QFT 2

Topic 3: Audy

SUPERSYMMETRY: INTRODUCTION BY ENGINEERING

$$S = \int d^4x \left(\mathcal{L}_{scalar} + \mathcal{L}_{fermion}\right)$$

$$\mathcal{L}_{scalar} = -\partial^\mu \phi^* \partial_\mu \phi \qquad \mathcal{L}_{fermion} = i\bar{\psi}\bar{\sigma}^\mu \partial_\mu \psi$$

TRANS: SC $\to$ FERM

TRY 1 $\quad \phi \mapsto \phi + \psi_\alpha \quad$ ✗ BAD IND'S.

TRY 2 $\quad \phi \mapsto \phi + \epsilon\psi \quad$ ✓ IND'S

NOTE: ϵ FERMIONIC TRANS. PARAM!

SIM: $\quad \phi^* \longmapsto \phi^* + \bar{\epsilon}\bar{\psi}$

$$\mathcal{L}_{scalar} \to \mathcal{L}_{scalar} - \epsilon\,\partial^\mu\psi\,\partial_\mu\phi^* - \bar{\epsilon}\,\partial^\mu\bar{\psi}\,\partial_\mu\phi$$

TRANS: FERM $\to$ SC

NEED: $\bar{\psi}_{\dot\alpha}$ TRANS TO GET ϵ, ∂

ψ_α TRANS TO GET $\bar{\epsilon}, \partial$

TAKE: $\quad \psi_\alpha \to \psi_\alpha - i(\sigma^\mu\bar{\epsilon})_\alpha\,\partial_\mu\phi$

$\bar{\psi}_{\dot\alpha} \to \bar{\psi}_{\dot\alpha} + i(\epsilon\sigma^\mu)_{\dot\alpha}\,\partial_\mu\phi^*$

$$\mathcal{L}_{fem} \to \mathcal{L}_{fem} \underbrace{-\epsilon\,\sigma^m\bar{\sigma}^n\partial_n\psi\,\partial_m\phi^* + \bar{\psi}\bar{\sigma}^\nu\sigma^m\bar{\epsilon}\,\partial_\mu\partial_\nu\phi.}$$

$$= \epsilon\,\partial^\mu\psi\,\partial_\mu\phi^* + \bar{\epsilon}\,\partial^\mu\bar{\psi}\,\partial_\mu\phi$$
$$- \partial_\mu\left(\epsilon\sigma^\nu\bar{\sigma}^m\psi\,\partial_\nu\phi^* + \epsilon\,\psi\,\partial^\mu\phi^* + \bar{\epsilon}\bar{\psi}\,\partial^\mu\phi\right)$$

CONCLUSION - $\mathcal{L}$ INVT, UP TO TOT. DERIVS!

- BOSON $\longleftrightarrow$ FERMION SYMMETRY

"SUPERSYMMETRY"

what do you know about it? if it exists in particle physics, how must it exist? (broken) "How much" susy?

$\underbrace{\text{"How much" susy?}}_{\text{amt symmetry} = \#\text{ gens}}$

TOPIC OUTLINE

$d=4$ $\mathcal{N}=1$ SUSY

1. 2 COMP. SPINORS
2. SUSY ALG. & REPS
3. COMPONENT FIELDS

   somewhere in here by spring break

4. SUPERSPACE & SUPERFIELDS
5. CHIRAL & VECTOR SUPERFIELDS
6. GAUGE INV. INTERACTIONS
   - start 2/21 or 2/22 in 2018
   - classes left: makeups WEDS' 2/21, 2/28, 3/21, 4/4

     Mondays: 2/26, ~~3/12~~ → OX, 3/19, ~~3/26~~ → MUN, 4/2, ~~4/9~~ → LON

     Thursdays: 2/22, 3/1, 3/15, 3/22, ~~3/29~~ → MUN, 4/5, 4/12
7. ~~SUSY~~

(1–7) Wess & Bagger

8. MSSM & SUSY PHENO ⟹ Martin's "A Supersymmetry Primer"

SOME MOTIVATION

NOTATION & 2 COMP. SPINORS

NOTATION 4-VECTORS w/ LOWERCASE LATIN IND. (eg A_m, NOT A_μ)

RECALL - 4-COMP ψ SPINORS $\Longrightarrow \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$, $\psi_{L,R}$ 2-COMP

- $\Gamma^\mu \sim \left(\begin{array}{c|c} 2\times2 & 2\times2 \\ \hline 2\times2 & 2\times2 \end{array}\right) \Longrightarrow \psi_{L,R}$ ACTED ON BY 2×2's
- **INSPECTION:** 2×2's ARE

$$\sigma^0 = \begin{pmatrix} -1 & \\ & -1 \end{pmatrix} \quad \& \quad \text{PAULI MATRICES}$$

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

CALL $\sigma^m = (\sigma^0, \vec{\sigma})$

FACT: σ^m GENERATE $SL(2,\mathbb{C})$ = DOUBLE COVER OF LORENTZ GROUP

(know this: $SU(2) \subseteq SL(2,\mathbb{C})$, 2-cover $\downarrow$ to $SO(3) \subseteq SO(3,1) = LOR.$, with $SL(2,\mathbb{C}) \downarrow$ 2-cover $SO(3,1)$)

$$\begin{array}{ccc} SU(2) & \subseteq & SL(2,\mathbb{C}) \\ {\scriptstyle 2\text{-cover}}\downarrow & & \downarrow{\scriptstyle 2\text{-cover}} \\ SO(3) & \subseteq & SO(3,1) = LOR. \end{array}$$

2-COMP. SPINOR TYPES

$M \in SL(2,\mathbb{C})$

M, M^*, $(M^T)^{-1}$, $(M^\dagger)^{-1}$ ALL REP $SL(2,\mathbb{C}) \Longrightarrow 4$ 2-COMP. SPINOR REPS

$$\boxed{\psi'_\alpha = M_\alpha{}^\beta \psi_\beta \qquad \bar{\psi}'_{\dot\alpha} = M^*_{\dot\alpha}{}^{\dot\beta} \bar{\psi}_{\dot\beta} \qquad \psi'^\alpha = (M^{-1})_\beta{}^\alpha \psi^\beta \qquad \bar{\psi}^{\dot\alpha\prime} = (M^{*-1})_{\dot\beta}{}^{\dot\alpha} \bar{\psi}^{\dot\beta}}$$

COMPARE TO OLD NOTATION $\psi_\alpha = \psi_L$ $\quad \bar{\psi}^{\dot\alpha} = \psi_R$

HERMITIAN 2×2 P $\Longrightarrow$ CAN WRITE $P = P_m \sigma^m = \begin{pmatrix} -P_0 + P_3 & P_1 - iP_2 \\ P_1 + iP_2 & -P_0 - P_3 \end{pmatrix}$, $P_m \in \mathbb{R}$

P HERM $\Longrightarrow P' = MPM^\dagger$ HERM $\Longrightarrow \sigma^m P'_m = M_\alpha{}^\beta \sigma^m P_m M^\dagger_{\dot\alpha}{}^{\dot\beta}$

SOME LORENTZ INVARIANTS: $\psi^\alpha \psi_\alpha =: \psi\psi$ $\quad \bar{\psi}_{\dot\alpha} \bar{\psi}^{\dot\alpha} =: \bar{\psi}\bar{\psi}$

$\psi^\alpha \sigma^m_{\alpha\dot\alpha} \bar{\psi}^{\dot\alpha} =: \psi \sigma^m \bar{\psi}$

NOTE $\dot\alpha$ $\dot\alpha$ ORDER.

