

QFT 1

Part 3: CM Field Theory



COMMENTS

i) WHAT MAKES QFT QFT?

not relativity.

instead Creation & annih. ops, renormalization,
phase transitions

- ii) MANY BODY PHYSICS $\xrightarrow{\text{EMERGENCE}}$ SIMPLE LOW E PHENOMENA
- iii) SYMMETRY BREAKING

Critical Exponents (read for Matt on Wilson-Fisher)

$$t = \frac{T - T_c}{T_c}$$

$$G(x) = \langle s(x)s(0) \rangle \sim \exp[-tx]^{\zeta}$$

away from crit

@ CRIT: correlation diverges

$$\xi \sim |t|^{-\nu} \quad G(x) \sim \frac{1}{|x|^{d-2+\eta}} \quad C_H \sim |t|^{-\alpha}$$

$$M \sim |t|^{\beta} \quad X \sim |t|^{-\gamma}$$

2-loop O(N) Wilson-Fisher: Pearson table 13.1

N=1 liquid-gas Xe, CO_2 OR Cu-Zn alloy " β -brass"

QFT: $\nu = 0.63$

EXPT $\nu = .625$ (binary liquid)
.65 (β -brass)

N=2 SUPERFLUID 4He

QFT: $\nu = .670$

EXPT: = .674

N=3 FERRO & ANTI-FERRO

QFT $\nu = .705$

EXPT $\nu = .70$
.704

EuO, EuS ^{ferro}
 $RbMnF_3$ \leftarrow ^{anti} _{zero}

WILSONIAN RG & NON-REL FT.

SUPERFLUIDS, SUPERCONDUCTORS, & SYMMETRY BREAKING

2 LECTURES

TOPOLOGICAL DEFECTS

CHERN-SIMONS, FQHE, & PARTICLE VORTEX DUALITY

FERROMAGNETS & O(3) NLSM

CFT

LIE THEORY

NON-REL F.T.

$$\mathcal{L} = \partial \bar{\phi}^+ \partial \bar{\phi} - m^2 \bar{\phi}^+ \bar{\phi} - \lambda (\bar{\phi}^+ \bar{\phi})^2$$

contains NR physics

CLASSICAL SOLUTIONS

EOM: $(\partial^2 + m^2) \bar{\phi} = 0$

PLANE WAVE SOL'N $\bar{\phi} \sim e^{i \vec{p} \cdot \vec{x}} \sim e^{-i E t}$

Q: what properties does E have in NR limit?

$$\vec{P}^M = (E, 0, 0, |\vec{p}|) \quad (\text{in some frame})$$

$$\vec{P} \cdot \vec{P} = E^2 - (\vec{p})^2 = m^2 \quad \text{if desired}$$

$$\Rightarrow E = \sqrt{m^2 + |\vec{p}|^2} = m \sqrt{1 + \frac{|\vec{p}|^2}{m^2}} \stackrel{\text{NON-REL}}{\approx} m \left(1 + \frac{|\vec{p}|^2}{2m^2} \right) = m + \frac{|\vec{p}|^2}{2m}$$

$$= : m + \varepsilon$$

\downarrow REST MASS
 \uparrow KINETIC ENERGY

KEY: REQ'D $\varepsilon \ll m$

$$\bar{\phi} \sim e^{-i E t} = e^{-i m t} e^{-i \varepsilon t}$$

\uparrow FAST OSC. \uparrow SLOW OSC.

$$\Rightarrow \text{WRITE } \bar{\phi}(x^\mu) = e^{-i m t} \psi(x^\mu)$$

\uparrow SLOW OSC $\sim \varepsilon$,

$$\psi = A(\vec{x}) e^{-i \varepsilon t}$$

$$\Rightarrow 0 = (\partial_t^2 + m^2) \bar{\Phi} = (\partial_t^2 - \nabla^2) e^{-imt} \varphi + m^2 \bar{\Phi}$$

$$= \partial_t [e^{-imt} (-im\varphi + \partial_t \varphi)] - \nabla^2 \varphi e^{-imt} + m^2 \bar{\Phi}$$

$$= [-im e^{-imt} (-im\varphi + \partial_t \varphi) + e^{-imt} (-im \partial_t \varphi + \partial_t^2 \varphi)] - \nabla^2 \varphi e^{-imt} + m^2 \bar{\Phi}$$

$$= -m^2 \bar{\Phi} - 2im e^{-imt} \partial_t \varphi + \partial_t^2 \varphi - \nabla^2 \varphi e^{-imt} + m^2 \bar{\Phi}$$

$$= -ie^{-imt} \partial_t \varphi + \frac{\partial_t^2 \varphi}{m} e^{-imt} - \frac{\nabla^2 \varphi}{2m} e^{-imt}$$

$\frac{1}{2m} \cdot \cancel{\partial_t \varphi} \sim \frac{e}{m} \varphi \quad \text{REL. } \frac{e}{m} \Rightarrow \text{drops } \partial_t^2 \text{ TERM!}$

$$\cancel{\frac{\partial_t^2 \varphi}{m}} \sim \frac{e^2}{m} \varphi$$

$$\Rightarrow 0 \approx [-i \partial_t \varphi - \frac{\nabla^2 \varphi}{2m}] e^{-imt}$$

$$\Rightarrow \boxed{i \partial_t \varphi = -\frac{\nabla^2 \varphi}{2m}}$$

E: KGE \Rightarrow SCHROD. IN NON-REL LIMIT
✓ convenience

NON-REL L

$$\bar{\Phi} = \frac{i}{\sqrt{2m}} e^{-imt} \varphi$$

✓ take down

$$\underbrace{\frac{\partial \bar{\Phi}^+}{\partial t} \frac{\partial \bar{\Phi}}{\partial t} - m^2 \bar{\Phi}^+ \bar{\Phi}}_{\text{APPEARS IN } \mathcal{L}_{\Phi^4}} \Rightarrow \frac{i}{2m} \left\{ \left[\left(im + \frac{\partial}{\partial t} \right) \varphi^+ \right] \left[\left(-im + \frac{\partial}{\partial t} \right) \varphi \right] - m^2 \varphi^+ \varphi \right\}$$

$$\text{APPEARS IN } \mathcal{L}_{\Phi^4} \underset{\cdot}{=} \frac{1}{2} i \left(\varphi^+ \frac{\partial \varphi}{\partial t} - \frac{\partial \varphi^+}{\partial t} \varphi \right)$$

$$\text{IBP} \rightarrow \mathcal{L}_{\Phi^4} = i \varphi^+ \partial_0 \varphi - \frac{i}{2m} \underbrace{\partial_i \varphi^+ \partial_i \varphi}_{\text{APPEARS IN } \mathcal{L}_{\Phi^4}} - g^2 (\varphi^+ \varphi)^2$$

$$g^2 = \frac{\lambda}{4m^2}$$

NB:

①

SPACE-TIME SPLIT

②

ROT. INV., NOT LOR.

③

LINEAR IN ∂_t !

NOETHER'S CURRENT

$$J_\mu = i(\bar{\Phi}^* \partial_\mu \bar{\Phi} - \partial_\mu \Phi^* \bar{\Phi})$$

$$J_0 = i(e^{imt} \varphi^* [-imt \varphi + \partial_0 \varphi] e^{-imt} - [imt \varphi^* + \partial_0 \varphi^*] e^{imt} e^{-imt} \varphi)$$

$$\stackrel{IBP}{=} \varphi^* \varphi$$

PROBABILITY

DENSITY

ρ

$$\therefore J_i = \frac{i}{2m} (\varphi^* \partial_i \varphi - \partial_i \varphi^* \varphi)$$

PROBABILITY
CURRENT

\vec{J}

Note: ρ and \vec{J} look very different!

$$\therefore \text{CONJ. MOM. } \Pi = \frac{\delta \mathcal{L}}{\delta \dot{\varphi}} = i\psi^+$$

\Rightarrow EQ. TIME COMM.

$$[\psi^+(\vec{x}, t), \Pi(\vec{x}', t)] = -i\delta^{(0)}(x - x')$$

$$\Rightarrow [\psi^+(\vec{x}, t), \psi(\vec{x}', t)] = -\delta^{(0)}(x - x')$$

REWRITE $\psi = \sqrt{\rho} e^{i\theta}$

$$\mathcal{L} = \frac{i}{2} \partial_0 \rho - \rho \partial_0 \theta - \frac{1}{2m} \left[\rho (\partial_i \theta)^2 + \frac{1}{4\rho} (\partial_i \rho)^2 \right] - g^2 \rho^2$$

$$\Rightarrow \Pi_\theta = \frac{\delta \mathcal{L}}{\delta \partial_0 \theta} = -\rho \quad \Rightarrow \quad [\rho(\vec{x}, t), \theta(\vec{x}', t)] = i\delta^{(0)}(\vec{x} - \vec{x}')$$

$$N := \int d^D x \rho(\vec{x}, t) = \# \text{ BOSONS}$$

$$\text{ABOVE} \Rightarrow [N, \theta] = i$$

NB: NUMBER CONJ. TO PHASE $\not\propto$!

