QM Lecture 1 & 2

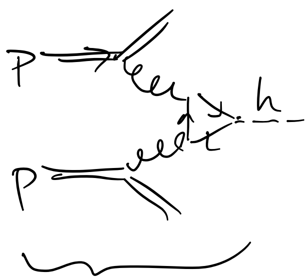
③ THIS: (the startling stuff, right away)

PROBABILITY & SUPERPOSITION

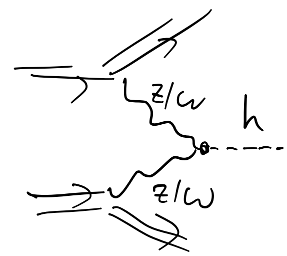
PROB. EX.

- HALF-LIFE
- HIGGS BOSON

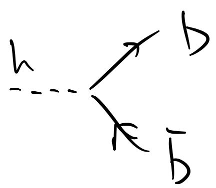
PRODUCE:



MOST OF THE TIME

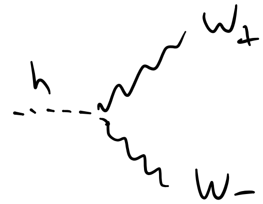


DECAY: (not inc $\tau\bar{\tau}$, $c\bar{c}$, ZZ, etc)



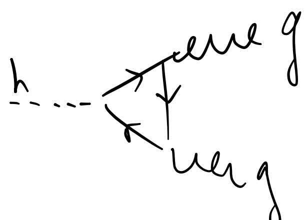
$h \to b\bar{b}$

$\sim 60\%$



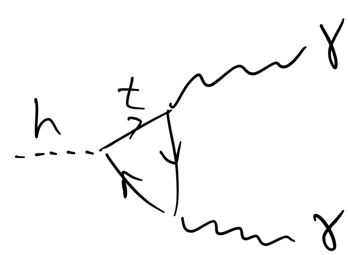
$h \to WW$

$\sim 20\%$



$h \to gg$

$\sim 10\%$



$h \to \gamma\gamma$

$\sim .2\%$

(can the Higgs just not make up its mind!?)

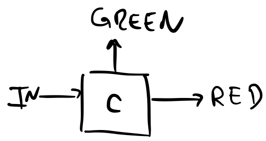
EXPERIMENTS FOR PROB & SUP.

- ELECTRONS w/ PROPERTIES:

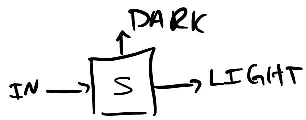
  COLOR & SHADE

- FACT: e GREEN OR RED
  LIGHT OR DARK

COLOR BOX:

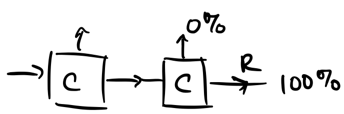


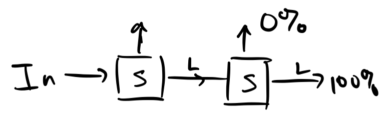
SHADE BOX:



- WON'T MATTER WHAT'S INSIDE

- EX 1

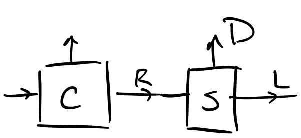




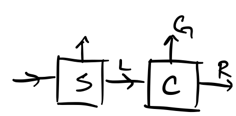
* REPEAT SAME BOX, PROPERTY PERSISTS

- EX 2
Q: MIGHT C+S BE RELATED?

SUPPOSE e R. DET S?



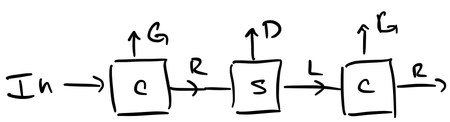
50% D
50% L



50% G
50% R

* C ~~DET~~ S, S ~~DET~~ C.

