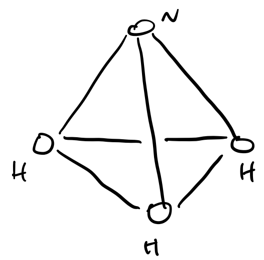
QM Lecture 8

EXAMPLE AMMONIA MOLECULE & AMMONIA MASER

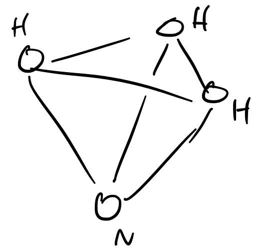
* previous, $\hat{H}$ was nearly diag, but small perturbation $(B_1 \neq 0)$ allowed transitions between the would be $\hat{H}$ eigenstates.

  Ammonia is another famous example.

$|1\rangle$



$|2\rangle$



* WANT: QM DESCRIPTION OF SYSTEM.

* N SAME DIST. FROM 3H-PLANE

* SYMMETRY $\Longrightarrow$ $\langle 1|\hat{H}|1\rangle \simeq \langle 2|\hat{H}|2\rangle =: E_0$, WHATEVER $\hat{H}$ IS.

* FACT: SOMETIMES NH_3 "FLIPS" ORIENTATION

  (Q: what does this imply about $\hat{H}$?)

$$\Rightarrow \langle 1|\hat{H}|2\rangle \neq 0 \quad \langle 2|\hat{H}|1\rangle \neq 0$$

* $H^\dagger = H \Longrightarrow \langle 1|\hat{H}|2\rangle = \langle 2|\hat{H}|1\rangle^*$ TAKE $= -A \in \mathbb{R}$

  (choice of sign for later convenience)

$$\hat{H} \xrightarrow{1,2 \text{ BAS}} \begin{pmatrix} \langle 1|\hat{H}|1\rangle & \langle 1|\hat{H}|2\rangle \\ \langle 2|\hat{H}|2\rangle & \langle 2|H|2\rangle \end{pmatrix} = \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix}$$

Q: WHAT ARE ENERGY E-VALS?

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

$$\begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix}\begin{pmatrix} \langle 1|\psi\rangle \\ \langle 2|\psi\rangle \end{pmatrix} = E\begin{pmatrix} \langle 1|\psi\rangle \\ \langle 2|\psi\rangle \end{pmatrix}$$

$$\det\begin{pmatrix} E_0-E & -A \\ -A & E_0-E \end{pmatrix} = 0 \Rightarrow (E_0-E)^2 - A^2 = 0 \Rightarrow E_0 - E = \pm A \Rightarrow \boxed{E = E_0 \pm A}$$

*DEFINE $|I\rangle$, $\hat{H}|I\rangle = (E_0 - A)|I\rangle$

$$\begin{pmatrix} E_0-E_0+A & -A \\ -A & E_0-E_0+A \end{pmatrix}\begin{pmatrix} \langle 1|I\rangle \\ \langle 2|I\rangle \end{pmatrix} = 0 \Rightarrow A\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} \langle 1|I\rangle \\ \langle 2|I\rangle \end{pmatrix} = 0 \Rightarrow \langle 1|I\rangle = \langle 2|I\rangle$$

$$\Rightarrow \boxed{|I\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)}$$

* SIMILARLY: DEF. $|II\rangle$ $\hat{H}|II\rangle = (E+A)|II\rangle$

$$\boxed{|II\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle)}$$

*NB ① EXISTENCE OF TRANSITION $\Leftrightarrow A \neq 0 \Leftrightarrow$ ENERGIES SPLIT

② EN. E-STATES ARE LIN. COMB. OF $|1\rangle, |2\rangle$

③ $|I\rangle$ SYMM. UNDER $|1\rangle \to |2\rangle$, $|2\rangle \to |1\rangle$

$|II\rangle$ ANTISYMM. (IE $|II\rangle \to |-II\rangle$)

④ EXPERIMENT $E_0 \sim eV$ $A \sim 10^{-4}\,eV$

⑤ $E_{II} - E_I = 2A \Rightarrow A \in \mathbb{R}$ (justified our assumption)