(in non-relativistic theory)

FINITE DENSITY:

CONTINUUM LIMITS (already saw, but again)

1D CRYSTAL



ION POSITION R_I

$$\text{PHONON HAMILTONIAN} \quad \text{1D} \quad H = \sum_{I=1}^N \frac{P_I^2}{2M} + \frac{k_s}{2} (R_{I+1} - R_I - a)^2$$

(Hamiltonian of N pt-like particles connected by springs w/ spring const k_s)

$$\text{LAGRANGIAN: } L = T - U = \sum_{I=1}^N \left[\frac{M \dot{R}_I^2}{2} - \frac{k_s}{2} (R_{I+1} - R_I - a)^2 \right]$$

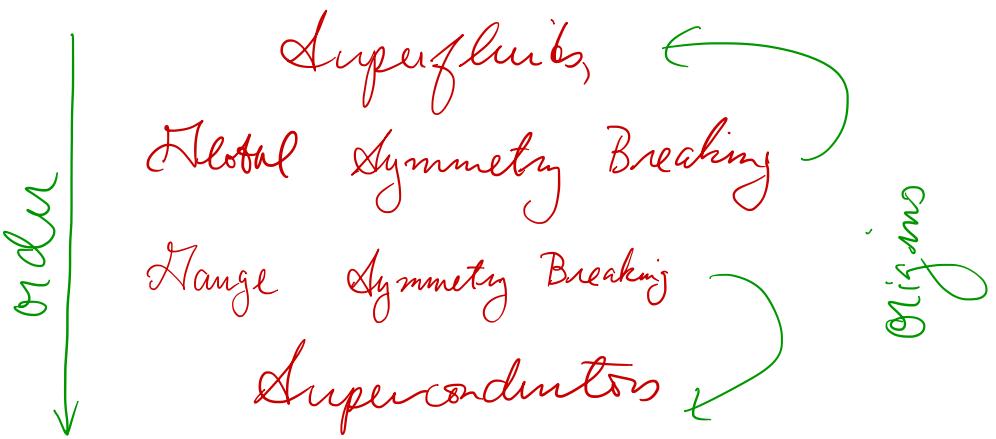
$\begin{matrix} \nearrow \text{kinetic} \\ \in \end{matrix}$ $\begin{matrix} \nearrow \text{potential} \\ \in \end{matrix}$

PERIODIC BOUNDARY $R_{N+1} = R_1$ (B.C. shouldn't matter @ large N)

LOW ENERGIES: $V \sim \begin{cases} 0 & R_1 \leq R \leq R_2 \\ \infty & \text{otherwise} \end{cases}$

$$R_I = \bar{R}_I + \phi_I \quad \Rightarrow \quad L = \sum_{I=1}^N \left[\frac{M \dot{\phi}_I^2}{2} - \frac{k_s}{2} (\phi_{I+1} - \phi_I)^2 \right]$$

$\begin{matrix} \uparrow \text{FLXLT. ABOUT MIN.} \\ \in \end{matrix}$



SUPERFLUIDS \Rightarrow quantum fluids w/ no viscosity,
just keep moving!

LANDAU'S ARGUMENT



$T=0$ QUANTUM FLUID, MASS M
e.g. LIQ He⁴

$$Mv = Mv' + \hbar k \quad (\text{momentum conservation})$$

$\frac{1}{2} Mv^2 > \frac{1}{2} Mv'^2 + \hbar \omega \quad (\text{Could lose energy e.g. to heat})$

$$\frac{(Mv' + \hbar k)^2}{2M} \approx \frac{(Mv')^2 (1 + 2 \frac{\hbar k}{Mv'})}{2M} = \frac{Mv'^2}{2} + \hbar k v'$$

$$\Rightarrow \hbar k v' > \hbar \omega \quad v' > \frac{\omega}{k} =: v_c \quad \begin{matrix} \text{critical} \\ \text{velocity} \\ \text{of fluid} \end{matrix}$$

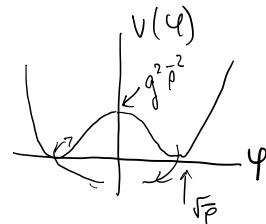
CONCLUSION: FLUID LOSES MOMENTUM UNTIL
 $v = v_c$, THEN SUPERFLUID

WHEN SUPER FLUID \exists QUANTA S.T. $\omega = v_c k$
linear dispersion relation $\xrightarrow{\text{smoking gun!}}$

F.T. OF SUPER FLUIDS

$$\mathcal{L} = i \psi^* \partial_0 \psi - \frac{1}{2m} \partial_i \psi^* \partial_i \psi - g^2 (\psi^* \psi - \bar{\rho})^2$$

$$V(\psi) = g^2 (\psi^* \psi - \bar{\rho})^2 \Rightarrow \psi = \sqrt{\bar{\rho}} e^{i\theta}$$



THEN $\mathcal{L} = -\rho \partial_0 \theta - \frac{1}{2m} \left[\frac{1}{4\rho} (\partial_i \rho)^2 + \rho (\partial_i \theta)^2 \right] - g^2 (\rho - \bar{\rho})^2$

EXPAND AROUND $\bar{\rho}$ $\sqrt{\rho} = \sqrt{\bar{\rho}} + h$

\uparrow field $\sqrt{\rho}(x)$ wants to be at value $\sqrt{\bar{\rho}}$

\uparrow small fluctuation, ie $\frac{h}{\sqrt{\bar{\rho}}} \ll 1$

NOW $\mathcal{L} = -2\sqrt{\bar{\rho}} \partial_0 \theta - \frac{\bar{\rho}}{2m} (\partial_i \theta)^2 - \frac{1}{2m} (\partial_i h)^2 - 4g^2 \bar{\rho} h^2 + \dots$

HMWK $= \bar{\rho} \partial_0 \theta \frac{1}{4g^2 \bar{\rho} - \frac{1}{2m} \partial_i^2} \partial_0 \theta - \frac{\bar{\rho}}{2m} (\partial_i \theta)^2 + \dots$

HMWK $= \frac{1}{4g^2} (\partial_0 \theta)^2 - \frac{\bar{\rho}}{2m} (\partial_i \theta)^2 + \dots$

$\hookrightarrow 4g^2 \bar{\rho} > \frac{\partial_i^2}{2m} \stackrel{\text{P-SP.}}{\downarrow} \frac{k^2}{2m} \Rightarrow k \ll \sqrt{8g^2 \bar{\rho} m}$

EOM FOR θ : $\left[\frac{\partial_0^2}{4g^2} - \frac{\bar{\rho}}{2m} \partial_i^2 \right] \theta = 0$

SOLN: $\theta = e^{-i\omega t} e^{i\vec{k} \cdot \vec{x}} \Rightarrow i^2 \left[\frac{\omega^2}{4g^2} - \frac{\bar{\rho} k^2}{2m} \right] = 0$

$\Rightarrow \omega = \sqrt{\frac{2\bar{\rho} g^2}{m} |\vec{k}|} \Rightarrow V_C = \sqrt{\frac{2\bar{\rho} g^2}{m}}$

CONCLUSION: @ CRITICALITY SUPERFLUIDS
 DESCRIBED BY $\cancel{\propto}$ DOF
 OF COMPLEX SCALAR
 \hookrightarrow systematic theory?