RAISE & LOWER INDICES USE $\varepsilon^{\alpha\beta}$ $\varepsilon_{\alpha\beta}$

$\varepsilon_{21} = \varepsilon^{12} = 1$ $\quad \varepsilon_{12} = \varepsilon^{21} = -1$, REST ZERO

$$\boxed{\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta \qquad \psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta \qquad \varepsilon_{\alpha\beta}\varepsilon^{\beta\gamma} = \delta_\alpha{}^\gamma}$$

$\bar{\sigma}^{m\,\dot{\alpha}\alpha} := \varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon^{\alpha\beta}\sigma^m_{\beta\dot{\beta}} \implies \bar{\sigma}^m = (\sigma^0, -\sigma^m)$ AS MATRICES

SOME IDENTITIES

INTEREST: $(\sigma^m\bar{\sigma}^n)_\alpha{}^\beta + (\sigma^n\bar{\sigma}^m)_\alpha{}^\beta$

$m=n=0 \to \sigma^0\sigma^0+\sigma^0\sigma^0 = 2\,\delta_\alpha{}^\beta$

$m=n=i,\ i=1,2,3 \implies \sigma^i(-\sigma^i) + \sigma^i(-\sigma^i) = -2\,\delta_\alpha{}^\beta$

$m=i\ n=j,\ j\neq i \implies \sigma^i(-\sigma^j) + \sigma^j(-\sigma^i) = -\{\sigma^i,\sigma^j\} = 0$ (anti-comm of Pauli props)

$m=0\ n=i \implies \sigma^0(-\sigma^i)+(\sigma^i)(\sigma^0) = -\mathbb{1}(-\sigma^i)+\sigma^i\sigma^0 = 0$

RESULT (& sim)

$$\boxed{\begin{aligned}(\sigma^m\bar{\sigma}^n+\sigma^n\bar{\sigma}^m)_\alpha{}^\beta &= -2\eta^{mn}\delta_\alpha{}^\beta = -2\eta^{mn}\mathbb{1}_{2\times2}\\ (\bar{\sigma}^m\sigma^n+\bar{\sigma}^n\sigma^m)^{\dot{\alpha}}{}_{\dot{\beta}} &= -2\eta^{mn}\delta^{\dot{\alpha}}{}_{\dot{\beta}} = -2\eta^{mn}\mathbb{1}_{2\times2}\end{aligned}}$$

(the above sounds familiar)

RECALL (P&S) $\gamma^0 = \begin{pmatrix}0 & \mathbb{1}\\ \mathbb{1} & 0\end{pmatrix}$ $\gamma^i = \begin{pmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{pmatrix}$

$$\{\gamma^m,\gamma^n\} = 2\eta^{mn}\,\mathbb{1}_{4\times4} = 2\eta^{mn}\begin{pmatrix}\mathbb{1}_{2\times2} & 0\\ 0 & \mathbb{1}_{2\times2}\end{pmatrix}$$

EVEN MORE

$$\boxed{\mathrm{Tr}\,\sigma^m\bar{\sigma}^n = -2\eta^{mn} \qquad \sigma_{\alpha\dot{\alpha}}{}^m\,\bar{\sigma}_m{}^{\dot{\beta}\beta} = -2\,\delta_\alpha{}^\beta\,\delta_{\dot{\alpha}}{}^{\dot{\beta}}}$$

LORENTZ GENS

Q: IF 4-COMP $\sim [\gamma^m,\gamma^n]$, WHAT 2-COMP? A: $\sim [\sigma^m,\bar{\sigma}^n]$

$$\sigma^{nm}{}_\alpha{}^\beta := \frac{1}{4}\left(\sigma_{\alpha\dot{\alpha}}{}^n\bar{\sigma}^{n\,\dot{\alpha}\beta} - \sigma_{\alpha\dot{\alpha}}{}^m\bar{\sigma}^{n\,\dot{\alpha}\beta}\right)$$

$$\bar{\sigma}^{nm\,\dot{\alpha}}{}_{\dot{\beta}} = \frac{1}{4}\left(\bar{\sigma}^{n\,\dot{\alpha}\alpha}\sigma_{\alpha\dot{\beta}}{}^m - \bar{\sigma}^{m\,\dot{\alpha}\alpha}\sigma_{\alpha\dot{\beta}}{}^n\right)$$

$$\boxed{\begin{aligned}\bar{\sigma}^a\sigma^b\bar{\sigma}^c - \bar{\sigma}^c\sigma^b\bar{\sigma}^a &= -2i\,\varepsilon^{abcd}\,\bar{\sigma}_d\\ \sigma^a\bar{\sigma}^b\sigma^c - \sigma^c\bar{\sigma}^b\sigma^a &= +2i\,\varepsilon^{abcd}\,\sigma_d\end{aligned}}$$

SUSY ALG

THEOREM: (COLEMAN-MANDULA)

ANY THY W/ SYMM. GROUP G A LIE ALGEBRA

i) S-MATRIX FROM LOCAL, REL. QFT

ii) BELOW ANY MASS M $\exists$ FINITE # PARTS p W $m_p < M$.

iii) $\exists$ ENERGY GAP BETW. $|0\rangle$ & ALL 1-PARTICLE STATES

HAS $G = \underbrace{\text{Poincaré}} \times \text{internal}$

$\sim \mathbb{R}^{3,1}$ trans $\times$ Lorentz

IE: SPACETIME & INT. SYMMETRIES CAN'T MIX!

Q: WHAT DO WE DO WITH THEOREMS?

A1: ASK WHETHER CONCLUSIONS ARE INTERESTING

A2: " " ASSUMPTIONS ARE REASONABLE

$\Rightarrow$ CONSIDER LOOPHOLES!

LOOPHOLE: TAKE G A GRADED LIE ALG. AKA SUPER ALGEBRA

SCHEMATIC:
- FERMIONIC GENS Q, Q', Q''
- BOSONIC GENS X, X', X''
- ALG $\sim$ $\{Q, Q'\} = X$ $\quad [X, X'] = X''$ $\quad [Q, X] = Q''$

THEOREM: (HAAG–LOPUSZANSKI–SOHNIUS)

- TAKE C-M ASSUMPTIONS, BUT G A GRADED L.A.
  W Q^L_α & $\bar{Q}^L_{\dot\alpha}$ (its Herm conjugate) IN ALG.

THEN: $\boxed{\{Q^L_\alpha, \bar{Q}_{\dot\beta M}\} = 2\sigma^m_{\alpha\dot\beta} P_m \delta^L{}_M , \quad \{Q^L_\alpha, Q_\beta{}^M\} = \varepsilon_{\alpha\beta} X^{LM}}$ ← antisymm.

