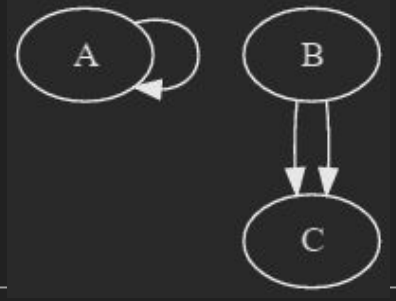
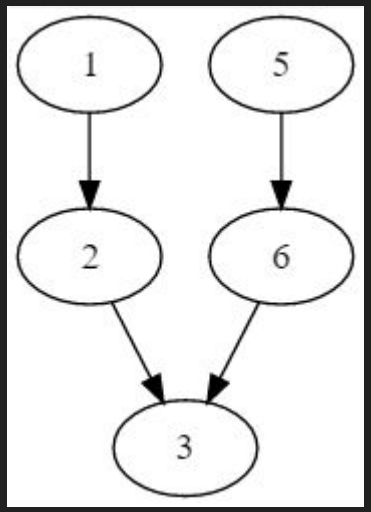
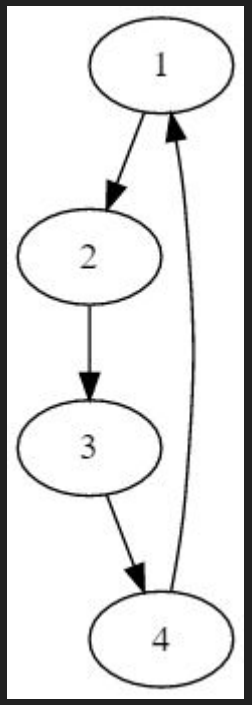
Competitive Programming Training

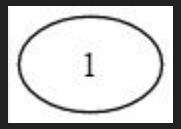
Classifying Graphs

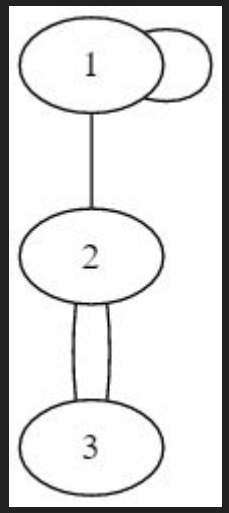
Property	**Description**
Directed or Undirected *(**bi**directional or **uni**directional)*	If the definition of edges involve direction (`a -> b` or `a -- b`)
Weighted or Unweighted	If there is a weight attached to each edge
Cyclic or Acyclic	If a cycle can be found in the graph
Simple / Multi Graph	

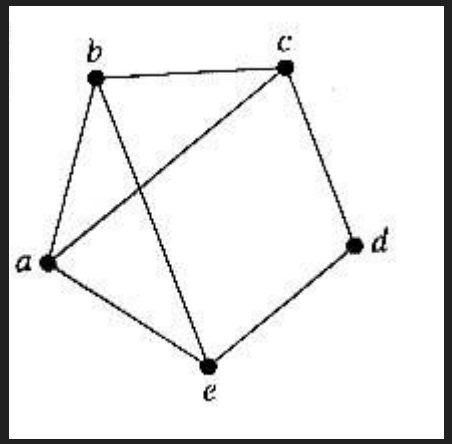
Classify the following graphs





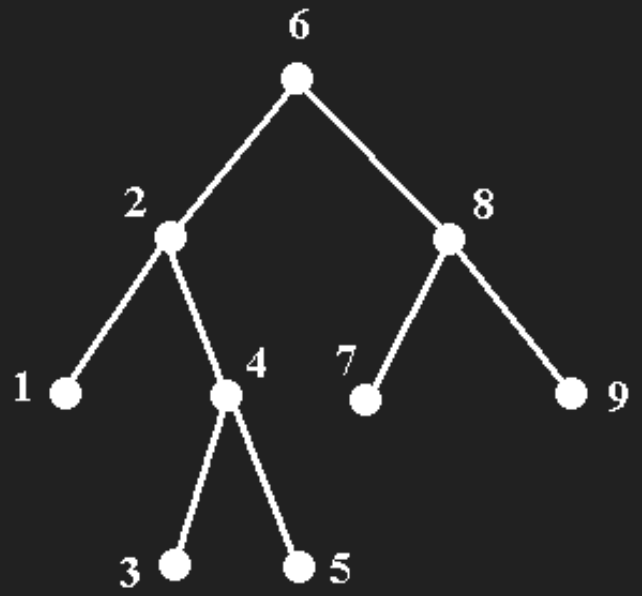






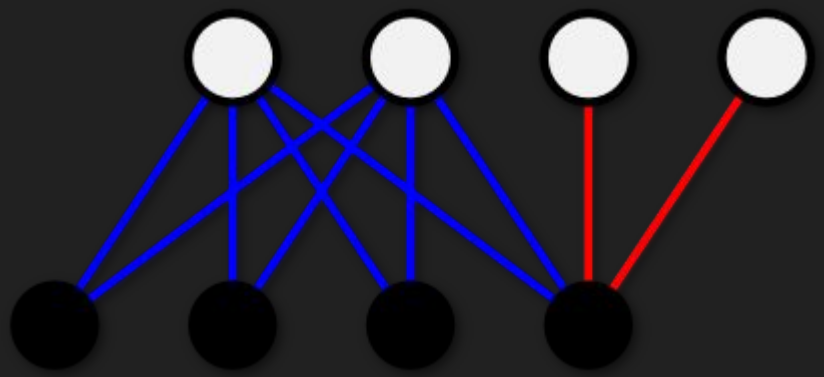
Trees

- ➔ M = N - 1 (exactly one less edges than nodes)
- ➔ 1 connected component
- ➔ acyclic (exactly 1 path between any pair of nodes)
- ➔ with exception of an empty graph



