

QFT 2

Topic 4: EFT



EFFECTIVE FIELD THEORY

- ① DIMENSIONAL ANALYSIS
- ② HEURISTIC IDEA
- ③ UV COMPLETIONS & INTEGRATING OUT (in @ \mathcal{L} -level)
- ④ EFT RULES
- ⑤ EXAMPLES
 - THE BLUE SKY
 - 4-FERMI THY
 - SCHRÖDINGER EQN
 - SM HIGGS EFFECTIVE THEORY
 - HIGGS PORTAL DM

RENORMALIZABLE

VS. NON-RENORMALIZABLE

"R"

"NR"

FACT: Λ -dependence dropped out.

FACT: Doesn't always happen!

Λ DROPS = R-THY

new generation: nothing wrong with these as effective theories

Λ DOESN'T DROP = NR-THY

(one way to characterize the issue)

DIMENSIONAL ANALYSIS

* $e^{iS} \Rightarrow S$ DIMENSIONLESS

$$S = \int d^4x \mathcal{L}$$

DIM LENGTH⁴ OR MASS⁻⁴

$$[.] = \text{MASS DIM OF } \cdot \Rightarrow [d^4x] = -4 \quad \text{THEN } [S] = 0 \Rightarrow [\mathcal{L}] = [S] - [d^4x] = 4$$

so

$$\boxed{[\mathcal{L}] = 4}$$

$$\text{similarly } [\partial^\mu] = [\partial_\mu] = 1$$

* KEY: USE KIN. TERMS TO DET' CLASSICAL FIELD DIM

SCALARS $\mathcal{L} \supset -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi \Rightarrow [\phi] = \frac{1}{2}(4 - 2[\partial^\mu]) = 1$

SPINORS $\mathcal{L} \supset i \bar{\psi} \gamma^\mu \psi \Rightarrow [\psi] = [\bar{\psi}] = \frac{1}{2}(4 - [\partial^\mu]) = \frac{3}{2}$

VECTORS $\mathcal{L} \supset -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \Rightarrow [A_\mu] = \frac{1}{2}(4 - 2[\partial^\mu]) = 1$

LORENTZ-INV'T INTERACTIONS

INTERACTION

$$m^2 \phi \phi \quad m \bar{\psi} \gamma^\mu \psi \quad m^2 A_\mu A^\mu$$

$$\lambda \phi^4$$

$$g \phi^3$$

$$y \phi \bar{\psi} \gamma^\mu \psi$$

$$e \bar{\psi} \gamma^\mu \psi$$

$$G \bar{\psi} \gamma^\mu \bar{\psi} \gamma^\nu$$

COUPLING DIMENSION

$$[m] = 1$$

$$[\lambda] = 0$$

$$[g] = 1$$

$$[y] = 0$$

$$[e] = 0$$

$$[G] = -2 \quad \text{FROM} \quad [G] + 4(\frac{3}{2}) = 4$$

$$\sqrt{G_N} h \partial h \partial h$$

$$[\sqrt{G_N}] = -1$$

$$(\sqrt{G_N} \sim \frac{1}{M_p})$$

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$$

$$S \sim \int d^4x (\partial h \partial h) + \sqrt{G_N} h \partial h \partial h$$

HEURISTIC IDEA IN EXAMPLES

$m @$ INC. PERT ORDERS, DIM ANALYSIS.

$$\lambda \phi^4 \quad m \sim \cancel{X} + \cancel{\lambda X} + \dots$$

$$\sim A\lambda + B\lambda^2 + \dots$$

FACT

$$[\lambda] = 0 \Rightarrow [A\lambda] = [A] + [\lambda] = [A] \quad \text{so} \quad [A] = [B]$$

$$[B\lambda^2] = [B] + 2[\lambda] = [B]$$

\Rightarrow SCALES $\tilde{\lambda}$ MUST APPEAR IN DIM'LESS COMBOS

\hookrightarrow EG \wedge OR S

$$G \bar{4} \gamma \bar{4} \gamma \quad [G] = -2$$

$$m \sim \cancel{X} + \cancel{\lambda} \overset{\sqrt{s}}{\circlearrowleft}$$

$$\sim AG + BG^2$$

$$\text{BUT NOW } [B] = [A] + [2] \Rightarrow B \sim C \tilde{\lambda}^2 \quad w/ \quad [C] = [A]$$

$$\text{SO } m \sim AG + C \tilde{\lambda}^2 G^2 = G(A + C \tilde{\lambda}^2 G + \dots)$$

