

# Risk preferences, intertemporal substitution, and business cycle dynamics

Presentation for Sveriges Riksbank, Stockholm

Jim Dolmas  
`jim.dolmas@dal.frb.org`

05/17/11

# What does this paper do?

- Study implications of alternative specifications of risk preferences—including non-EU first-order risk aversion—for business cycle behavior.
- Fairly standard business cycle model
  - RBC core of many DSGE models
  - Includes habit formation, capital adjustment costs
- Impact of alternative choices for EIS (including  $> 1$ )

# Findings

- Risk preferences matter a great deal for welfare
  - Costs as high as 1.3% of lifetime consumption
- Some first moment impact (precautionary capital accumulation, significant under FORA), **but no impact on average asset returns.**
- Negligible impact on second moments

# What *does* matter for second moments?

- EIS matters a great deal
  - In stripped down model(no habits),  $EIS > 1$  can lead to  $\text{corr}(Y, C) < 0$ .
  - Variable labor and nonseparable intratemporal preferences a factor.
- Habits matters, of course.
  - Smaller amplification, low volatility of hours, **positive response of hours to negative TFP shock.**

# Roadmap of talk

- 1 Introduction
- 2 Motivate FORA
- 3 The model
  - Preferences
  - Technology
- 4 Solution
  - Chebyshev approximation
  - Parameter values
- 5 Results
  - What are we interested in?
  - Basic settings
  - Stripped-down model
  - Full model
- 6 Conclusions

## 550 is the correct CRRA coefficient

### Example

Suppose an agent with initial wealth of \$30,000 faces a 0.00477 probability of losing \$55. This is a small risk—the standard deviation of the lottery  $\{\tilde{w}; p\} = \{(29945, 30000); (0.00477, 0.99523)\}$ , as a percent of mean wealth, is about 0.013%.

Would they pay 45 cents to insure against it?

Yes, if EU/CRRA coefficient is 550.

\$30,000 wealth is hypothetical, but expected loss, probability, price of insurance taken from Cicchetti & Dubin (JPE 1994) phone wire insurance study. 57% of customers bought the insurance.

# No, actually 50 is the correct CRRA coefficient

## Example

Suppose the agent with wealth equal to \$30,000 faces a 0.245 probability of losing \$182. The standard deviation of this gamble, as percent of mean wealth, is 0.26%.

Would they pay \$55 to insure against it?

Yes, if CRRA coefficient is around 50.

The loss, loss probability, and price of insurance again come from an empirical study: Cohen and Einav's (AER 2007) analysis of the choice of auto insurance deductibles in a large sample of Israeli drivers.

# No, really 4 is the correct CRRA coefficient

## Example

Suppose the agent, again with initial wealth of \$30,000, faces a 7% probability of suffering a \$5,000 loss. This represents a gamble with a standard deviation equal to 4.3% of mean wealth.

Would they be willing to pay \$500 to insure against it?

Yes, if CRRA coefficient is around 4.

The 7% probability and \$5,000 loss are roughly the US average homeowners' multi-peril insurance claim rate and claim intensity for the period 2000–2004. \$500 is in the neighborhood of the average 2004 premium.



# FORA can fit all three cases

- Allows plausible risk aversion over wide range of gamble sizes.
- Below, specify a two-parameter family of risk preferences. We'll see in a moment what  $\theta$  and  $\gamma$  represent, but for now, note benchmark FORA calibration I'll use ( $\gamma = 0.9, \theta = 1$ ) fits all three of the previous examples.



# Add leisure, external habit to EZ JME 1990

- $\rho$  governs intertemporal substitution,  $\psi$  governs allocation of time to work.
  - $\epsilon = 1/(1 - \rho)$  is EIS,  $\epsilon_c = 1/(1 - \psi\rho)$  is EIS in consumption.
- The habit stock is assumed to evolve (externally) according to

$$H_{t+1} = (1 - \delta_h)H_t + C_t^a.$$

# Certainty equivalent $\mu_t(U_{t+1})$

- Embodies both conventional EZ preferences and FORA. For  $\theta \geq 0, \theta \neq 1$

$$\mu_t(U_{t+1}) = (\hat{\mathbb{E}}_t[U_{t+1}^{1-\theta}])^{1/(1-\theta)}$$

or, for  $\theta = 1$

$$\mu_t(U_{t+1}) = \exp(\hat{\mathbb{E}}_t[\ln(U_{t+1})])$$

- Ignore the 'hat' over the expectations operator for a moment. Then  $\mu_t(\cdot)$  is conventional EZ.  $\theta$  is CRRA parameter, and  $1 - \theta = \rho$  gives time-separable EU.
  - Even if  $1 - \theta \neq \rho$ , still CRRA-EU for timeless gambles

# First-order risk aversion (FORA)

- What about the  $\hat{\mathbb{E}}_t$ ?
- Our FORA specification is non-linear in probabilities, so can think of as  $\hat{\mathbb{E}}_t \neq \mathbb{E}_t$ .
- Based on Yaari, Quiggin; applied by EZ to equity premium in Lucas tree economy.