SPONTANEOUS SYMMETRY BREAKING

"Let the system break the symmetry itself"

$$\mathcal{L} = \frac{1}{2} [(\partial\vec{\varphi})^2 + \mu^2 \vec{\varphi}^2] - \frac{\lambda}{4!} (\vec{\varphi}^2)^2 \quad \vec{\varphi} = (\varphi_1, \dots, \varphi_N)$$

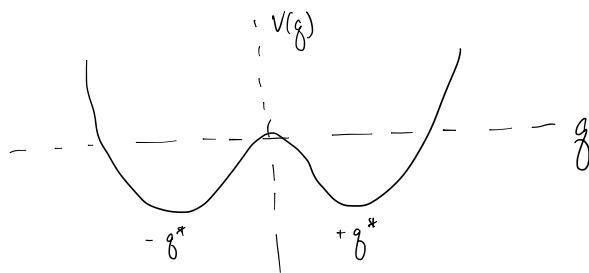
\uparrow NOTE SIGN DIFFERENCE!

$m^2 = -\mu^2 \Rightarrow$ "Tachyon" (not a problem if you know where it leads)

ANALOGY

ANHARMONIC OSC.

$$L = \frac{1}{2} (\dot{g}^2 + k g^2) - \frac{\lambda}{4} g^4 \quad V(g) = -\frac{1}{2} g^2 + \frac{\lambda}{4} g^4$$



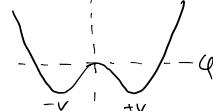
@ LOW ENERGIES, PICK MINIMUM $\pm g^*$, STUDY OSC. ABOUT $\pm g^*$

"PICK MINIMUM" \Rightarrow OSC. ABOUT STATE THAT BREAKS $g \rightarrow -g$ SYM

SPONTANEOUS SYM BREAKING: L HAS Sym, GROUND STATE BREAKS

$$N=1 \text{ CASE} \quad \mathcal{L} = \frac{1}{2} (\partial\varphi)^2 + \frac{\mu^2}{2} \varphi^2 - \frac{\lambda}{4} \varphi^4 \Rightarrow V(\varphi) = -\frac{\mu^2}{2} \varphi^2 + \frac{\lambda}{4} \varphi^4$$

$$\frac{\partial V}{\partial \varphi} = \varphi (-\mu^2 + \lambda \varphi^2) \Rightarrow \min_{V(\varphi)} \textcircled{a} \quad \varphi = 0 \quad \varphi = \pm \sqrt{\frac{\mu}{\lambda}} =: \pm v$$



LOCAL MAX OR MIN?

$$\frac{\partial^2 V}{\partial \varphi^2} = \frac{\partial}{\partial \varphi} (-\mu^2 \varphi + \lambda \varphi^3) = -\mu^2 + 3\lambda \varphi^2$$

$$\left. \frac{\partial^2 V}{\partial \varphi^2} \right|_{\varphi=0} = -\mu^2 \Rightarrow \text{LOCAL MAX}$$

$$\left. \frac{\partial^2 V}{\partial \varphi^2} \right|_{\varphi=\pm v} = -\mu^2 + 3v^2 = -\mu^2 + \frac{\mu^2}{\lambda} = \mu^2 \left(\frac{1}{\lambda} - 1 \right) \Rightarrow \begin{array}{l} \text{LOCAL} \\ \text{MIN FOR} \\ \lambda \text{ SMALL} \end{array}$$

EXPAND ABOUT $\varphi = +v$ GROUND STATE

$$\varphi = v + \delta \varphi =: v + \varphi'$$

$$\Rightarrow \mathcal{L} = \frac{\mu^4}{2\lambda} + \frac{1}{2} (\partial \varphi')^2 - \mu^2 \varphi'^2 - \mathcal{O}(\varphi')^3$$

$$\Rightarrow \varphi' \text{ HAS } m = \sqrt{2} \mu$$

QFT: G.S = VACUUM

HERE: 2 PHYSICALLY EQUIV. VACUA $|^{+v}\rangle$.

Lang: "vacuum expectation value"

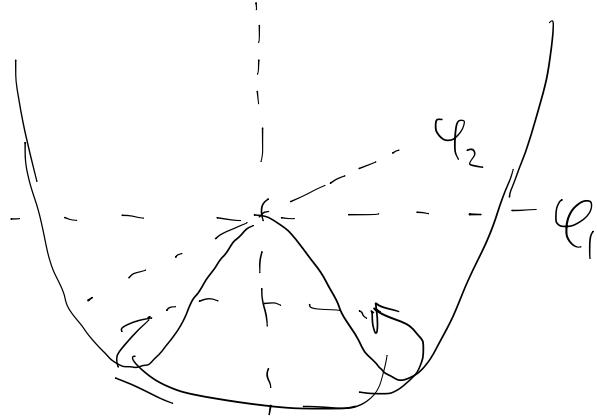
N=2 CASE

$$\mathcal{L} = \frac{1}{2} [(\partial_i \vec{\varphi})^2 + \mu^2 \vec{\varphi}^2] - \frac{\lambda}{4!} (\vec{\varphi}^2)^2$$

$$\vec{\varphi} = (\varphi_1, \varphi_2)$$

$$\begin{aligned} \frac{\partial V}{\partial \varphi_i} &= \frac{\partial}{\partial \varphi_i} \left(-\mu^2 \varphi_j \varphi_j + \frac{\lambda}{4!} \varphi_i \varphi_j \varphi_k \varphi_l \right) = -\mu^2 \varphi_i + \frac{\lambda}{2} \varphi_i \varphi_k \varphi_k + \frac{\lambda}{2} \varphi_j \varphi_j \varphi_i \\ &= \varphi_i (-\mu^2 + \lambda \vec{\varphi}^2) \Rightarrow \frac{\partial V}{\partial \varphi_i} = 0 @ \varphi_i = 0 \quad \vec{\varphi}^2 = \pm \sqrt{\frac{\mu}{\lambda}} \end{aligned}$$

$\vec{\varphi}^2$: ONLY FIXES MAG OF $\vec{\varphi}$! NOT $\vec{\varphi}$



NOTE: \exists "FLAT DIRECTION" @ MIN $V(\varphi) \Rightarrow$ CONTINUUM OF VACUA

Language: "Classical moduli space of vacua"

PICK A VACUUM, COMPUTE \mathcal{L}_{eff}

$$\varphi_1 = v + \varphi_1' \quad \varphi_2 = \varphi_2' \quad (\text{picked it in } \varphi_1 \text{ direction})$$

$$\mathcal{L}_{eff} = \frac{\mu^4}{4\lambda} + \frac{1}{2} [(\partial_1 \varphi_1')^2 + (\partial_2 \varphi_2')^2] - \mu^2 \varphi_1'^2 + \mathcal{O}(\varphi^3)$$

$$m_{\varphi_1'} = \sqrt{2} \mu \quad m_{\varphi_2'} = 0$$



φ_1 "climbing the wall"

φ_2 "rolling along the gutter"

Language: "Goldstone Boson"

Critical Difference

$N=1$ BROKE DISCRETE SYMMETRY

$N=2$ " CONTIN. " \Rightarrow MASSLESS BOSON

ANOTHER $N=2$ VIEW $\Psi = \frac{1}{\sqrt{2}}(\Psi_1 + i\Psi_2)$

$$\Rightarrow \mathcal{L} = \partial\Psi^+ \partial\Psi + \mu^2 \Psi^+ \Psi - \lambda (\Psi^+ \Psi)^2 \quad (\text{CMPLX SCALAR!})$$

$\Psi \rightarrow e^{i\alpha} \Psi$ INVARIANT, GLOBAL U(1) SYMMETRY

$$\text{REWRITE} \quad \Psi = \rho e^{i\Theta} \quad \Rightarrow \quad \partial_\mu \Psi = (\partial_\mu \rho + i\rho \partial_\mu \Theta) e^{i\Theta}$$

$$\Rightarrow \mathcal{L} = \rho^2 (\partial\Theta)^2 + (\partial\rho)^2 + \mu^2 \rho^2 - \lambda \rho^4$$

$$V(\rho) \text{ MIN } @ \quad v = \sqrt{\frac{\mu^2}{2\lambda}} \quad \Rightarrow \quad \rho = v + \chi$$

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= v^2 \partial\Theta^2 + \left[(\partial\chi)^2 - 2\mu^2 \chi^2 - 4\sqrt{\frac{\mu^2 \lambda}{2}} \chi^3 - \lambda \chi^4 \right] + \left(\sqrt{\frac{2\mu^2}{\lambda}} \chi + \chi^2 \right) (\partial\Theta)^2 \\ &= \mathcal{L}_{\Theta \text{ kin}} + \mathcal{L}_\chi + \mathcal{L}_{\chi, \Theta} \end{aligned}$$

NB $\Theta(x)$ MASSLESS SCALAR!