PHYSICAL THY $\tilde{\lambda}^2 = s$ H.O.T. IN $\tilde{\lambda}^2 G$
 $\hookrightarrow m = \text{PERT IN GS ONLY GOOD FOR } s \ll G^{-1}$

$$\Rightarrow \text{NEW PHYSICS} @ S \approx \frac{1}{G_F} \approx (90 \text{ GeV})^2 \approx M_{W/Z}^2$$

NR THYS ARE LOW ENERGY EFFECTIVE FIELD THEORIES "EFTS"

WHAT DOES THIS MEAN? EXAMPLE

WEAK INTERACTIONS

$$= (-ig)^2 \bar{u} \gamma^\mu u \frac{i}{k^2 - M_{W/Z}^2 + i\epsilon} \bar{u} \gamma_\mu u$$

LOW ENERGY $k^2 \ll M_{W/Z}^2 \Rightarrow$

$$\simeq -(-ig)^2 \bar{u} \gamma^\mu u \frac{i}{M_{W/Z}^2} \bar{u} \gamma_\mu u$$

* $\mathcal{L}_{int} = G_F \bar{u} \gamma^\mu u \bar{u} \gamma_\mu u$ w/ $G_F \approx \frac{g^2}{M^2}$ \Rightarrow SAME \mathcal{L} @ LOW ENERGIES

Language: "Effective Operator"

$$\simeq G_F \bar{u} \gamma^\mu u \bar{u} \gamma_\mu u$$

* \mathcal{L}_{int} FAILS @ $k^2 \gtrsim M_{W/Z}^2$. UV PHYSICS REPLACES EFFECTIVE INT.

CON: MANY UV MODELS COULD GIVE SAME EFF. INT.

PRO: "

IGNORANCE CAN BE BLISS!

(great that predecessors could focus on lower energy physics w/o worrying about sm details.)

UV COMPLETIONS & INTEGRATING OUT

(demonstrate the basic idea in a toy example, POINT: $\text{ETI}^{\text{c}} \text{ come from UV models, but are agnostic}$)
 (also: concretely see in LIGO)

TOY THY: $\mathcal{L} = \sum_{i=1}^2 \left(\frac{1}{2} \partial_\mu \phi_i \partial^\mu \phi_i \right) - \frac{M^2}{2} \phi_2^2 - \varepsilon \phi_1^3 \phi_2$

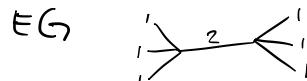
Q: WHAT DOES THY EFFECTIVELY LOOK LIKE @ $E \ll M$?

A: ϕ_2 EOM: $\partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi_2} - \frac{\delta \mathcal{L}}{\delta \phi_2} = 0$

$$\Rightarrow (\square + M^2) \phi_2 + \varepsilon \phi_1^3 = 0$$

$$\xrightarrow{k \text{-sp}} (k^2 + M^2) \phi_2 + \varepsilon \phi_1^3 = 0$$

PHYSICS FOR $\varepsilon \ll \mu_1$, ϕ_2 ONLY INTERNAL, $k^2 \ll M^2$



$$\Rightarrow \phi_2 = -\frac{\varepsilon \phi_1^3}{k^2 + M^2} = -\frac{\varepsilon \phi_1^3}{M^2} \left(\frac{1}{1 + \frac{k^2}{M^2}} \right) \simeq -\frac{\varepsilon \phi_1^3}{M^2} + \mathcal{O}\left(\frac{k^2}{M^2}\right)$$

\Rightarrow EFFECTIVE LAGRANGIAN

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 - \frac{M^2}{2} \left(\frac{\varepsilon^2}{M^4} \right) \phi_1^6 - \varepsilon \phi_1^3 \left(-\frac{\varepsilon}{M^2} \phi_1^3 \right) \\ &= \frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 - \phi_1^6 \left(\frac{\varepsilon^2}{2M^2} - \frac{\varepsilon^2}{M^2} \right) \end{aligned}$$

$$\boxed{\mathcal{L}_{\text{eff}} = \frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 + \frac{\varepsilon}{2M^2} \phi_1^6}$$

NOTE: - VALID EFFECTIVE DESC FOR $\frac{k^2}{M^2} \ll 1$

- ϕ_2 ABSENT, Language: "Integrated Out"