# First-order risk aversion (FORA)

- Generalizes the following two-state case: Imagine r.v.  $w$  that takes on two values,  $w_L < w_H$ , with probabilities  $p$  and  $1 - p$ . For  $0 < \gamma \leq 1$ , Yaari's certainty equivalent is

$$\hat{\mathbb{E}}(w) = p^\gamma w_L + (1 - p^\gamma) w_H$$

- Effectively, over-weights worse outcome when  $\gamma < 1$
- In our context, this gives

$$\mu(U) = [p^\gamma U_L^{1-\theta} + (1 - p^\gamma) U_H^{1-\theta}]^{1/(1-\theta)}$$

# Standard RBC model with capital adjustment costs

- Representative firm hires labor and capital from households to produce output according to

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha}$$

- Output divided between consumption, gross investment

$$Y_t \geq C_t + X_t$$

- Households' capital stocks following

$$K_{t+1} = (1 - \delta_k)K_t + K_t g(X_t/K_t)$$

# Standard RBC model with capital adjustment costs

- Adjustment cost function takes following form (as in Jermann, JME 1998):

$$g(z) = (b_0/b_2)z^{b_2} - b_1$$

where  $b_0, b_1, b_2 \geq 0$  and  $b_2 \leq 1$ .



# Equilibrium decision rules

- Equilibrium consists of a value function & decision rules that satisfy intratemporal FOC, intertemporal FOC (Euler equation), and Bellman equation.
  - External habit  $\Rightarrow$  Impose  $c = c^a$  after taking FOCs
- Really only need to find two maps:  $v(a, k, h)$  and  $N(a, k, h)$ . Everything else can be derived from these.
- Use Chebyshev collocation method described by Caldara, Fernandez-Villaverde, Rubio-Ramirez & Yao to approximate  $v(a, k, h)$  and  $N(a, k, h)$ .

# Approximations

- Assume TFP process  $\{a_t\}$  generated by finite state Markov chain
- Treat decision rules as vector-valued functions at each  $(k, h)$ :  
 $N_i(k, h) = N(a_i, k, h)$
- Then approximate as tensor product of Chebyshev polynomials in  $k, h$ :

$$N_i(k, h) \approx \sum_{l=0}^{O_k} \sum_{m=0}^{O_h} D_N^i(l, m) T_l(\iota(k)) T_m(\iota(h)) \equiv \mathcal{N}(k, h; D_N^i)$$

$$v_i(k, h) \approx \sum_{l=0}^{O_k} \sum_{m=0}^{O_h} D_v^i(l, m) T_l(\iota(k)) T_m(\iota(h)) \equiv \mathcal{V}(k, h; D_v^i)$$

# Calibration is mostly—OK, sort of—standard

	Value	Remarks
Technology parameters:		
$\alpha$	0.4	Standard
$\delta_k$	0.0127	10% annual
$\eta$	1.0045	1.8 % annual
Habit formation parameters:		
$\phi$	0.5	Habit strength
$\delta_h$	1	$h_t = c_{t-1}$
Capital adjustment cost function:		
$b_0$	0.1312	$g(\bar{z}) = \bar{z}, Dg(\bar{z}) = 1$
$b_1$	0.0172	$g(\bar{z}) = \bar{z}, Dg(\bar{z}) = 1$
$b_2$	0.5	Q elasticity of $z = 2$



# Markov chain for TFP

- Approximate an AR(1) with persistence 0.95 and residual standard deviation of 0.07 (Cooley & Prescott)
- Use Rouwenhorst's method to approximate with 9-state Markov chain (See Kopecky & Suen, RED 2010 for advantages of Rouwenhorst's method, compared to, say Tauchen's)
- Last step is set mean level of TFP such that deterministic s.s. output is one.

# Several objects of interest

- First moments:
  - Precautionary accumulation (stochastic s.s.  $k$  versus deterministic s.s.  $k$ );
  - Average returns on physical capital, hypothetical riskless asset
- Second moments:
  - Standard volatility measures (absolute, relative to  $y$ , relative to TFP)
  - Impulse responses
- Welfare cost of volatility
  - Number of ways one could calculate this

# Common features of the numerical experiments

- Two EIS settings,  $\epsilon = 0.5$  and  $\epsilon = 15$  (corresponds to  $\epsilon_c$ 's around 3/4 and 1.5)
- Given EIS, three cases for risk preferences
  - EU:  $\gamma = 1$ ,  $\theta = 1/\epsilon$
  - 'High CRRA':  $\gamma = 1$ ,  $\theta = 100$  (like Tallarini)
  - FORA:  $\gamma = 0.9$ ,  $\theta = 1$
- Except for impulse responses, use same draw of 10,100 disturbances for all runs; discard first 100 observations.
- Standard deviations and correlations are for HP-filtered model data
- Impulse responses calculated similar to Caldara et al.