← "SUSY ALG"

WITH $L = 1, \dots, \mathcal{N}$, P_m EN-MOM. OPS

NB: - SUSY GENS $Q\,\bar{Q}$ ANTI COMM TO SPACETIME TRANS (P_m)
- X^{LM} "CENTRAL CHARGES"
  - NONE IF $\mathcal{N}=1$ (Q: why? antisymmetry)
  - ONE IF $\mathcal{N}=2$ (Q: why? X^{LM} 2×2 antisymm)

CONSERVED CHARGE ∃ SUSY QFT'S WHERE

$$Q_\alpha = \int d^3x\, J^0_\alpha \quad \text{HAS} \quad \frac{dQ_\alpha}{dt} = 0 \quad \text{B/C} \quad \partial_m J^m_\alpha = 0$$

REPS OF SUSY ALG (Q: what are ireps of susy alg?)

$|M>0|$ M = mass

REST FRAME $P_m = (M,0,0,0)$

ALGEBRA: $\{Q^A_\alpha, \bar{Q}_{\dot\beta B}\} = 2M\,\delta_{\alpha\dot\beta}\,\delta^A_B$ (↓why? $P_i = 0$ & $\sigma^0_{\alpha\dot\beta} = \delta_{\alpha\dot\beta}$) $\{Q,Q\} = \{\bar{Q},\bar{Q}\} = 0$ (↓ignore central ch. for now)

RESCALE: $a^A_\alpha := \frac{1}{\sqrt{2M}} Q^A_\alpha \qquad (a^A_\alpha)^\dagger := \frac{1}{\sqrt{2M}} \bar{Q}_{\dot\alpha A}$

S.T. $\{a^A_\alpha, (a^B_\beta)^\dagger\} = \delta^\beta_\alpha\, \delta^A_B \qquad \{a,a\} = \{a^\dagger, a^\dagger\} = 0$

NB: $2N$ (↓from spinor index) PAIRS OF FERM CREAT. & ANNI. OPS

$\Rightarrow \exists\ |\Omega\rangle \mid a^A_\alpha |\Omega\rangle = 0 \quad \forall \alpha, A$

REP: (as with $\mathfrak{so}^{(s)}$ spinor) ACT w/ $(a^A_\alpha)^+$ IN ALL POSS WAYS

$\Rightarrow$ yes/NO ON EACH, $\underbrace{A,\alpha}_{}$ — 2N CHOICES

$\Rightarrow 2^{2N}$ STATES

EXAMPLES $\quad N=1 \quad \underbrace{|\Omega\rangle}_{\text{spin }0},\ \underbrace{(a_\alpha)^+|\Omega\rangle}_{\text{spin-}\frac{1}{2}},\ \underbrace{\tfrac{1}{\sqrt{2}}\epsilon^{\alpha\beta}(a_\alpha)^+(a_\beta)^+|\Omega\rangle}_{\text{spin }0}$

I.E. 2 FERM, 2 BOS

FACT: $|\Omega\rangle$ CAN HAVE SPIN j, $\Rightarrow |\Omega_j\rangle$

TABLE

SPIN	Ω_0	$\Omega_{\frac{1}{2}}$	Ω_1	$\Omega_{3/2}$
0	2	1		
1/2	1	2	1	
1		1	2	1
3/2			1	2
2				1
Σ states	$4 = 1\cdot 2^2$	$8 = 2\cdot 2^N$	$12 = 3\times 2^N$	$16 = 4\times 2^N$

$\Rightarrow$ coeff = # of m's for $|\Omega_j\rangle$

(names of first two: massive chiral multiplet, massive vector mult)

$\boxed{M=0}$ $\quad P^2=0$, NO REST FRAME, $P_m=(-E,0,0,E)$

$\Rightarrow$ ALG IS $\{Q^A_\alpha, \bar{Q}_{\dot\beta B}\} = 2E(\sigma^0-\sigma^3)_{\alpha\dot\beta}\,\delta^A{}_B \overset{\text{plug }\sigma^0,\sigma^3}{=} 2\begin{pmatrix} 2E & 0 \\ 0 & 0\end{pmatrix}\delta^A{}_B$

$\{Q,Q\} = \{\bar{Q},\bar{Q}\}=0$

NB ON M=0 STATES $\{Q^A_2, \bar{Q}^A_{\dot 2}\} = \{Q^A_2, (Q^A_2)^+\}=0.$

BUT ANY $\{O,O^+\}$ IS POS. SEMI-DEF, $\Rightarrow Q_2{}^A = \bar{Q}_{\dot 2}{}^A = 0$ ON THESE M=0 STATES

NB: ONLY $\alpha=1$ NON-TRIVIAL $\quad a^A := \frac{1}{2\sqrt{E}}Q_1{}^A \quad (a^A)^+ := \frac{1}{2\sqrt{E}}\bar{Q}_{\dot 1}{}^A$

$\Rightarrow \{a^A, a^{+B}\} = \delta^A{}_B \quad \{a,a\}=\{a^+,a^+\}=0$

$\Rightarrow$ N (NOT 2N) CREAT. ANNI. OPS $\Rightarrow 2^N$

w/ CPT, 2×2^N

CASE N=1 TABLE

N=1 M=0 HELICITY (recall: massless particle in Poincaré rep det'd by "helicity", which is half integer or integer)

helicity of $|\Omega\rangle$ OR CPT $|\Omega\rangle$

	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
2								1
3/2							1	1
1						1	1	
1/2					1	1		
0				1	1			
-1/2			1	1				
-1		1	1					
-3/2	1	1						
-2	1							

chiral mult: 2 hel 0, hel ±1

vector multiplet:
hel ±1 ⇒ vector
hel ±1/2 ⇒ fermion

GENERAL RESULT: $\boxed{\text{SUSY REPS HAVE \# BOSONS = \# FERMIONS!}}$

COMPONENT FIELDS

Q: HOW DOES SUSY ACT ON FIELDS?

NEED: SPINOR SUSY PARAMS $\xi^\alpha, \bar\xi_{\dot\alpha}$ (recall $\epsilon, \bar\epsilon$ from previous engineering)

$$\{\xi^\alpha,\xi^\beta\} = \{\xi^\alpha, Q_\beta\} = \dots = [P_m, \xi^\alpha] = 0$$

SUSY ALG $\Rightarrow$

i) $[\xi Q, \bar\xi\bar Q] = 2\xi\sigma^m\bar\xi P_m$

ii) $[\xi Q, \xi Q] = [\bar\xi\bar Q, \bar\xi\bar Q] = 0$

iii) $[P^m, \xi Q] = [P^m, \bar\xi\bar Q] = 0$

aside
$$\theta\eta = \theta^\alpha\eta_\alpha = \theta^\alpha\epsilon_{\alpha\beta}\eta^\beta = -\epsilon_{\beta\alpha}\theta^\alpha\eta^\beta = -\theta_\beta\eta^\beta = -\theta_\alpha\eta^\alpha$$

PROOF:
$$[\xi Q, \bar\xi\bar Q] = \xi Q\bar\xi\bar Q - \bar\xi\bar Q\xi Q = \xi^\alpha Q_\alpha\bar\xi_{\dot\alpha}\bar Q^{\dot\alpha} - \bar\xi_{\dot\alpha}\bar Q^{\dot\alpha}\xi^\alpha Q_\alpha$$
$$= -\xi^\alpha\bar\xi_{\dot\alpha}Q_\alpha\bar Q^{\dot\alpha} \overset{=(-)(-)}{-} \xi^\alpha\bar\xi_{\dot\alpha}\bar Q^{\dot\alpha}Q_\alpha = -\xi^\alpha\bar\xi_{\dot\alpha}\{Q_\alpha, \bar Q^{\dot\alpha}\}$$
$$= \xi^\alpha\bar\xi^{\dot\alpha}\{Q_\alpha, \bar Q_{\dot\alpha}\} = \xi^\alpha\bar\xi^{\dot\alpha}\, 2\sigma^m_{\alpha\dot\alpha}P_m = 2\xi\sigma^m\bar\xi P_m \quad \checkmark$$

