

QFT 2

Topic 3: Duty



SUPERSYMMETRY: INTRODUCTION By ENGINEERING

$$S = \int d^4x (L_{\text{scalar}} + L_{\text{fermion}})$$

$$L_{\text{scalar}} = -\partial^\mu \phi^* \partial_\mu \phi \quad L_{\text{fermion}} = i \bar{\psi} \sigma^\mu \partial_\mu \psi$$

TRANS: SC \rightarrow FERM

TRY 1 $\phi \mapsto \phi + \gamma_\alpha$ X BAD IND'S.

TRY 2 $\phi \mapsto \phi + \varepsilon \psi$ ✓ IND'S

NOTE: ε FERMIONIC TRANS. PARAM!

SIM: $\phi^* \mapsto \phi^* + \bar{\varepsilon} \bar{\psi}$

$$L_{\text{scalar}} \rightarrow L_{\text{scalar}} - \varepsilon \partial^\mu \gamma_\alpha \partial_\mu \phi^* - \bar{\varepsilon} \partial^\mu \bar{\psi} \partial_\mu \phi$$

TRANS: FERM \rightarrow SC

NEED: $\bar{\psi}_\alpha$ TRANS TO GET ε, ∂

γ_α TRANS TO GET $\bar{\varepsilon}, \partial$

TAKE: $\gamma_\alpha \rightarrow \gamma_\alpha - i (\sigma^\mu \bar{\varepsilon})_\alpha \partial_\mu \phi$

$$\bar{\gamma}_\alpha \rightarrow \bar{\gamma}_\alpha + i (\varepsilon \sigma^\mu)_\alpha \partial_\mu \phi^*$$

$$L_{\text{ferm}} \rightarrow L_{\text{ferm}} - \underbrace{\varepsilon \sigma^\mu \bar{\sigma}^\nu \partial_\mu \gamma_\nu \partial_\nu \phi^* + \bar{\psi} \bar{\sigma}^\nu \sigma^\mu \bar{\varepsilon} \partial_\mu \partial_\nu \phi}_{= \varepsilon \partial^\mu \gamma_\alpha \partial_\mu \phi^* + \bar{\varepsilon} \partial^\mu \bar{\psi} \partial_\mu \phi}$$

$$= \varepsilon \partial^\mu \gamma_\alpha \partial_\mu \phi^* + \bar{\varepsilon} \partial^\mu \bar{\psi} \partial_\mu \phi$$

$$- \partial_\mu (\varepsilon \sigma^\nu \bar{\sigma}^\mu \gamma_\nu \partial_\nu \phi^* + \bar{\varepsilon} \gamma^\mu \partial_\mu \phi^* + \bar{\varepsilon} \bar{\psi} \partial^\mu \phi)$$

CONCLUSION - L INV, UP TO TOT. DERIVS!

- BOSON \leftrightarrow FERMIon SYMMETRY

"SUPERSYMMETRY"

what do you know about it? if it exists in particle physics,
how must it exist? (broken) "How much" susy?

amt symmetry = # gens

TOPIC OUTLINE

$d=4$ $N=1$ susy

① 2 COMP. SPINORS

② SUSY ALG. & REPS

③ COMPONENT FIELDS

+ somewhere in here by Spivak Beck

Wess & Bagger

④ SUPERSPACE & SUPERFIELDS

⑤ CHIRAL & VECTOR SUPERFIELDS

⑥ GAUGE INV. INTERACTIONS

- start 2/21 or 2/22 IN 2018

- classes left: ~~makeups~~ WEDS' 2/21, 2/28, 3/21, 4/4

~~Mondays:~~ 2/26, 3/12, 3/19, 3/26, 4/2, 4/9 ~~MON~~

~~Thursdays:~~ 2/22, 3/1, 3/15, 3/22, 3/29, 4/5, 4/12

⑦ ~~SUSY~~

⑧ MSSM & SUSY PHENO \Rightarrow Martins'

"A Supersymmetry Primer"

SOME MOTIVATION

NOTATION & 2 COMP. SPINORS

NOTATION 4-VECTORS w/ LOWERCASE LATIN IND. (eg A_m , not A_μ)

RECALL - 4-COMP ψ SPINORS $\Rightarrow \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$, $\psi_{L,R}$ 2-COMP

- $\Gamma^m \begin{pmatrix} 2 \times 2 & 2 \times 2 \\ 2 \times 2 & 2 \times 2 \end{pmatrix} \Rightarrow \psi_{L,R}$ ACTED ON BY 2×2 'S

- INSPECTION: 2×2 'S ARE

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \vec{\sigma} \quad \text{PAULI MATRICES}$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$(\text{ALL } \sigma^m = (\sigma^0, \vec{\sigma}))$$

FACT: σ^m GENERATE $SL(2, \mathbb{C})$ = DOUBLE COVER OF LORENTZ GROUP
 (know this: $SU(2) \subseteq SL(2, \mathbb{C})$
 \downarrow cover \downarrow 2-cover
 $SO(3) \subseteq SO(3, 1) = LOR.$)

2-COMP. SPINOR TYPES

$$M \in SL(2, \mathbb{C})$$

$M, M^*, (M^\top)^{-1}, (M^+)^{-1}$ ALL REP $SL(2, \mathbb{C}) \Rightarrow$ 4 2-COMP. SPINOR REPS

$$\psi_\alpha' = M_\alpha^\beta \psi_\beta \quad \bar{\psi}_\alpha' = M_\alpha^\beta \bar{\psi}_\beta \quad \psi'^\alpha = (M^{-1})^\alpha_\beta \psi^\beta \quad \bar{\psi}'^\alpha = (M^{*-1})^\alpha_\beta \bar{\psi}^\beta$$

COMPARE TO OLD NOTATION $\psi_\alpha = \psi_L \quad \bar{\psi}^\alpha = \psi_R$

HERMITIAN 2×2 P \Rightarrow CAN WRITE $P = P_m \sigma^m = \begin{pmatrix} -P_0 + P_3 & P_1 - iP_2 \\ P_1 + iP_2 & -P_0 - P_3 \end{pmatrix}, P_m \in \mathbb{R}$

$$P \text{ HERM} \Rightarrow P^* = M P M^\dagger \text{ HERM} \Rightarrow \sigma^m P_m' = M_\alpha^\beta \sigma^m P_m M_\beta^\alpha$$

SOME LORENTZ INVARIANTS: $\psi^\alpha \psi_\alpha =: \psi \psi \quad \bar{\psi}_\alpha \bar{\psi}^\alpha =: \bar{\psi} \bar{\psi}$
 $\psi^\alpha \sigma^m \bar{\psi}^\alpha =: \psi \sigma^m \bar{\psi}$ $\xrightarrow{\text{NOTE}} \alpha^\alpha \bar{\alpha}^\alpha$ ORDER.

RAISE & LOWER INDICES USE $\epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$

$$\epsilon_{\alpha\beta} = \epsilon^{\alpha\beta} = 1 \quad \epsilon_{12} = \epsilon^{21} = -1, \text{ REST ZERO}$$

$$\boxed{\psi^\alpha = \epsilon^{\alpha\beta} \psi_\beta \quad \psi_\alpha = \epsilon_{\alpha\beta} \psi^\beta \quad \epsilon_{\alpha\beta} \epsilon^{\beta\gamma} = \delta_\alpha^\gamma}$$

$$\bar{\sigma}^{m \alpha \alpha} := \epsilon^{\dot{\alpha} \beta} \epsilon^{\alpha \beta} \sigma_{\beta \dot{\alpha}}^m \quad \Rightarrow \quad \bar{\sigma}^m = (\sigma^0, -\sigma^m) \quad \text{AS MATRICES}$$

SOME IDENTITIES

$$\text{INTEREST: } (\bar{\sigma}^m \bar{\sigma}^n)_\alpha^\beta + (\bar{\sigma}^n \bar{\sigma}^m)_\alpha^\beta$$

$$m=n=0 \rightarrow \sigma^0 \sigma^0 + \sigma^0 \sigma^0 = 2 \delta_\alpha^\beta$$

$$m=n=i \underset{i=1,2,3}{\Rightarrow} \sigma^i (-\sigma^i) + \sigma^i (-\sigma^i) = -2 \delta_\alpha^\beta$$

$$m=i, n=j \underset{j \neq i}{\Rightarrow} \sigma^i (-\sigma^j) + \sigma^j (-\sigma^i) = -\{ \sigma^i, \sigma^j \} = 0 \quad (\text{from anti-comm of Pauli pages})$$

$$m=0, n=i \Rightarrow \sigma^0 (-\sigma^i) + (\sigma^i) (\sigma^0) = -\{ \sigma^0, \sigma^i \} = 0$$

RESULT ($\in \mathbb{M}_4$)

$$\boxed{(\bar{\sigma}^m \bar{\sigma}^n + \bar{\sigma}^n \bar{\sigma}^m)_\alpha^\beta = -2 \gamma^{mn} \delta_\alpha^\beta = -2 \gamma^{mn} \mathbb{I}_{2 \times 2}}$$

$$\boxed{(\bar{\sigma}^m \sigma^n + \bar{\sigma}^n \sigma^m)_{\dot{\alpha}}^{\dot{\beta}} = -2 \gamma^{mn} \delta_{\dot{\alpha}}^{\dot{\beta}} = -2 \gamma^{mn} \mathbb{I}_{2 \times 2}}$$

