRU*).

What do I find?

- DA does complement RD: Can get good results with disaster probabilities and sizes one-half or one-fourth the average sizes in Barro's *QJE* paper.
- Quality of model's fit depends on assumption about EIS.
- For a low EIS (0.10), can match almost exactly the means and standard deviations of risk-free rate and equity return.
- For larger EIS (1 or 1.5), fit is less satisfactory, but still broadly similar to results obtained by Gourio (2010, *FRL*).
  - The one big departure is that our model doesn't generate *enough* volatility in the risk-free rate.
  - Not often viewed as a problem.

Roadmap of the talk

1. Introduction and motivation
2. Background
3. Model
4. Numerical experiments

Background: Rare disasters

- One of the earliest responses to Mehra-Prescott (Rietz 1988 *JME*), but lost favor.
  - Mehra-Prescott's response (1988 *JME*): disasters are too big; risk aversion, discount factor too high; risk-free asset not likely risk-free in a crisis.
  - Merits just a footnote mention in Kocherlakota's 1996 *JEL* survey.
- Interest revived by empirical work of Barro (2006 *QJE*), theoretical contributions by Gabaix, Gourio, others.
- Disaster risk provides potential resolution to equity premium puzzle by providing a channel for holding down risk-free rate.

More

Background: Disappointment aversion

- As with Epstein-Zin preferences, uncertain future utility reduced to a certainty equivalent value. Disappointment aversion enters through the form of the certainty equivalent operator.
  - Disappointing outcomes get extra weight.
- But what outcomes are disappointing?
- Gul (1991 *Ecta*), certainty equivalent obeys a consistency requirement—threshold for disappointment is certainty equivalent itself.
  - So certainty equivalent only defined implicitly.

Background: Disappointment aversion

- Very roughly paraphrased: disappointment averse agents take expectations over-weighting outcomes that would fall below expectations.
- Used by Routledge and Zin (2010 *JF*) in a Mehra-Prescott framework; Campanale, Castro, Clementi (2010 *RED*) in a model with capital accumulation.
    - Routledge and Zin use a 'generalized' form of DA that allows them some control over which states are disappointing.

Disappointment aversion: Formally

- More formally, the agent's intertemporal preferences have the Epstein-Zin form

$$U_t = [(1-\beta)c_t^{1-\rho} + \beta\mu_t(U_{t+1})^{1-\rho}]^{1/(1-\rho)}$$

  for $\rho \geq 0$, $\rho \neq 1$.
- The agent's EIS is ρ^{-1}.
- Conditionality in μ_t eventually will come from Markov structure of consumption growth; for now, just ignore it.
  - I'll drop the t for now to avoid confusion.

Disappointment aversion: Formally

- Let z be a discrete random variable taking the value z_i with probability p_i (given the state at t). The certainty equivalent of z is defined implicitly by

$$\mu(z)^{1-\alpha} = \frac{\sum_i p_i z_i^{1-\alpha}[1 + \theta I\,(z_i < \mu(z))]}{1 + \theta \sum_i p_i I\,(z_i < \mu(z))}$$

- $I(\,\cdot\,)$ is an indicator function, taking the value one when $z_i < \mu(z)$ (z_i disappoints), and zero otherwise.
- $\alpha \geq 0$, $\alpha \neq 1$ and $\theta \geq 0$ are parameters.

How might DA interact with RD?

- Suppose state $i = 1$ is the only disappointing state. Then:

$$\mu(z)^{1-\alpha} = \frac{1+\theta}{1+\theta p_1} p_1 z_1^{1-\alpha} + \frac{1}{1+\theta p_1} \sum_{i \neq 1} p_i z_i^{1-\alpha}$$

- This is precisely the form of a standard, Epstein-Zin certainty equivalent, with distorted probabilities $\hat{p}$,

$$\hat{p}_1 = \frac{p_1 + \theta p_1}{1+\theta p_1}$$
$$\hat{p}_j = \frac{p_j}{1+\theta p_1} \quad (j > 1)$$

- Think of state 1 as a rare disaster. Then, $p_1 = .01$ and $\theta = 1$ gives same c.e. as $p_1 = .02$ and $\theta = 0$.