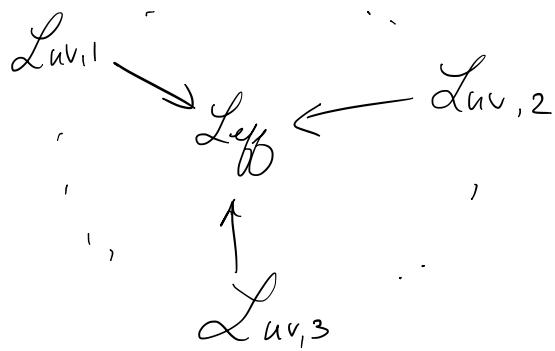
- IN PLACE OF ϕ_2 , $\varepsilon \phi_1^3 \phi_2$, $\exists \phi_1^6$

IE @ LOW E,  looks like 

UV COMPLETION

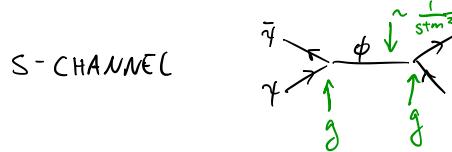
IDEA: $\mathcal{L}_{\text{uv Thy}}$ $\xrightarrow[\text{int. out}]{\text{low en}} \mathcal{L}_{\text{eff}}$

EFT IS UV-AGNOSTIC: $\exists \mathcal{L}_{\text{eff}}$ SUCH THAT



in multiple
UV-completions of
same effective theory!

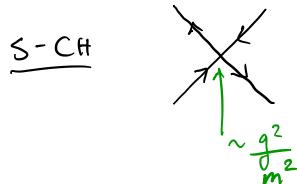
EG $\mathcal{L}_{\text{uv,1}} = i\bar{\gamma}\not{\partial}\gamma + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - g\phi\bar{\gamma}\gamma$



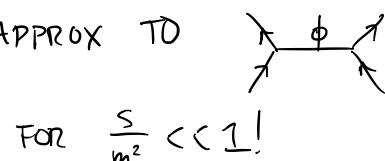
LOW E: $\frac{s}{m^2} \ll 1$?

$$\phi \text{ EOM } \approx m^2\phi + g\bar{\gamma}\gamma = 0 \implies \phi = -g\frac{\bar{\gamma}\gamma}{m^2} + \mathcal{O}\left(\frac{s}{m^2}\right)$$

$$\begin{aligned} \mathcal{L}_{\text{eff},1} &= i\bar{\gamma}\not{\partial}\gamma - \frac{m^2}{2}\frac{q^2}{m^2}\bar{\gamma}\gamma\bar{\gamma}\gamma - g\left(-\frac{q^2}{m^2}\right)\bar{\gamma}\gamma\bar{\gamma}\gamma \\ &= i\bar{\gamma}\not{\partial}\gamma + \frac{q^2}{m^2}\bar{\gamma}\gamma\bar{\gamma}\gamma \end{aligned}$$



GOOD APPROX TO

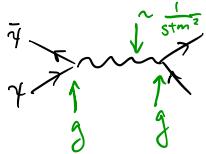


FOR $\frac{s}{m^2} \ll 1$!

(one might say this
description in effective)

$$\underline{\text{EG}} \quad \mathcal{L}_{\text{UV},1} = i\bar{\psi}\gamma^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu - g\bar{\psi}A\gamma^4$$

S-CHANNEL

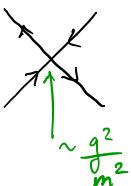


$$\underline{\text{LOW } E:} \quad \frac{s}{m^2} \ll 1 ?$$

$$A_\mu \text{ EOM } \simeq m^2 A_\mu + g\bar{\psi}\gamma_\mu\gamma^4 \implies A_\mu = -\frac{g}{m^2}\bar{\psi}\gamma_\mu\gamma^4 + \mathcal{O}\left(\frac{s}{m^2}\right)$$

$$\mathcal{L}_{\text{eff}} = i\bar{\psi}\gamma^4 - \frac{m^2}{2} \frac{g^2}{m^4} \bar{\psi}\gamma_\mu\gamma^4\bar{\psi}\gamma_\mu\gamma^4 - g\left(-\frac{1}{m^2}\right) \bar{\psi}\gamma^\mu\bar{\psi}\gamma_\mu\gamma^4\gamma^4$$