(the above sounds familiar)

$$\text{RECALL } (\text{P.S.}) \quad \gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\{ \gamma^m, \gamma^n \} = 2 \gamma^{mn} \mathbb{I}_{4 \times 4} = 2 \gamma^{mn} \begin{pmatrix} \mathbb{I}_{2 \times 2} & 0 \\ 0 & \mathbb{I}_{2 \times 2} \end{pmatrix}$$

EVEN MORE

$$\boxed{\text{Tr } \bar{\sigma}^m \bar{\sigma}^n = -2 \gamma^{mn} \quad \sigma_{\alpha \dot{\alpha}}^m \bar{\sigma}_m^{\dot{\alpha} \beta} = -2 \delta_\alpha^\beta \delta_{\dot{\alpha}}^\beta}$$

Lorentz GENs

$$\text{Q: If 4-comp } \sim [\gamma^m, \gamma^n], \text{ what 2-comp? A: } \sim [\sigma_i^m, \sigma_i^n]$$

$$\sigma_{\alpha \dot{\alpha}}^m{}^\beta := \frac{1}{4} (\sigma_{\alpha \dot{\alpha}}^m \bar{\sigma}^n \dot{\alpha} \beta - \bar{\sigma}_{\alpha \dot{\alpha}}^m \bar{\sigma}^n \dot{\alpha} \beta)$$

$$\bar{\sigma}_{\dot{\alpha} \dot{\beta}}^m{}^\alpha = \frac{1}{4} (\bar{\sigma}_{\dot{\alpha} \dot{\beta}}^m \sigma_{\alpha \dot{\alpha}}^n - \bar{\sigma}_{\dot{\alpha} \dot{\beta}}^m \sigma_{\alpha \dot{\alpha}}^n)$$

$$\boxed{\bar{\sigma}^a \sigma^b \bar{\sigma}^c - \bar{\sigma}^c \sigma^b \bar{\sigma}^a = -2i \epsilon^{abcd} \bar{\sigma}_d \quad \sigma^a \bar{\sigma}^b \sigma^c - \sigma^c \bar{\sigma}^b \sigma^a = +2i \epsilon^{abcd} \sigma_d}$$

SUSY ALG

THEOREM: (COLEMAN-MANDLIA)

ANY THY W/ SYMM. GROUP G A LIE ALGEBRA

- i) S-MATRIX FROM LOCAL, REL. QFT
- ii) BELOW ANY MASS M \exists FINITE # PARTS p w/ $m_p < M$.
- iii) \exists ENERGY GAP BETW. 10s \in ALL 1-PARTICLE STATES

HAS $G = \underbrace{\text{Poincaré}}_{\sim \mathbb{R}^3 \text{ trans}} \times \text{internal}$
 $\sim \mathbb{R}^3 \text{ trans} \times \text{Lorentz}$

IE: SPACETIME & INT. SYMMETRIES CAN'T MIX!

Q: WHAT DO WE DO WITH THEOREMS?

A1: ASK WHETHER CONCLUSIONS ARE INTERESTING

A2: " " ASSUMPTIONS ARE REASONABLE

\Rightarrow CONSIDER LOOPTHOLE!

LOOPTHOLE: TAKE G A GRADED LIE ALG. AKA SUPER ALGEBRA

SCHEMATIC: - FERMIONIC GENS Q, Q', Q''

- BOSONIC GENS X, X', X''

- ALG $\sim \{Q, Q'\} = X \quad [X, X'] = X'' \quad [Q, X] = Q''$

THEOREM: (HAAG - LOPUSZANSKI - SOHNIES)

- TAKE C-M ASSUMPTIONS, BUT G A GRADED L.A.
W $Q_\alpha^L \in \bar{Q}_\alpha^L$ (its Hermit conjugate) IN ALG.

THEN:

$$\{Q_\alpha^L, \bar{Q}_{\beta M}\} = 2\sigma_{\alpha\beta}^m P_m \delta_M^L, \quad \{Q_\alpha^L, Q_\beta^M\} = \epsilon_{\alpha\beta} X^{LM}$$

WITH $L = 1, \dots, N$, P_m EN-MOM. OPS

↑ antisymm.

↔ "susy ALG"

NB: - SUSY GENS $Q \bar{Q}$ ANTI COMM TO SPACETIME TRANS (P_m)

- X^{LM} "CENTRAL CHARGES"

• NONE IF $N=1$ (Q: why? antisymmetry)

• ONE IF $N=2$ (Q: why? X^{LM} 2×2 antisym)

CONSERVED CHARGE \exists SUSY QFT'S WHERE

$$Q_\alpha = \int d^3x J_\alpha^0 \text{ HAS } \frac{dQ_\alpha}{dt} = 0 \text{ B/C } \partial_m J_\alpha^m = 0$$

REPS OF SUSY ALG (Q: what are imp's of susy alg?)

|M>0| $M = \text{mass}$

REST FRAME $P_m = (M, 0, 0, 0)$

$$\text{ALGEBRA: } \{Q_\alpha^A, \bar{Q}_\beta^B\} = 2M \delta_{\alpha\beta} \delta_A^B \quad \{Q, Q\} = \{\bar{Q}, \bar{Q}\} = 0$$

↓ whg? $P_i = 0 \Rightarrow \sigma_{\alpha\beta}^0 = \delta_{\alpha\beta}$ ignore ch. of non-central

$$\text{RESCALE: } a_\alpha^A := \frac{1}{\sqrt{2M}} Q_\alpha^A \quad (a_\alpha^A)^+ := \frac{1}{\sqrt{2M}} \bar{Q}_{\alpha A}$$

$$\text{s.t. } \{a_\alpha^A, (a_\beta^B)^+\} = \delta_\alpha^B \delta_A^B \quad \{a, a\} = \{a^+, a^+\} = 0$$

NB: 2N PAIRS OF FERM CREAT. & ANNI. OPS
↓ from spinor index

$$\Rightarrow \exists |L\rangle \mid a_\alpha^A |L\rangle = 0 \quad \forall \alpha, A$$

REP: (as with $S^{(s)}$ spinor) ACT w/ $(a_\alpha^*)^+$ IN ALL POSS WAYS
 \Rightarrow YES/NO ON EACH, $\boxed{A, \alpha}$ $\rightarrow 2^N$ CHOICES
 $\Rightarrow 2^{2N}$ STATES

EXAMPLES

$N=1$

$$\text{spin } 0 \quad \text{spin } \frac{1}{2} \quad \text{spin } 0$$

$$|1\Omega\rangle, (a_\alpha^*)^+ |1\Omega\rangle, \frac{1}{\sqrt{2}} \sum^\oplus (a_\alpha^*)^+ (a_\beta^*)^+ |1\Omega\rangle$$

I.E. 2 FERM, 2 BOS

FACT: $|1\Omega\rangle$ CAN HAVE SPIN $j_i \Rightarrow |1\Omega_j\rangle$

TABLE

SPIN	Ω_0	$\Omega_{\frac{1}{2}}$	Ω_1	$\Omega_{\frac{3}{2}}$
0	2	1		
$\frac{1}{2}$	1	2	1	
1		1	2	1
$\frac{3}{2}$			1	2
2				1

$$\sum \text{states} = \begin{cases} 4 \\ = 1 \cdot 2^2 \end{cases} \quad \begin{cases} 8 \\ = 2 \cdot 2^3 \end{cases} \quad \begin{cases} 12 \\ = 3 \cdot 2^4 \end{cases} \quad \begin{cases} 16 \\ = 4 \cdot 2^5 \end{cases} \Rightarrow \text{coeff} = \# \text{ of } m \text{ for } |\Omega_j\rangle$$

(names of first two: massive chiral multiplet, massive vector mult)

$M=0$ $P^2=0$, NO REST FRAME, $P_\mu = (-E, 0, 0, E)$

$$\Rightarrow \text{ALG IS } \{Q_\alpha^A, \bar{Q}_\beta^B\} = 2E(\delta^0 - \delta^3)_{\alpha\beta} \delta^A_B \stackrel{\text{plug } \sigma^0, \sigma^3}{=} 2 \begin{pmatrix} \epsilon & 0 \\ 0 & 0 \end{pmatrix} \delta^A_B$$

$$\{Q, Q\} = \{\bar{Q}, \bar{Q}\} = 0$$

NB ON $M=0$ STATES $\{Q_2^A, \bar{Q}_i^k\} = \{Q_2^A, (Q_2^k)^+\} = 0.$

BUT ANY $\{Q, Q^+\}$ IS POS. SEMI-DEF, \Rightarrow

$Q_2^A = \bar{Q}_i^k = 0$ ON THESE $M=0$ STATES

NB: ONLY $\alpha=1$ NON-TRIVIAL $a^A = \frac{1}{2\sqrt{E}} Q_1^A \quad (a^A)^+ = \frac{1}{2\sqrt{E}} \bar{Q}_i^A$

$$\Rightarrow \{a^A, a^B\} = \delta^A_B \quad \{a, a\} = \{a^+, a^+\} = 0$$

$\Rightarrow N$ (NOT $2N$) CREAT. ANNI. OPS $\Rightarrow 2^N$

w/ CPT, 2×2^N

CASE $N=1$ TABLE

		HELIPLICITY (null: massless particle in Poincaré rep det'd by "helicity", which is half integer or integer)							
		helicity of $ L\rangle$ OR CPT $ L\rangle$							
		-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
2									1
$\frac{3}{2}$								1	1
1								1	
$\frac{1}{2}$									
0									
$-\frac{1}{2}$			(1)	(1)	(1)	(1)	(1)		
-1			(1)	(1)	(1)	(1)	(1)		
$-\frac{3}{2}$	1	1							
-2	1								

(1) (1) (1) (1) (1) (1) (1) (1)

chiral mult: 2 hel 0, hel ± 1
 vector multiplet
 $\text{hel } \pm 1 \Rightarrow \text{vector}$
 $\text{hel } \pm \frac{1}{2} \Rightarrow \text{fermion}$

GENERAL RESULT:

SUSY REPS HAVE #BOSONS = # FERMIONS!