W&B's bad scalar notation. We'll stick w/ it for consistency

COMPONENT FIELDS (A, ψ)

INFINITESIMAL FIELD TRANS

$$A \longmapsto (1 + \xi Q + \bar\xi\bar Q)A \qquad \text{SO} \qquad \delta_\xi A = (\xi Q + \bar\xi\bar Q)A$$

SIM: $\delta_\xi\psi = (\xi Q + \bar\xi\bar Q)\psi$

SUCCESIVE TRANS? $\delta_\eta\delta_\xi - \delta_\xi\delta_\eta$?

overall sign error when I did it, just keep track since not right

$$(\delta_\eta\delta_\xi - \delta_\xi\delta_\eta) = -(\xi Q + \bar\xi\bar Q)(\eta Q + \bar\eta\bar Q) + (\eta Q + \bar\eta\bar Q)(\xi Q + \bar\xi\bar Q)$$
$$= -(\xi Q\bar\eta\bar Q + \bar\xi\bar Q\eta Q - \eta Q\bar\xi\bar Q - \bar\eta\bar Q\xi Q)$$
$$= -(\xi^\alpha Q_\alpha\bar\eta_{\dot\alpha}\bar Q^{\dot\alpha} - \bar\eta_{\dot\alpha}\bar Q^{\dot\alpha}\xi^\alpha Q_\alpha + \bar\xi_{\dot\alpha}\bar Q^{\dot\alpha}\eta^\alpha Q_\alpha - \eta^\alpha Q_\alpha\bar\xi_{\dot\alpha}\bar Q^{\dot\alpha})$$
$$= -(- - \xi^\alpha\bar\eta^{\dot\alpha}Q_\alpha\bar Q_{\dot\alpha} - - - - \xi^\alpha\bar\eta^{\dot\alpha}\bar Q_{\dot\alpha}Q_\alpha + - - - \eta^\alpha\bar\xi^{\dot\alpha}\bar Q_{\dot\alpha}Q_\alpha - - - \eta^\alpha\bar\xi^{\dot\alpha}Q_\alpha\bar Q_{\dot\alpha})$$
$$= -(\xi^\alpha\bar\eta^{\dot\alpha}\{Q_\alpha, \bar Q_{\dot\alpha}\} - \eta^\alpha\bar\xi^{\dot\alpha}\{Q_\alpha, \bar Q_{\dot\alpha}\}) = 2(\eta\sigma^m\bar\xi - \xi\sigma^m\bar\eta)P_m$$

Q: why do all other terms die?
A: $\{Q, Q\}, \{\bar Q, \bar Q\}$

WILL NEED, CALL (*)

Q: SO HOW DOES SUSY ACT ON FIELDS?

MASS DIMENSION

$\{Q, \bar{Q}\} \sim \sigma^m P_m \implies [Q] = [\bar{Q}] = \frac{1}{2}$ SINCE $[P^m] = 1$

$\therefore [\delta_\xi A] = \frac{3}{2}$ & $[\delta_\eta \delta_\xi A] = 2$ FOR SCALAR A

DEFINE ψ_α S.T. IT'S THE DIM-$\frac{3}{2}$ FIELD $\propto \delta_\xi A$

IE $\delta_\xi A = \sqrt{2}\, \xi \psi$ (factor of $\sqrt{2}$ for convenience)

THEN $\delta_\xi \psi$ HAS $[\delta_\xi \psi] = 2 \quad \therefore$

CAN BE MADE FROM SOME NEW $F \mid [F] = 2$,

OR $\partial_m A$ SINCE $[\partial_m A] = 2$

IE $\delta_\xi \psi = \underbrace{i}\sqrt{2}\, \sigma^m \bar{\xi}\, \partial_m A + \sqrt{2}\, \xi F$

↑ chosen, by freedom to def F

Hmwk: i) fixed by req ⊛ satisfied

ii) $\delta_\xi F$ fixed by ⊛ on ψ, ie $(\delta_\eta \delta_\xi - \delta_\xi \delta_\eta)\psi$

SUMMARY

$$\boxed{\begin{aligned} \delta_\xi A &= \sqrt{2}\, \xi \psi \\ \delta_\xi \psi &= i\sqrt{2}\, \sigma^m \bar{\xi}\, \partial_m A + \sqrt{2}\, \xi F \\ \delta F &= i\sqrt{2}\, \bar{\xi}\, \bar{\sigma}^m \partial_m \psi \end{aligned}}$$

BOS $\implies$ FER

FER $\implies$ BOS + AUX F

AUX F $\implies$ FER

PURPOSE OF F: ENSURE SUSY-$\mathcal{L}$ OFF-SHELL, IE W/O USING EOM

INVARIANT ACTION $\quad \mathcal{L}_0 = i\, \partial_n \bar{\psi}\, \bar{\sigma}^n \psi + A^* \square A + F^* F + m\left(AF + A^* F^* - \frac{1}{2}\psi\psi - \frac{1}{2}\bar{\psi}\bar{\psi}\right)$

↳ up to total derivs

Q: TERM PURPOSES?

Q: $\exists\ \psi\psi$ MASS TERM. A^*A MASS TERM!? NO. HMWK.

Q: HOW TO SYSTEMATICALLY WRITE SUSY-INV $\mathcal{L}$'s?

SUPERFIELDS

POINT: PACKAGE COMPS IN NICE SUSY OBJECTS

$$\begin{tikzcd} \text{COMPONENTS} \arrow[rr, bend left, "\text{Construction}"] & & \text{SUPERFIELDS} \arrow[ll, bend left, "\text{power series}"] \end{tikzcd}$$

SUPER ALGEBRA: $1 + i\left(-x^m P_m + \theta Q + \bar\theta \bar Q\right) + \ldots$

$$\downarrow$$

SUPERGROUP: $G(x, \theta, \bar\theta) = e^{i\{-x^m P_m + \theta Q + \bar\theta \bar Q\}}$

group element.

Q: what is SU(N) analogy?

"SUPERSPACE": COORDS $(x, \theta, \bar\theta)$ (indices implicit)

ASIDE: $\theta^\alpha \theta^\beta \neq 0$ (convince yourself)

BUT $\theta^\alpha \theta^\beta \theta^\gamma = 0$!

PROOF $\alpha, \beta, \gamma \in \{1, 2\}$,

WRITE $\theta^\alpha\theta^\beta\theta^\gamma = \alpha\beta\gamma$, $2^3 = 8$ POSS

$(\theta^1)^2 = (\theta^2)^2 = 0$ (due to anti-com).

$\Rightarrow$ $111 = 0$ $\quad 112 = 0$ $\quad 211 = 0$

$222 = 0$ $\quad 221 = 0$ $\quad 122 = 0$

2 MORE $\quad 121 = -112 = 0$, $\quad 212 = -221 = 0$ ✓

CONC: ANYTHING w > 2 θ's or > 2 $\bar\theta$'s VANISHES

SUPERFIELD: LABEL LORENTZ-TYPE!