- EX3

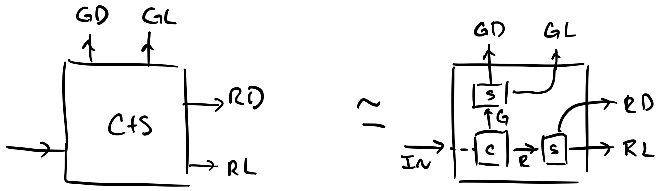


"INT." PRED: 100% R
0% G

FACT: 50/50

* PRESENCE OF S BOX ALTERS C
(hidden variables? not as far as we can tell)
* INTRINSICALLY PROBABILISTIC

* CAN'T BUILD C+S BOX



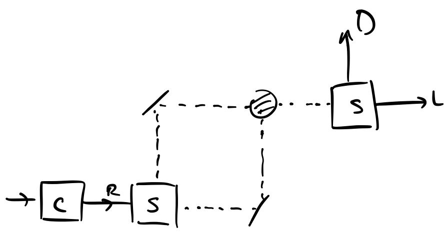
(why doesn't this work?)
(A: e.g. put $\boxed{C}$ after RL, get 50/50 R/G! bad source of red electrons)

* POINT: "R & L", "R & D", "G & L", "G & D"

DO NOT EXIST

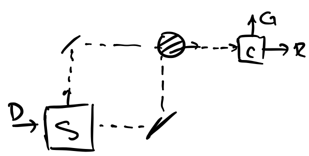
UNCERTAINTY PRINCIPLE

* EX 4.1 In R MEASURE S @ OUT



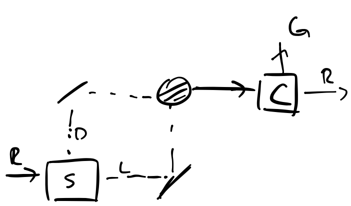
D: 50 ?
L: 50 ? ✓

EX 4.2 In D MEASURE C @ OUT



G: 50 ?
R: 50 ✓

EX 4.3 In R, MEASURE C @ OUT

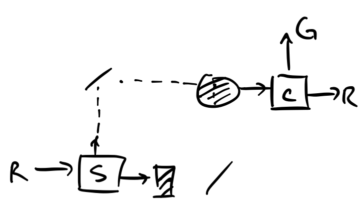


G: 50%
R: 50% ? ✗

FACT
G: 0%
R: 100%

* WHAT!?

EX 4.4 PUT IN BARRIERS IN R, C @ OUT, WALL @ L



PRED: - OUT PUT DOWN 50% ✓
- ALL R. ✗
↳ FACT 50% R
50% G (just EX 3)

(how could the electron know!?)

Q: WHAT DOES EX 4.3 TELL US ABOUT THE ELECTRON!?

WHAT ROUTE ~~DID~~ IT TAKE?

DARK PATH? __NO__ BY 4.4

LIGHT PATH? __NO__ BY 4.4 w/ WALL SWAPPED

BOTH? __NO__ (put detectors on each path, check for "split" never find. Always at most one hit.)

NEITHER? __NO__ (put barrier in both paths, nothing comes out!)

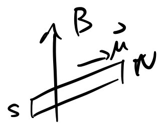
__WHAT!?__ = THIS COURSE

(so what are electrons doing!?)
↳ something we've never thought of, need to develop language / theory for this

$\boxed{\text{ELECTRON IN SUPERPOSITION}}$

STERN-GERLACH EXP.: BUILD A BOX!

* MAGNET w MOMENT $\vec{\mu} = \frac{q}{2mc}\vec{L}$ IN $\vec{B}$-FIELD



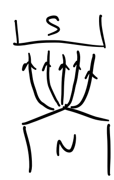
POTENTIAL ENERGY
$$U = -\vec{\mu}\cdot\vec{B}$$
(sign a convention)

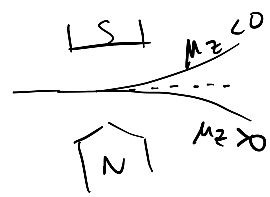
$\exists$ FORCE: $\vec{F} = -\nabla U = \nabla(\vec{\mu}\cdot\vec{B})$

* TAKE $\vec{\mu}$ = CONST, $\vec{B} = B_z\hat{k}$ VARIES IN z ONLY

$$\vec{F} = \mu_z \frac{\partial B_z}{\partial z}\hat{k}$$

* BEAM OF MAGNETS? THROUGH BIG ONE





$\frac{\partial B_z}{\partial z} < 0$ (field line den. less near S)

DEFLECTION!