COMPONENT FIELDS Q: HOW DOES SUSY ACT ON FIELDS?

NEED: SPINOR SUSY PARAMS $\xi^\alpha, \bar{\xi}_\alpha$ (recall $\epsilon, \bar{\epsilon}$ from previous engineering)
 $\{\xi^\alpha, \xi^\beta\} = \{\bar{\xi}_\alpha, \bar{\xi}_\beta\} = \{Q_\alpha, Q_\beta\} = \dots = [P_m, \xi^\alpha] = 0$

SUSY ALG \Rightarrow i) $[\xi Q, \bar{\xi} \bar{Q}] = 2 \xi^\alpha \bar{\xi}_\alpha \bar{Q}^m P_m$
ii) $[\xi Q, \xi Q] = [\bar{\xi} \bar{Q}, \bar{\xi} \bar{Q}] = 0$
iii) $[P_m, \xi Q] = [P_m, \bar{\xi} \bar{Q}] = 0$

wide $\Theta \eta = \Theta^\alpha \eta_\alpha = \Theta^\alpha \epsilon_{\alpha\beta} \eta^\beta$
 $= -\epsilon_{\alpha\beta} \Theta^\beta \eta^\alpha = -\Theta_\alpha \eta^\alpha$
 $= -\Theta_\alpha \eta^\alpha$

PROOF: $[\xi Q, \bar{\xi} \bar{Q}] = \xi Q \bar{\xi} \bar{Q} - \bar{\xi} \bar{Q} \xi Q = \xi^\alpha Q_\alpha \bar{\xi}_\alpha \bar{Q}^m - \bar{\xi}_\alpha \bar{Q}^\alpha \xi^\alpha Q_\alpha$
 $= -\xi^\alpha \bar{\xi}_\alpha Q_\alpha \bar{Q}^\alpha \stackrel{= (+)(-)}{=} -\xi^\alpha \bar{\xi}_\alpha \{Q_\alpha, \bar{Q}^\alpha\}$
 $= \xi^\alpha \bar{\xi}^\alpha \{Q_\alpha, \bar{Q}_\alpha\} = \xi^\alpha \bar{\xi}^\alpha 2 \sigma_{\alpha\dot{\alpha}}^m P_m = 2 \xi^\alpha \bar{\xi}^\alpha P_m \checkmark$

COMPONENT FIELDS

(A, γ)

W&B's bad scalar notation. We'll stick w/ it for consistency

INFINITESIMAL FIELD TRANS

$$A \mapsto (1 + \xi Q + \bar{\xi} \bar{Q}) A \quad \text{so} \quad \delta_\xi A = (\xi Q + \bar{\xi} \bar{Q}) A$$

SIM: $\delta_\xi \gamma = (\xi Q + \bar{\xi} \bar{Q}) \gamma$

SUCCESSIONAL TRANS?

$\delta_\eta \delta_\xi - \delta_\xi \delta_\eta ?$
 $(\delta_\eta \delta_\xi - \delta_\xi \delta_\eta) = -(\xi Q + \bar{\xi} \bar{Q})(\eta Q + \bar{\eta} \bar{Q}) + (\eta Q + \bar{\eta} \bar{Q})(\xi Q + \bar{\xi} \bar{Q})$

Q: why do all other terms die?
A: $\xi Q, Q \bar{Q}, \bar{Q} \bar{Q}$

$\rightarrow -(\xi Q \bar{\eta} \bar{Q} + \bar{\xi} \bar{Q} \eta Q - \eta Q \bar{\xi} \bar{Q} - \bar{\eta} \bar{Q} \xi Q)$
 $= -(\xi^\alpha Q_\alpha \bar{\eta}_\alpha \bar{Q}^\alpha - \bar{\eta}_\alpha \bar{Q}^\alpha \xi^\alpha Q_\alpha + \bar{\xi}_\alpha \bar{Q}^\alpha \eta^\alpha Q_\alpha - \eta^\alpha Q_\alpha \bar{\xi}_\alpha \bar{Q}^\alpha)$
 $= -(-\xi^\alpha \bar{\eta}^\alpha Q_\alpha \bar{Q}^\alpha - \xi^\alpha \bar{\eta}^\alpha \bar{Q}^\alpha Q_\alpha + \eta^\alpha \bar{\xi}^\alpha \bar{Q}^\alpha Q_\alpha - \eta^\alpha \bar{\xi}^\alpha Q_\alpha \bar{Q}^\alpha)$
 $= -(\xi^\alpha \bar{\eta}^\alpha \{Q_\alpha, \bar{Q}^\alpha\} - \eta^\alpha \bar{\xi}^\alpha \{Q_\alpha, \bar{Q}^\alpha\}) = 2(\eta^\alpha \bar{\xi}^\alpha - \xi^\alpha \bar{\eta}^\alpha) P_m$

WILL NEED, CALL \circledast

Q: SO HOW DOES SUSY ACT ON FIELDS?

MASS DIMENSION

$$\{Q, \bar{Q}\} \sim \sigma^m P_m \Rightarrow [Q] = [\bar{Q}] = \frac{1}{2} \text{ SINCE } [P^m] = 1$$

$$\therefore [S_3 A] = \frac{3}{2} \quad \because [S_3 S_3 A] = 2 \quad \text{FOR SCALAR } A$$

DEFINE ψ ST. IT'S THE $\text{DIM-}\frac{3}{2}$ FIELD $\propto S_3 A$

IE $S_3 A = \sqrt{2} \psi$ (Factor of $\sqrt{2}$ for convenience)

THEN $S_3 \psi$ HAS $[S_3 \psi] = 2$ \therefore

CAN BE MADE FROM SOME NEW F | $[F] = 2$,

OR $\partial_m A$ SINCE $[\partial_m A] = 2$

IE $S_3 \psi = i\sqrt{2} \sigma^m \bar{\epsilon} \partial_m A + \sqrt{2} \bar{\epsilon} F$

\uparrow chosen, by freedom to def. F

Hmwk: i) ψ fixed by $\psi \circledast$ satisfied

ii) $S_3 F$ fixed by \circledast on F , in $(S_3 S_3 - S_3 S_3) F$

SUMMARY

$$\boxed{\begin{aligned} S_3 A &= \sqrt{2} \psi \\ S_3 \psi &= i\sqrt{2} \sigma^m \bar{\epsilon} \partial_m A + \sqrt{2} \bar{\epsilon} F \\ S_3 F &= i\sqrt{2} \bar{\epsilon} \sigma^m \partial_m \psi \end{aligned}}$$

BOS \Rightarrow FER

FER \Rightarrow BOS + AUX F

AUX F \Rightarrow FER

PURPOSE OF F: ENSURE SUSY- \mathcal{L} OFF-SHELL, IE
w/o USING EOM

INVARIANT ACTION $\mathcal{L}_0 = i \partial_n \bar{\psi} \bar{\sigma}^n \psi + A^* \square A + F^* F + m(AF + A^* F^* - \frac{1}{2} \bar{\psi} \psi - \frac{1}{2} \bar{F} \bar{F})$

\hookrightarrow up to total derivs

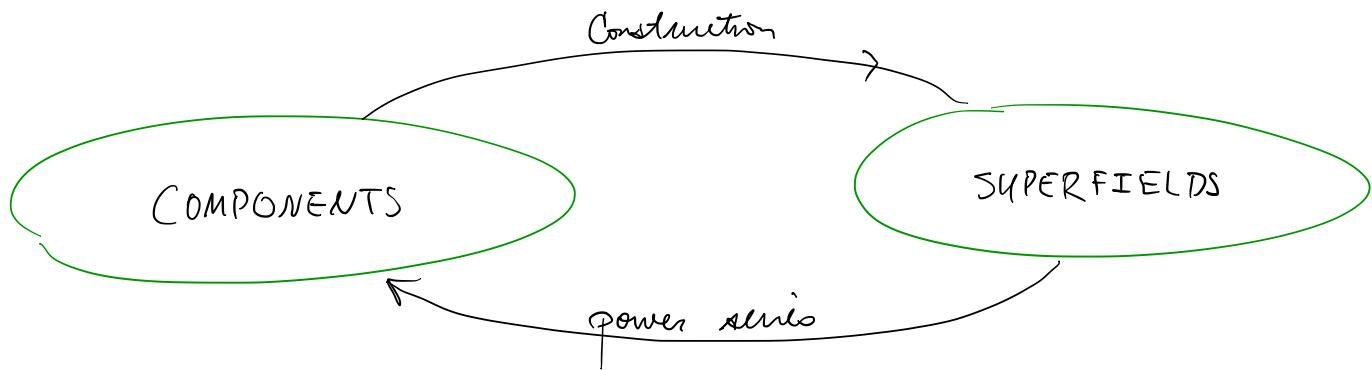
Q: TERM PURPOSES?

Q: $\exists \psi \psi$ MASS TERM. A MASS TERM!? NO. Hmwk.