DA and asset-pricing

- What does the SDF look like?
- Assume log consumption growth, x_{t+1}, follows a Markov chain, $\{x_i, P_{i,j}\}$.
- The SDF has the form:

$$m_{i,j} = \beta e^{-\rho x_j} \left(\frac{\phi_j e^{x_j}}{\mu_i(\phi e^x)} \right)^{\rho-\alpha} \frac{1 + \theta I_{i,j}}{1 + \theta \sum_j P_{i,j} I_{i,j}},$$

  which is similar to the version given by Campanale *et al.*
- ϕ is lifetime utility scaled by consumption (v_t/c_t), and $I_{i,j} = 1$ if state j is disappointing conditional on state i today.

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Consumption process

- Starting point is Mehra and Prescott's Markov chain—two states, with gross consumption growth y_L and y_H, and symmetric transition matrix Q.
- Add a distribution of N disaster states D_i with *relative* frequencies f_i and *relative* sizes z_i.
  - $\sum_i f_i = 1$ and $\sum_i f_i z_i = 1$.
- If average disaster size is b, then size of disaster in state D_i is bz_i.
- If overall disaster probability is p, then probability of state D_i is pf_i.

Consumption process

- Modify transition matrix as follows:
    - If the state today is either L or H, then the D_i state occurs with probability pf_i.
    - If today's state is a disaster state, growth returns next period to $\{y_L, y_H\}$ with probabilities $\{1/2, 1/2\}$.
    - There is zero probability of remaining in a disaster state or transitioning from one disaster state to another.
- Unlike Rietz, I assume Mehra-Prescott Markov chain describes "normal" (non-disaster) times.
    - Keep y_L, y_H, Q fixed as we vary p, b.

Markov chain

Assets

- I consider claims to a one-period 'riskless' asset, paying one unit of consumption next period (if no disaster), and two forms of 'equity.'
- Riskless asset is 'riskless' because I allow partial default in the disaster state. Call its return the bill rate to avoid confusion.

  Default
- Two equity claims:
  - One is standard claim to aggregate consumption
  - The other (a 'dividend' claim) is a claim to a leveraged version of aggregate consumption. Leverage
- Returns are defined in the usual way. Returns

Roadmap of the talk

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Structure of the experiments

- Vary the environment by varying p, disaster probability, and b, the average disaster size. Find parameters of certainty equivalent—θ, strength of DA, and α, curvature—that get us close to first and second moments of returns data (from Gourio 2008 *FRL*).
  - I use discrete grids for θ and α.
  - $\theta \in \{0, 0.1, \dots, 3\}$, $\alpha \in \{0, 0.1, \dots 5\}$.
  - The $\theta \leq 3$ restriction keeps us in the empirically plausible range.

▸ Choosing θ

- Model moments I compare to data are for non-disaster samples (average behavior in 'normal times').
- Focus on bill rate and dividend (leveraged consumption) return.

Structure of the experiments

- Starting point is p and average b from Barro's *QJE* paper:

$$p = 0.017$$
$$b = 0.29$$

- Consider in turn (and in combinations) disaster probabilities and disaster sizes that are $1/2$ or $1/4$ the averages from Barro's data. I.e., p and b such that

$$p \in \{0.017, 0.0085, 0.00425\}$$
$$b \in \{0.29, 0.145, 0.0725\}.$$

Fixing other parameters

- Some parameters fixed throughout experiments—discounting, leverage, default fraction, EIS, *relative* sizes and frequencies of disaster states.
- In all the environments, data prefer a very low EIS, around 0.1. But we know there are also good reasons to consider EIS bigger than one. Simulate model for EIS of 0.1, 1.0, 1.5.
- I follow Gourio (2008 *FRL*), in setting fraction of bill payoff defaulted on in disaster (η_i) to be 40% of the disaster size.

Basic parameters

ρ^{-1}	β	λ	η_i	(f_i, z_i)
$\{0.10, 1.0, 1.5\}$	0.97	3	$0.4bz_i$	Derived from Barro

Frequency & size data

Results: Low EIS case

- The model's best fits are for the low EIS case.

Results for ($\rho^{-1} = 0.10$)

p (%)	b (%)	θ	α	$\mathrm{E}(R^f)$	$\mathrm{E}(R^d)$	$\sigma(R^f)$	$\sigma(R^d)$
1.70	29.0	0.3	2.9	1.02	9.58	4.37	15.17
0.85	29.0	0.9	2.8	1.01	9.33	3.97	14.81
0.43	29.0	1.6	2.1	1.04	9.09	3.71	14.55
1.70	14.5	1.2	3.3	1.04	9.32	3.75	14.64
1.70	7.25	1.9	1.5	1.06	9.04	3.56	14.42
0.85	14.5	1.7	2.6	1.04	9.21	3.62	14.51
0.43	7.25	2.4	0.3	1.03	8.97	3.52	14.37
0	0	2.5	0.5	1.04	9.07	3.50	14.36
Data	–	–	–	1.03	8.91	4.36	15.04

- Data are from Gourio (2008 *FRL*), based on 1926–2004 sample used in Cochrane (2008 *RFS*).

