OAT homing with an accelerometer.

Consider an MPU6050 accelerometer/gyro unit mounted horizontally on the side of the camera bar such that the X axis is into the bar, Y axis is pointing along the bar in the camera viewing direction and the Z axis is pointing down. With the bar aligned along the polar axis, rotation about Y is roll (rotation of the RA wheel), that of the DEC wheel is pitch. Angular offsets of the accelerometer axes with respect to the bar are assumed negligible.

Pitch (DEC), angle θ, is a rotation about the X axis and roll (RA), angle ϕ, is a rotation about the Y axis. The polar axis is taken as the reference axis for angular measurements, pointing up normal to the plane of the RA wheel. When the bar is level transverse plane (RA), and aligned along the polar axis, the gravitational acceleration components along the MPU axes are

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g\cos\delta \end{bmatrix}$$

where δ is the build angle of the mount (30° here) and g is the acceleration due to gravity.

Now consider the bar in an arbitrary orientation with (offset corrected) acceleration components

$x = g\cos\alpha$, $y = g\cos\beta$, $z = g\cos\gamma$, where the roll angle $\phi = atn(x/z)$.

Applying a rotation about Y of ϕ leads to the new components of:

$$\begin{aligned}\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} &= g\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}\begin{bmatrix} cos\alpha \\ \cos\beta \\ \cos\gamma \end{bmatrix} \\ &= \begin{bmatrix} \cos\phi cos\alpha + \sin\phi\cos\gamma \\ \cos\beta \\ -\sin\phi cos\alpha + \cos\phi\cos\gamma \end{bmatrix}\end{aligned}.$$

Hence $x' = 0$ if $\tan\phi = -\dfrac{\cos\alpha}{\cos\gamma} = -\dfrac{x}{z}$, i.e. rotate by the negative of the roll angle.

The pitch of the bar will be returned to the build angle (bar level) by rotating about Y by an angle

$$\theta = -atn\left(\frac{y}{-x\sin\phi + z\cos\phi}\right).$$

Calibration

Accurate homing requires that the offsets of each of the accelerometer channels must be calibrated by setting the bar level and reading the X,Y and Z values. Bubble levels clamped into the side-slot of the 2020 bar were used for determining the X and Y channel offsets. The MPU6050 was moved to the top slot to calibrate the Z channel offset, with a bubble mounted across the bar. Pitch and roll angles are given by $\operatorname{atan}(\{x, y\}/z)$. The sensitivity coefficient for tangent *vs* angle is $\frac{d}{dx}\tan x = \frac{1}{1+x^2} \approx 1 \text{ for } x \approx 0$. At the level position, z ~1 g and the uncertainty in angle is given directly in radians by the uncertainty of the offset. The short term stability of the accelerometer is quite good – averaging 1000 readings spaced by 4 mS gave standard deviations typically of 0.02 mg. These are much smaller than the quoted shift of ± 35mg over the temperature range of 0°C to 70°C, corresponding to an uncertainty in angle of ± 2.1°.

The temperature drift can be handled by calibrating the offsets over a range of temperatures (the device includes a temperature channel) and applying a correction. Alternatively, once the bar is leveled to an approximate level position, searching for a minimum in the square of the accelerometer signal in the Z and Y channels (this is quadratic in angle about the offset value if the temperature is stable).

Offsets were measured with the bar level (both pitch and roll). The Z axis offset was measured by mounting the accelerometer on top of the bar (X down). The standard deviation of 1000 readings 4 mS apart was 3.10-4 g (equivalent to 0.002°). Repeat sets over a showed a larger variation (reported temperatures were in the range 25°C - 28°C) as shown in the table, along with the offsets for each axis:

	mg	SD	Degrees	SD
X both:	0.0495	0.0010	3.0	0.06
Y both:	0.0003	0.0006	0.0	0.03
Z:	-0.0521	0.0003	-3.1	0.02

Provided the bar can be driven to an approximate level position, 3 equally spaced readings Δ apart can be used to find the minimum of a quadratic dependence:

$$
\begin{aligned}
y_1 &= a + b + c^2 \\
y_0 &= a \\
y_{-1} &= a - b + c^2
\end{aligned}
$$

From which

$$
\begin{aligned}
b &= (y_1 - y_{-1})/2 \\
c &= (y_1 + y_{-1} - y_0)/2
\end{aligned}
$$

The minimum of this quadratic occurs at a setting of $(-b\Delta/2c)^{\circ}$. This can be iterated by reducing the step size and repeating measurements if necessary (applying backlash compensation if applicable).

Setting to latitude

If the bar pitch is set to zero (DEC leveled) with the 2020 base level, the DEC wheel can be moved by the difference between the latitude and build angle, then the rear base screw adjusted until the bar is again level. Useful in the Southern Hemisphere for initial alignment.

If a 4-wire stepper with a TMC2209 driver is used for the DEC wheel, stops fitted to limit the travel of the wheels can be used to home both the DEC and RA wheels using the StallGuard option. By offsetting the DEC wheel as in the previous paragraph, setting the bar level as indicated by a bar-mounted bubble level, provides a reasonably accurate initial declination alignment.

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