Q: HOW TO SYSTEMATICALLY WRITE SUSY-INV \mathcal{L}' ?

SUPERFIELDS

POINT: PACKAGE COMPS IN NICE SUSY OBJECTS



SUPER ALGEBRA: $1 + i(-x^m P_m + \theta Q + \bar{\theta} \bar{Q}) + \dots$

SUPERGROUP: $G(x, \theta, \bar{\theta}) = e^{i\{-x^m P_m + \theta Q + \bar{\theta} \bar{Q}\}}$

↑ group element.

Q: what is $SU(n)$ analogy?

"SUPERSPACE": COORDS $(x, \theta, \bar{\theta})$ (indices implicit)

ASIDE: $\theta^\alpha \theta^\beta \neq 0$ (convince yourself)

BUT $\theta^\alpha \theta^\beta \theta^\gamma = 0$!

PROOF $\alpha, \beta, \gamma \in \{1, 2\}$,

WRITE $\theta^\alpha \theta^\beta \theta^\gamma = \alpha \beta \gamma$, $2^3 = 8$ POSS

$(\theta')^2 = (\bar{\theta}')^2 = 0$ (due to antisym).

$$\Rightarrow 111 = 0 \quad 112 = 0 \quad 211 = 0$$

$$222 = 0 \quad 221 = 0 \quad 122 = 0$$

$$2 \text{ MORE} \quad 121 = -112 = 0, \quad 212 = -221 = 0 \quad \checkmark$$

CONC: ANYTHING w/ $> 2 \theta'$ or $> 2 \bar{\theta}'$ VANISHES

SUPERFIELD: LABEL Lorentz-type!

$$F(x, \theta, \bar{\theta}) \stackrel{\text{POW IN } \theta, \bar{\theta}}{=} f(x) + \overset{\text{SC}}{\theta} \phi(x) + \overset{\text{SP}}{\bar{\theta}} \overset{\text{SP}}{\bar{x}}(x)$$

$$+ \theta \theta \overset{\text{SC}}{m}(x) + \bar{\theta} \bar{\theta} \overset{\text{SC}}{n}(x) + \theta \sigma^m \bar{\theta} \overset{\text{VEC}}{v}_m(x)$$

$$+ \theta \theta \bar{\theta} \overset{\text{SP}}{\lambda}(x) + \bar{\theta} \bar{\theta} \theta \overset{\text{SP}}{\gamma}(x) + \theta \theta \bar{\theta} \bar{\theta} \overset{\text{SC}}{d}(x)$$

OUR ASIDE \Rightarrow ALL HIGHER POWERS VANISH!

SUSY TRANS

$$F \rightarrow e^{i(\theta Q + \bar{\theta} \bar{Q})} F \simeq F + i \underbrace{(\theta Q + \bar{\theta} \bar{Q})}_{=: S_\theta F} F + \dots$$

Q : GO ON FOREVER
OR TRUNCATE?

$$S_\xi F = S_\xi f + \theta S_\xi \phi + \bar{\theta} S_\xi \bar{x} + \theta \theta S_\xi m + \bar{\theta} \bar{\theta} S_\xi n$$

$$+ \theta \sigma^m \bar{\theta} S_\xi v_m + \theta \theta \bar{\theta} S_\xi \bar{\lambda} + \bar{\theta} \bar{\theta} \theta S_\xi \gamma + \theta \theta \bar{\theta} \bar{\theta} S_\xi d = (\xi Q + \bar{\xi} \bar{Q}) F$$

$Q = ?$ ON F ? SOME DIFFERENTIAL OP

REQUIRE: $(S_\eta S_\xi - S_\xi S_\eta) F = -2i (\eta \sigma^m \bar{\xi} - \xi \sigma^m \bar{\eta}) \partial_m F$

\uparrow remember: \circledast from few pages back

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i \sigma_{\alpha\dot{\alpha}}{}^m \bar{\theta}^{\dot{\alpha}} \partial_m \quad \bar{Q}_{\dot{\alpha}} = - \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \theta^\alpha \sigma_{\alpha\dot{\alpha}}{}^m \partial_m$$

HWK: show these give \circledast

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2i \underbrace{\sigma_{\alpha\dot{\alpha}}{}^m}_{\text{P}_m} \partial_m \quad \{Q, Q\} = \{\bar{Q}, \bar{Q}\} = 0$$

$P_m = i \partial_m$
 \Rightarrow gen SUSY algebra

\exists OTHER NICE OPERATORS (will see why)

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma_{\alpha\dot{\alpha}}^m \bar{\theta}^{\dot{\alpha}} \partial_m \quad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i \theta^\alpha \sigma_{\alpha\dot{\alpha}}^m \partial_m$$

$\{ D_\alpha, \bar{D}_{\dot{\alpha}} \} = -2i \sigma_{\alpha\dot{\alpha}}^m \partial_m$ $\{ D, D \} = \{ \bar{D}, \bar{D} \} = \{ D, Q \} = \{ D, \bar{Q} \} = \{ \bar{D}, Q \} = \{ \bar{D}, \bar{Q} \} = 0$

only difference from $\{ Q, \bar{Q} \}$

CHIRAL SUPERFIELDS

DEF'N: A SUPERFIELD Φ SATIS. $\bar{D}_\alpha \Phi = 0$ IS A CHIRAL SUPERFIELD

NICE WAY TO EXPRESS

CHANGE OF COORDS

$$y^m = x^m + i\theta \sigma^m \bar{\theta}$$

$$\bar{D}_\alpha = -\frac{\partial}{\partial \bar{\theta}} - i\theta^\alpha \sigma_{\alpha\dot{\alpha}} \partial_{\dot{\alpha}}$$

$$\begin{aligned} \Rightarrow \bar{D}_\alpha (x^m + i\theta \sigma^m \bar{\theta}) &= -i\theta^\alpha \sigma_{\alpha\dot{\alpha}} \partial_{\dot{\alpha}} x^m - \frac{\partial}{\partial \bar{\theta}} (i\theta^\beta \sigma_{\beta\dot{\beta}} \bar{\theta}^{\dot{\beta}}) \\ &= -i\theta^\alpha \sigma_{\alpha\dot{\alpha}} \delta_{\dot{\alpha}}^m + i\theta^\beta \sigma_{\beta\dot{\beta}} \delta_{\dot{\beta}}^{\dot{m}} = 0 \end{aligned}$$

so $\boxed{\bar{D}_\alpha(y^m) = \bar{D}_\alpha \theta = 0}$

why this one?

\therefore ANY $f(y^m, \theta)$ IS CH. SUP. Q: why?

WRITE $\boxed{\Phi = A(y) + \sqrt{2}\theta \psi(y) + \theta\bar{\theta} F(y)}$

CPLX SC = 2 BOSONS

2 CMPT SPINOR = 2 FERMIONS

note: most general soln
to $\bar{D}_\alpha \Phi = 0$

ALSO: $\Rightarrow \bar{\Phi} = A(x) + i\theta \sigma^m \bar{\theta} \partial_m A(x) + \frac{1}{4}\theta\bar{\theta} \bar{\theta}\bar{\theta} \square A(x)$
 $+ \sqrt{2}\theta \psi(x) - \frac{i}{\sqrt{2}}\theta\bar{\theta} \partial_m \psi(x) \sigma^m \bar{\theta} + \theta\bar{\theta} F(x)$

$\bar{\Phi}^+$: SATISFIES $D_\alpha \bar{\Phi}^+ = 0$ (ANTI-CHIRAL SUP)

$$\bar{\Phi}^+ = A^*(y^+) + \sqrt{2}\bar{\theta} \bar{\psi}(y^+) + \bar{\theta}\bar{\theta} F^*(y^+)$$

TRANSFORMATION CHECKS

(whole point of superfields
was to show trans. be automatic,
need to check!)

$$\text{GENERAL : } \mathcal{D}_{\bar{\theta}} F(x, \theta, \bar{\theta}) = (\nabla Q + \bar{\nabla} \bar{Q}) F$$

$$\therefore \text{CHIRAL: } S_3 F(y, \theta) = \left(\xi^{\alpha} Q_{\alpha} + \bar{\xi}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}} \right) F(y, \theta) \quad (\text{recall } y^m = x^m + i \theta \sigma^m \bar{\theta})$$

$$\text{IN } y - \text{VARS} \quad Q_\alpha = \frac{\partial}{\partial \theta^\alpha} \quad \bar{Q}_\alpha = - \frac{\partial}{\partial \bar{\theta}^\alpha} + 2i \theta^\alpha \tau_{\alpha\bar{\alpha}}^m \partial_m$$

$$\begin{aligned}
 \text{THEN } S_{\xi} \bar{\Phi} &= (\xi Q + \bar{\xi} \bar{Q})(A + \sqrt{2}\theta Y + \bar{\theta} \bar{Y}) \\
 &= \xi(\sqrt{2}Y_x + 2\theta_x F) - \bar{\xi}^* 2i\theta^* \sigma_{xx}^m \partial_m (A + \sqrt{2}\theta Y + 0) \\
 &\quad \uparrow \text{due to up-down change} \\
 &= \sqrt{2} \xi Y + \sqrt{2} \theta^* (\sqrt{2} \xi_x F + i\sqrt{2} \sigma_{xx}^m \bar{\xi}^* \partial_m A) + 2\sqrt{2} i\theta^* \sigma_{xx}^m \bar{\xi}^* \theta \partial_m Y \\
 &\quad \underbrace{\qquad\qquad\qquad}_{= i\sqrt{2} \theta \theta^* \bar{\xi}^* \sigma^m \partial_m Y} \\
 &\quad \text{some work!} \\
 &= \underbrace{\sqrt{2} \xi Y}_{= S_{\xi} A} + \underbrace{\sqrt{2} \theta^* (\sqrt{2} \xi_x F + i\sqrt{2} \sigma^m \bar{\xi} \partial_m A)}_{= S_{\xi} Y} + \underbrace{\theta \theta^* (i\sqrt{2} \bar{\xi} \bar{\sigma}^m \partial_m Y)}_{= S_{\xi} F}
 \end{aligned}$$

NOTE: MATCHES KNOWN COMPONENT FIELD TRANS!

