QFT 2

Topic 3: study

SUPERSYMMETRY: INTRODUCTION BY ENGINEERING

$$S = \int d^4x \left(\mathcal{L}_{scalar} + \mathcal{L}_{fermion}\right)$$

$\mathcal{L}_{scalar} = -\partial^\mu \phi^* \partial_\mu \phi$ $\qquad\qquad$ $\mathcal{L}_{fermion} = i\bar{\psi}\bar{\sigma}^\mu \partial_\mu \psi$

TRANS: SC $\to$ FERM

TRY 1 $\quad \phi \mapsto \phi + \psi_\alpha$ $\quad$ X BAD IND'S.

TRY 2 $\quad \phi \mapsto \phi + \epsilon\psi$ $\quad$ ✓ IND'S

NOTE: ϵ FERMIONIC TRANS. PARAM!

SIM: $\quad \phi^* \longmapsto \phi^* + \bar{\epsilon}\bar{\psi}$

$$\mathcal{L}_{scalar} \to \mathcal{L}_{scalar} - \epsilon\,\partial^\mu\psi\,\partial_\mu\phi^* - \bar{\epsilon}\,\partial^\mu\bar{\psi}\,\partial_\mu\phi$$

TRANS: FERM $\to$ SC

NEED: $\bar{\psi}_{\dot\alpha}$ TRANS TO GET ϵ, ∂

ψ_α TRANS TO GET $\bar{\epsilon}$, ∂

TAKE: $\psi_\alpha \to \psi_\alpha - i(\sigma^\mu\bar{\epsilon})_\alpha\,\partial_\mu\phi$

$\bar{\psi}_{\dot\alpha} \to \bar{\psi}_{\dot\alpha} + i(\epsilon\sigma^\mu)_{\dot\alpha}\,\partial_\mu\phi^*$

$$\mathcal{L}_{fem} \to \mathcal{L}_{fem} \underbrace{-\epsilon\,\sigma^m\bar{\sigma}^n\partial_n\psi\,\partial_m\phi^* + \bar{\psi}\bar{\sigma}^\nu\sigma^m\bar{\epsilon}\,\partial_\mu\partial_\nu\phi.}$$

$$= \epsilon\,\partial^\mu\psi\,\partial_\mu\phi^* + \bar{\epsilon}\,\partial^\mu\bar{\psi}\,\partial_\mu\phi$$
$$-\partial_\mu\left(\epsilon\sigma^\nu\bar{\sigma}^m\psi\,\partial_\nu\phi^* + \epsilon\,\psi\,\partial^\mu\phi^* + \bar{\epsilon}\bar{\psi}\,\partial^\mu\phi\right)$$

CONCLUSION - $\mathcal{L}$ INVT, UP TO TOT. DERIVS!

- BOSON $\longleftrightarrow$ FERMION SYMMETRY

"SUPERSYMMETRY"

what do you know about it? if it exists in particle physics, how must it exist? (broken) "How much" susy?

amt symmetry = # gens

TOPIC OUTLINE

$d=4$ $\mathcal{N}=1$ SUSY

1. 2 COMP. SPINORS
2. SUSY ALG. & REPS
3. COMPONENT FIELDS

   somewhere in here by spring break

4. SUPERSPACE & SUPERFIELDS
5. CHIRAL & VECTOR SUPERFIELDS
6. GAUGE INV. INTERACTIONS

   - start 2/21 OR 2/22 IN 2018
   - classes left: makeups WEDS' 2/21, 2/28, 3/21, 4/4

     Mondays: 2/26, ~~3/12~~ → OX, 3/19, ~~3/26~~ → MUN, 4/2, ~~4/9~~ → LON

     Thursdays: 2/22, 3/1, 3/15, 3/22, ~~3/29~~ → MUN, 4/5, 4/12

7. ~~SUSY~~

Wess & Bagger

8. MSSM & SUSY PHENO ⟹ Martin's "A Supersymmetry Primer"

SOME MOTIVATION

NOTATION & 2 COMP. SPINORS

NOTATION 4-VECTORS w/ LOWERCASE LATIN IND. (eg A_m, NOT A_μ)