POW IN $\theta, \bar\theta$

$$F(x,\theta,\bar\theta) = \overset{SC}{f}(x) + \theta \overset{SP}{\phi}(x) + \bar\theta \overset{\dot{SP}}{\bar\chi}(x) + \theta\theta \overset{SC}{m}(x) + \bar\theta\bar\theta \overset{SC}{n}(x) + \theta\sigma^m\bar\theta \overset{VEC}{v_m}(x) + \theta\theta\bar\theta \overset{\dot{SP}}{\bar\lambda}(x) + \bar\theta\bar\theta\theta \overset{SP}{\psi}(x) + \theta\theta\bar\theta\bar\theta \overset{SC}{d}(x)$$

OUR ASIDE $\Rightarrow$ ALL HIGHER POWERS VANISH!

SUSY TRANS

Q: GO ON FOREVER OR TRUNCATE?

$$F \to e^{i(\theta Q + \bar\theta \bar Q)} F \simeq F + \underbrace{i(\theta Q + \bar\theta\bar Q) F}_{=: \delta_\theta F} + \ldots$$

$$\delta_\xi F = \delta_\xi f + \theta \delta_\xi \phi + \bar\theta \delta_\xi \bar\chi + \theta\theta \delta_\xi m + \bar\theta\bar\theta \delta_\xi n + \theta\sigma^m\bar\theta \delta_\xi v_m + \theta\theta\bar\theta \delta_\xi \bar\lambda + \bar\theta\bar\theta\theta \delta_\xi \psi + \theta\theta\bar\theta\bar\theta \delta_\xi d = (\xi Q + \bar\xi \bar Q) F$$

Q = ? ON F? SOME DIFFERENTIAL OP

REQUIRE: $(\delta_\eta \delta_\xi - \delta_\xi \delta_\eta) F = -2i(\eta\sigma^m\bar\xi - \xi\sigma^m\bar\eta)\partial_m F$

remember: (*) from few pages back

$$\boxed{Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i \sigma_{\alpha\dot\alpha}{}^m \bar\theta^{\dot\alpha} \partial_m \qquad \bar Q_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\theta^\alpha \sigma_{\alpha\dot\alpha}{}^m \partial_m}$$

HMWK: show these give (*)

$$\{Q_\alpha, \bar Q_{\dot\alpha}\} = \underbrace{2i\sigma_{\alpha\dot\alpha}{}^m \partial_m}_{} \qquad \{Q,Q\} = \{\bar Q, \bar Q\} = 0$$

$P_m = i\partial_m$

$\Rightarrow$ gen susy algebra

$\exists$ OTHER NICE OPERATORS (will see why)

$$\boxed{D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\,\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m \qquad \bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m}$$

$\dot{\mathcal{E}}$ $\{D_\alpha, \bar D_{\dot\alpha}\} = -2i\,\sigma^m_{\alpha\dot\alpha}\partial_m$ $\qquad \{D,D\} = \{\bar D,\bar D\} = \{D,Q\} = \{D,\bar Q\} = \{\bar D,Q\} = \{\bar D,\bar Q\} = 0$

only difference from $\{Q,\bar Q\}$

CHIRAL SUPERFIELDS

DEF'N: A SUPERFIELD Φ SATIS. $\bar{D}_{\dot\alpha}\Phi = 0$ IS A CHIRAL SUPERFIELD

NICE WAY TO EXPRESS

CHANGE OF COORDS $\quad y^m = x^m + i\theta\sigma^m\bar\theta$

$$\bar{D}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\sigma_{\alpha\dot\alpha}{}^m\partial_m$$

$$\Rightarrow \bar{D}_{\dot\alpha}(x^m + i\theta\sigma^m\bar\theta) = -i\,\theta^\alpha\sigma_{\alpha\dot\alpha}{}^n\partial_n x^m - \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\left(i\,\theta^\beta\sigma^m_{\beta\dot\beta}\bar\theta^{\dot\beta}\right)$$

$$= -i\theta^\alpha\sigma^n_{\alpha\dot\alpha}\delta^m_n + i\,\theta^\beta\sigma^m_{\beta\dot\beta}\,\delta^{\dot\beta}_{\dot\alpha} = 0$$

SO $\boxed{\bar{D}_{\dot\alpha}(y^m) = \bar{D}_{\dot\alpha}\theta = 0}$

why this one?

$\therefore$ ANY $F(y^m, \theta)$ IS CH. SUP. $\quad$ Q: why?

CPLX SC = 2 BOSONS

WRITE $\boxed{\Phi = A(y) + \sqrt{2}\,\theta\psi(y) + \theta\theta F(y)}$ $\quad$ note: most general sol'n to $\bar{D}_{\dot\alpha}\Phi = 0$

2 CMPT SPINOR = 2 FERMIONS

ALSO: $\Rightarrow$
$$\Phi = A(x) + i\theta\sigma^m\bar\theta\,\partial_m A(x) + \tfrac{1}{4}\theta\theta\bar\theta\bar\theta\,\Box A(x)$$
$$+\sqrt{2}\,\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\,\partial_m\psi(x)\sigma^m\bar\theta + \theta\theta F(x)$$

$\Phi^\dagger$: SATISFIES $D_\alpha\Phi^\dagger = 0$ (ANTI-CHIRAL SUP)

$$\Phi^\dagger = A^*(y^\dagger) + \sqrt{2}\,\bar\theta\bar\psi(y^\dagger) + \bar\theta\bar\theta F^*(y^\dagger)$$

TRANSFORMATION CHECKS (whole point of superfields was to have trans. be automatic, need to check!)

GENERAL: $\delta_\xi F(x,\theta,\bar\theta) = (\xi Q + \bar\xi \bar Q)\, F$

$\therefore$ CHIRAL: $\delta_\xi F(y,\theta) = (\underset{\xi^\alpha Q_\alpha}{\xi Q} + \underset{\bar\xi_{\dot\alpha}\bar Q^{\dot\alpha}}{\bar\xi \bar Q})\, F(y,\theta)$ (recall $y^m = x^m + i\theta\sigma^m\bar\theta$)

IN y-VARS $\quad Q_\alpha = \dfrac{\partial}{\partial\theta^\alpha} \qquad \bar Q_{\dot\alpha} = -\dfrac{\partial}{\partial\bar\theta^{\dot\alpha}} + 2i\theta^\alpha \sigma^m_{\alpha\dot\alpha}\partial_m$

THEN $\delta_\xi \Phi = (\xi Q + \bar\xi\bar Q)(A + \sqrt2\,\theta\psi + \theta\theta F)$

$= \xi^\alpha(\sqrt2\,\psi_\alpha + 2\theta_\alpha F) - \bar\xi^{\dot\alpha}\, 2i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m(A + \sqrt2\,\theta\psi + \overset{\text{why? } \theta\theta\theta}{0})$

↑ due to up-down change

$= \sqrt2\,\xi\psi + \sqrt2\,\theta^\alpha(\sqrt2\,\xi_\alpha F + i\sqrt2\,\sigma^m_{\alpha\dot\alpha}\bar\xi^{\dot\alpha}\partial_m A) + \underbrace{2\sqrt2\, i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\bar\xi^{\dot\alpha}\theta\partial_m\psi}_{= i\sqrt2\,\theta\theta\,\bar\xi\bar\sigma^n\partial_n\psi \text{ (some work!)}}$

$= \underbrace{\sqrt2\,\xi\psi}_{=\delta_\xi A} + \sqrt2\,\theta^\alpha\underbrace{(\sqrt2\,\xi F + i\sqrt2\,\sigma^m\bar\xi\partial_m A)}_{=\delta_\xi\psi} + \theta\theta\underbrace{(i\sqrt2\,\bar\xi\bar\sigma^n\partial_n\psi)}_{=\delta_\xi F}$

NOTE: MATCHES KNOWN COMPONENT FIELD TRANS!