SUSY L-TERMS FROM CHIRAL SUPERFIELDS

- i) NOTE: $\theta\bar{\theta}$ COMPONENT OF CHIRAL SUPERFIELD TRANS TO TOTAL DERIV
- ii) PRODUCTS OF CHIRAL SUP. ARE CHIRAL SUP.

$$\Rightarrow W(\Phi) = \sum_{k=1}^N g_{i_1 \dots i_k}^{(k)} \Phi_{i_1} \dots \Phi_{i_k}$$

$$= g_{i_1}^{(1)} \Phi_{i_1} + g_{i_1 i_2}^{(2)} \Phi_{i_1} \bar{\Phi}_{i_2} + g_{i_1 i_2 i_3}^{(3)} \Phi_{i_1} \bar{\Phi}_{i_2} \bar{\Phi}_{i_3} + \dots$$

IS CHIRAL, $\therefore \theta\bar{\theta}$ COMP. TRANS TO TOT. DER.

? CAN BE PUT IN SUSY LAGRANGIAN!

DEF'N: $w(\Phi)$ IS THE SUPERPOTENTIAL.

PROPERTIES: i) HOLOMORPHIC, IE FN OF $\bar{\Phi}^j$, NOT Φ^j 'S
 ii) NOT RENORMALIZED IN PERT. THY

FIRST FEW TERMS

$$i) \bar{\Phi}_i \bar{\Phi}_j = \theta\bar{\theta} [A_i F_j + A_j F_i - \gamma_i \gamma_j] + \text{NON-}\theta\bar{\theta}$$

\downarrow mass term!

$$iii) \bar{\Phi}_i \bar{\Phi}_j \bar{\Phi}_k = \theta\bar{\theta} [F_i A_j A_k - \gamma_i \gamma_j A_k + \text{cyclic permutations}]$$

\uparrow Yukawa coupling

CONC: $w(\Phi)$ CAN GIVE MASS TERMS ? YUKAWA COUPLINGS.

Q: WHAT'S MISSING? KINETIC TERMS!

\rightarrow will easily understand once we have vector superfields

FACT: $\bar{\Phi}_i \bar{\Phi}_j^+$ HAS $\bar{\theta} \bar{\theta} \bar{\theta} \bar{\theta}$ TERM TRANS INTO TOT DER.

\therefore CAN USE IN \mathcal{L} !

$$\bar{\Phi}_i^+ \bar{\Phi}_j = \bar{\theta} \bar{\theta} \bar{\theta} \bar{\theta} \left[F_i^* F_j + \frac{1}{4} A_i^{* \alpha} \square A_j^\alpha + \frac{1}{4} \square A_i^* A_j - \frac{1}{2} \partial_m A_i^* \bar{\sigma}^m A_j + \frac{i}{2} \partial_m \bar{\psi}_i \bar{\sigma}^m \bar{\psi}_j - \frac{i}{2} \bar{\psi}_i \bar{\sigma}^m \partial_m \bar{\psi}_j \right] + \dots$$

function of x

HWK

NOTE: KIN TERMS FOR A 'S, ψ 'S, NOT F 'S

IF $i=j$

PUT IT TOGETHER

{ most general SUSY renorm. Lag.
involving only chiral sup. }

$$\mathcal{L} = \bar{\Phi}_i^+ \bar{\Phi}_i \Big|_{\bar{\theta} \bar{\theta} \bar{\theta} \bar{\theta} \text{ comp}} + \left[(\lambda_i \bar{\Phi}_i + \frac{1}{2} m_{ij} \bar{\Phi}_i \bar{\Phi}_j + \frac{1}{3} g_{ijk} \bar{\Phi}_i \bar{\Phi}_j \bar{\Phi}_k) \Big|_{\bar{\theta} \bar{\theta}} \right] + h.c.$$

$$\begin{aligned} \mathcal{L}_{\text{comp}} &= i \partial_m \bar{\psi}_i \bar{\sigma}^m \psi_i + A_i^* \square A_i + F_i^* F_i \\ &+ [m_{ij} (A_i F_j - \frac{1}{2} \bar{\psi}_i \psi_j) + g_{ijk} (A_i A_j F_k - \bar{\psi}_i \bar{\psi}_j A_k) + \lambda_i F_i + h.c.] \end{aligned}$$

SCALAR POTENTIAL

AUXILIARY FIELDS F

$$\text{EOM: } 0 = \frac{\delta \mathcal{L}}{\delta F_k^*} = F_k + \lambda_k^* + m_k^* A_i^* + g_{ijk}^* A_i^* A_j^* \quad \text{SIM FOR } \frac{\delta \mathcal{L}}{\delta F_k} = 0$$

NOTE: i) SOLVE $\Rightarrow F_k \in F_k^*$ FUNC(SCALARS)

ii) PLUG INTO $F_k^* F_k \Rightarrow \boxed{V(A, A^*)}$

\hookrightarrow Language F-term
Scalar potential

iii) SINCE $V(A, A^*)$ FROM $F_k^* F_k$,

HAVE ABS. MIN. IF $F_k = 0 \forall k$

R-SYMMETRIES SYMMS THAT ROTATE Θ 'S (alt. susy charges)

SUPPOSE $\Theta \mapsto e^{-i\alpha} \Theta$, $\therefore \Theta \Theta \mapsto e^{-2i\alpha} \Theta \Theta$

THEN $\Phi_i \bar{\Phi}_j \theta \theta$ INV IF $\Phi_i \mapsto e^{g_{\Phi_i} i\alpha} \bar{\Phi}_i$
 $\bar{\Phi}_j \mapsto e^{g_{\bar{\Phi}_j} i\alpha} \bar{\Phi}_j$

AND $g_{\Phi_i} + g_{\bar{\Phi}_j} = 2$

Language: $g_{\Phi_i}, g_{\bar{\Phi}_j}$ are R-charges

BUT $\Phi_i \bar{\Phi}_j \bar{\Phi}_k \theta \theta$ REQ'S $g_{\Phi_i} + g_{\bar{\Phi}_j} + g_{\bar{\Phi}_k} = 2$.

OVERRCONSTRAINED SYSTEM? NO R-SYMMETRY

EXAMPLE: 2 CH. SUPS. Φ_1 Φ_2

$$W(\Phi) = m_1 \Phi_1^2 + m_2 \Phi_2^2 + g \Phi_1 \Phi_1 \Phi_2$$
$$\begin{array}{l} \text{---} \\ g_1 + g_1 = 2 \\ \text{---} \end{array} \quad \begin{array}{l} \text{---} \\ g_2 + g_2 = 2 \\ \text{---} \end{array} \quad \begin{array}{l} \text{---} \\ g_1 + g_1 + g_2 = 2 \\ \text{---} \end{array}$$

NO SOL'N \Rightarrow NO R-SYMM

OIOH: IF $g \rightarrow 0$, $g_1 = g_2 = 1$ SOL'N \Rightarrow R-SYMM

VECTOR SUPERFIELDS

DEF'N: A SUPERFIELD V SATIS. $V^\dagger = V$ IS A VECTOR SUPERFIELD

FACT: $\theta\bar{\theta}\bar{\theta}\bar{\theta}$ COMPONENT OF V TRANSFORMS INTO TOT DER.

AHA! IF Φ CHIRAL, $(\bar{\Phi}^+\Phi)^+ = \bar{\Phi}\bar{\Phi}^+ = \bar{\Phi}^+\bar{\Phi}$
 $\Rightarrow \bar{\Phi}^+\bar{\Phi}$ VECTOR \Rightarrow USE $\theta\bar{\theta}\bar{\theta}\bar{\theta}$ COMP OF $\bar{\Phi}^+\bar{\Phi}$ IN \mathcal{L}

EXPANSION

$$\begin{aligned} V(x, \theta, \bar{\theta}) = & C + i\theta x - i\bar{\theta}\bar{x} + \frac{i}{2}\theta\theta[M+iN] - \frac{i}{2}\bar{\theta}\bar{\theta}[M-iN] \\ & - \theta\sigma^m\bar{\theta}v_m + i\theta\theta\bar{\theta}\left[\bar{x} + \frac{i}{2}\bar{\sigma}^m\partial_m x\right] - i\bar{\theta}\bar{\theta}\theta\left[x + \frac{i}{2}\sigma^m\partial_m \bar{x}\right] \\ & + \frac{i}{2}\theta\theta\bar{\theta}\bar{\theta}[D + \frac{1}{2}\square C] \end{aligned}$$

NOTE i) $V^\dagger = V$ REQ C, D, M, N, v_m REAL

ii) \exists VECTOR FIELD v_m (hence the name)

GAUGE TRANSFORMATIONS

WANT: $V \rightarrow V + \text{FUNC(SUPER-PARMS)}$

S.T. $v_m \rightarrow v_m + \partial_m \alpha$ Q: why focusing on this comp?