RECALL – 4-COMP ψ SPINORS $\Longrightarrow \begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix}$, $\psi_{L,R}$ 2-COMP

– $\Gamma^\mu \sim \left(\begin{array}{c|c} 2\times2 & 2\times2 \\ \hline 2\times2 & 2\times2 \end{array}\right) \Longrightarrow \psi_{L,R}$ ACTED ON BY 2×2's

– **INSPECTION:** 2×2's ARE

$$\sigma^0 = \begin{pmatrix} -1 & \\ & -1 \end{pmatrix} \quad \& \quad \text{PAULI MATRICES}$$

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

CALL $\sigma^m = (\sigma^0, \vec{\sigma})$

FACT: σ^m GENERATE $SL(2,\mathbb{C})$ = DOUBLE COVER OF LORENTZ GROUP

(know this: $SU(2) \subseteq SL(2,\mathbb{C})$, $SU(2) \xrightarrow{\text{2-cover}} SO(3)$, $SL(2,\mathbb{C}) \xrightarrow{\text{2-cover}} SO(3,1)$, $SO(3) \subseteq SO(3,1) = LOR.$

2-COMP. SPINOR TYPES

$M \in SL(2,\mathbb{C})$

M, M^*, $(M^T)^{-1}$, $(M^\dagger)^{-1}$ ALL REP $SL(2,\mathbb{C}) \Longrightarrow$ 4 2-COMP. SPINOR REPS

$$\boxed{\psi_\alpha' = M_\alpha{}^\beta \psi_\beta \qquad \bar{\psi}_{\dot\alpha}' = M^*_{\dot\alpha}{}^{\dot\beta} \bar{\psi}_{\dot\beta} \qquad \psi'^\alpha = (M^{-1})_\beta{}^\alpha \psi^\beta \qquad \bar{\psi}^{\dot\alpha\prime} = (M^{*-1})_{\dot\beta}{}^{\dot\alpha} \bar{\psi}^{\dot\beta}}$$

COMPARE TO OLD NOTATION $\psi_\alpha = \psi_L$ $\bar{\psi}^{\dot\alpha} = \psi_R$

HERMITIAN 2×2 P $\Longrightarrow$ CAN WRITE $P = P_m \sigma^m = \begin{pmatrix} -P_0 + P_3 & P_1 - iP_2 \\ P_1 + iP_2 & -P_0 - P_3 \end{pmatrix}$, $P_m \in \mathbb{R}$

P HERM $\Longrightarrow P' = MPM^\dagger$ HERM $\Longrightarrow \sigma^m P_m' = M_\alpha{}^\beta \sigma^m P_m M^\dagger_{\dot\alpha}{}^{\dot\beta}$

SOME LORENTZ INVARIANTS: $\psi^\alpha \psi_\alpha =: \psi\psi$ $\quad \bar{\psi}_{\dot\alpha} \bar{\psi}^{\dot\alpha} =: \bar{\psi}\bar{\psi}$

$\psi^\alpha \sigma^m_{\alpha\dot\alpha} \bar{\psi}^{\dot\alpha} =: \psi \sigma^m \bar{\psi}$ $\quad$ NOTE $_{\dot\alpha}$ $^{\dot\alpha}$ ORDER.

RAISE & LOWER INDICES USE $\varepsilon^{\alpha\beta}$ $\varepsilon_{\alpha\beta}$

$\varepsilon_{21} = \varepsilon^{12} = 1$ $\quad \varepsilon_{12} = \varepsilon^{21} = -1$, REST ZERO

$$\boxed{\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta \qquad \psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta \qquad \varepsilon_{\alpha\beta}\varepsilon^{\beta\gamma} = \delta_\alpha{}^\gamma}$$

$\bar\sigma^{m\,\dot\alpha\alpha} := \varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\alpha\beta}\sigma^m_{\beta\dot\beta} \quad \Rightarrow \quad \bar\sigma^m = (\sigma^0, -\sigma^m)$ AS MATRICES