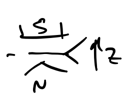
* SUBATOMIC ANG. MOM "SPIN"

m mass
q el. ch.

$$\vec{\mu} = g\left(\frac{q}{2mc}\right)\vec{S}$$

- $\vec{S}$: SPIN. ANG. MOM. $|\vec{S}| = \frac{\hbar}{2}$ $\hbar = \frac{h}{2\pi}$

- POINT PART.: $g = 2$

* CALL  SG_z, SIM  $\uparrow x$ SG_x

* DEF UP, $S_z = \frac{\hbar}{2}$, CALL $|+z\rangle$

DEF DOWN, $S_z = -\frac{\hbar}{2}$ CALL $|-z\rangle$

SIM: $S_x = \pm\frac{\hbar}{2}$ are $|\pm x\rangle$

key point:

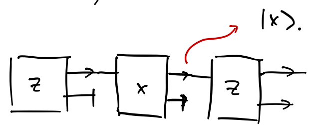
$\boxed{c}$ now $\boxed{z}$

$\boxed{s}$ now $\boxed{x}$

GENERAL STATE VECTORS

NEED: THY THAT ACCTS FOR THIS! IDEA: QM $\simeq$ LIN ALG. PROB BEHAVES WELL

#3



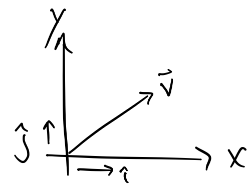
SOME PROB z_+, z_- $\Rightarrow$ WRITE $|x\rangle = \tilde{c}_+|+z\rangle + \tilde{c}_-|-z\rangle$

$|+z\rangle$, $|-z\rangle$ A BASIS, $\Rightarrow$ ANY STATE $|\psi\rangle = c_+|+z\rangle + c_-|-z\rangle$

↳ *lin ind set that spans V*

Q: HOW TO DET c_+ AND c_-?

ANALOGY: $\vec{v} \in \mathbb{R}^2$



N.B. $\hat{\imath}\cdot\hat{\imath} = 1$ $\hat{\imath}\cdot\hat{j} = 0$
$\hat{j}\cdot\hat{j} = 1$

ANY $\vec{v} = v_x\,\hat{\imath} + v_y\,\hat{j}$

DET COEFF BY $\cdot$!

$\hat{\imath}\cdot\vec{v} = v_x\,\hat{\imath}\cdot\hat{\imath} + v_y\,\cancel{\hat{\imath}\cdot\hat{j}}^{\,0} = v_x$ $\hat{j}\cdot\vec{v} = v_y$

Q: *what props of the basis were critical for this?*

MORE GEN:

DEFINE: A SET OF BASIS VECTORS $\{\hat{v}_i\}$ IS "ORTHONORMAL"

IF $\hat{v}_i\cdot\hat{v}_i = 1$ & $\hat{v}_i\cdot\hat{v}_j = 0$

THEN ANY $v = \sum_i c_i\,\hat{v}_i$ & $\hat{v}_j\cdot v = \sum_i c_i\,\hat{v}_j\cdot\hat{v}_i = c_j\,\hat{v}_j\cdot\hat{v}_j = c_j$

BACK TO QM:

"$|\psi\rangle\cdot|\psi\rangle$", WRITE $\langle\psi|\psi\rangle$

$|\psi\rangle$ "KET" $\langle\psi|$ "BRA"

$\Downarrow$

$\langle\psi|\psi\rangle$ "BRA-KET"

BRA $\in$ DUAL V.S. V^*

↳ "DUAL VECTOR" TAKES VEC, GIVES #

$V_D: V \longrightarrow \#$

$\underline{EX}$: $V = \mathbb{R}^2$. $\quad V_{COL} \in V \qquad V_{ROW} \cdot V_{COL} \geq 0$

$V_{ROW} \in V^* \qquad \mathbb{R}_{\geq 0}$

↳ row vector associated to V_{COL}

$\underline{eg}$ $V_C = \begin{pmatrix} a \\ b \end{pmatrix} \Rightarrow V_R = (a\ b)$

$(a\ b)\begin{pmatrix} a \\ b \end{pmatrix} = a^2 + b^2 \geq 0$

$V_R: V \Rightarrow \mathbb{R}$ TAKE VEC, GET #

$\underline{ANY}$ $v = \begin{pmatrix} c \\ d \end{pmatrix}$ $\quad (a\ b)\begin{pmatrix} c \\ d \end{pmatrix} = ac + bd \in \mathbb{R}$