RECALL IN x-COORDS $\bar{\Phi} = i\theta\sigma^m\bar{\theta}\partial_m A + \dots$

$$\bar{\Phi}^+ = -i\theta\sigma^m\bar{\theta}\partial_m A^* + \dots$$

$$\therefore V \rightarrow V + \bar{\Phi} + \bar{\Phi}^+$$

$$\text{HAS } v_m \rightarrow v_m - i\partial_m(A - A^*) = v_m - 2i\partial_m \text{Im}(A) = v_m + \partial_m \alpha \quad \checkmark$$

$$\alpha := -2i\text{Im}(A)$$

FULL TRANS. IF $V \rightarrow V + \bar{\Phi} + \bar{\Phi}^+$

THEN $C \rightarrow C + A + A^*$

$\chi \rightarrow \chi - i\sqrt{2}\gamma$

$M+iN \rightarrow M+iN-2iF$

$V_m \rightarrow V_m - i\partial_m(A - A^*)$

$\lambda \rightarrow \lambda$

$D \rightarrow D$

WEISS-ZUMINO GAUGE

CHOOSE TRANS PARAMS A, γ, F | $C = \chi = \mu = N = 0$

THEN
$$V_{wz} = -\theta \sigma^m \bar{\theta} v_m + i\theta \bar{\theta} \bar{\lambda} - i\bar{\theta} \bar{\theta} \theta \chi + \frac{1}{2} \theta \bar{\theta} \bar{\theta} \bar{\theta} D$$

SPINOR, 2 FERM, GAUGINOS

VECTOR FIELD, 2 BOSONS

AUXILIARY

NOTE $V^2 = \theta \sigma^m \bar{\theta} v_m \theta \sigma^m \bar{\theta} v_m = -\frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} v^m v_m$

$V^3 = 0$ Q: why do tons of terms drop?

FIELD STRENGTHS

(we see V_m , but where is $F_{\mu\nu}$?)

$$W_\alpha := -\frac{1}{4} \bar{D} \bar{D} D_\alpha V$$

$$\bar{W}_\alpha := -\frac{1}{4} D D \bar{D}_\alpha V$$

CLAIM: THESE ARE i) CHIRAL ? ii) GAUGE INV.

$$i) \bar{D}_\beta W_\alpha = 3 \bar{D}'_\alpha (\) = 0 \quad D_\beta \bar{W}_\alpha = 3 D' \bar{\beta} (\) = 0 \quad \checkmark$$

$$ii) W_\alpha \mapsto -\frac{1}{4} \bar{D} \bar{D} D_\alpha (V + \Phi + \bar{\Phi}^+) = -\frac{1}{4} \bar{D} \bar{D} D_\alpha (V + \Phi)$$

$$= W_\alpha - \frac{1}{4} \bar{D} \bar{D} D_\alpha \Phi - \underbrace{\frac{1}{4} \bar{D} D_\alpha \bar{D} \bar{\Phi}}_{\text{adding } 0 \text{ since } \bar{D} \bar{\Phi} = 0} = W_\alpha - \frac{1}{4} \bar{D} \{ \bar{D}, D_\alpha \} \bar{\Phi} = W_\alpha \quad \checkmark$$

SIM FOR \bar{W}_α

IN W -Z GAUGE

$$W_\alpha = -i \lambda_\alpha + \left[S_\alpha^\beta - \frac{i}{2} (\bar{\sigma}^m \bar{\sigma}^n)_\alpha^\beta (\partial_m v_n - \partial_n v_m) \right] \partial_\beta + \theta \sigma_{\alpha i}{}^m \partial_m \bar{\lambda}^i$$

$$\bar{W}_\alpha = i \bar{\lambda}_\alpha + \left[\bar{S}_{\dot{\alpha}}{}^{\dot{\beta}} D + \frac{i}{2} \epsilon_{\dot{\alpha}\dot{\beta}} (\bar{\sigma}^m \sigma^n)^{\dot{\alpha}}{}_{\dot{\beta}} (\partial_m v_n - \partial_n v_m) \right] \bar{\partial}^{\dot{\beta}} - \epsilon_{\dot{\alpha}\dot{\beta}} \bar{\theta} \bar{\theta} \bar{\sigma}^{m\dot{\beta}} \partial_m \lambda_\alpha$$

HUWK

NOTE

$$D W = \bar{D} \bar{W}$$

$W^\alpha W_\alpha |_{\partial\theta}$ ALLOWED

LAGRANGIANS W^α CHIRAL \Rightarrow $W^\alpha W_\alpha$ CHIRAL \Rightarrow IN $\cancel{\times}$

$W^\alpha W_\alpha$ not for F^{mn}

$$W^\alpha W_\alpha |_{\partial\theta} = -2i \lambda \sigma^m \partial_m \bar{\lambda} - \frac{1}{2} V^{mn} V_{mn} + D^2 + \frac{i}{4} V^{mn} V^{lk} \epsilon_{mnlk}$$

$$\mathcal{L} = \frac{1}{4} \left(w^\alpha w_\alpha |_{\theta\bar{\theta}} + \bar{w}_i \bar{w}^i |_{\bar{\theta}\bar{\theta}} \right)$$

↓

$$\int d^4x \quad \mathcal{L}_{\text{component}} = \int d^4x \quad \left\{ \frac{1}{2} D^2 - \frac{1}{4} V^{mn} V_{mn} - i \lambda \sigma^m \partial_m \bar{\chi} \right\}$$

↓ some partial integ

1 AUX FIELD

2 PROP. BOSONS hel ± 1

2 PROP. FERMIS hel $\pm \frac{1}{2}$

GAUGE INV'T INTERACTIONS (in superfield form.)

FIRST: ABELIAN INT

THEN: NON-ABELIAN INT

ABELIAN INTERACTIONS

SIMP. METHOD: GLOBAL \rightarrow LOCAL (remind me what this means?)

GLOBAL U(1)

$$\bar{\Phi}_\ell^i = e^{-it_\ell \lambda} \bar{\Phi}_\ell, \quad t_\ell, \lambda \in \mathbb{R}$$

$$\mathcal{L} = \mathcal{L}_{K.E.} + \mathcal{L}_{P.E.}$$

$$\mathcal{L}_{K.E.} = \bar{\Phi}_\ell^+ \bar{\Phi}_\ell \Big|_{\theta \bar{\theta} \bar{\theta} \bar{\theta}}$$

$$\mathcal{L}_{P.E.} = \underbrace{\left[\frac{1}{2} m_{ij} \bar{\Phi}_i^+ \bar{\Phi}_j + \frac{1}{3} g_{ijk} \bar{\Phi}_i^+ \bar{\Phi}_j \bar{\Phi}_k \right]}_{\text{SUPER POT. } W(\bar{\Phi})!} \Big|_{\theta \bar{\theta}} + \text{h.c.}$$

INV'C REQ'S

$$m_{ij} = 0 \quad \text{IF} \quad t_i + t_j \neq 0$$

$$g_{ijk} = 0 \quad \text{IF} \quad t_i + t_j + t_k \neq 0$$

LOCAL U(1) $\lambda \mapsto \Lambda(x, \theta, \bar{\theta})$

$$\bar{\Phi}_\ell^i = e^{-it_\ell \lambda} \bar{\Phi}_\ell$$

$$O \stackrel{\Phi_{ch.}}{=} \bar{D}_x \bar{\Phi}_\ell^i = \bar{D}_x \bar{\Phi}_\ell^i e^{-it_\ell \lambda} - it_\ell \bar{D}_x \Lambda \bar{\Phi}_\ell$$

$$\Rightarrow \bar{D}_x \Lambda = 0, \quad \boxed{\text{GAUGE PARAM } \Lambda \text{ IS CH. SUP}}$$

NOTE $\mathcal{L}_{P.E.}$ INV'T! Q: Why?

HOWEVER $\mathcal{L}_{K.E.}$ NOT INV'T SINCE

$$\bar{\Phi}_\ell^i + \bar{\Phi}_\ell^i = \bar{\Phi}_\ell^i \bar{\Phi}_\ell^i e^{it_\ell (\Lambda^+ - \Lambda)}$$

Q: what would we do in the non-SUSY case at this juncture?