TIME EVOLUTION

RECALL: $|1\rangle$ NOT $\hat{H}$ E-STATE. $\Rightarrow$ TIME EVOLVES

$$|\psi(0)\rangle = |1\rangle = \frac{1}{\sqrt{2}}\left(|I\rangle + |II\rangle\right)$$

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}\,\frac{1}{\sqrt{2}}\left(|I\rangle + |II\rangle\right) = \frac{1}{\sqrt{2}}\left(e^{-i(E_0-A)t/\hbar}|I\rangle + e^{-i(E_0+A)t/\hbar}|II\rangle\right)$$

$$= e^{-i(E_0-A)t/\hbar}\left(\frac{1}{\sqrt{2}}|I\rangle + \frac{e^{-2iAt/\hbar}}{\sqrt{2}}|II\rangle\right)$$

* TIME DEP. REL STATE $\Rightarrow$ NOT STATIONARY STATE.
* IF $A \to 0$, BECOMES STATIONARY STATE

(Q: why is this expected?)

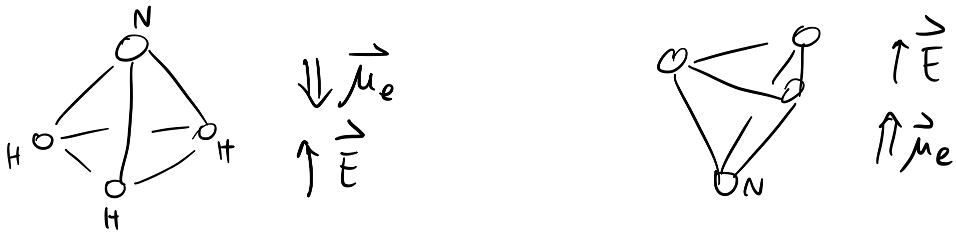
NOTE $\quad |\psi(0)\rangle = |1\rangle$, BUT $\left|\psi\left(\frac{\hbar\pi}{2A}\right)\right\rangle = (\text{phase})\left(\frac{1}{\sqrt{2}}|I\rangle - \frac{1}{\sqrt{2}}|II\rangle\right)$

$$= (\text{phase})\,|2\rangle$$

$\Rightarrow$ $\boxed{\text{AMMONIA FLIPPING BETWEEN } |1\rangle, |2\rangle\text{!}}$

(empirically, the freq. is 24 GHz)

PERTURBATION: STATIC EXTERNAL ELECTRIC FIELD



$$U = -\vec{\mu}_e \cdot \vec{E}$$

* SYMMETRY BROKEN BY $|\vec{E}| \neq 0$!

$$\Rightarrow \langle 1|\hat{H}|1\rangle \neq \langle 2|\hat{H}|2\rangle$$

* IE DIAG. ENTRIES CHANGE!

$$\hat{H} \xrightarrow{1,2\ \text{BAS}} \begin{pmatrix} E_0 + \mu_e|\vec{E}| & -A \\ -A & E_0 - \mu_e|\vec{E}| \end{pmatrix}$$ (Q: why these signs?)

$\Rightarrow$ EN. E-VALS

$$E = E_0 \pm \sqrt{(\mu_e|\vec{E}|)^2 + A^2}$$

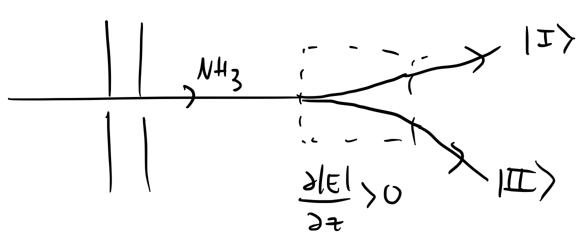
* WE TURNED ON $\vec{E}$-FIELD. CAN MAKE $|\vec{E}|$ SMALL

$$\Rightarrow E = E_0 \pm A\left[1 + \frac{(\mu_e|\vec{E}|)^2}{2A}\right] = (E_0 \pm A) \pm \frac{1}{2}\frac{(\mu_e|\vec{E}|)^2}{A}$$