SOME IDENTITIES

INTEREST: $(\sigma^m\bar\sigma^n)_\alpha{}^\beta + (\sigma^n\bar\sigma^m)_\alpha{}^\beta$

$m=n=0 \rightarrow \sigma^0\sigma^0+\sigma^0\sigma^0 = 2\,\delta_\alpha{}^\beta$

$m=n=i \Rightarrow \sigma^i(-\sigma^i)+\sigma^i(-\sigma^i) = -2\,\delta_\alpha{}^\beta$, $i=1,2,3$

$m=i\ n=j,\ j\neq i \Rightarrow \sigma^i(-\sigma^j)+\sigma^j(-\sigma^i) = -\{\sigma^i,\sigma^j\} = 0$ (from anti-comm of Pauli props)

$m=0\ n=i \Rightarrow \sigma^0(-\sigma^i)+(\sigma^i)(\sigma^0) = -\mathbb{1}(-\sigma^i)+\sigma^i\sigma^0 = 0$

RESULT (ε dim)

$$\boxed{\begin{aligned}(\sigma^m\bar\sigma^n+\sigma^n\bar\sigma^m)_\alpha{}^\beta &= -2\eta^{mn}\delta_\alpha{}^\beta = -2\eta^{mn}\mathbb{1}_{2\times2}\\ (\bar\sigma^m\sigma^n+\bar\sigma^n\sigma^m)^{\dot\alpha}{}_{\dot\beta} &= -2\eta^{mn}\delta^{\dot\alpha}{}_{\dot\beta} = -2\eta^{mn}\mathbb{1}_{2\times2}\end{aligned}}$$

(the above sounds familiar)

RECALL (P&S) $\gamma^0 = \begin{pmatrix}0 & \mathbb{1}\\ \mathbb{1} & 0\end{pmatrix}$ $\gamma^i = \begin{pmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{pmatrix}$

$$\{\gamma^m,\gamma^n\} = 2\eta^{mn}\mathbb{1}_{4\times4} = 2\eta^{mn}\begin{pmatrix}\mathbb{1}_{2\times2} & 0\\ 0 & \mathbb{1}_{2\times2}\end{pmatrix}$$

EVEN MORE

$$\boxed{\mathrm{Tr}\,\sigma^m\bar\sigma^n = -2\eta^{mn} \qquad \sigma_{\alpha\dot\alpha}{}^m\,\bar\sigma_m{}^{\dot\beta\beta} = -2\,\delta_\alpha{}^\beta\,\delta_{\dot\alpha}{}^{\dot\beta}}$$

LORENTZ GENS

Q: IF 4-COMP $\sim [\gamma^m,\gamma^n]$, WHAT 2-COMP? A: $\sim [\sigma^m,\bar\sigma^n]$

$$\sigma^{nm}{}_\alpha{}^\beta := \frac14\left(\sigma_{\alpha\dot\alpha}{}^n\bar\sigma^{n\,\dot\alpha\beta} - \sigma_{\alpha\dot\alpha}{}^m\bar\sigma^{n\,\dot\alpha\beta}\right)$$

$$\bar\sigma^{nm\,\dot\alpha}{}_{\dot\beta} = \frac14\left(\bar\sigma^{n\,\dot\alpha\alpha}\sigma_{\alpha\dot\beta}{}^m - \bar\sigma^{m\,\dot\alpha\alpha}\sigma_{\alpha\dot\beta}{}^n\right)$$

$$\boxed{\begin{aligned}\bar\sigma^a\sigma^b\bar\sigma^c - \bar\sigma^c\sigma^b\bar\sigma^a &= -2i\,\varepsilon^{abcd}\,\bar\sigma_d\\ \sigma^a\bar\sigma^b\sigma^c - \sigma^c\bar\sigma^b\sigma^a &= +2i\,\varepsilon^{abcd}\,\sigma_d\end{aligned}}$$