S-CH



GOOD APPROX TO



$$\text{FOR } \frac{s}{m^2} \ll 1 !$$

(one might say this
description is effective)

POINT TWO 4-FERMI EFT'S FROM VERY DIFF \mathcal{L}_{UV} 'S

(this was a simple example but in fact can get identical \mathcal{L}_{eff}
from diff \mathcal{L}_{UV})

EX OF EXACTLY SAME \mathcal{L}_{eff} : SEIBERG DUALITY

$$d=4 \quad N=1$$

$$G = SU(N_c)$$

$$N_f \quad (\square \quad \square)$$

CHIRALS

$$d=4 \quad N=1$$

$$G = SU(N_f - N_c)$$

$$N_f \quad \square \quad \square \quad \text{CHIRALS}$$

$$\mathcal{L}_{\text{eff}}$$

note: different gauge groups, identical low-energy physics!

EFT RULES

- CONSIDER LIGHT DOF
- ALLOW θ' 's w/ $[\theta] > d$ ($= 4$, for us)

KEY: SENSITIVE TO NEW UV PHYSICS,
AGNOSTIC ABOUT WHAT IT IS!

(this is the power of effective field theory)

"RULES"

① CONSIDER \mathcal{L} w/ LIGHT DOF

↪ SYMMETRY GROUP G

② DET. MAX $[\theta]$ DESIRED

(always allowed in practice because
increasing θ is increasingly irrelevant)

③ WRITE DOWN ALL θ UP TO $[\theta]_{\max}$

CONSISTENT w/ SYMMETRIES,

SUPPRESSED BY APPROP A-POWERS

↪ WITH DIM-LESS "WILSON COEFFS"

$$\text{IE } \mathcal{L} = \mathcal{L}_{\text{kin}} + \sum_{k=1}^{[\theta]_{\max}} \sum_{\substack{\theta_{k,i} \\ [\theta_k]=k}} \frac{c_{k,i}}{N^{k+1}} \theta_{k,i}$$

(in of whole thing = 4, more gen, d)

EXAMPLES

i) THE BLUE SKY



(see see VIII.3 for rel. treat)

① DOF / INGREDIENTS: PHOTONS $\Rightarrow \mathcal{L}_{eff} \supset -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
 NEUTRAL AIR $\Rightarrow \mathcal{L}_{eff} \supset \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2$
 (model as spin 0, same conc holds for $\frac{1}{2}$)

Symm: (11) GAUGE $\not\vdash$ LORENTZ

② $[\Theta_{max}]$: WHATEVER LEADING ORDER γ -AIR INT. IS

③ $A^\mu \phi \phi$? not \mathcal{L}_0 inv, not G-inv

$A^\mu A_\mu \phi \phi$? not G-inv

$\phi \phi F_{\mu\nu} F^{\mu\nu}$? G.I. $\not\vdash$ L.I. ✓

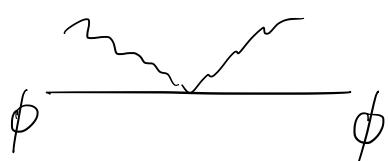
NOTHING ELSE AT $[\phi \phi F^2] = 6$

NOTE By RULE, must also INCLUDE DIM ≤ 6 ϕ -INT

$$\Rightarrow \mathcal{L}_{eff} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 + C_3 \Lambda \phi^3 + C_4 \phi^4 + \underbrace{C_5 \phi^5}_{\Lambda} + \frac{C_6}{\Lambda^2} \phi F^2 + \frac{C_{6,1}}{\Lambda^2} \phi^6 + \frac{C_{6,2}}{\Lambda^2} \phi \phi F_{\mu\nu} F^{\mu\nu} + H.O.T.$$

just saw wrote it

γ -AIR SCATTERING



Q: STEPS TO COMPUTE \mathcal{M} ?