GOLDSTONE'S THEOREM

* Q CONSERVED, GENS A SYMM $\Rightarrow [H, Q] = 0$

* $|0\rangle$ IS VACUUM

* $H \rightarrow H + c$ SO $H|0\rangle = 0$

* SYMMETRY BREAKING

SUPPOSE $|0\rangle$ BREAKS SYMM. Q , IE $Q|0\rangle \neq 0$

CONSIDER $|1\rangle := Q|0\rangle$

WHAT IS ITS ENERGY?

$$H|1\rangle = HQ|0\rangle = [H, Q]|0\rangle = 0$$

$\Rightarrow \exists$ ANOTHER STATE $|1\rangle$ w/ SAME ENERGY AS $|0\rangle$!

QFT $Q = \int d^D x J^0(\vec{x}, t)$

CONSIDER $|s\rangle := \int d^D x e^{-i\vec{k} \cdot \vec{x}} J^0(\vec{x}, t) |0\rangle$

$\lim_{\vec{k} \rightarrow 0} |s\rangle = Q|0\rangle = 0$ ENERGY STATE

\Rightarrow BEHAVIOR EXPECTED OF MASSLESS PARTICLE!

(energy $\rightarrow 0$ as momentum $\rightarrow 0$)

\downarrow more rigor
 \downarrow exists

$\Rightarrow \exists$ massless spin 0 boson associated w each
broken, continuous, global symmetry

Language: Goldstone Boson

Language: Goldstone's Theorem

HOW MANY GOLDSTONE BOSONS?

SUPPOSE $G \longrightarrow H$ SPONTANEOUSLY

WRITE $\dim(G)$ G-GENS = $\{A_1, A_2, \dots, A_{\dim(H)}, \underbrace{B_1, \dots, B_{\dim(G)-\dim(H)}}_{\text{G-GENS NOT H-GENS}}\}$

$$\Rightarrow B_i|0\rangle \neq 0 \quad \forall i=1, \dots, \dim(G) - \dim(H)$$

$\Rightarrow \exists \dim(G) - \dim(H)$ MASSLESS GOLDSTONE BOSONS.

AKA NAMBU-GOLDSTONE BOSONS

\exists "PSEUDO-NAMBU GOLDSTONE BOSONS" TOO

- MASSIVE DUE TO SPONTANEOUS \nvdash EXPLICIT BREAKING

- EG PION: QCD \Rightarrow SSB OF CHIRAL sym $\overset{C}{\Rightarrow}$ π GOLDSTONE QUARK MASSES \Rightarrow EXPLICIT \Rightarrow MASS TO CH. S.B. GOLDSTONE π

ANDERSON - HIGGS MECHANISM: GAUGE SYMMETRY BREAKING

WHAT: BREAK GAUGE, NOT GLOBAL, SYMM.

GLOBAL \Rightarrow GOLDSTONE

GAUGE \Rightarrow GOLDSTONE GETS "EATEN" BY VECTOR

ABELIAN HIGGS MODEL

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (\partial\varphi)^+ \partial\varphi + \mu^2 \varphi^+ \varphi - \lambda (\varphi^+ \varphi)^2$$

$$\varphi^+ = \varphi^* \quad D_\mu = \partial_\mu - ieA_\mu \quad \exists \text{ U(1) GAUGE SYMMETRY}$$

$$\left. \begin{array}{c} \dot{\varphi} \\ \varphi \end{array} \right\} \Rightarrow \langle 0 | \dot{\varphi} | 0 \rangle \neq 0 \quad \text{SO POLAR ALLOWED}$$

$$\text{POLAR} \quad \varphi = \rho e^{i\theta} \quad \Rightarrow \quad D_\mu \varphi = [\partial_\mu \rho + i\rho (\partial_\mu \theta - e A_\mu)] e^{i\theta}$$

$$\Rightarrow \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \rho^2 (\partial_\mu \theta - e A_\mu)^2 + (\partial_\rho)^2 + \mu^2 \rho^2 - \lambda \rho^4$$

ASIDE U(1) GLOBAL \Rightarrow NO GAUGE FIELD

$$\mathcal{L}_{\text{GLOBAL}} = \rho^2 (\partial_\mu \theta)^2 + (\partial_\rho)^2 + \mu^2 \rho^2 - \lambda \rho^4$$

$$\mathcal{L} - \mathcal{L}_{\text{GLOBAL}} = \underbrace{-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}}_{\text{PHOTON PROPAGATES}} + \underbrace{\rho^2 (\partial_\mu \theta - e A_\mu)^2}_{\text{CRITICAL TERM!}}$$

NB: GAUGE TRANS:

$$\varphi \mapsto e^{i\alpha} \varphi \quad \Rightarrow \quad \theta + \alpha \quad \text{THEN} \quad b_\mu = \partial_\mu \theta - e A_\mu$$

$$A_\mu \mapsto A_\mu + \frac{i}{e} \partial_\mu \alpha \quad \mapsto \partial_\mu \theta + \partial_\mu \alpha - e A_\mu - \partial_\mu \alpha$$

$\Rightarrow B_\mu$ GAUGE INVARIANT!

$$F_{\mu\nu}^B = \partial_\mu B_\nu - \partial_\nu B_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{1}{e} \partial_\mu \partial_\nu \theta + \frac{1}{e} \partial_\nu \partial_\mu \theta = F_{\mu\nu}^A$$

$$\begin{array}{lll} \text{SSB} & \rho = \frac{1}{\sqrt{2}}(v + \chi) & v = \sqrt{\frac{\mu^2}{\lambda}} \\ & \xrightarrow{\text{VEV}} & \xrightarrow{\text{MIN}} v(\ell) \end{array}$$

$$\begin{array}{c} \xrightarrow{\text{SSB} + B_\mu + \text{ALG}} \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^B F^{B\mu\nu} + \frac{1}{2} (\partial \chi)^2 - \mu^2 \chi^2 - \sqrt{\lambda} \mu \chi^3 - \frac{\lambda}{4!} \chi^4 + \frac{\mu^4}{4 \lambda} \\ + \frac{1}{2} M^2 B_\mu B^\mu + e^2 v \chi B_\mu^2 + \frac{1}{2} e \chi^2 B_\mu^2 \\ \Downarrow \quad \Downarrow \quad \Downarrow \\ \begin{array}{c} B_\mu \text{ MASS} \\ M = e v \end{array} \quad \begin{array}{c} 3\text{-PT HIGGS} \\ \text{COUPLING} \\ \chi \text{---} \chi \end{array} \quad \begin{array}{c} 4\text{-PT HIGGS} \\ \text{COUPLING} \\ \chi \text{---} \chi \end{array} \end{array}$$

- NB:
- (1) MASSIVE B_μ , NO A_μ OR θ
 A_μ "ATE" θ , GAINED MASS, "IS NOW" B_μ
 - (2) IF GLOBAL, θ WOULD BE GOLDSTONE BOSON
 - (3) θ EATEN, BUT MASSIVE SCALAR χ LEFTOVER
Language: "Higgs Boson"
 - (4) 4 DOF = 1 EATEN + 1 HIGGS $2 = 1 + 1$

