Type families and data kinds

Advanced functional programming - Lecture 8

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Last lecture

In the previous lecture, we saw some examples of programming with GADTs.

- ▶ This allowed us to define rich types, enforcing all kinds of properties.
- ▶ But we also ran into some limitations.

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Today

- A bit more on the `Equal` data type
- Kinds beyond *
- Programming with types

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Equality type

We introduce `Equal` as a proof that two types are equal:

```
data Equal :: * -> * -> * where
  Refl :: Equal a a
```

We could even 'prove' some properties of the relation:

```
refl  :: Equal a a
sym   :: Equal a b -> Equal b a
trans :: Equal a b -> Equal b c -> Equal a c
```

Expressive power of equality

Equality proofs and phantom types are enough to 'implement' GADTs:

```
data Expr a =
    LitI   (Equal a Int)  Int
  | LitB   (Equal a Bool) Bool
  | IsZero (Equal a Bool) (Equal b Int)
  | Plus   (Equal a Int)  (Expr Int)  (Expr Int)
  | If     (Expr Bool)    (Expr a)    (Expr a)
```

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Safe vs unsafe coercions

Using our equality function we can safely coerce between types:

```
coerce :: Equal a b -> a -> b
coerce Refl x = x
```

Question

Why does this type check?

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Safe vs unsafe coercions

Using our equality function we can safely coerce between types:

```
coerce :: Equal a b -> a -> b
coerce Refl x = x
```

Question

Why does this type check?

Question

What about this definition:

```
coerce :: Equal a b -> a -> b
coerce p x = x
```

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Aside: irrefutable patterns

Haskell also allows irrefutable patterns:

```
lazyHead ~(x:xs) = x
```

This does not force the list to weak head normal form.

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Aside: irrefutable patterns

In tandem with GADTs this is particularly dangerous:

```
coerceL :: Equal a b -> a -> b
coerceL ~Refl x = x
```

Question

How could this cause well-typed program to crash with a type error?

Aside: irrefutable patterns

In tandem with GADTs this is particularly dangerous:

```
coerceL :: Equal a b -> a -> b
coerceL ~Refl x = x
```

Question

How could this cause well-typed program to crash with a type error?

```
foo :: Bool -> Int
foo b = coerceL undefined b
```

Apparently unrelated language features may interact in unexpected ways!

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Outlook generic programming

We can even use GADTs to *reflect* types themselves as data:

```
data Type :: * -> * where
  INT  :: Type Int
  BOOL :: Type Bool
  LIST :: Type a -> Type [a]
  PAIR :: Type a -> Type b -> Type (a,b)
```

Safe dynamically typed values

We can define dynamically typed values by packing up a type representation with a value:

```
data Dynamic :: * where
  Dyn :: Type a -> a -> Dynamic
```

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```

To unwrap these values safely, we check whether the types line up as expected:

```
coerce :: Type a -> Dynamic -> Maybe a
coerce t (Dyn t' x) =
  case eqType t t'
    Just Refl -> Just x
    _          -> Nothnig
```

Generic programming

We can also define new functions *by induction on the type structure*:

```
f :: Type a -> ... a ...
```

In this way, we can define our own versions of functions such as show, read, equality, etc.

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System FC

Haskell's core language, System FC, is a typed lambda calculus, extended with data types and pattern matching.

One of its more distinct features is *coercions* and *casts*.

- ▸ Coercions play the same role as our `Equal` data type;
- ▸ If two types are coercible, one can be cast to the other:

```haskell
isZero :: (a ~ Int) => a -> Bool
```

There is quite a lot of work necessary to guarantee that this does not accidentally make the type system unsound!

Pattern matching on GADTs introduces such coercions in the individual branches.

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Problems with GADTs

```
toVec :: [a] -> Vec a ???
```

To define this function, we needed to reify natural numbers on the type level – defining a *singleton type* SNat.

```
vappend :: Vec a n -> Vec a m -> Vec a ???
```

To define this function, we needed to construct an explicit relation describing how to add two types, n and m.