SUSY $\mathcal{L}$-TERMS FROM CHIRAL SUPERFIELDS

i) NOTE: $\theta\theta$ COMPONENT OF CHIRAL SUPERFIELD TRANS TO TOTAL DERIV

ii) PRODUCTS OF CHIRAL SUP. ARE CHIRAL SUP.

reps all ch. sup.

$$\Rightarrow \quad W(\Phi) = \sum_{k=1}^{N} g^{(k)}_{i_1 \dots i_k} \Phi_{i_1} \cdots \Phi_{i_k}$$

$$= g^{(1)}_{i_1} \Phi_{i_1} + g^{(2)}_{i_1 i_2} \Phi_{i_1} \Phi_{i_2} + g^{(3)}_{i_1 i_2 i_3} \Phi_{i_1} \Phi_{i_2} \Phi_{i_3} + \dots$$

IS CHIRAL, $\therefore$ $\theta\theta$ COMP. TRANS TO TOT. DER.

& CAN BE PUT IN SUSY LAGRANGIAN!

DEF'N: $W(\Phi)$ IS THE SUPERPOTENTIAL.

PROPERTIES: i) HOLOMORPHIC, IE FN OF Φ's, NOT Φ^{+}'s

ii) NOT RENORMALIZED IN PERT. THY

FIRST FEW TERMS

as function of y

i) $\Phi_i \Phi_j = \theta\theta[A_i F_j + A_j F_i - \psi_i \psi_j] + \text{NON-}\theta\theta$

mass term!

iii) $\Phi_i \Phi_j \Phi_k = \theta\theta[F_i A_j A_k - \psi_i \psi_j A_k + \text{cyclic permutations}]$

Yukawa coupling

CONC: $W(\Phi)$ CAN GIVE MASS TERMS & YUKAWA COUPS.

Q: WHAT'S MISSING? KINETIC TERMS!

will easily understand once we have vector superfields

FACT: $\Phi_i \Phi_j^\dagger$ HAS $\theta\theta\bar\theta\bar\theta$ TERM TRANS INTO TOT DER.

$\therefore$ CAN USE IN $\mathcal{L}$!

as function of x.

$$\Phi_i^\dagger \Phi_j = \theta\theta\bar\theta\bar\theta\Big[F_i^* F_j + \tfrac{1}{4} A_i^* \Box A_j + \tfrac{1}{4} \Box A_i^* A_j - \tfrac{1}{2}\partial_m A_i^* \partial^m A_j + \tfrac{i}{2}\partial_m \bar\psi_i \bar\sigma^m \psi_j - \tfrac{i}{2}\bar\psi_i \bar\sigma^m \partial_m \psi_j \Big] + \dots$$

HMWK

NOTE: KIN TERMS FOR A'S, ψ'S, NOT F'S

IF $i=j$

PUT IT TOGETHER

most general susy renorm. Lag. involving only chiral sup.

$$\boxed{\mathcal{L} = \Phi_i^\dagger \Phi_i \Big|_{\theta\theta\bar\theta\bar\theta \text{ comp}} + \Big[\Big(\lambda_i \Phi_i + \tfrac{1}{2} m_{ij} \Phi_i \Phi_j + \tfrac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k \Big)\Big|_{\theta\theta} + h.c. \Big]}$$

$\Downarrow$

$$\mathcal{L}_{comp} = i\partial_m \bar\psi_i \bar\sigma^m \psi_i + A_i^* \Box A_i + F_i^* F_i + \Big[m_{ij}\Big(A_i F_j - \tfrac{1}{2}\psi_i\psi_j\Big) + g_{ijk}\Big(A_i A_j F_k - \psi_i \psi_j A_k\Big) + \lambda_i F_i + h.c. \Big]$$

SCALAR POTENTIAL

AUXILIARY FIELDS F

EOM: $0 = \frac{\delta \mathcal{L}}{\delta F_k^*} = F_k + \lambda_k^* + m_{ik}^* A_i^* + g_{ijk}^* A_i^* A_j^*$ SIM FOR $\frac{\delta \mathcal{L}}{\delta F_k} = 0$

NOTE: i) SOLVE $\Rightarrow$ F_k & F_k^* FUNC(SCALARS)

ii) PLUG INTO $F_k^* F_k \Rightarrow \boxed{V(A, A^*)}$

$\hookrightarrow$ Language: F-term scalar potential

iii) SINCE $V(A, A^*)$ FROM $F_k^* F_k$, HAVE ABS. MIN. IF $F_k = 0 \;\forall k$

R-SYMMETRIES

SYMMS THAT ROTATE θ's (alt: susy charges)

SUPPOSE $\theta \longmapsto e^{-i\alpha}\theta$, & $\therefore$ $\theta\theta \longmapsto e^{-2i\alpha}\theta\theta$

THEN $\Phi_i \Phi_j \theta\theta$ INV'T IF $\Phi_i \longmapsto e^{g_{\Phi_i} i\alpha} \Phi_i$

$\Phi_j \longmapsto e^{g_{\Phi_j} i\alpha} \Phi_j$

AND $g_{\Phi_i} + g_{\Phi_j} = 2$

Language: g_{Φ_i} & g_{Φ_j} are R-charges

BUT $\Phi_i \Phi_j \Phi_k \theta\theta$ REQ'S $g_{\Phi_i} + g_{\Phi_j} + g_{\Phi_k} = 2$.

OVERCONSTRAINED SYSTEM? NO R-SYMMETRY

EXAMPLE: 2 CH. SUPS. Φ_1 Φ_2

$$W(\Phi) = m_1 \Phi_1^2 + m_2 \Phi_2^2 + g\, \Phi_1 \Phi_1 \Phi_2$$

$$q_1 + q_1 = 2 \qquad q_2 + q_2 = 2 \qquad q_1 + q_1 + q_2 = 2$$

NO SOL'N $\Longrightarrow$ NO R-SYMM

OTOH: IF $g \to 0$, $q_1 = q_2 = 1$ SOL'N $\Longrightarrow$ R-SYMM

VECTOR SUPERFIELDS

DEF'N: A SUPERFIELD V SATIS. $V^{\dagger}=V$ IS A VECTOR SUPERFIELD

FACT: $\theta\theta\bar{\theta}\bar{\theta}$ COMPONENT OF V TRANSFORMS INTO TOT DER.