REAL WORLD: $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$

- (1) - w_+ w_- z ARE B_μ
- EACH ATE A DOF
- (2) 1 MASSIVE SCALAR LEFTOVER \Rightarrow ACTUAL HIGGS BOSON OF SM
 $m = 125$ GeV
- (3) $\underbrace{4 \text{ DOF}}_{\text{COMPLEX HIGGS FIELD DOUBLET}} = \underbrace{3 \text{ EATEN}}_{w_+ z} + \underbrace{1 \text{ HIGGS}}_{\text{REAL ONE}}$

CONDENSED MATTER APPLICATION: SUPERCONDUCTORS

BCS THEORY (see 6.4 of Altland and Simons)

$$\hat{H} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} \hat{n}_{\vec{k}\sigma} - \frac{g}{V} \sum_{\vec{k}, \vec{k}', g} C_{\vec{k}+g}^+ C_{-\vec{k}'}^+ \gamma_L C_{\vec{k}'} \gamma_L$$

note: \exists different kinds of superconductors

ATTRACTIVE, PHONON-INDUCED
INTERACTION

- $T < T_c$: ① ELECTRONS BIND INTO COOPER PAIRS, $Q = 2e$
 ② RESISTIVITY $\Rightarrow 0$
 ③ \vec{B} FIELDS EXPelled

\hookrightarrow Language "Meissner effect"

FIELD THEORY

ABELIAN HIGGS MODEL w/ CHARGE 2 FIELD φ

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (\partial\varphi)^+ \partial\varphi + \mu^2 \varphi^+ \varphi - \lambda (\varphi^+ \varphi)^2$$

Q: RECOVER MEISSNER?

NOTE: $\lim_{M \rightarrow \infty} \mathcal{L}$, K.E. $\rightarrow 0$ (particle so massive, doesn't want to move)

$$\Rightarrow \text{EOM} \simeq \frac{\delta}{\delta B_\mu} \left[\frac{1}{2} M^2 B_\mu B^\mu + e^2 v X B_\mu^2 + \frac{1}{2} e X^2 B_\mu^2 \right]$$

PART OF \mathcal{L} w/ B_μ , $-B_\mu$ KIN. TERM.

$$= B^\mu [M^2 + 2e^2 v X + e X^2] = 0$$

$$\Rightarrow B^\mu = 0 \quad \text{SOLVES APPROX E.O.M}$$

RECALL $B_\mu = \partial_\mu \theta - e A_\mu$.

normal magnetic field

THEN $B_\mu = 0 \Rightarrow \partial_\mu \theta = e A_\mu \Rightarrow \vec{B} = \vec{\nabla} \times \vec{A} = \vec{\nabla} \times \frac{\vec{\nabla} \theta}{e} = 0$
 $\vec{\nabla} \theta = e \vec{A}$

NO \vec{B} -FIELD IN SUPERCONDUCTOR!

MEISSNER EFFECT

Solitons & Topological Defects

WHAT?

NON-TRIVIAL SOLNS TO EOM

C.M.: TONS OF
SYSTEMS

(superfluids, superconductors,
ferromagnets, KT, Top. insulators)
...

H.E.: QCD,
COSMOLOGY

① LOCAL, NOT GLOBAL, ENERGY MIN.

② TOPOLOGICAL CHARGE

③ TOPOLOGICALLY STABLE

④ ENERGETICS LOWER BOUNDS

(very important in susy gauge theory
& string theory)

ORIGIN = Symmetry

KINKS 1+1 DIMS, REAL SCALAR

$$\mathcal{L} = \frac{1}{2} (\partial \varphi)^2 - V(\varphi) \quad V(\varphi) = \frac{\lambda}{4} (\varphi^2 - v^2)^2$$

- MINIMA / VACUA AT $\varphi = \pm v$
- $\varphi = v + X$, small osc X about v $\Rightarrow m_X = (\lambda v^2)^{\frac{1}{2}}$ HOW!?

$$\text{E.O.M. } (\partial_t^2 - \partial_x^2)\varphi + \lambda \varphi (\varphi^2 - v^2) = 0$$

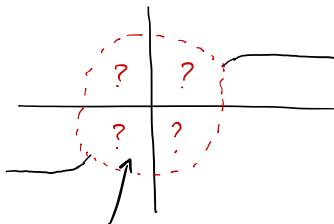
USUAL SOLNS $\varphi = \pm v$ (expand as above)

NON-TRIVIAL SOLUTIONS

Q: CAN YOU INTERPOLATE?

$$\phi(+\infty) = v$$

$$\phi(-\infty) = -v$$



$$\frac{\sqrt{\lambda}}{2\sqrt{3}} \rightarrow \frac{\sqrt{\lambda} \sqrt{6}}{2\sqrt{3}} = \sqrt{\frac{\lambda}{2}}$$

REGION w/ NON-MIN. E!

\Rightarrow NOT GLOBAL MIN.

\exists LOCAL?

<u>"KINK" SOLUTION</u>	
$\varphi = v \tanh(\sqrt{\frac{\lambda}{2}} v x)$	Language: "kink" in 2d, "domain wall" in higher d

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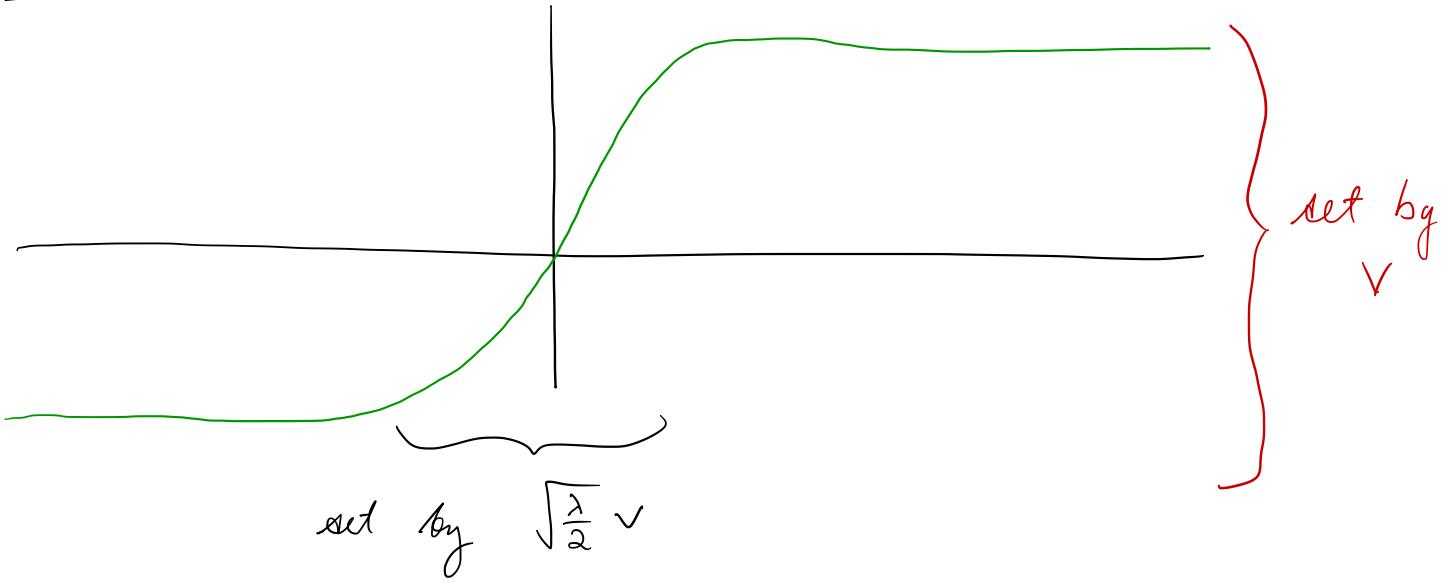
$$\text{CHECK: } \frac{d}{dx} \tanh(x) = 1 - \tanh^2 x \quad A := \sqrt{\frac{\lambda}{2}} v$$

$$\begin{aligned} \partial_x^2 \varphi &= \partial_x \left[(1 - \tanh^2 Ax) Av \right] = -Av \cdot 2 \tanh Ax \cdot A [1 - \tanh^2 Ax] \\ &= -2A^2 v \tanh Ax [1 - \tanh^2 Ax] = -2 \left(\frac{\lambda v^2}{2} \right) v \tanh \left(\sqrt{\frac{\lambda}{2}} v x \right) [1 - \tanh^2 \left(\sqrt{\frac{\lambda}{2}} v x \right)] \\ &= -\lambda \varphi (\varphi^2 - v^2) \end{aligned}$$

$$\text{NEED } (\partial_t^2 - \partial_x^2) \varphi + \lambda \varphi (\varphi^2 - v^2) = 0$$

$$\partial_t^2 \varphi = 0 \quad ? \quad \text{BY EOM} \Rightarrow \checkmark$$

SKETCH



ENERGY OF KINK

$$\text{CONT. MOM. } \Pi = \frac{\delta \mathcal{L}}{\delta \dot{\varphi}} = \partial_0 \varphi$$

$$\implies \dot{\mathcal{H}}(x, t) = \Pi \dot{\varphi} - \mathcal{L} = \frac{1}{2} [(\partial_0 \varphi)^2 + (\partial_x \varphi)^2] + \frac{\lambda}{4} (\varphi^2 - v^2)^2$$