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Passing explicit Sums

In the last lecture, we saw how to pass explicit an explicit argument, explaining how to add two 'type-level' natural numbers:

```
data Sum :: * -> * -> * -> * where
  SumZero :: Sum Zero n n
  SumSucc :: Sum n m s -> Sum (Succ n) m (Succ s)
```

But constructing this evidence by hand is tedious…

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Multi-parameter type classes

We can automate this through a *multi-parameter type class*

```
class Summable a b c | a b -> c where
  makeSum :: Sum a b c

instance Summable Zero n n where
  makeSum = SumZero

instance Summable n m s =>
         Summable (Succ n) m (Succ s) where
  makeSum = SumSucc makeSum

append :: Sum n m s
       => Vec a n -> Vec a m -> Vec a s
append = vappend makeSum
```

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Multi-parameter type classes

Type classes define *relations* between types:

- `Eq` defines a subset of all types that support equality;
- `MonadState` (from `mtl`) defines a subset of pairs of types `s` and `m`, where `m` supports read/write operations on a state of type `s`.

The `Summable` type class is special case of such relations – it is really defining a *function* between types.

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Multi-parameter type classes

For some time, multi-parameter type classes with functional dependencies were the *only* way in Haskell to define such type-level computations.

But there has been a flurry of research in the last decade exploring alternative language extensions.

```
... the interaction of functional dependencies
with other type-level features such as
existentials and GADTs is not well understood
and possibly problematic.
```

Kiselyov, Peyton Jones, Shan in *Fun with type families*

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Associated types and type families

Type classes let you capture an *interface* – such as monads (supporting return and bind), or monoids (supporting zero and addition).

These interfaces can describe *functions*.

But what if we would like them to describe *types*.

Associated types

Associated types let you declare a type in a class declaration:

```
class Collects c where
  type Elem c -- Associated type synonym
  empty  :: c
  insert :: Elem c -> c -> c
  toList :: c -> [Elem c]
```

Any instance of the `Collects` class must choose a type of elements, together with definitions for the functions.

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Associated types – examples

```
instance Eq e => Collects [e] where
  type Elem [e] = e
  empty = []
  ...

instance Collects BitSet where
  type Elem BitSet = Char
  ...
```

Addition through association

We can use such associated types to replace the functional dependencies we saw previously:

```
class Summable n m where
  type TheSum n m
  makeSum :: Sum n m (TheSum n m)

instance Summable Zero n where
  type TheSum Zero m = m
  ...

instance Summable n m => Summable (Succ n) m where
  type TheSum (Succ n) m = Succ (TheSum n m)
  ...
```

Associated types or multiparameter type?

Both approaches are similar in expressive power.

Multiparameter type classes are no longer fashionable

- ▶ They can make type class resolution unpredictable.

Associated types have gained traction in other languages

- ▶ Like Rust or Apple's Swift.

Type families

Associated types always require a class definition – even if we're only interested in the types.

Type families build upon the technology that associated types provide, enabling you to write:

```
type family Sum n m
type instance Sum Zero     n = n
type instance Sum (Succ n) m = Succ (Sum n m)
```

This looks much more like regular programming…

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Closed type families

If we piggyback on the associated type machinery, however, all our type families are *open* – we can add bogus definitions:

```
type instance Sum n Zero = Zero
```

Closed type families

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```
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```

The more modern *closed type families* allow you to define a function between types using pattern matching:

```
type family Sum n m where
  Sum Zero     n = n
  Sum (Succ n) m = Succ (Sum n m)
```

GHC will try to match a given type against the patterns one by one, taking the first branch that matches successfully.

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Apartness for closed type families

GHC only moves to the next branch if it know that the previous one may never match.

- ► The types are *apart* from the pattern.

```
> :kind! Sum n (Succ m)
-- does not reduce further!
```

Furthermore, all our 'type level' code is essentially untyped.

```
type instance Sum Bool Int = Char
```

We want "type-level types", as we have in the term-level.

Kinds

So far we have seen that two different forms of kinds:

- ▶ all types have kind `*`
- ▶ given two kinds `k1` and `k2`, we can form the kind `k1 -> k2` – corresponding to the type constructor taking something of kind `k1` to produce a type of kind `k2`.

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Kinds

So far we have seen that two different forms of kinds:

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As soon as we do richer programming with types, however, we would like stronger guarantees about the safety of the *type level* computations that we write.

```
data Zero
data Succ n

data Nat = Zero | Succ Nat
```

How can we ensure all numbers in our types to be built from Zero and Succ?

Promotion

Using the `DataKinds` language extension we can introduce new kinds and automatically *promote* data constructors into their type-level variants:

```
{-# LANGUAGE DataKinds #-}

data Nat = Zero | Succ Nat
```

This declaration introduces:

- ▶ a new kind `Nat`
- ▶ a type `'Zero :: Nat`
- ▶ a type `'Succ :: Nat -> Nat`

If you want to promote a GADT, you need `TypeInType`.