AHA! IF Φ CHIRAL, $(\Phi^{\dagger}\Phi)^{\dagger}=\Phi\Phi^{\dagger}=\Phi^{\dagger}\Phi$

$\Longrightarrow$ $\Phi^{\dagger}\Phi$ VECTOR $\Longrightarrow$ USE $\theta\theta\bar{\theta}\bar{\theta}$ COMP OF $\Phi^{\dagger}\Phi$ IN $\mathcal{L}$

EXPANSION

$$
\begin{aligned}
V(x,\theta,\bar{\theta}) = {} & C + i\theta\chi - i\bar{\theta}\bar{\chi} + \tfrac{i}{2}\theta\theta[M+iN] - \tfrac{i}{2}\bar{\theta}\bar{\theta}[M-iN] \\
& - \theta\sigma^m\bar{\theta}V_m + i\theta\theta\bar{\theta}[\bar{\lambda} + \tfrac{i}{2}\bar{\sigma}^m\partial_m\chi] - i\bar{\theta}\bar{\theta}\theta[\lambda + \tfrac{i}{2}\sigma^m\partial_m\bar{\chi}] \\
& + \tfrac{1}{2}\theta\theta\bar{\theta}\bar{\theta}[D + \tfrac{1}{2}\Box C]
\end{aligned}
$$

NOTE i) $V^{\dagger}=V$ REQ C, D, M, N, V_m REAL

ii) $\exists$ VECTOR FIELD V_m (hence the name)

GAUGE TRANSFORMATIONS

WANT: $V \longrightarrow V + \text{FUNC}(\text{SUPER-PARAMS})$

S.T. $V_m \longrightarrow V_m + \partial_m \alpha$

Q: why focusing on this comp?

RECALL IN X-COORDS $\Phi = i\theta\sigma^m\bar{\theta}\partial_m A + \ldots$

$\Phi^{\dagger} = -i\theta\sigma^m\bar{\theta}\partial_m A^* + \ldots$

$\therefore$ $V \longrightarrow V + \Phi + \Phi^{\dagger}$ $\quad \alpha := -2i\,\mathrm{Im}(A)$

HAS $V_m \longrightarrow V_m - i\partial_m(A - A^*) = V_m - 2i\partial_m \mathrm{Im}(A) = V_m + \partial_m\alpha$ ✓

FULL TRANS. IF $V \to V + \Phi + \Phi^\dagger$

THEN
$$C \to C + A + A^*$$
$$\chi \to \chi - i\sqrt{2}\psi$$
$$M + iN \to M + iN - 2iF$$
$$v_m \to v_m - i\partial_m (A - A^*)$$
$$\lambda \to \lambda$$
$$D \to D$$

WESS-ZUMINO GAUGE

CHOOSE TRANS PARAMS $A, \psi, F \mid C = \chi = M = N = 0$

SPINOR, 2 FERM, GAUGINOS

THEN
$$\boxed{V_{WZ} = -\theta\sigma^m\bar{\theta} v_m + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta} D}$$

VECTOR FIELD, 2 BOSONS

AUXILIARY

NOTE
$$V^2 = \theta\sigma^m\bar{\theta} v_m \theta\sigma^n\bar{\theta} v_n = -\frac{1}{2}\theta\theta\bar{\theta}\bar{\theta} v^m v_m$$
$$V^3 = 0$$

Q: why do tons of terms drop?

FIELD STRENGTHS (we see v_m, but when is $F_{\mu\nu}$?)

$$W_\alpha := -\frac{1}{4}\bar{D}\bar{D}D_\alpha V \qquad \bar{W}_{\dot\alpha} = -\frac{1}{4}DD\bar{D}_{\dot\alpha}V$$

CLAIM: THESE ARE i) CHIRAL & ii) GAUGE INV.

i) $\bar{D}_{\dot\beta}W_\alpha = 3\ \bar{D}'s\ (\ ) = 0 \qquad D_\beta \bar{W}_{\dot\alpha} = 3\ D's\ (\ ) = 0$ ✓

ii) $W_\alpha \longmapsto -\frac{1}{4}\bar{D}\bar{D}D_\alpha(V+\Phi+\Phi^\dagger) = -\frac{1}{4}\bar{D}\bar{D}D_\alpha(V+\Phi)$

$$= W_\alpha - \frac{1}{4}\bar{D}\bar{D}D_\alpha\Phi \underbrace{-\frac{1}{4}\bar{D}D_\alpha\bar{D}\Phi}_{\text{adding 0 since } \bar{D}\Phi = 0} = W_\alpha - \frac{1}{4}\bar{D}\{\bar{D},D_\alpha\}\Phi$$

$$= W_\alpha \quad ✓$$

SIM FOR $\bar{W}_{\dot\alpha}$

IN W-Z GAUGE

$$W_\alpha \overset{\text{no func of } Y}{=} -i\lambda_\alpha + \Big[\delta_\alpha{}^\beta - \frac{i}{2}(\sigma^m\bar\sigma^n)_\alpha{}^\beta \overbrace{(\partial_m v_n - \partial_n v_m)}^{F_{mn}\ !}\Big]\theta_\beta + \theta\theta\,\sigma_{\alpha\dot\alpha}{}^m\partial_m\bar\lambda^{\dot\alpha}$$

$$\bar{W}_{\dot\alpha} = i\bar\lambda_{\dot\alpha} + \Big[\varepsilon_{\dot\alpha\dot\beta}D + \frac{i}{2}\varepsilon_{\dot\alpha\dot\gamma}(\bar\sigma^m\sigma^n)^{\dot\gamma}{}_{\dot\beta}(\partial_m v_n - \partial_n v_m)\Big]\bar\theta^{\dot\beta} - \varepsilon_{\dot\alpha\dot\beta}\bar\theta\bar\theta\bar\sigma^{m\dot\beta\alpha}\partial_m\lambda_\alpha$$

HMWK

NOTE $DW = \bar{D}\bar{W}$

LAGRANGIANS W^α CHIRAL $\Rightarrow W^\alpha W_\alpha$ CHIRAL $\Rightarrow W^\alpha W_\alpha|_{\theta\theta}$ ALLOWED IN $\mathcal{L}$

W&B not for F^{mn}

$$W^\alpha W_\alpha|_{\theta\theta} = -2i\lambda\sigma^m\partial_m\bar\lambda - \frac{1}{2}v^{mn}v_{mn} + D^2 + \frac{i}{4}v^{mn}v^{lk}\varepsilon_{mnlk}$$

$$\mathcal{L} = \frac{1}{4}\left(W^\alpha W_\alpha \big|_{\theta\theta} + \bar{W}_{\dot\alpha} \bar{W}^{\dot\alpha} \big|_{\bar\theta\bar\theta} \right)$$

$$\Downarrow$$

some partial integ

$$\int d^4x \; \mathcal{L}_{\text{component}} = \int d^4x \left\{ \underbrace{\frac{1}{2} D^2 - \frac{1}{4} V^{mn} V_{mn} - i \lambda \sigma^m \partial_m \bar{\lambda}} \right\}$$

1 AUX FIELD

2 PROP. BOSONS hel ± 1

2 PROP FERMS hel $\pm \frac{1}{2}$

GAUGE INV'T INTERACTIONS (in superfield form.)

FIRST: ABELIAN INT

THEN: NON-ABELIAN INT

ABELIAN INTERACTIONS

SIMP. METHOD: GLOBAL $\longrightarrow$ LOCAL (remind me what this means?)