KINK EN. PER UNIT LENGTH

$$\mathcal{E}(x) = \mathcal{H}(x, t) - \frac{1}{2} (\partial_0 \varphi)^2 = \frac{1}{2} (\partial_x \varphi)^2 + \frac{\lambda}{4} (\varphi^2 - v^2)^2$$

\uparrow B/C time indep soln

$$M := \int dx \mathcal{E}(x) = \frac{2\sqrt{\lambda}}{3} \sqrt{x} v^3$$

$$\text{RECALL } m_\chi = (\lambda v^2)^{\frac{1}{2}} = \sqrt{\lambda} v$$

$$\implies M = \frac{2\sqrt{\lambda}}{3} m_\chi v^2 = \frac{2\sqrt{\lambda}}{3} m_\chi \left(\frac{m_\chi^2}{\lambda} \right) \quad \begin{matrix} \text{(dimensions work b/c } [\lambda]=2 \\ \text{in d=2)} \end{matrix}$$

KEY: χ F.T. PERTURBATIVE $\Rightarrow \lambda \ll 1 \Rightarrow M \gg m_\chi$, HEAVY KINKS!

ALSO KEY: P.T. IN λ NEVER FINDS KINK

TOPOLOGICAL CHARGE

Q: DO \exists CONTINUOUS SYMM?

A: NO \Rightarrow NO NOETHER'S CURRENT

(but Noether \Rightarrow conserved current,
not nec^c other way around)

$$J^{\mu} = \frac{1}{2\sqrt{v}} \epsilon^{\mu\nu} \partial_{\nu} \varphi \quad \text{IS CONSERVED!} \quad \partial_{\mu} J^{\mu} = \frac{1}{2\sqrt{v}} \epsilon^{\mu\nu} \partial_{\mu} \partial_{\nu} \varphi = 0$$

↓ contracted symm. & anti-symm. tensors



$$Q = \int_{-\infty}^{\infty} dx J^0(x) \quad \text{CONSERVED IF} \quad \frac{dQ}{dt} = 0$$

$$\int_{-\infty}^{\infty} dx \frac{1}{2\sqrt{v}} \epsilon^{01} \partial_1 \varphi = \int_{-\infty}^{\infty} dx \frac{1}{2\sqrt{v}} \partial_x \varphi = \frac{1}{2\sqrt{v}} [\varphi(+\infty) - \varphi(-\infty)] = 1$$

ANTI KINK

$$\varphi_{AK} = -\varphi_K$$



$$Q = -1$$

BOGOMOL'NYI BOUND

USE $a^2 + b^2 \geq 2|ab|/|b|$

$$\mu = \sqrt{v}$$

$$\begin{aligned} M &= \int dx \left[\frac{1}{2} (\partial_x \varphi)^2 + \frac{\lambda}{4} (\varphi^2 - v^2)^2 \right] \geq \int dx 2 \sqrt{\frac{1}{2} (\partial_x \varphi)^2} \sqrt{\frac{\lambda}{4} (\varphi^2 - v^2)^2} \\ &= \int dx \sqrt{\frac{\lambda}{2}} \left| (\partial_x \varphi)(\varphi^2 - v^2) \right| = \sqrt{\frac{\lambda}{2}} \int dx \left| \left[\partial_x \left(\frac{\varphi^3}{3} \right) - v^2 \partial_x \varphi \right] \right| \\ &= \sqrt{\frac{\lambda}{2}} \left| \left[\frac{\varphi^2}{3} - v^2 \varphi \right]_{-\infty}^{+\infty} \right| = \sqrt{\frac{\lambda}{2}} \left[\frac{1}{3} \varphi^3 \Big|_{-\infty}^{+\infty} - v^2 \varphi \Big|_{-\infty}^{+\infty} \right] = \left| \frac{4}{3\sqrt{2}} m_X \left(\frac{m_X^2}{\lambda} \right) Q \right| \end{aligned}$$

algebra

$$\text{so } \frac{M}{\frac{4}{3\sqrt{2}} m_X \left(\frac{m_X^2}{\lambda} \right)} \gg |Q|$$

in units of $\frac{4}{3\sqrt{2}} m_X \left(\frac{m_X^2}{\lambda} \right)$

$M \gg |Q|$

Language: "Bogomol'nyi Bound"
or "BPS Bound"

KEY: KINK MASS BOUNDED BY ITS CHARGE

VORTICES & MONOPOLIES (will do in a little less detail, same idea)

$$d = 2 + 1$$

$$\mathcal{L} = \partial_i \varphi^* \partial_i \varphi - \lambda (\varphi^* \varphi - v^2)^2$$

Q: we saw this recently. where? conclusions?
 - global sym. break.
 - $\text{J} \bar{\psi} \psi - \text{Goldstone Boson}$

Q: DOES THIS THEORY HAVE TOPOLOGICAL DEFECTS?

OBTAINING FINITE ENERGY

$$\mathcal{H}(x) = [\partial_i \varphi^* \partial_i \varphi + \lambda (\varphi^* \varphi - v^2)^2]$$

$$E = \int d^2 x [\partial_i \varphi^* \partial_i \varphi + \lambda (\varphi^* \varphi - v^2)^2]$$

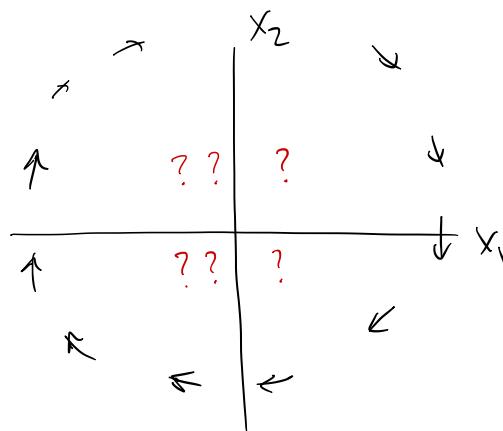
FINITE E REQ $\lim_{r \rightarrow \infty} \mathcal{H}(x) \rightarrow 0$ HOW? EACH TERM $\rightarrow 0$

2nd TERM

$$\text{ANSATZ: } \lim_{r \rightarrow \infty} \varphi = v e^{i\theta}$$

$$\text{THEN: } \lim_{r \rightarrow \infty} \lambda (\varphi^* \varphi - v^2) = \lambda (v e^{-i\theta} v e^{i\theta} - v^2) = 0 \quad \checkmark$$

VORTEX $\lim_{r \rightarrow \infty} \varphi = v e^{i\theta} \Rightarrow \varphi$ WHIRS AROUND $\theta \infty$!