# First, no habits or adjustment costs

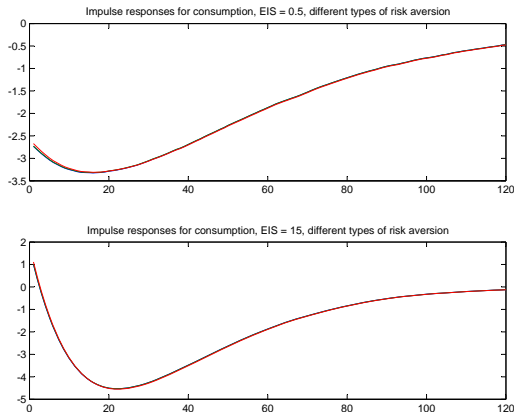
- First moment implications

	$\epsilon = 0.50$		
	$\%(k/\bar{k})$	$R^e$	$R^f$
EU, $\theta = 1/\epsilon, \gamma = 1$	-0.0675	5.2054	5.2019
High CRRA, $\theta = 100, \gamma = 1$	0.7372	5.1614	5.2063
FORA, $\theta = 1, \gamma = 0.9$	1.4267	5.1560	5.2019

	$\epsilon = 15$		
	$\%(k/\bar{k})$	$R^e$	$R^f$
EU, $\theta = 1/\epsilon, \gamma = 1$	-0.0386	5.2037	5.2007
High CRRA, $\theta = 100, \gamma = 1$	0.5568	5.1711	5.1965
FORA, $\theta = 1, \gamma = 0.9$	0.8359	5.1560	5.2007



# This picture kills a couple birds with one stone



## Other points from stripped-down model

- High EIS case makes you wonder what “long-run risk” folks will find when they incorporate production with variable work effort
- Complementarity between consumption and leisure is important for strange behavior when EIS high (can check approximate impulse response for  $mu_c$ )
- Amplification on order of 1.4–1.8 (ratio  $\sigma(y)/\sigma(a)$ )
- Welfare cost of volatility:
  - EU nil
  - High CRRA roughly 0.3–0.4% of lifetime consumption
  - FORA 1.1–1.3%.

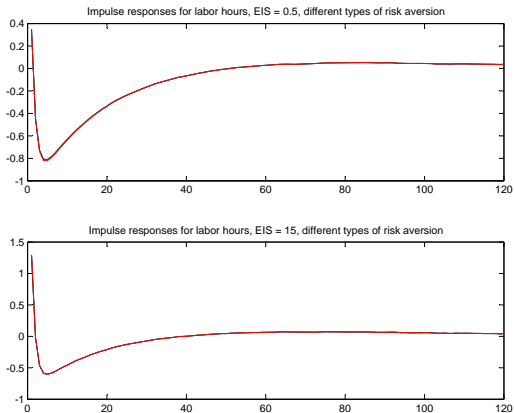
# Habits matter, not much else does

- Adding habit & capital adjustment cost changes results substantially, but conditional on presence of habit **neither risk preferences nor EIS matter that much.**
- First moments:
  - Less precautionary capital accumulation, none in higher EIS case
  - Significantly, average equity premium still minute, doesn't vary significantly with either risk preferences or EIS choice
  - That's a sharp contrast to Jermann (fixed labor model)

## Second moments with habits & adjustment costs

- Compared to stripped-down model, behavior of labor hours is key difference
- Hours are significantly less volatile, less highly correlated with output
  - $100\sigma(\ln(N))$  now 0.1–0.2, was 0.6–1.3
  - $\text{corr}(\ln(N), \ln(Y))$  now 0.1–0.6, was basically 1.0
  - Impulse responses have hours up at impact in response to negative shock
- 2nd moments still virtually identical across risk preference specifications

# Odd hours behavior in model with habits



## Other points from the full model

- Hours jump maybe not that surprising, given need to maintain  $c_t > \phi c_{t-1}$ . (See Graham, JnlMacro 2008)
- Lower hours volatility means much less amplification ( $\sigma(Y)/\sigma(a) \approx 1$ )
- Welfare costs: Similar orders of magnitude as before
  - EU  $\approx 0$
  - High CRRA  $\approx 0.3\%$  of consumption
  - FORA  $\approx 1.3\%$

# Conclusions

- Risk preferences don't matter for dynamics
  - Caveat: Not exhaustive. What about disappointment aversion? (Campanale, Castro, Clementi RED 2010)
- EIS matters a great deal
  - And elastic labor, intratemporal preferences play important role
- Volatility potentially very costly
  - Question: What's the correct thought experiment? (Compare current results w/earlier version)