Results: Large EIS case

Results for ($\rho^{-1} = 1.5$)

p (%)	b (%)	θ	α	$E(R^f)$	$E(R^d)$	$\sigma(R^f)$	$\sigma(R^d)$
1.70	29.0	0.0	4.0	1.11	10.23	0.25	10.58
0.85	29.0	0.0	4.8	1.07	9.74	0.24	10.52
0.43	29.0	0.8	5.0	1.14	10.89	0.14	10.86
1.70	14.5	3.0	5.0	2.14	12.20	0.12	11.33
1.70	7.25	3.0	5.0	2.48	10.84	0.16	11.08
0.85	14.5	3.0	5.0	2.35	11.21	0.14	11.14
0.43	7.25	3.0	5.0	2.55	10.32	0.17	10.97
Data	–	–	–	1.03	8.91	4.36	15.04

- Results are less satisfactory—especially volatility of the bill rate.
- Even so, apart from standard deviation of bill rate, results are broadly similar to those obtained by Gourio (even as we greatly reduce disaster risk).
- Bang-bang nature of optimal θ choice also interesting—either very small or at upper bound.

Results: Large EIS case

- The problem is really the volatility; the means are easily correctible by choice of β.
- So far, we've set $\beta = 0.97$, but nothing really to pin down β other than level of risk free rate.
- Here's the $p = 1.7\%$, $b = 7.25\%$ line from previous table; $\beta = 0.984$ is the value that matches the bill rate:

Case: $\rho^{-1} = 1.5$, $p = 0.017$, $b = 0.0725$

β	θ	α	$E(R^f)$	$E(R^d)$	$\sigma(R^f)$	$\sigma(R^d)$
0.970	3.0	5.0	2.48	10.84	0.16	11.08
0.984	3.0	5.0	1.03	9.25	0.15	10.91
Data	–	–	1.03	8.91	4.36	15.04

Conclusions, directions

- Smaller disaster risks can still generate realistic asset returns, when agent is endowed with empirically plausible degree of disappointment aversion.
- Obvious directions for further work:
  - Time-varying disaster probability (useful for getting times series properties of the data, but problems imposing empirical discipline).
  - Generalized disappointment aversion (as in Routledge-Zin); can more control over which states disappointment help with the large-EIS results?
  - Changing the shape of the distribution of disasters, rather than just joint disaster probability p, average size b.

The current version of the paper and Matlab code for solving the model are available at:
`www.jimdolmas.net/economics/current-work`

I would value your feedback; email me at
`jim@jimdolmas.net`

RD and the risk-free rate

- Recall that in representative agent, CRRA, lognormal case, risk-free rate obeys:

$$R^f = -\log(\beta) + \alpha\gamma - \frac{\alpha^2\sigma^2}{2}$$

where α is the CRRA coefficient, γ and σ are mean and standard deviation of log consumption growth.

- Well-known that for typical estimates of γ and σ, the first-order term $\alpha\gamma$ dominates as α increases.
  - A high value of α, necessary to get a non-negligible equity risk premium, thus produces a counterfactually high risk-free rate.

RD and the risk-free rate

- Let a disaster add $\log(1-b) < 0$ to $\log(c_{t+1}/c_t)$, and occur with probability p (independently of the log-normal component). Then, the log risk-free rate becomes

$$R^f = -\log(\beta) + \alpha\gamma - \frac{\alpha^2\sigma^2}{2} - \log(1 - p + p(1-b)^{-\alpha}).$$

- For typical values of γ and σ, and for small values of p, we can generate low values of R^f even for large values of α, and with $\beta < 1$.