SCALING

$$\phi \phi F_{\mu\nu} F^{\mu\nu} \Rightarrow \text{VERT } \omega / \frac{P^\mu P^\nu}{\lambda^2}$$

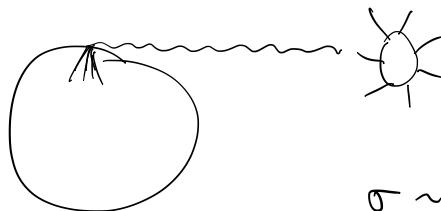
$$\Rightarrow M \sim \frac{P^\mu P^\nu}{\lambda^2} \sim \frac{E^2}{\lambda^2} \Rightarrow \sigma \sim |m|^2 \sim \frac{E^4}{\lambda^4} \sim \frac{\omega^4}{\lambda^4} \quad (\text{note: not get dimensionally correct})$$

CAREFULLY TAKE NR $(\frac{E}{m} \ll 1)$ (in 3 DIMS) $\Rightarrow \sigma = A \frac{\omega^4}{\lambda^6}$

DEFINE $d := \frac{1}{\lambda} \Rightarrow \sigma \sim d^6 \omega^4$ RAYLEIGH'S σ

NOTE:

①



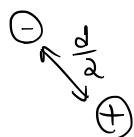
$\sigma \sim \omega^4 \Rightarrow$ BLUE LIGHT
SCATTERS MORE
THAN RED

\Rightarrow SKY IS BLUE

② CUTOFF IS PHYSICAL

$$\lambda \rightarrow \infty \Rightarrow d \rightarrow 0, \sigma \rightarrow 0$$

Q: WHAT IS d ?



A: ATOM SIZE!

(as $d \rightarrow 0$, charge density $\rightarrow 0$)

\Rightarrow photon coupling $\rightarrow 0$

$\Rightarrow \sigma(\text{att} \rightarrow 0)$

(ii) 4-FERMI THY FOR β -DECAY

① DOF INGREDIENTS: MASSIVE FERMION
 $\Rightarrow \mathcal{L} \supset \bar{\psi}(i\gamma^\mu)\psi$

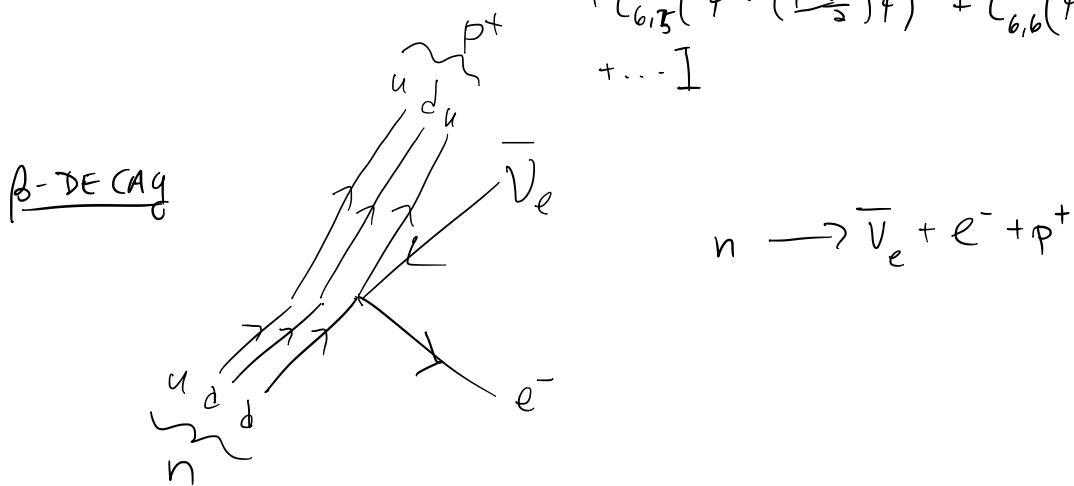
$G = \text{LORENTZ}$

② $[O]_{\max}$ SET BY LEAD INT

③ RECALL BILINEAR: $\bar{\psi}\psi$ $\bar{\psi}\gamma^5\psi$ $\bar{\psi}\gamma^\mu\psi$ $\bar{\psi}\gamma^\mu\gamma^5\psi$

SQUARE ANY \Rightarrow LOR. INV, P-INV

$$\Rightarrow \mathcal{L}_{\text{eff}} = \bar{\psi}(i\gamma^\mu)\psi + \frac{1}{\Lambda^2} [C_{6,1}(\bar{\psi}\psi)^2 + C_{6,2}(\bar{\psi}\gamma^5\psi)^2 + C_{6,3}(\bar{\psi}\gamma^\mu\psi)^2 + C_{6,4}(\bar{\psi}\gamma^\mu\gamma^5\psi)^2 + C_{6,5}(\bar{\psi}\gamma^\mu(1-\gamma^5)\psi)^2 + C_{6,6}(\bar{\psi}(1-\gamma^5)\psi)^2 + \dots]$$