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Example: booleans

```haskell
-- the usual definition of booleans
data Bool = True | False

-- Not function on values
not :: Bool -> Bool
not True = False
not False = True

-- Not function on types
type family Not (a :: Bool) :: Bool where
  Not True = False
  Not False = True
```

GHC takes the idea of programming with types quite far.

It has added support and syntax for:

- ▶ type-level strings;
- ▶ type-level lists;
- ▶ type-level integers;

List membership

Member x xs should be true if x is a member of the type-level list xs.

First attempt: type class

```
class    Member x xs
instance Member x (x ': xs)
instance Member x xs => Member x (y ': ys)
```

Why is this code rejected?

List membership

Member x xs should be true if x is a member of the
type-level list xs.

First attempt: type class

```
class    Member x xs
instance Member x (x ': xs)
instance Member x xs => Member x (y ': ys)
```

Why is this code rejected?

```
instance {-# OVERLAPS #-}
         Member x (x ': xs)
instance {-# OVERLAPPABLE #-}
         Member x xs => Member x (y ': ys)
```

The case against overlapping instances

1. Overlapping instances make type resolution brittle.
   - Which instance is selected depends on how much we know about a type.

2. Overlapping instances are not modular.
   - Adding a new instance that overlaps may render previous resolution wrong.

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List membership with a closed type family

```
type family Member' x xs where
  Member' x '[]        = 'False
  Member' x (x ': xs) = 'True
  Member' x (y ': xs) = Member' x xs

type Member x xs = Member' x xs ~ 'True
```

- ► The second branch is *non-linear*, x is repeated.
  - ► Not allowed in term-level pattern matching.
  - ► Fine with closed type families.
- ► `Member` defines a synonym for a constraint.
  - ► In GHC, constraints are just types of the special `Constraint` kind.

Polymorphic kinds

Which is the kind of `Member'`?

```
Member' :: *   -> [*]   -> Bool
Member' :: Nat -> [Nat] -> Bool
```

Polymorphic kinds

Which is the kind of `Member'`?

```
Member' :: *   -> [*]   -> Bool
Member' :: Nat -> [Nat] -> Bool
```

Kinds may be *polymorphic*, as types are!

```
Member' :: k -> [k] -> Bool
```

Type-level application

```
data Apply f a = MkApply (f a)
```

Question

What is the kind of `Apply`?

```
> :set -XPolyKinds
> :info Apply
type role Apply representational nominal
data Apply (f :: k -> *) (a :: k) = MkApply (f a)
```

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Proxy data types

Consider the `read` function:

```haskell
read :: Read a => String -> a
```

How do we fix the type we want to get?

Proxy data types

Consider the `read` function:

```haskell
read :: Read a => String -> a
```

How do we fix the type we want to get?

First solution, annotate it

```haskell
read "123" :: Int
```

In many cases, the type can be inferred instead.

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Proxy data types

Second solution, fix it via other parameter

```haskell
-- Wrap it with an additional argument
read' :: Read a => a -> String -> a
read' _ = read
-- And use it like this
read' (undefined :: Int) "123"
```

- ▶ The first argument is never touched.
- ▶ Gives valuable information to the compiler.

These are called *proxy* arguments.

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Proxy data types

```haskell
-- Polymorphic kind!
data Proxy (a :: k) = Proxy
-- The proxy is also ignored
read' :: Read a => Proxy a -> String -> a
read' _ = read
-- To use it you create a Proxy value
read (Proxy :: Proxy Int) "123"
```

Question

What is the benefit of using `Proxy`?

(No) proxy data types

Third solution, explicit type application

```
read @Int "123"
```

Only from GHC 8 with `TypeApplications` enabled.

Outlook: writing webserver with Servant

Servant is a library for describing web APIs. From such a description, it will generate documentation, a simple webserver, a JavaScript client, etc.

► Instead of describing the APIs using Haskell values – you describe the API as a (complex) Haskell type.
► And then *generate* any desired functionality from it.

```
type HackageAPI =
      "users" :> Get '[JSON] [UserSummary]
 :<|> "user" :> Capture "username" Username
           :> Get '[JSON] UserDetailed
 :<|> "packages" :> Get '[JSON] [Package]
```

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Recap: GADTs

GADTs give you more power to define interesting types.

- ▶ We can decorate our types with *more specific information*.

But we still cannot do any interesting *computation*.

- ▶ We need to use type classes or type families.
- ▶ We end up mirroring expression-level concepts on the type level (e.g. natural numbers).

The 'value language' and 'type language' live in very different worlds…

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