Homework: 3) 2pt 5) 3pt 10) 3pt

SUSY ALG

THEOREM: (COLEMAN-MANDULA)

ANY THY W/ SYMM. GROUP G A LIE ALGEBRA

i) S-MATRIX FROM LOCAL, REL. QFT

ii) BELOW ANY MASS M $\exists$ FINITE # PARTS p W $m_p < M$.

iii) $\exists$ ENERGY GAP BETW. $|0\rangle$ & ALL 1-PARTICLE STATES

HAS $G = \underbrace{\text{Poincare}}_{\sim \mathbb{R}^{3,1} \text{ trans} \times \text{Lorentz}} \times \text{internal}$

IE: SPACETIME & INT. SYMMETRIES CAN'T MIX!

Q: WHAT DO WE DO WITH THEOREMS?

A1: ASK WHETHER CONCLUSIONS ARE INTERESTING

A2: " " ASSUMPTIONS ARE REASONABLE

$\overset{\text{"}}{\Longrightarrow}$ CONSIDER LOOPHOLES!

LOOPHOLE: TAKE G A GRADED LIE ALG. AKA SUPERALGEBRA

SCHEMATIC: - FERMIONIC GENS Q, Q', Q''

- BOSONIC GENS X, X', X''

- ALG $\sim$ $\{Q, Q'\} = X$ $\quad [X, X'] = X''$ $\quad [Q, X] = Q''$

THEOREM: (HAAG-LOPUSZANSKI-SOHNIUS)

- TAKE C-M ASSUMPTIONS, BUT G A GRADED L.A. W Q^L_α & $\bar{Q}^L_{\dot\alpha}$ (its Herm conjugate) IN ALG.

THEN: $$\boxed{\{Q^L_\alpha, \bar{Q}_{\dot\beta M}\} = 2\sigma^m_{\alpha\dot\beta} P_m \delta^L{}_M, \quad \{Q^L_\alpha, Q_\beta{}^M\} = \varepsilon_{\alpha\beta} X^{LM}}$$

antisymm.

"SUSY ALG"

WITH $L = 1, \ldots, \mathcal{N}$, P_m EN-MOM. OPS

NB: - SUSY GENS $Q\,\bar{Q}$ ANTI COMM TO SPACETIME TRANS (P_m)

- X^{LM} "CENTRAL CHARGES"
  - NONE IF $\mathcal{N}=1$ (Q: why? antisymmetry)
  - ONE IF $\mathcal{N}=2$ (Q: why? X^{LM} 2×2 antisymm)

CONSERVED CHARGE $\exists$ SUSY QFT'S WHERE

$$Q_\alpha = \int d^3x\, J^0_\alpha \quad \text{HAS} \quad \frac{dQ_\alpha}{dt} = 0 \quad \text{B/C} \quad \partial_m J^m_\alpha = 0$$

REPS OF SUSY ALG (Q: what are ireps of susy alg?)

$\underline{M>0}$ M = mass

REST FRAME $P_m = (M, 0, 0, 0)$

why? $P_i = 0$ & $\sigma^0_{\alpha\dot\beta} = \delta_{\alpha\dot\beta}$ ignore central ch. for now

ALGEBRA: $\{Q^A_\alpha, \bar{Q}_{\dot\beta B}\} = 2M\,\delta_{\alpha\dot\beta}\,\delta^A_B$ $\quad \{Q,Q\} = \{\bar{Q},\bar{Q}\} = 0$

RESCALE: $a^A_\alpha := \frac{1}{\sqrt{2M}} Q^A_\alpha \quad (a^A_\alpha)^+ := \frac{1}{\sqrt{2M}} \bar{Q}_{\dot\alpha A}$

S.T. $\{a^A_\alpha, (a^B_\beta)^+\} = \delta^\beta_\alpha\, \delta^A_B \quad \{a,a\} = \{a^+, a^+\} = 0$

from spinor index

NB: $2N$ PAIRS OF FERM CREAT. & ANNI. OPS

$\Rightarrow \exists\, |\Omega\rangle \mid a^A_\alpha |\Omega\rangle = 0 \quad \forall \alpha, A$