$\Rightarrow$ ENERGIES SPLIT FURTHER

* NOTE: CAN USE $\vec{E}$-FIELD TO SPLIT

$$F_z \cong -\frac{\partial}{\partial z}\left[\pm \frac{(\mu|\vec{E}|)^2}{2A}\right]$$ (sim. to SG dev)



INDUCED TRANSITIONS

(ammonia maser: induce transitions with a time dependent electric field)

PUT $\hat{H}$ IN $|I\rangle, |II\rangle$ BASIS

$$(\hat{H})_{I\,II} = S^{+}(\hat{H})_{12}S \qquad (\hat{H})_{1,2} = \begin{pmatrix} E_0 + \mu_e|\vec{E}| & -A \\ -A & E_0 - \mu_e|\vec{E}| \end{pmatrix}$$

$$S = \begin{pmatrix} \langle 1|I\rangle & \langle 1|II\rangle \\ \langle 2|I\rangle & \langle 2|II\rangle \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\Rightarrow \hat{H} \xrightarrow{I,II} \left(\frac{1}{\sqrt{2}}\right)^2 \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} E_0 + \mu_e|\vec{E}| & -A \\ -A & E_0 - \mu_e|\vec{E}| \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} E_0 - A & \mu_e|\vec{E}| \\ \mu_e|\vec{E}| & E_0 + A \end{pmatrix}$$

INTRODUCE $\vec{E} = \vec{E}(t) = \vec{E}_0 \cos\omega t$

$$(\hat{H})_{I\,II} = \begin{pmatrix} E_0 - A & \mu_e|\vec{E}_0|\cos\omega t \\ \mu_e|\vec{E}_0|\cos\omega t & E_0 + A \end{pmatrix}$$

* THIS $\hat{H}$ HAS SAME FORM AS $\hat{H}_{\text{MAG RES.}}$ (so just use that soln)

DICTIONARY

$$\hbar\omega_0/2 \longleftrightarrow E_0 - A$$
$$-\hbar\omega_0/2 \longleftrightarrow E_0 + A$$
$$\hbar\omega_1/2 \longleftrightarrow \mu_e|\vec{E}_0|$$

* $\omega \simeq \omega_0 = \frac{2(E_0 - A)}{\hbar}$ RESONANCE

$$|\psi(t)\rangle \simeq \begin{pmatrix} \cos\left(\frac{\omega_1 t}{4}\right) e^{-i(E_0 - A)t/\hbar} \\ -i\sin\left(\frac{\omega_1 t}{4}\right) e^{i(E_0 + A)t/\hbar} \end{pmatrix} \Longrightarrow \begin{array}{l} |\langle I|\psi(t)\rangle|^2 \simeq \cos^2 \frac{\mu_e|\vec{E}_0|t}{2\hbar} \\ |\langle II|\psi(t)\rangle|^2 \simeq \sin^2 \frac{\mu_e|\vec{E}_0|t}{2\hbar} \end{array}$$

BUILD AMMONIA MASER (Microwave Amplification by Stimulated Emission of Radiation)

(1) AMMONIA BEAM THROUGH CONST $\vec{E}_1$ FILTER $|II\rangle$ (higher energy st)

(2) $|II\rangle$ THROUGH RES. TUNED $\vec{E}$-FIELD

(3) WAIT ONE PERIOD

(4) $\Rightarrow$ $|II\rangle \xrightarrow{\text{trans}} |I\rangle_1$ ENERGY RELEASE

(energy split $\simeq 2A + O(|\vec{E}|^2)$ corr.)

(microwaves $\lambda \sim 1$ cm)

$\Rightarrow A \sim 10^{-4}$ eV

ENERGY UNCERTAINTY

$|1\rangle$ NOT EN. EIG-STATE.

$\Rightarrow$ WHAT IS EN. UNC.?

$$(\Delta H)_1 = (\langle H^2\rangle - \langle H\rangle^2)^{\frac{1}{2}} = \left\{\tfrac{1}{2}(E_0 - A)^2 + \tfrac{1}{2}(E_0+A)^2\right] - \left[\tfrac{1}{2}(E_0 - A) + \tfrac{1}{2}(E_0+A)\right]^2\Big\}^{\frac{1}{2}}$$

$$= A$$

(makes sense based on level splitting)