$\underline{EX}$ $V = \mathbb{C} \qquad z = x + iy$, $\quad z \in \mathbb{C}$, $x \in \mathbb{R}$, $y \in \mathbb{R}$

Q: WANT $z_D \cdot z \geq 0$. HOW DOES z_D REL. z?

$$z \cdot z_D = \underbrace{(xx_D - yy_D)}_{\text{WANT } \geq 0} + i\,\underbrace{(xy_D + yx_D)}_{\text{WANT } = 0}$$

FOR PROB.

$\underline{GUESS}$: $z_D = z$ IE $x_D = x$ $y_D = y$ ~~WANTS~~?

$\underline{GUESS}$ $z_D = z^*$ IE $x_D = x$ $y_D = -y$. $z_D \cdot z = x^2 + y^2 \geq 0$ WORKS!

$V^* = \mathbb{C}^*$, THEN "$\bullet = \times$" $v_D \in V^*$ $v \in V$ $v_D \cdot v = v^* \cdot v \geq 0$ $\forall v_D, v$

$\underline{EX}$ $V = \mathbb{C}^2 \qquad V_D = V^T$? $\quad (z_1\ z_2)\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = z_1^2 + z_2^2 \in \mathbb{C}$ NOT $\mathbb{R}_{\geq 0}$ $\boxed{NO}$

$V_D = V^{T*} \quad (z_1^*\ z_2^*)\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = z_1^* z_1 + z_2^* z_2 = |z_1|^2 + |z_2|^2 \in \mathbb{R}_{\geq 0}$

$\boxed{KEY}$ CONJUGATE & TRANSPOSE. (will see often)

$\underline{Gen}$ $\quad v_2 \in V \quad v_2 = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad v_D(v_2) = v^{*T} v_2 = (z_1^*\ z_2^*)\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = z_1^* a_1 + z_2^* a_2 \in \mathbb{C}$

QM: WANT $\langle\psi|\psi\rangle$ TO REP. PROB, SO NEED $\in \mathbb{R}^+$

- NB IF $|+z\rangle, |-z\rangle$ FORM ORTHONORMAL BASIS IE $\langle +z|+z\rangle = \langle -z|-z\rangle = 1$, $\langle +z|-z\rangle = \langle -z|+z\rangle = 0$

$|\psi\rangle = c_+|+z\rangle + c_-|-z\rangle$ HAS $\langle +z|\psi\rangle = c_+\underbrace{\langle +z|+z\rangle}_{1} + c_-\cancel{\langle +z|-z\rangle}^{0} = c_+$

SIM $\langle -z|\psi\rangle = c_-$

$$\therefore \quad |\psi\rangle = \langle +z|\psi\rangle\, |+z\rangle + \langle -z|\psi\rangle\, |-z\rangle = \left(|+z\rangle\langle +z| + |-z\rangle\langle -z| \right)|\psi\rangle$$

FORM? $v = Av \ \forall v$, A MAPS VECTORS TO VECTORS?

$\Rightarrow A = \mathbb{I}$ = IDENTITY MATRIX

$$\mathbb{I} = |+z\rangle\langle +z| + |-z\rangle\langle -z|$$

MORE GEN $|v_i\rangle$ ORTHNML

$|\psi\rangle = \sum c_i |v_i\rangle \qquad \langle v_i|\psi\rangle = c_i \underbrace{\langle v_i|v_i\rangle}_{1} + \sum_{i\neq j} c_j \cancel{\langle v_i|v_j\rangle}^{0}$

$$\boxed{\langle v_i|\psi\rangle = c_i}$$

SO $|\psi\rangle = \sum_i \langle v_i|\psi\rangle |v_i\rangle = \left(\sum_i |v_i\rangle\langle v_i|\right)|\psi\rangle$

$$\Rightarrow \boxed{\mathbb{I} = \sum_i |v_i\rangle\langle v_i|}$$

Q: WHAT IS $\langle\psi|$, GIVEN $|\psi\rangle = c_+|+z\rangle + c_-|-z\rangle$

NEED $\langle\psi|\psi\rangle \in \mathbb{R}^+$

GUESS $\langle\psi| = c_+\langle z| + c_-\langle z|$ THEN $\langle\psi|\psi\rangle = c_+^2 + c_-^2 \in \mathbb{C}$