__REP:__ (as with $\mathfrak{g}^{(s)}$ spinor) ACT w/ $(a^A_\alpha)^+$ IN ALL POSS WAYS

$\Rightarrow$ YES/NO ON EACH, $\underbrace{A, \alpha}_{2N \text{ CHOICES}}$

$\Rightarrow$ 2^{2N} CHOICES

__EXAMPLES__ $\quad N=1 \quad \underbrace{|\Omega\rangle}_{\text{spin } 0}, \underbrace{(a_\alpha)^+|\Omega\rangle}_{\text{spin-}\frac{1}{2}}, \underbrace{\frac{1}{\sqrt{2}}(a_\alpha)^+(a_\beta)^+|\Omega\rangle}_{\text{spin } 0}$

I.E. 2 FERM, 2 BOS

__FACT:__ $|\Omega\rangle$ CAN HAVE SPIN j, $\Rightarrow |\Omega_j\rangle$

__TABLE__

SPIN	Ω_0	$\Omega_{1/2}$	Ω_1	$\Omega_{3/2}$
0	2	1		
1/2	1	2	1	
1		1	2	1
3/2			1	2
2				1
Σ states	$4 = 1\cdot 2^2$	$8 = 2\cdot 2^N$	$12 = 3\times 2^N$	$16 = 4\times 2^N$

$\Rightarrow$ coeff = # of m's for $|\Omega_j\rangle$

(names of first two: massive chiral multiplet, massive vector mult)

$\boxed{M=0}$ $\quad P^2=0$, NO REST FRAME, $P_m = (-E, 0, 0, E)$

$\Rightarrow$ ALG IS $\quad \{Q^A_\alpha, \bar{Q}_{\dot\beta B}\} = 2E(\sigma^0 - \sigma^3)_{\alpha\dot\beta}\,\delta^A{}_B \overset{\text{plug } \sigma^0, \sigma^3}{=} 2\begin{pmatrix} 2E & 0 \\ 0 & 0 \end{pmatrix}\delta^A{}_B$

$\{Q, Q\} = \{\bar{Q}, \bar{Q}\} = 0$

__NB__ ON $M=0$ STATES $\{Q^A_2, \bar{Q}^A_{\dot 2}\} = \{Q^A_2, (Q^A_2)^+\} = 0$.

BUT ANY $\{\mathcal{O}, \mathcal{O}^+\}$ IS POS. SEMI-DEF, $\Rightarrow$ $Q_2{}^A = \bar{Q}_{\dot 2}{}^A = 0$ ON THESE $M=0$ STATES

__NB:__ ONLY $\alpha=1$ NON-TRIVIAL $\quad a^A := \frac{1}{2\sqrt{E}} Q_1{}^A \quad (a^A)^+ := \frac{1}{2\sqrt{E}} \bar{Q}_{\dot 1}{}^A$

$\Rightarrow$ $\{a^A, a^{+B}\} = \delta^A{}_B \quad \{a, a\} = \{a^+, a^+\} = 0$

$\Rightarrow$ N (NOT $2N$) CREAT. ANNI. OPS $\Rightarrow 2^N$

w/ CPT, 2×2^N

CASE $N=1$ TABLE

$N=1$ $M=0$ HELICITY (recall: massless particle in Poincaré rep det'd by "helicity", which is half integer or integer)

helicity of $|\Omega\rangle$ OR CPT $|\Omega\rangle$

	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
2								1
$3/2$							1	1
1						1	1	
$1/2$					1	1		
0				1	1			
$-1/2$			1	1				
-1		1	1					
$-3/2$	1	1						
-2	1							

chiral mult: 2 hel 0, hel $\pm\frac{1}{2}$

vector multiplet:

hel $\pm 1 \Rightarrow$ vector

hel $\pm\frac{1}{2} \Rightarrow$ fermion

GENERAL RESULT: $\boxed{\text{SUSY REPS HAVE \# BOSONS = \# FERMIONS!}}$