NATURE V-A CURRENT $\frac{(1-\gamma^5)}{2}$ CORRECT

$$\underline{\text{EXP}} \quad \frac{c}{\Lambda^2} =: G_F = \frac{1.17}{10^5 \text{ GeV}^2}$$

CUTOFF IS PHYSICAL

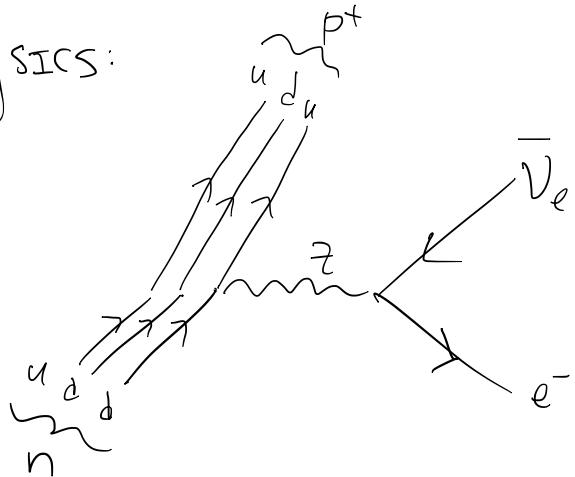
- \mathcal{L}_{eff} ONLY VALID FOR $e^2 G_F \ll 1$

- IF COLLIDER E HAS $e^2 G_F \sim \mathcal{O}(1)$

PROBE NEW PHYSICS!

COLLIDER + EXP: 1983, S_{PPS} § UA2, $E_{\text{com}} \approx 600 \text{ GeV}$

NEW PHYSICS:



(this is β^- decay.
W in β^+ decay
 $p^+ + n \rightarrow \bar{\nu}_e + e^+$)

$$G_F = \frac{\sqrt{2}}{8} \frac{q^2}{m_W^2} \quad (\sin, M_Z^2)$$

POINT: ① WEAK INTERACTIONS UV-COMPLETE FERMI THY
 ② GET FERMI BACK AFTER INTEGRATING OUT $\omega, z \Rightarrow G_F = f(M_W, M_Z)$, if
 $M_W \sim M_Z$ SETS CUTOFF!

EFT & BSM PHYSICS

- 0) SM OPS
- 1) "HIGGS PORTALS" TO DARK SECTORS
- 2) HIGGS EFT (sensitivity to new Higgs physics)

- 0) SM OPS (from a T-shirt, suppresses detailed index structure, but all the types of operators are there!)

$$\mathcal{L} = -\frac{1}{4} \overbrace{F_{\mu\nu} F^{\mu\nu}}^4 + i \overbrace{\bar{\psi} \not{D} \psi}^4 + h.c. + y_{ij} \overbrace{\bar{\psi}_i \psi_j}^4 h + h.c. + \underbrace{|D_\mu h|^2}_{4} + m^2 \underbrace{h^+ h^-}_{2} - \lambda \underbrace{(h^+ h^-)^2}_{4}$$

DIM IN RED

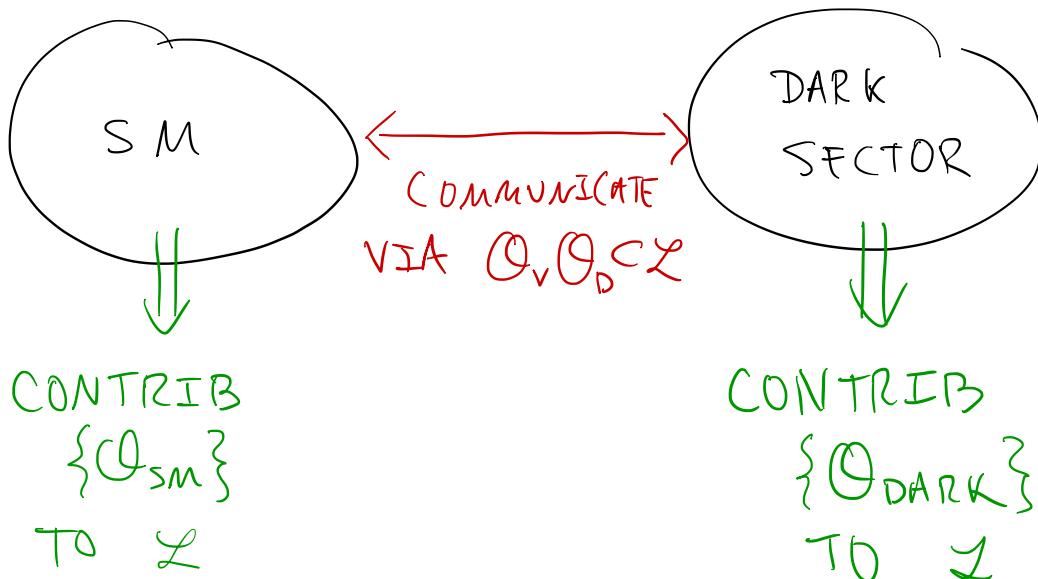
$$[\mathcal{O}_{\text{kin}}] = [\mathcal{O}_{\text{ Yukawa }}] = [\mathcal{O}_{\text{ Higgs self-loop }}] = 4$$