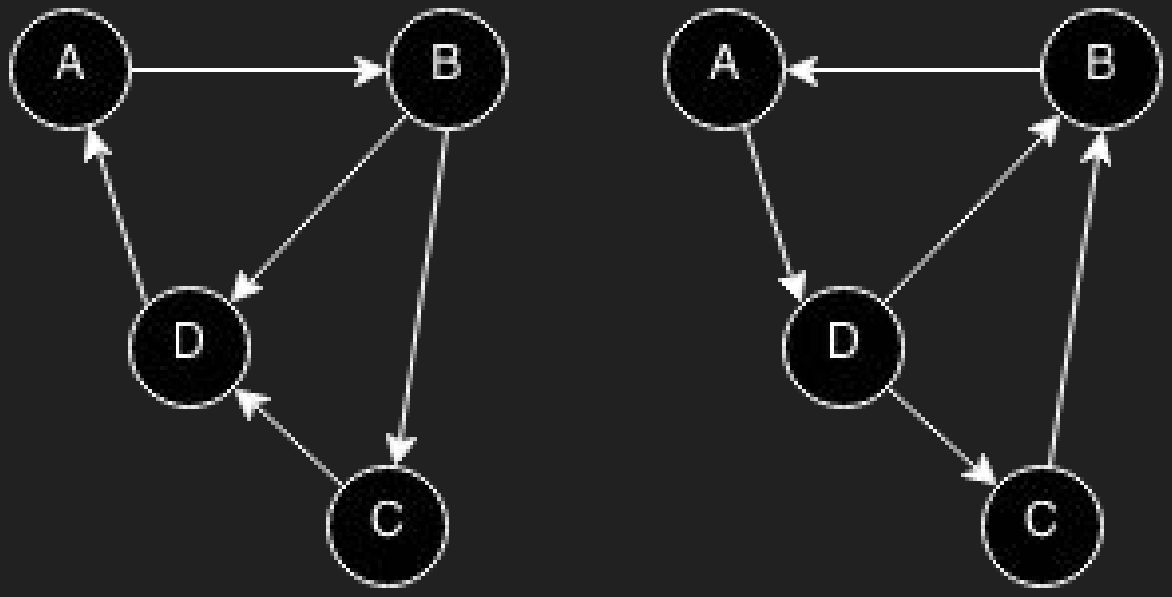
Bipartite Graphs

- A connected graph that can be colored with only 2 colors (typically red-black) such that:
  - Every node is colored with either color
  - No adjacent nodes (connected with an edge) have the same colors



Strongly-Connected Graph

- ➔ Typically defined for **directed graphs**
- ➔ There is at least one path between any pair of nodes



Questions

- ➔ Are all trees graphs?
- ➔ Are all trees simple graphs?
- ➔ Are all graphs trees?
- ➔ Are all undirected graphs strongly connected?
- ➔ Are all trees strongly connected?
- ➔ Is there such bipartite graph that is strongly connected?
- ➔ Is there such trees that is bipartite?

Are all trees graphs?

Yes.

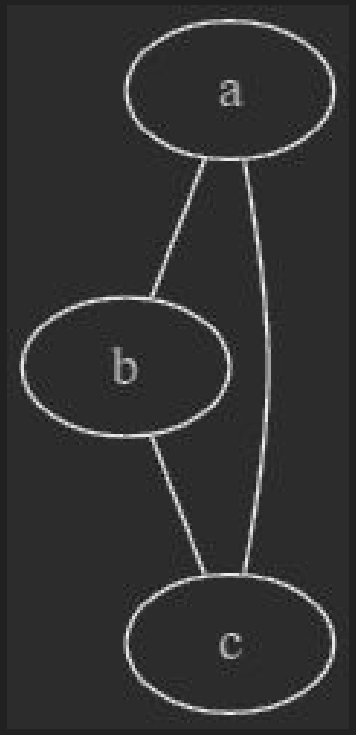
Are all trees simple graphs?

Yes.

Are all graphs trees?

No.

This is a graph, but not a tree:



Proof by counterexample.

Are all undirected graphs strongly connected?

No.

Undirected graphs that are disconnected is not strongly connected.

Proof by counterexample.

Are all trees strongly connected?

Yes.

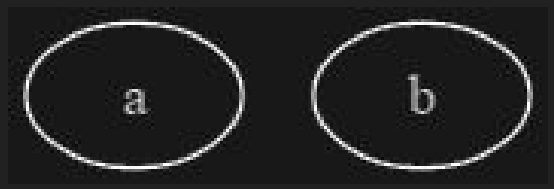
Trees are **connected**, **undirected**, acyclic graphs.

Hence, they are strongly connected.

Is there such bipartite graph that is strongly connected?

No.

This is a bipartite graph, but not strongly connected:



Proof by counterexample.

(bipartite graphs don’t need to be connected)

Is there such trees that is bipartite?

Yes.

The following tree is bipartite: