More Condensed Matter

2018: got here in class # 8

TOPIC 2: MORE CONDENSED MATTER (ie things in CM systems, but also in HE)

i) ASYMPTOTIC FREEDOM IN NLSM

ii) DEFECTS, CHERN-SIMONS, 2+1 PARTICLE VORTEX-DUALITY

iii) GENERALIZED FERMIONS (ie different types in different dims)

ASYMPTOTIC FREEDOM IN NLSM & FERROMAGNETS

ASIDE: SCALAR FIELD DIMENSION

$$[S_\phi] = 0 = \left[\int d^d x \, \partial_\mu \phi \partial^\mu \phi + \cdots\right] = d[dx] + 2[\partial_\mu] + 2[\phi]$$

$$[\partial_\mu] = 1 \qquad [dx] = -1 \qquad \left(M \sim \frac{1}{L}\right)$$

$$\Rightarrow -d + 2 + 2[\phi] = 0 \qquad \text{SO} \qquad [\phi] = \frac{d-2}{2}$$

2D SCALAR FIELDS (1+1)

$$\mathcal{L} = \underbrace{f_{ij}(\phi^i)}_{\text{ANY FUNCTION}} \partial_\mu \phi^i \partial^\mu \phi^j$$

ANY FUNCTION $\Rightarrow [f_{ij}\,\partial\phi\,\partial\phi]$ (use symmetry to constrain)

O(N) SYMMETRY

$$\phi_i \longrightarrow n_i(x) \;\Big|\; n^i n^i = 1$$

$$\overset{\text{restricts } \mathcal{L}}{\Longrightarrow} \quad \mathcal{L} = \frac{1}{2g^2} |\partial_\mu \vec{n}|^2 \quad \text{S.T.} \quad n^i n^i = 1$$

IMPOSE CONSTRAINT $\quad n^i = (\pi^1, \ldots, \pi^{N-1}, \sigma) \overset{n^i n^i = 1}{\Longrightarrow} \sigma = (1 - \pi^2)^{\frac{1}{2}}$

$$|\partial_\mu n^i|^2 \supset \partial_\mu \sigma \partial^\mu \sigma = \partial_\mu\left[(1-\vec{\pi}^2)^{\frac{1}{2}}\right]\partial^\mu\left[(1-\vec{\pi}^2)^{\frac{1}{2}}\right] = \left(\frac{-2\left(\frac{1}{2}\right)\vec{\pi}\cdot\partial_\mu\vec{\pi}}{\sqrt{1-\vec{\pi}^2}}\right)^2 = \frac{(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2}{1-\vec{\pi}^2}$$

$$\mathcal{L} = \frac{1}{2g^2}\left[|\partial_\mu \vec{\pi}|^2 + \frac{(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2}{1-\pi^2}\right]$$

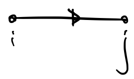
NOTE: $\vec{\pi}$ MASSLESS! CONSISTENT W/ GOLDSTONE OF O(N) BREAKING

expand in π^k

$$\mathcal{L} = \frac{1}{2g^2}|\partial_\mu \vec{\pi}|^2 + \frac{1}{2g^2}(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2 + \cdots$$

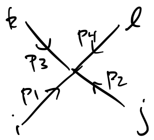
higher point interactions there but not important for us

PROPAGATOR $\quad \frac{1}{2g^2}|\partial_\mu \vec{\pi}|^2 \overset{\text{IBP}}{=} \vec{\pi}\cdot\underbrace{\left(-\frac{1}{2g^2}\Box\right)}_{\text{invert for prop}}\cdot\vec{\pi}$

$\Longrightarrow$  $= \frac{ig^2}{p^2}\delta_{ij}$

INTERACTION $\quad \frac{1}{2g^2}(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2$

PROPERTIES: 4PTS, 2 p's, $\sim \frac{1}{g^2}$



$= \frac{-i}{g^2}\Big[\underbrace{(p_1+p_2)\cdot(p_3+p_4)\,\delta^{ij}\delta^{kl} + (p_1+p_3)\cdot(p_2+p_4)\,\delta^{ik}\delta^{jl} + (p_1+p_4)\cdot(p_2+p_3)\,\delta^{il}\delta^{jk}}_{\text{all possible index pairings}}\Big]$

CALLAN SYMANZIK

RECALL: CONSTRAINTS ON GREEN'S FUNCTIONS

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + n\gamma(g)\right]G^{(n)} = 0$$

NEED: SOME $G^{(n)}$'s (enough to determine β, γ fcns)

FIRST $\quad G^{(1)}(x) = \langle\sigma(x)\rangle = \langle 0|T(\sigma(x))|0\rangle$

BUT $\quad \sigma(x) = (1-\pi^2)^{\frac{1}{2}} = 1 - \frac{1}{2}\pi^2 + \dots$

$\Rightarrow G^{(1)}(0) = 1 - \frac{1}{2}\langle\pi^2(0)\rangle + \dots = 1 - \frac{1}{2}$ 

$\langle\pi^k(0)\,\pi^l(0)\rangle =$  $= \int\frac{d^dk}{(2\pi)^d}\,\frac{ig^2}{k^2-\mu^2}\,\delta^{kl}$ to handle infrared div.

$= \frac{g^2}{(4\pi)^{d/2}}\,\frac{\Gamma(1-\frac{d}{2})}{(\mu^2)^{1-d/2}}\,\delta^{kl}$

STEPS HERE = ?

USE IN $\langle\sigma\rangle$ & RENORMALIZATION COND.

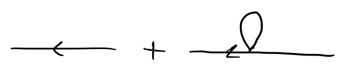
$$\Rightarrow \langle\sigma\rangle = 1 - \tfrac{1}{2}(N-1)\underbrace{\frac{g^2}{(4\pi)^{d/2}}\Gamma\left(1-\tfrac{d}{2}\right)}_{\sim \frac{2}{\varepsilon}+\dots}\left(\underbrace{\frac{1}{(\mu^2)^{1-d/2}}}_{(\,)\varepsilon+\dots} - \underbrace{\frac{1}{(M^2)^{1-d/2}}}_{\text{"subtraction" from ren. P.T., counterterm, etc.}}\right) + \mathcal{O}(g^4)$$

$$\overset{d=2}{=} 1 - \frac{g^2(N-1)}{8\pi}\log\frac{M^2}{\mu^2} + \mathcal{O}(g^4)$$

C-S $\quad 0 = \left(M\frac{\partial}{\partial M} + \beta\frac{\partial}{\partial g} + \gamma\right)\left(1 - \frac{g^2(N-1)}{8\pi}\log\frac{M^2}{\mu^2} + \dots\right)$

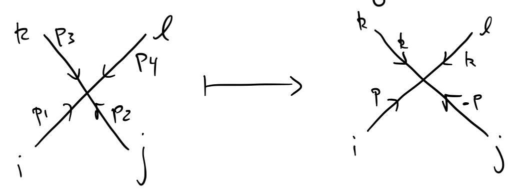
$$= \left(-\frac{g^2(N-1)}{8\pi}\cdot 2 + \underbrace{\beta\frac{\partial}{\partial g}G^{(1)}}_{\text{H.O.T. IN } g} + \gamma\left(1 - g^2(\,)\dots\right)\right)$$

$$\Rightarrow \gamma = \frac{g^2(N-1)}{4\pi} + \mathcal{O}(g^4)$$

SECOND $\quad \langle \pi^k(p)\pi^l(-p)\rangle =$  $+ \dots$

$$= \frac{ig^2}{p^2}\delta^{kl} + \frac{ig}{p^2}\underbrace{\left(-i\Pi^{kl}\right)}_{\text{effectively a definition of the thing in the loop}}\frac{ig}{p^2}$$

Π^{kl}: TWO CONSTRACTION TYPES @ VERTICES



i) i WITH j (one possibility)
ii) i WITH other two poss)

NEED $\quad \langle \partial_\mu\pi^k(0)\partial^\mu\pi^l(0)\rangle = \int\frac{d^dk}{(2\pi)^d}\frac{ig^2k^2}{k^2-\mu^2}\delta^{kl}$

$$= -\frac{g^2}{(4\pi)^{d/2}}\frac{\frac{d}{2}\Gamma\left(-\frac{d}{2}\right)}{(\mu^2)^{-d/2}}\delta^{kl}$$

$$\Pi^{kl}(p) \overset{\text{HMMM?}}{=} -\delta^{kl}p^2\frac{1}{(4\pi)^{d/2}}\frac{\Gamma\left(1-\frac{d}{2}\right)}{(\mu^2)^{1-d/2}}$$

$$\overset{\text{W R.C. \& } d=2}{\Rightarrow} \langle\pi^k(p)\pi^l(-p)\rangle = \frac{ig^2}{p^2}\delta^{kl} + \frac{ig}{p^2}\left(ip^2\frac{1}{4\pi}\log\frac{M^2}{\mu^2}\right)\frac{ig^2}{p^2}\delta^{kl} + \dots$$

$$= \frac{i}{p^2}\delta^{kl}\left(g^2 - \frac{g^4}{4\pi}\log\frac{M^2}{\mu^2}\right) + \mathcal{O}(g^6)$$

CALLAN-SYMANZIK

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}+2\gamma\right]\langle \pi^k(p)\,\pi^\ell(-p)\rangle=0$$

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}+2\gamma\right]\frac{i}{p^2}\delta^{k\ell}\left(g^2-\frac{g^4}{4\pi}\log\frac{M^2}{\mu^2}\right)$$

$$=\frac{i\delta^{k\ell}}{p^2}\cdot\Big[\underbrace{2\left(-\frac{g^4}{4\pi}\right)}_{g^4}+\underbrace{2g\beta}_{\mathcal{O}(g\beta)}+\underbrace{\#_1 g^3\beta}_{\mathcal{O}(g^3\beta)}+\underbrace{2g^2\gamma}_{\mathcal{O}(g^4)}+\underbrace{\#_2\, g^4\gamma}_{\mathcal{O}(g^6)}\Big]$$

$\gamma \rightarrow \mathcal{O}(g^2)$

NEED $$-\frac{g^4}{2\pi}+2g\beta+2g^2\,\frac{g^2(N-1)}{4\pi}=0$$

$$\Longrightarrow\ \beta=\frac{1}{2g}\left[-2g^2\,\frac{g^2(N-1)}{4\pi}+\frac{g^4}{2\pi}\right]$$

$$=\frac{1}{2g}\left[-2\left[\frac{g^2(N-1)}{4\pi}-\frac{g^2}{4\pi}\right]\right]$$

$$\boxed{\beta=M\frac{\partial g}{\partial M}=-\frac{g^3}{4\pi}(N-2)+\mathcal{O}(g^5)}$$

CASES: i) N=2: $\beta=0$

CHANGE VARIABLES $\pi^1=\sin\theta$ $\quad\sigma=\cos\theta$

$$\Longrightarrow\ \mathcal{L}=\frac{1}{2g^2}(\partial_\mu\theta)^2$$

Q: WHY $\beta=0$?

A: FREE THEORY

ii) N>2 $\quad\beta<0\ \Longrightarrow$ ASYMPTOTICALLY FREE (in UV)

see Peskin for another "more physical" derivation using Wilsonian momentum slicing.