(note: ? can be det, but not wronging about it here)

CHARGE m $\lim_{r \rightarrow \infty} \varphi = v e^{im\theta} \Rightarrow \varphi$ ROTATES m TIMES
 UNDER $\theta \rightarrow \theta + 2\pi$

CAVEAT

$$\text{1ST TERM} \quad \partial_i \varphi \sim \frac{1}{r} \frac{\partial}{\partial \theta} \Rightarrow \text{LOG DIV. @ } \infty!$$

SINGLE VORTEX D.N.E. IN GLOBAL $\ell(1)$

SOL'NS:

① ADD SOMETHING ELSE

$$\text{VORTEX} \quad \lim_{r \rightarrow \infty} \varphi = v e^{i\theta}$$

$$\text{ANTI VORTEX} \quad \lim_{r \rightarrow \infty} \varphi = v e^{-i\theta}$$

FACT: REL SIGN MAKES $E(\text{VORTEX} + \text{ANTIVORTEX})$ FINITE

VORTEX + ANTI VORTEX PAIRS EXIST

(what does this sound like? K-T transition from Collog!)

② MODIFY THE THEORY (add gauge field & coupling)

$$\mathcal{L} = D^* \varphi^* D \varphi - \lambda (\varphi^* \varphi - v^2)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad D_i \varphi = \partial_i \varphi - ie A_i \varphi$$

Q: WHAT CAN SOLVE E-PROB? (ask class)

PROB WAS $\partial_i \varphi^* \partial^i \varphi$ TERM $\partial_i \varphi \sim \frac{1}{r}$

NOW $D_i \varphi^* D^i \varphi$ \exists ANOTHER FIELD!

$$\text{IF} \quad \lim_{r \rightarrow \infty} A_i = \frac{-i}{e} \left(\frac{1}{|\varphi|^2} \right) \varphi^* \partial_i \varphi = \frac{1}{e} \partial_i \theta$$

THEN CANCEL, FINITE E

$$B\text{-FLUX } @ \infty = \int d^2x F_{12} = \oint_C dx_i A_i = \frac{2\pi}{e}$$

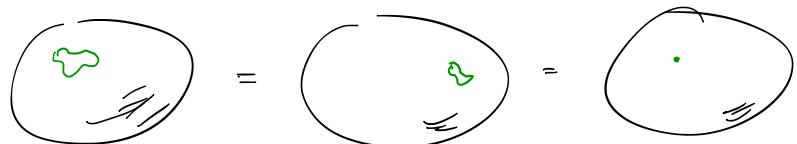
MAGNETICALLY CHARGED VORTEX! (type II superconductors)

Language "Abrikosov-Gelfan-Oleinik vortex"

DEFECT ORGANIZING PRINCIPLE: HOMOTOPY (more next semester!)

GOOD ENOUGH DEF'N: The n^{th} homotopy group $\pi_n(M)$ of a manifold M is the space of topologically inequivalent maps $F: S^n \rightarrow M$.

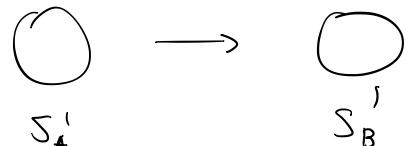
EXAMPLE $\pi_1(S^2)$



CIRCLE: EMBEDS S^1 IN S^2

NB: ALL S^1 IN S^2 ARE = $\Rightarrow \pi_1(S^2) = \mathbb{O}$

EXAMPLE $\pi_1(S^1)$



$$F: S_A^1 \rightarrow S_B^1$$

DEFINE EXPLICIT MAP

$$F_k: S_A^1 \rightarrow S_B^1 \quad \text{ST.} \quad F_k(\theta) = k\theta$$

F_k WRAPS S_A^1 k TIMES ON S_B^1 , CAN'T CONTRACT

$\Rightarrow \pi_1(S^1) = \mathbb{Z}$ (characterize by # wrappings)

VORTEX

$S_A^1 = \text{CIRCLE } @ \infty$

$S_B^1 = \underbrace{\text{GROUND STATE}}_{\text{MFLD}}$

Recall: each θ gave ground state!

$$\pi_1(S^1) = \mathbb{Z}$$

Q: why all the integers?

HEDGEHOGS \vdash T HOOFT-POLAKOV MONOPOLES

$$\mathcal{L} = \frac{1}{2} \partial \vec{\varphi} \cdot \partial \vec{\varphi} - V(\vec{\varphi} \cdot \vec{\varphi})$$

CHOOSE $V = \lambda (\vec{\varphi}^2 - v^2)^2$

Q: GROUND STATE?

A: $\vec{\varphi}$ w $\vec{\varphi}^2 = v^2$ IN UNIF. DIR

Q: $G \xrightarrow{\text{BREAK}} H$, $G = ?$ $H = ?$

A: $G = SO(3)$ $H = SO(2) = U(1)$

Q: # GOLDSTONES?

A: 2 $(\dim(G) - \dim(H) = 3 - 1 = 2)$

Q: DEFECTS?

long exact sequence in homotopy

$\pi_1(U(1)) \cong \mathbb{Z}$

A: $\pi_2(SO(3)/U(1)) = \pi_2(S^2) = \mathbb{Z}$
 (map S^2 around S^2)

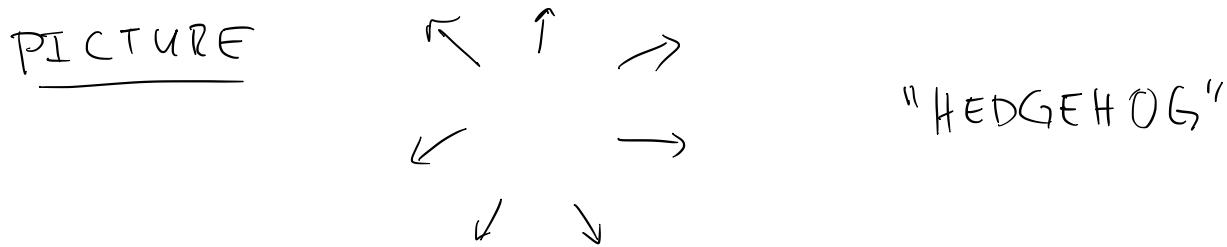
ENERGETICS

$$H = \int d^3x \left[\frac{1}{2} (\vec{\nabla} \psi)^2 + \lambda (\vec{\psi}^2 - v^2)^2 \right] \quad \text{how do you get this?}$$

FINITE $\epsilon \Rightarrow \vec{\psi}^2 = v^2$ @ $r = \infty, \Rightarrow$ ON S^2 INDEED!

SOLN: $\psi^a \xrightarrow[r \rightarrow \infty]{} \propto \frac{x^a}{r}$

CHECK $\vec{\psi}^2 = \propto^2 \frac{x^2 + y^2 + z^2}{r^2} = v^2 \checkmark$



CAVEAT $\left(\partial_x \left(\frac{1}{\sqrt{A}} \right) = \frac{\partial A}{\partial x} \partial_A \frac{1}{\sqrt{A}} = -\frac{1}{2} \frac{\partial A}{\partial x} \frac{1}{A^{3/2}} \right)$

$$(\partial \vec{\psi})^2 \Big|_{\psi^a \sim \frac{x^a}{r}} \sim \partial_\mu \left(\frac{x^a}{r} \right) \partial^\mu \left(\frac{x^a}{r} \right) = \vec{\nabla} \left(\frac{x^a}{\sqrt{x^2 + y^2 + z^2}} \right) \cdot \vec{\nabla} \left(\frac{x^a}{\sqrt{x^2 + y^2 + z^2}} \right)$$

$$= \left(\frac{1}{r} - \frac{1}{2} \frac{x(2x)}{(r^2)^{3/2}}, \sin, \sin \right) \cdot \text{same}$$

$$= \frac{3}{r^2} + \dots \Rightarrow \int d^3x (\nabla \psi^a)^2 > \int d\Omega \int \frac{3r^2 dr}{r^2} \Rightarrow \text{DIVERGES}$$

SOL'N. (similar to vortex case)

GAUGE THE SYMMETRY!

$$\mathcal{L} = -\frac{1}{4g^2} G_{\mu\nu}^a G^{\mu\nu a} + \frac{1}{2} (D_\mu \varphi^a) (D^\mu \varphi^a) - \lambda (\varphi^a \varphi^a - v^2)^2$$

$$D_\mu \varphi^a = \partial_\mu \varphi^a + e \epsilon^{abc} A^b \varphi^c$$

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc} A_\mu^b A_\nu^c \quad \phi = \phi^a \frac{x^a}{2}$$

↑ Non-abelian gauge field strength. QFT 2!

$$A_i^b \xrightarrow[r \rightarrow \infty]{} \frac{1}{e} \epsilon^{bij} \frac{x^j}{r^2}$$

HAVE: ① $\vec{\varphi}$ NON-TRIVIAL, RADIALLY OUT
 ② NON TRIVIAL $A^b = \frac{1}{e} \epsilon^{bij} \frac{x^j}{r^2}$
 AS $r \rightarrow \infty$

\Rightarrow UNBROKEN U(1) \vec{B} -FIELD

RADIALLY OUT!