BETTER $\langle\psi| = c_+^*\langle z| + c_-^*\langle -z| \Rightarrow \langle\psi|\psi\rangle = |c_+|^2 + |c_-|^2 \in \mathbb{R}^+$ ✓

GEN $\langle\psi| = \sum c_i^* \langle v_i|$ SO $|\psi\rangle = \sum |c_i|^2$

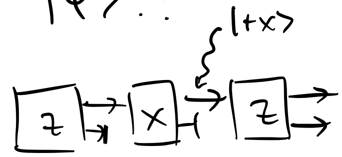
SUPERPOSITIONS AND PROBABILITIES

DEFINE: $\langle \varphi | \psi \rangle$ = PROB. AMPLITUDE FOR PART. IN STATE $|\psi\rangle$ TO BE FOUND IN STATE $|\varphi\rangle$.

objections ① "amplitude"? give it a second

② how can $|\psi\rangle$ be in $|\varphi\rangle$!?

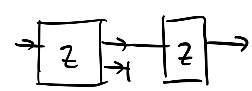
recall TE #3.



DEFINE: $|\langle \varphi | \psi \rangle|^2$ PROB. SYS. IN $|\psi\rangle$ IS FOUND IN STATE $|\varphi\rangle$.

SANITY CHECKS FOR DEFINITIONS

* $\langle +z | +z \rangle = 1 \Rightarrow |\langle +z | +z \rangle|^2 = 1$



(TE 1: stays in $|+z\rangle$) SIM $|\langle -z | -z \rangle|^2 = 1$

MAKES SENSE FOR BASIS STATES! ✓

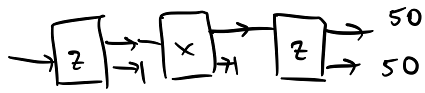
* $\langle \psi | \psi \rangle = \sum_i |c_i|^2 \in \mathbb{R}^+$ BUT GEN $\langle \varphi | \psi \rangle \in \mathbb{C}$

$\Rightarrow \langle \varphi | \psi \rangle$ NOT "PROBABILITY"

* $|\langle \psi | \psi \rangle|^2$: PROB $|\psi\rangle$ IN $|\psi\rangle$ NEEDS TO BE 1 FOR PROB. INTERP $\Rightarrow \boxed{\sum_i |c_i|^2 = 1}$

<u>Comment</u>: $\langle\varphi|\psi\rangle$ measures the "φ-ness" of $|\psi\rangle$,
<u>not</u> (for now) whether it has become $|\varphi\rangle$.

<u>TE #3</u>: DOES THIS FORMALISM EXPLAIN IT?



* $|+x\rangle$ IS A STATE.
* $|+z\rangle, |-z\rangle$ IS BASIS

$\longrightarrow \quad |+x\rangle = C_+|+z\rangle + C_-|-z\rangle$

* FINAL $SG_z \Rightarrow |\langle +z|+x\rangle|^2 = \frac{1}{2} = |C_+|^2$

$$|\langle -z|+x\rangle|^2 = \frac{1}{2} = |C_-|^2$$

* <u>NOTE</u>: $\not\Rightarrow C_+ = C_- = \frac{1}{\sqrt{2}}$ UNLESS $C_+, C_- \in \mathbb{R}$

$|C_+| = |C_-| = \frac{1}{\sqrt{2}}$, BUT THERE MAY BE A <u>PHASE</u>.

RECALL $z \in \mathbb{C}$ HAS $z = x + iy$

BUT IN POLAR $z = re^{i\phi}$

w $|z|^2 = r^2 \quad e^{i\phi} = \cos\phi + i\sin\phi$

$\Rightarrow$ TE 3 DET r, NOT ϕ

IF $\boxed{C_+ = \frac{1}{\sqrt{2}} e^{i\delta_+} \qquad C_- = \frac{1}{\sqrt{2}} e^{i\delta_-}}$

<u>Q</u>: CAN $\delta_\pm$ MATTER!?

(alt: can $C_\pm$ <u>complex</u> really matter?)