ADD VEC. SUP. V S.T. $V' = \underbrace{V + X + X^+}_{\text{recall: this is gauge trans form of } V. \text{ sup, where } X \text{ is ch. sup.}}$

WRITE $X = i\Lambda$ THEN $V' = V + i(\Lambda - \Lambda^+)$

$$\begin{aligned} \text{THEN } \bar{\Phi}_l^+ e^{t_\ell V} \bar{\Phi}_l &\mapsto \bar{\Phi}_l^+ e^{it_\ell \Lambda^+} e^{t_\ell V} e^{it_\ell (\Lambda - \Lambda^+)} e^{-it_\ell \Lambda} \bar{\Phi}_l \\ &= \bar{\Phi}_l^+ e^{t_\ell V} \bar{\Phi}_l \quad \underline{\text{INV!}} \end{aligned}$$

ADD KIN. TERMS FOR V

$$\Rightarrow \boxed{\mathcal{L} = \frac{1}{4} (W^\alpha W_\alpha|_{\theta\theta} + \bar{W}_\beta \bar{W}^\beta|_{\theta\theta}) + \bar{\Phi}_l^+ e^{t_\ell V} \bar{\Phi}_l|_{\bar{\theta}\bar{\theta}\bar{\theta}\bar{\theta}} + \left[\frac{1}{2} (m_{ij} \bar{\Phi}_i \bar{\Phi}_j + \frac{1}{3} g_{ijk} \bar{\Phi}_i \bar{\Phi}_j \bar{\Phi}_k) \Big|_{\bar{\theta}\bar{\theta}} + \text{h.c.} \right]}$$

(*)

IN W-Z GAUGE $V^3 = 0$, GET

$$\begin{aligned} \bar{\Phi}_l^+ e^{t_\ell V} \bar{\Phi}_l|_{\theta\theta\bar{\theta}\bar{\theta}} &= F^* F + A \square A^* + i \partial_n \bar{\psi} \bar{\gamma}^n \psi \quad \left. \right\} \text{kinetic terms} \\ &+ t v^n \left(\frac{1}{2} \bar{\psi} \bar{\gamma}^n \psi + \frac{i}{2} A^* \partial_n A - \frac{i}{2} \partial_n A^* A \right) \\ &- \frac{i}{\sqrt{2}} t (A \bar{\lambda} \bar{\psi} - A^* \lambda \psi) + \frac{1}{2} (t D - \frac{1}{2} t^2 v_n v^n) A^* A \quad \left. \right\} \text{couplings to gauge bos. \& gauginos} \end{aligned}$$

SUPER QED super-electron & super-position what physically does this theory have?

→ ABOVE, BUT 2 CHIRAL SUP $\bar{\Phi}_+$ $\bar{\Phi}_-$

$$\text{WHERE } \bar{\Phi}'_+ = e^{-ie\Lambda} \bar{\Phi}_+ \quad \bar{\Phi}'_- = e^{ie\Lambda} \bar{\Phi}_-$$

$$\mathcal{L}_{\text{SUPER-QED}} \supset -m (\bar{\psi}_+ \psi_+ + \bar{\psi}_- \psi_-) = -m \bar{\Psi}_{\text{Dir}} \Psi_{\text{Dir}}$$

point ψ_+, ψ_- can be rewritten as
four-comp Dual spinor

NON-ABELIANINTERACTIONS

HAD: $\bar{\Phi}_e^i = e^{-it_e \Lambda} \bar{\Phi}_e$ (abelian case)

NOW: $\bar{\Phi}_i^i = e^{-i\Lambda_{ij}} \bar{\Phi}_j$ $\bar{\Phi}_j^+ = \bar{\Phi}_i^+ e^{i\Lambda_{ij}^+}$ (non-abelian gen)

WHERE Λ IS MATRIX $\Lambda_{ij} = T_{ij}^a \Lambda_a$

T_{ij}^a MATRIX-REP OF GENS OF GAUGE GROUP
*remind me:
what does this mean?*

NORM: $\text{Tr } T^a T^b = k \delta^{ab} \quad k > 0$

$$[T^a, T^b] = i \epsilon^{abc} T^c$$

NON-ABELIAN LAG.

\mathcal{L} IS INV'T WITH A FEW MOD'S.

i) $e^{V_{ij}^i} = e^{-i\Lambda_{ii}^+} V_{ij} e^{i\Lambda_{jj}^+}$

$$V_{ij} = T_{ij}^a V_a \quad (\text{how is this exp like in the non-SUSY theory?})$$

ASIDE: $V^i = V + i \underbrace{(\Lambda - \Lambda^\dagger)}_{\text{leading term}} + \dots$

V -INDEP
 \Rightarrow can think of the w 's

\exists NON-AB. w ? GAUGE w/ $V^3 = 0$

$$(i) \quad W_\alpha = -\frac{1}{4} \bar{\partial} \bar{\partial} e^{-V} D_\alpha e^V$$

HMK: This and invariance of
kinetic term

$$W'_\alpha = e^{-i\Lambda} W_\alpha e^{i\Lambda}$$

Summary

$$\boxed{\begin{aligned} \mathcal{L} = & \frac{1}{16k^2} \text{Tr} \left(W^\alpha W_\alpha /_{\partial\bar{\partial}} + \bar{W}_\alpha \bar{W}^\alpha /_{\bar{\partial}\bar{\partial}} \right) + \bar{\Phi}^+ c^V \Phi /_{\partial\bar{\partial}\bar{\partial}\bar{\partial}} \\ & + \left[\frac{1}{2} m_{ij} \bar{\Phi}_i \bar{\Phi}_j + \frac{1}{3} g_{ijk} \bar{\Phi}_i \bar{\Phi}_j \bar{\Phi}_k \right] /_{\partial\bar{\partial}} + h.c. \end{aligned}}$$

GENERAL N=1 D=4 SUSY L

REWRITE REPLACE $|_{\theta\bar{\theta}} \mapsto \int d^2\theta$, $|_{\theta\bar{\theta}\bar{\theta}\bar{\theta}} \mapsto \int d^2\theta d^2\bar{\theta}$

$$\boxed{L = \int d^2\theta d^2\bar{\theta} K(\Phi, \bar{\Phi}) + \left[\int d^2\theta G(w) + \int d^2\bar{\theta} W(\bar{\Phi}) + h.c \right]}$$

K = KÄHLER POTENTIAL

w = SUPERPOTENTIAL

G = GAUGE KINETIC FUNCTION

EACH CAN RECEIVE CERTAIN CORRECTIONS

PREV EX: $K = \bar{\Phi}^\dagger e^v \Phi$ (canonical kin terms)

$w = \frac{1}{2} m_{ij} \bar{\Phi}_i \bar{\Phi}_j + \frac{1}{3} g_{ijk} \bar{\Phi}_i \bar{\Phi}_j \bar{\Phi}_k$ (renormalizable. Could have higher!)

$G = \text{Tr } w_i w^i$ (canonical kin terms)

SCALAR POTENTIALS

$V = V_F + V_D$ N.B. TWO POS-DEF PIECES, ONE FOR EACH AUX. FIELD TYPE.

$V_F = F_k^* F_k = w_i^* w_i$ SINCE EOM $\Rightarrow F_k = -w_i^*$ $F_k^* = -w_i$
 $w_i = \frac{\partial w}{\partial \Phi}|_{\Phi^0}$ (in se. piece)

$V_D = \frac{1}{2} D^a D^a = \frac{1}{2} g_i^2 (\phi^* T^a \phi)^2$ $D^a = -g(\phi^* T^a \phi)$

\exists abelian subtlety:
Fayet-Iliopoulos term

SPONTANEOUS SUSY BREAKING

Q: Why?

A1: BECAUSE you CAN

A2: NO MASS-DEGEN. BOSE-FERMI PAIRS IN SM

(i.e., if \exists susy completion of the SM, must break susy)

SPONTANEOUS SUSY: (Lagrangian is susy, vacuum is not)

$$Q_\alpha |0\rangle \neq 0 \quad \bar{Q}_\alpha |0\rangle \neq 0$$

SIMPLE LITMUS TEST: $\langle 0 | H | 0 \rangle$

$$\text{WHY?} \quad H = P^0, \quad H = -P_0$$

$$\{ Q_\alpha, \bar{Q}_\alpha \} = 2 \sigma_{\alpha\dot{\alpha}}^m P_m$$

$$\Rightarrow \{ Q_\alpha, \bar{Q}_\alpha \} \bar{\sigma}^{n\dot{\alpha}\alpha} = 2 \sigma_{\alpha\dot{\alpha}}^m \bar{\sigma}^{n\dot{\alpha}\alpha} P_m = 2 \text{Tr}(\sigma^m \bar{\sigma}^n) P_m = 2(-2\eta^{mn}) P_m = -4 P^n$$

$$\Rightarrow P^n = -\frac{1}{4} \{ Q_\alpha, \bar{Q}_\alpha \} \bar{\sigma}^{n\dot{\alpha}\alpha}$$

$$\text{SO } H = P^0 = -\frac{1}{4} \{ Q_\alpha, \bar{Q}_\alpha \} (-\delta_{\alpha\dot{\alpha}}) = \frac{1}{4} (Q_1 \bar{Q}_1 + \bar{Q}_1 Q_1 + Q_2 \bar{Q}_2 + \bar{Q}_2 Q_2)$$

SUSY VACUA HAVE $Q_\alpha |0\rangle = \bar{Q}_\alpha |0\rangle = 0$, $\therefore \langle 0 | H | 0 \rangle = 0$

SUSY VACUA HAVE $Q_\alpha |0\rangle \neq 0, \bar{Q}_\alpha |0\rangle \neq 0$, $\therefore \langle 0 | H | 0 \rangle \neq 0$

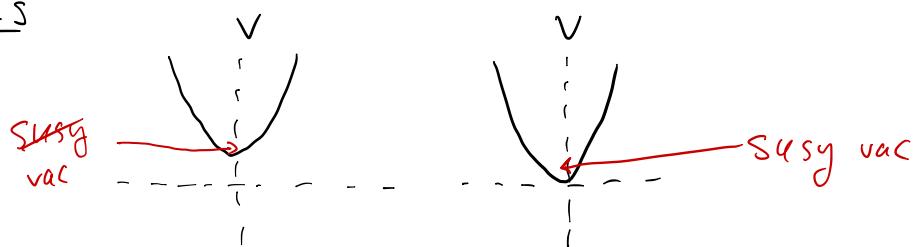
ALSO $\langle 0 | H | 0 \rangle > 0$

$$\text{SINCE} \quad \langle 0 | H | 0 \rangle = \frac{1}{4} (|\bar{Q}_1 |0\rangle|^2 + |Q_1 |0\rangle|^2 + \dots + |Q_2 |0\rangle|^2)$$