$$[\mathcal{O}_{\text{ Higgs mass }}] = 2$$

COLLECTIVELY CALL " \mathcal{O}_{SM} " TO REP.
ALL OPS IN SM.

I) HIGGS PORTALS TO DARK SECTORS

GENERAL



Q: WHAT ARE LEADING SM-DARK INTS?

A: FROM LOWEST DIM OPS!

$$O_{sm} O_{dark} \text{ S.T. } [O_{sm}] = \min [\{O_{sm}\}]$$

$$[O_{dark}] = \min [\{O_{dark}\}]$$

NOTE: $\Rightarrow O_{sm} = h^+ h_1$ SINCE
IT'S THE MIN-DIM
SM OP.

$\Rightarrow h^+ h_1 O_{dark}$ IS LEADING OP

Language "Higgs Portal" to dark sector!

SCALAR DM

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{2} \lambda \phi^2 h^+ h^-$$

FERMION DM

$$\mathcal{L} = \bar{\chi} (i\gamma^\mu - m_\chi) \chi - \frac{\lambda}{\Lambda} h^+ h^- \bar{\chi} \chi$$

VECTOR DM

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 V_\mu V^\mu \\ & - \frac{\lambda_{HV}}{2} V_\mu V^\mu h^+ h^- - \frac{\lambda_V}{4!} (V_\mu V^\mu)^2 \end{aligned}$$

DETECTION STRATEGY

- ① CONSTRAIN PARAMS w/ $\Omega_{\text{obs}} h^2$
- ② DETERMINE SM σ_0
- ③ CONSTRAIN w/ LHC, DD, IOD

3) SU EFT @ DIM 6

\exists MANY!

SOME SIMPLE ONES RELATED TO HIGGS @ LHC

OPERATOR

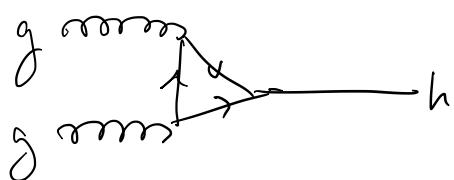
COEFF

CONSTRAINT

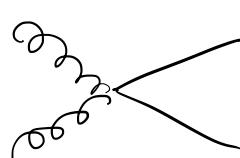
$$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \gamma^\alpha (D^\nu H) W_{\mu\nu}^\alpha \quad \frac{m_w^2}{\Lambda^2} C_{HW} \quad (-0.012, 0.008)$$

$$\mathcal{O}_{HB} = ig^{-1} (D^\mu H)^\dagger (D^\nu H) B_{\mu\nu} \quad \frac{m_w^2}{\Lambda^2} C_{HB} \quad (-0.053, 0.044)$$

$$\mathcal{O}_g = g_s^{-2} |H|^2 G_{\mu\nu}^A G^{A\mu\nu} \quad \frac{m_w^2}{\Lambda^2} C_g \quad (0, 3.0) \times 10^{-5}$$



vs



large
coeff \Rightarrow
large di-Higgs
production

$$\mathcal{O}_\gamma = g^{-2} |H|^2 B_{\mu\nu} B^{\mu\nu} \quad \frac{m_w^2}{\Lambda^2} C_\gamma \quad (-4.0, 2.3) \times 10^{-4}$$