MAGNETIC
MONOPOLE!

$$\begin{aligned} & \xrightarrow[r \rightarrow \infty]{} \epsilon^{kmi} \partial_m \left[\epsilon^{bij} \frac{x^j}{r^2} \right] = \frac{\epsilon^{kmi} \epsilon^{bij}}{e} \partial_m \left[\frac{x^j}{r^2} \right] \\ &= \frac{\epsilon^{kmi} \epsilon^{bij}}{e} \left[\frac{\delta_m^j}{r^2} - x^j \partial_m \left(\frac{1}{r^2} \right) \right] \\ &= \frac{\epsilon^{kmi} \epsilon^{bij}}{er^2} - \frac{\epsilon^{kmi} \epsilon^{bij}}{e} x^j \partial_m \left(\frac{1}{r^2} \right) \\ &\underset{\circlearrowleft}{=} - \frac{\epsilon^{kmi} \epsilon^{bij}}{e} x^j (2x^m) \left(-\frac{2}{r^3} \right) \end{aligned}$$

Language 't Hooft Polyakov Monopole (one you get from symmetry breaking)

BOGOMOL'NYI BOUND

$$E = \int d^3x \left[\frac{1}{4} (\vec{F}_{ij})^2 + \frac{1}{2} (D_i \vec{\Phi})^2 + V(\vec{\Phi}) \right] \quad (\text{how would } \downarrow \text{ get this?})$$

NB

$$\frac{1}{4} (\vec{F}_{ij})^2 + \frac{1}{2} (D_i \vec{\Phi})^2 = \underbrace{\frac{1}{4} (\vec{F}_{ij} - \epsilon_{ijk} D_k \vec{\Phi})^2}_{\text{POS DEF}} + \frac{1}{2} \epsilon_{ijk} \vec{F}_{ij} \cdot D_k \vec{\Phi}$$

$$M \geq \int d^3x \left[+ \frac{1}{2} \epsilon_{ijk} \vec{F}_{ij} \cdot \vec{D}_k \vec{\Phi} + V(\vec{\Phi}) \right] \geq 4\pi g \sim \frac{m_w}{e^2}$$

g : MONOPOLE B-CHARGE

m_w : "W-BOSON" MASS OF $SO(3) \Rightarrow U(1)$ BREAKING

NB: PERT. THY $\Rightarrow e \ll 1 \Rightarrow m_{\text{monopole}} \gg m_w$

(but maybe only $0(100)$ times greater?)

GUT MONOPOLIES & INFLATION

IDEA: STRONG + WEAK + E-M \Rightarrow ONE "UNIFIED" FORCE
GRAND UNIFIED THY (GUT)

$$H = SU(3) \times SU(2) \times U(1) \xleftarrow{\text{BREAK}} G = SU(5) \quad (\text{most common})$$

GUT MONOPOLIES $\pi_2\left(\frac{G}{H}\right) = \pi_2\left(\frac{SU(5)}{SU(3) \times SU(2) \times U(1)}\right) \neq 0!$

$$M \simeq \frac{M_{XY}}{\alpha_{GUT}} \simeq 10^{16} \text{ GeV}$$

PROBLEM: WHY DON'T WE SEE THEM!?

COSMOLOGY HOMOGENEOUS & ISOTROPIC

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$$

ALT $g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & a(t) & & \\ & & a(t) & \\ & & & a(t) \end{pmatrix}$

Friedmann Language
- Friedmann
- Robertson
- Walker

(classical F.T.)

GENERAL RELATIVITY $\mathcal{L} = \int d^4x \sqrt{-g} R$

EOM = Einstein's Equations $\boxed{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$

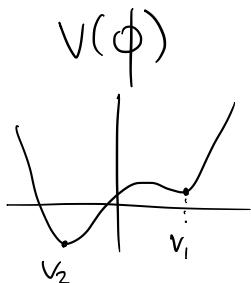
INFLATION $S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$

FRW + "SLOW-ROLL" $a \sim e^{Ht} \rightarrow \text{exponential expansion}$

CAN DILUTE GUT MONOPOLIES! (also does other more important things. agrees w/ data)

MULTIVERSE: IT'S SCIENTIFICALLY CONSERVATIVE
 (ie follows from principles we already have)
 (if this is true, most important thing we could learn about fund phys.)

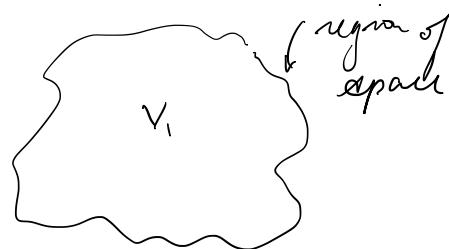
WHAT:



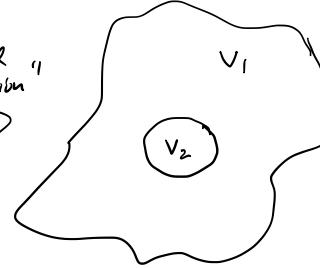
Q: CAN SPACE REGIONS TUNNEL FROM v_1 TO v_2 ?

A: YES COLEMAN-DELUCCIA

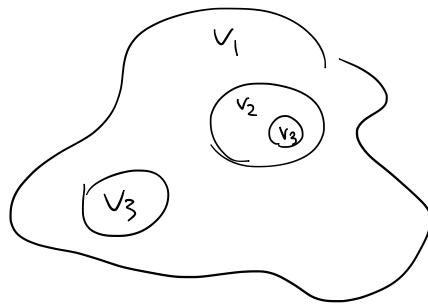
THEN



"Bubble Nucleation"
 \Rightarrow



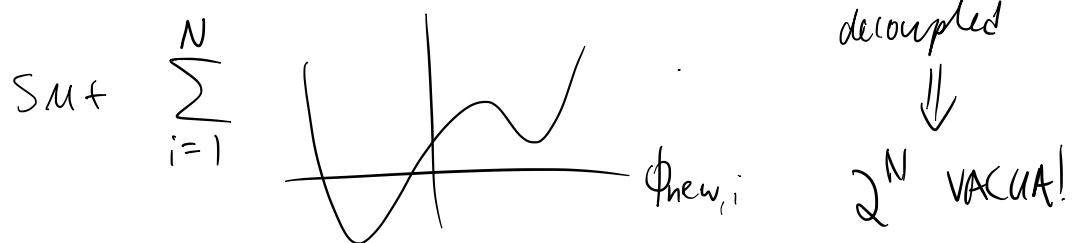
MORE & MORE
 \Rightarrow



"MULTIVERSE"

LAWS OF PHYSICS CAN BE DIFF. IN EACH BUBBLE!

Why ① SCALARS EXIST (and probably more!)



SM: # FERMS = 90 # GAUGE BOSONS = 24

SCALARS = 4 $\Rightarrow N \approx 50-100!$?
 (STRINGS:
 $N \sim 100-1000$)

② QUANTUM MECHANICS TUNNELING HAPPENS!

(if time, accordingly interesting & important)

HEISENBERG MODEL (for magnets)

$$\hat{H} = J \sum_{\langle i,j \rangle} \vec{n}_i \cdot \vec{n}_j$$

\vec{n}_i : LOCAL MAG. MOM. @ SITE i

$\langle i,j \rangle$: NEAREST NEIGHBOR SUM

<u>SIGN(j)</u>	<u>PREF</u>	<u>NAME</u>
-	II	FERROMAGNET
+	II	ANTI "