GLOBAL U(1) $\quad \Phi_\ell' = e^{-it_\ell \lambda} \Phi_\ell \qquad t_\ell, \lambda \in \mathbb{R}$

$\mathcal{L} = \mathcal{L}_{K.E.} + \mathcal{L}_{P.E.}$

$$\mathcal{L}_{K.E.} = \Phi_\ell^+ \Phi_\ell \Big|_{\theta\theta\bar\theta\bar\theta} \qquad \mathcal{L}_{P.E.} = \Big[\overbrace{\tfrac{1}{2} m_{ij} \Phi_i \Phi_j + \tfrac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k}^{\text{SUPERPOT. } W(\Phi)!} \Big]\Big|_{\theta\theta} + h.c.$$

INV'C REQ'S $\quad m_{ij} = 0 \quad$ IF $\quad t_i + t_j \neq 0$

$\qquad\qquad\qquad g_{ijk} = 0 \quad$ IF $\quad t_i + t_j + t_k \neq 0$

LOCAL U(1) $\qquad \lambda \longmapsto \Lambda(x, \theta, \bar\theta)$

$$\Phi_\ell' = e^{-it_\ell \Lambda} \Phi_\ell$$

$$0 \overset{\Phi \text{ ch.}}{=} \bar D_\alpha \Phi_\ell' = \cancelto{0}{\bar D_\alpha \Phi_\ell} \, e^{-it_\ell \Lambda} - i t_\ell \bar D_\alpha \Lambda \; \Phi_\ell$$

$\Longrightarrow \bar D_\alpha \Lambda = 0, \quad \boxed{\text{GAUGE PARAM } \Lambda \text{ IS CH. SUP}}$

NOTE $\mathcal{L}_{P.E.}$ INV'T! Q: WHY?

HOWEVER $\mathcal{L}_{KE}$ NOT INV'T SINCE

$$\Phi_\ell'^+ \Phi_\ell' = \Phi_\ell^+ \Phi_\ell \, e^{it_\ell(\Lambda^+ - \Lambda)}$$

Q: what would we do in the non-SUSY case at this juncture?

ADD VEC. SUP. V S.T. $V' = \underbrace{V + X + X^{\dagger}}$

recall: this is gauge trans form for V. sup, where X is ch. sup.

WRITE $X = i\Lambda$ THEN $V' = V + i(\Lambda - \Lambda^{\dagger})$

THEN $\Phi_\ell^{\dagger} e^{t_\ell V} \Phi_\ell \mapsto \Phi_\ell^{\dagger} e^{+it_\ell \Lambda^{\dagger}} e^{t_\ell V} e^{it_\ell(\Lambda - \Lambda^{\dagger})} e^{-it_\ell \Lambda} \Phi_\ell$

$$= \Phi_\ell^{\dagger} e^{t_\ell V} \Phi_\ell \qquad \text{INV'T!}$$

ADD KIN. TERMS FOR V

$$\Rightarrow \boxed{\begin{aligned} \mathcal{L} = {} & \tfrac{1}{4}\left(W^{\alpha} W_{\alpha}\big|_{\theta\theta} + \bar{W}_{\dot\alpha} \bar{W}^{\dot\alpha}\big|_{\bar\theta\bar\theta}\right) + \Phi_\ell^{\dagger} e^{t_\ell V} \Phi_\ell \big|_{\theta\theta\bar\theta\bar\theta} \\ & + \left[\tfrac{1}{2}\left(m_{ij} \Phi_i \Phi_j + \tfrac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k\right)\Big|_{\theta\theta} + h.c.\right] \end{aligned}}$$

$(*)$

IN W-Z GAUGE $V^3 = 0$, GET

$$\Phi^{\dagger} e^{tV} \Phi\big|_{\theta\theta\bar\theta\bar\theta} = F^* F + A \Box A^* + i\partial_n \bar\psi \bar\sigma^n \psi \quad \Big\}\ \text{kinetic terms}$$

$$\left.\begin{aligned} & + t v^n\left(\tfrac{1}{2}\bar\psi \bar\sigma^n \psi + \tfrac{i}{2} A^* \partial_n A - \tfrac{i}{2}\partial_n A^* A\right) \\ & - \tfrac{i}{\sqrt{2}} t\left(A \bar\lambda \bar\psi - A^* \lambda \psi\right) + \tfrac{1}{2}\left(tD - \tfrac{1}{2} t^2 v_n v^n\right) A^* A \end{aligned}\right\}\ \text{couplings to gauge bos. \& gaugino}$$

SUPER QED super-electron & super-positron

what physically does this theory have?

$\hookrightarrow$ ABOVE, BUT 2 CHIRAL SUP Φ_+ Φ_-

WHERE $\Phi_+' = e^{-ie\Lambda} \Phi_+ \qquad \Phi_-' = e^{ie\Lambda} \Phi_-$

$$\mathcal{L}_{\text{SUPER-QED}} \supset -m\left(\psi_+ \psi_- + \bar\psi_+ \bar\psi_-\right) = -m\, \bar\Psi_{Dir} \Psi_{Dir}$$

point ψ_+, ψ_- can be rewritten as four-comp Dirac spinor

NON-ABELIAN INTERACTIONS

HAD: $\Phi_e' = e^{-it_e\Lambda}\Phi_e$ (abelian case)

NOW: $\Phi_i' = e^{-i\Lambda_{ij}}\Phi_i$ $\quad \Phi_j^{+\prime} = \Phi_i^+ e^{i\Lambda_{ij}^+}$ (non-abelian gen)

WHERE Λ IS MATRIX $\quad \Lambda_{ij} = T_{ij}^a \Lambda_a$

T_{ij}^a $\underbrace{\text{MATRIX-REP}}$ OF GENS OF GAUGE GROUP

remind me:
what does this mean?

NORM: $\quad \mathrm{Tr}\, T^a T^b = k\delta^{ab} \quad k>0$

$$[T^a, T^b] = i t^{abc} T^c$$

NON-ABELIAN LAG.

(*) IS INV'T WITH A FEW MOD'S.

i) $e^{V'_{ij}} = e^{-i\Lambda^+_{ii'}} e^{V_{i'j'}} e^{i\Lambda_{j'j}}$

$V_{ij} = T^a_{ij} V_a$ (how is this exp like in the non-SUSY theory?)

ASIDE: $\quad V' = V + i\underbrace{(\Lambda - \Lambda^+)} + \dots$

leading term V-INDEP

$\Longrightarrow$ can think of WZ

$\exists$ NON-AB WZ GAUGE w/ $V^3 = 0$

(ii) $$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}\, e^{-V} D_\alpha e^{V}$$

$$W'_\alpha = e^{-i\Lambda} W_\alpha e^{i\Lambda}$$

HMWK: this and invariance of kinetic term

SUMMARY

$$\boxed{\begin{aligned}\mathcal{L} = \frac{1}{16kg^2}\,\mathrm{Tr}\left(W^\alpha W_\alpha\big|_{\theta\theta} + \bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}\big|_{\bar\theta\bar\theta}\right) + \bar{\Phi}^\dagger e^V \Phi\,\big|_{\theta\theta\bar\theta\bar\theta} \\ + \left[\frac{1}{2} m_{ij}\,\Phi_i\Phi_j + \frac{1}{3} g_{ijk}\,\Phi_i\Phi_j\Phi_k\right)\Big|_{\theta\theta} + h.c.\Big]\end{aligned}}$$