Return

Consumption process: Markov chain

- Mehra and Prescott's two-state Markov chain for gross consumption growth y_{t+1} is:

$$y_{t+1} \in \begin{bmatrix} y_L \\ y_H \end{bmatrix} = \begin{bmatrix} 0.982 \\ 1.054 \end{bmatrix}$$

with the transition matrix

$$Q = \begin{bmatrix} Q_{LL} & Q_{LH} \\ Q_{HL} & Q_{HH} \end{bmatrix} = \begin{bmatrix} 0.43 & 0.57 \\ 0.57 & 0.43 \end{bmatrix}$$

Consumption process: Markov chain

- The modified Markov chain is given by the set of states

$$y_{t+1} \in \begin{bmatrix} \mathbf{1} - b\mathbf{z} \\ 0.982 \\ 1.054 \end{bmatrix}$$

and the transition matrix

$$P = \begin{bmatrix} \mathbf{0} & \frac{1}{2}[\mathbf{1}, \mathbf{1}] \\ p[\mathbf{f}, \mathbf{f}]^\top & (1-p)Q \end{bmatrix}$$

- $\mathbf{z}$ denotes the z_i's, $\mathbf{f}$ denotes the f_i's, $\mathbf{0}$ a matrix of 0's, $\mathbf{1}$ a vector of 1's.

Return

The 'riskless' asset

- The 'riskless' asset here is a one-period zero-coupon bond (in zero net supply) whose price at t is q_t and whose payoff at $t+1$ is F_{t+1}, where

$$F_{t+1} = \begin{cases} 1-\eta_i & \text{if } y_{t+1} = y_{D,i} \\ 1 & \text{if } y_{t+1} \in \{y_L, y_H\} \end{cases}$$

- In the experiments, I'll follow Gourio (2010 *FRL*) in making η_i a fraction of the disaster size in state D_i.

Return

Dividend claim

- The ‘dividend’ claim is a claim to a process (‘dividends’) whose log growth rate is a multiple of log consumption growth.
- Letting y^d_{t+1} denote gross dividend growth from t to $t+1$,

$$y^d_{t+1} = (y_{t+1})^{\lambda},$$

where $\lambda \geq 1$ is the “leverage” parameter.

- This dividend formulation is now pretty standard—used, for example, in Routledge and Zin, Gourio, Bansal-Yaron, *etc.*

Return

Asset returns

- Returns are defined in the standard way.
- If q is the price of the 'riskless asset,' and w^c and w^d denote the price-dividend ratios for the two equity claims, then the key pricing equations are

$$\begin{aligned} q_t &= \mathrm{E}_t[m_{t+1}F_{t+1}] \\ w_t^c &= \mathrm{E}_t\left[m_{t+1}y_{t+1}\left(1+w_{t+1}^c\right)\right] \\ w_t^d &= \mathrm{E}_t\left[m_{t+1}y_{t+1}^d\left(1+w_{t+1}^d\right)\right], \end{aligned}$$

Asset returns

- Then, for Markov states i today and j tomorrow:

$$R^f_{i,j} = \frac{F_j}{q_i} \qquad \text{(Risk-free rate)}$$

$$R^c_{i,j} = \frac{y_j(w^c_j + 1)}{w^c_i} \qquad \text{(Consumption claim return)}$$

$$R^d_{i,j} = \frac{y^d_j(w^d_j + 1)}{w^d_i} \qquad \text{(Dividend claim return)}$$

where F_j is $1 - \eta_j$ if $y_j = y_{D,j}$, 1 otherwise.

Return

What's a plausible amount of DA?

- Experimental evidence on size of θ:
  - Choi, Fisman, Gale and Kariv (2007 *AER*) estimate mean around 0.3, with ninety percent of estimates for individual subjects between 0 and around 1.4.
  - Camerer and Ho (1994, *JRU*) synthesize data from nine experimental choice studies, estimate $\theta = 3$ treating the choice data as coming from a single representative agent.
- How does our $\theta \in [0, 3]$ compare with calibrations in other work on disappointment aversion and asset pricing?
  - Campanale *et al.* rely on Choi *et al.*, consider $\theta \in [0, 0.4]$.
  - Routledge and Zin use θ's from 9 (mostly) to 24 (in one case). Given their generalized form of DA, hard to say what experimental evidence implies for their θ.

Return

Distribution of disaster sizes

Relative frequencies and sizes are taken from Barro's (2006) histogram.

Calibration of z_i's, f_i's

Size	Nobs.	z_i	f_i
0.17	20	0.59	0.33
0.22	13	0.77	0.22
0.27	3	0.94	0.05
0.32	9	1.11	0.15
0.37	5	1.29	0.08
0.42	0	1.46	0.00
0.47	2	1.63	0.03
0.52	3	1.81	0.05
0.57	3	1.98	0.05
0.62	2	2.16	0.03

Return