FACT: IN ABS. OF x^m -DEP EFFECTS \in FERMI. CONDENSATES

$$\langle 0 | H | 0 \rangle = \langle 0 | V | 0 \rangle$$

\Rightarrow PICTURES



SUSY \Leftrightarrow SCALAR POTENTIALS

$$V = V_F + V_D = F_k^* F_k + \frac{1}{2} D^\alpha D^\alpha \quad \text{BOTH POS. DEF}$$

\therefore SUSY VAC $\Rightarrow V_F = 0 \Leftrightarrow V_D = 0$

\therefore MINIMA w/ $V_F \neq 0$ or $V_D \neq 0 \Rightarrow$ SUSY VAC

Lang F-TERM BREAKING $V_F \neq 0 \Leftrightarrow F_k \neq 0$ FOR SOME k
 D-TERM BREAKING $V_D \neq 0 \Leftrightarrow D^\alpha \neq 0$ FOR SOME α

F-TERM SUSY

$$V_F = F_k^* F_k = W_i^* W_i$$

VACUA: LOCAL MIN OF $V = V_F$ \downarrow HERE

SUSY VACUA: REQ $V_F = 0 \Rightarrow F_k = 0 \forall k$

Q: DO SUCH SOLUTIONS EXIST?

$$\text{EXAMPLE 1} \quad W(\bar{\Phi}) = \lambda_i \bar{\Phi}_i + m_{ij} \bar{\Phi}_i \bar{\Phi}_j + g_{ijk} \bar{\Phi}_i \bar{\Phi}_j \bar{\Phi}_k$$

$$\Rightarrow F_k^* = -(\lambda_k + m_{ik} A_i + g_{ijk} A_i A_j) = 0 \quad \forall k$$

NOTE: DEPENDS ON # FIELDS & PARAMS

EXAMPLE 2 $i=1,2$ 2 FIELDS

$$F_1^* = F_2^* = 0 \Rightarrow \lambda_1 + m_{11} A_1 + m_{21} A_2 + g_{111} A_1 A_1 + (g_{121} + g_{211}) A_1 A_2 + g_{221} A_2 A_1 \\ \lambda_2 + m_{22} A_2 + m_{12} A_1 + g_{112} A_1 A_1 + (g_{122} + g_{212}) A_1 A_2 + g_{222} A_2 A_2$$

EXAMPLE

O'RAIFEAR TAIGH

$$W = \lambda \bar{\Phi}_0 + m \bar{\Phi}_1 \bar{\Phi}_2 + g \bar{\Phi}_0 \bar{\Phi}_1 \bar{\Phi}_2$$

$$\begin{aligned} F_0 = F_1 = F_2 = 0 \Rightarrow & \quad \left. \begin{aligned} \lambda + g A_1^2 &= 0 \\ m A_2 + 2g A_0 A_1 &= 0 \\ m A_1 &= 0 \end{aligned} \right\} \\ & \quad \text{*} \end{aligned}$$

DOES \exists SOLN?

NO: EQ 1 & EQ 3 INCOMPATIBLE, SO $\not\in$ SUSY VAC

DO \exists VACUA?

$$V = V_F + V_D = |(\lambda + g A_1)|^2 + |m|^2 (A_2 A_2^* + A_1 A_1^*)$$

$$\frac{\partial V}{\partial A_1} = 2g (\lambda^* + g^* A_1^*) + |m|^2 A_1^* \quad \frac{\partial V}{\partial A_2} = |m|^2 A_2^*$$

$$\frac{\partial V}{\partial A_1^*} = 2g^* (\lambda + g A_1) + |m|^2 A_1 \quad \frac{\partial V}{\partial A_2^*} = |m|^2 A_2$$

$$\text{SOLN @} \quad A_2 = A_2^* = 0 \quad A_1 = \frac{-2g^* \lambda}{2|g|^2 + |m|^2} \quad A_1^* = \frac{-2g \lambda^*}{2|g|^2 + |m|^2}$$

SO \exists VAC (technically need to check derive for, too)

$$V|_{\text{soln}} = \left| \left(\lambda + g \left(\frac{-2g^* \lambda}{2|g|^2 + |m|^2} \right) \right) \right|^2 + |m|^2 \frac{4|g|^2 |\lambda|^2}{(2|g|^2 + |m|^2)^2}$$

$\neq 0$ UNLESS $\lambda = 0$. BUT THEN * HAS SOLN!

fix this

D-TERM SUSY

$$V_D = \frac{1}{2} D^a D^a$$

NON-ABELIAN: $D^a = -g(A^* T^a A)$

NOTE: \Rightarrow SCALAR QUARTIC

w/ COEFF. DET BY g (why might we care?)

ABELIAN CASE:

\rightarrow Lang: Fayet-Iliopoulos Param (aka FI)

SUBTLETY: $2\chi \int d^2\theta d^2\bar{\theta} V$ ALLOWED

$$\Rightarrow \mathcal{L} \supset 2KD \quad \text{WHERE } D = \int d^2\theta d^2\bar{\theta} V \quad (\text{not highest comp})$$

↑ w/ B_1 , some book don't have α , $\Rightarrow \alpha \frac{1}{2}$ in D

$$\Rightarrow K \text{ APPEARS IN D-TERM EOM} \Rightarrow \text{IN } V_D$$

$$\text{EOM} \Rightarrow \boxed{D = -K + \frac{g}{2} \sum g_i |A_i|^2}$$

EXAMPLE

$$\begin{aligned} \mathcal{L} = & \int d^2\theta \frac{1}{4}(W^\alpha W_\alpha + \bar{W}_\alpha \bar{W}^\alpha) + \int d^2\theta d^2\bar{\theta} \left(\Phi_1^+ e^{eV} i + \bar{\Phi}_2^+ \bar{e}^{-eV} \bar{\Phi}_2^- \right) \\ & + \left(\int d^2\theta m \bar{\Phi}_1 \Phi_2 + \text{h.c.} \right) + \int d^2\theta d^2\bar{\tau} 2K V \end{aligned}$$

$$V = \frac{1}{2} D^2 + F_1^* F + F_2^* F$$

$$\begin{aligned} \text{EOM} \Rightarrow D = & -K + \frac{g}{2} (A_1^* A_1 - A_2^* A_2) = 0 \\ F_1^* = -W_1^* = -m^* A_2^* = 0 \\ F_2^* = -W_2^* = -m^* A_1^* = 0 \end{aligned} \quad \left. \right\} \text{NO SOLN} \Rightarrow \text{NO SUSY VAC}$$

HOWEVER IF $K=0$, \exists SOLN.

$\Rightarrow K$ CONTROLS SUSY IN THIS MODEL

$$V = \frac{1}{2} K^2 + \left(m^2 + \frac{1}{2} eK \right) A_1 A_1^* + \left(m^2 - \frac{1}{2} eK \right) A_2^* A_2 + \frac{1}{8} e^2 (A_1^* A_1 - A_2^* A_2)^2$$

CASE 1 $m^2 > \frac{1}{2} eK$

$$\text{MIN } @ \quad A_1 = A_2 = 0 \quad \Rightarrow \quad V|_{\min} = \frac{K^2}{2} \quad \Rightarrow \quad \text{susy}$$

PARTICLE CONTENT

$\psi_1 \& \psi_2$: MASS m

$A_1 \& A_2$: MASS $m \pm \frac{1}{2} eK$

v^m, λ : MASS 0

KEY: BOSE-FERMI MASS DEGEN
BROKEN BY K

$$\lim_{K \rightarrow 0} m_{A_1, A_2} = m_{\psi_1, \psi_2} \quad \& \quad \lim_{K \rightarrow 0} V = 0$$

\Rightarrow susy AS $K \rightarrow 0$ (Both Bose-Fermi degen
 $\&$ $\langle 0 | H | 0 \rangle = 0$)

NOTE: MASSLESS FERM. ASSOC w/ susy
IS λ Language: "Goldstone"

CASE 2

$m^2 < \frac{1}{2} e K \Rightarrow A_1 = A_2 = 0$ NO LONGER MIN
 (easy way to see this?)

MIN

$$\text{SOLVE } \frac{\partial V}{\partial A_1^*} = \frac{\partial V}{\partial A_2^*} = 0$$

$$\text{SOLN } A_1 = 0 \quad A_2 = v \quad w/ \quad \frac{1}{4} e^2 v^2 + \left(m^2 - \frac{1}{2} e K \right) = 0$$

↳ real by gauge trans

EXP AROUND MIN

$$A = A_1 \quad \tilde{A} = A_2 - v$$

$$V = \frac{2m}{e^2} (eK - m^2) + 2m^2 A^* A$$

$$+ \frac{1}{2} \left(\frac{1}{2} e^2 v^2 \right) \left(\frac{1}{\sqrt{2}} (\tilde{A} + \tilde{A}^*) \right)^2 + \frac{1}{2} \left(\frac{1}{2} e^2 v^2 \right) v^m v_m + \dots$$

NB: i) const $\frac{2m}{e^2} (eK - m^2)$ pos \Rightarrow ~~SUSY~~

ii) v_m GOT MASS BY EATING
 GOLDSTONE $(\tilde{A} - \tilde{A}^*) / \sqrt{2}$

~~SUSY~~ ? ~~YES~~ !

EX

RANDOM MODEL

ask for fields, specify most gen ren ω ,
 solve for m_i , etc