QM Lecture 21

QM IN YOUR FACE!

(credit to Sydney Coleman on the title & basic idea)

TODAY

1. ENTANGLEMENT & EPR
2. GHZ & EXPERIMENTAL EPR REFUTATION

Q: WHAT BOTHERS YOU ABOUT QM?

- PROBABILITY? SHOULDN'T. ROULETTE.
- PARTICLES DON'T HAVE DEFINITE PROPERTIES!

EG $|+z,+x\rangle$ DOES NOT EXIST.

(lack of definite properties follows from non-commuting observables.)

①

ENTANGLEMENT

* 2 SPIN $\frac{1}{2}$ PARTICLES
* ONE BASIS: $|\uparrow\uparrow\rangle\ |\uparrow\downarrow\rangle\ |\downarrow\uparrow\rangle\ |\downarrow\downarrow\rangle$ $\quad \uparrow,\downarrow$ WRT. Z-DIR
* RECALL $\vec{S} = \vec{S}_1 + \vec{S}_2$.

$$\vec{S}^2 = \vec{S}_1^2 + \vec{S}_2^2 \qquad S_z = S_{1z} + S_{2z} \qquad [\vec{S}^2, S_z] = 0$$

$\Rightarrow$ SIMULTANEOUS E-STATES $|s,m\rangle$

$$S_z|s,m\rangle = m\hbar\,|s,m\rangle$$
$$\vec{S}^2|s,m\rangle = s(s+1)\hbar^2|s,m\rangle$$

SPIN "TRIPLET" s=1

$$\begin{cases} |1\,1\rangle = |\uparrow\uparrow\rangle \\ |1\,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \\ |1\,{-1}\rangle = |\downarrow\downarrow\rangle \end{cases}$$

SPIN "SINGLET": s=0

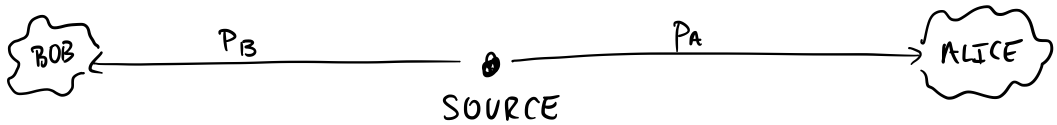
$$|0\,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

IN EACH STATE: SPINS ARE "ENTANGLED". QUANTUM CORRELATED

THE EINSTEIN PODOLSKY ROSEN "PARADOX" (EPR, 1935)

(not a paradox) (paper title: "Can the quantum mechanical description of reality be considered complete?")

* SPIN 0 ⟶ TWO SPIN $\frac{1}{2}$ IN $|0\,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$



EXPT #	ALICE SEES	BOB SEES
1	↑	↓
2	↓	↑
3	↑	↓
4	↑	↓
5	↑	↓
6	↓	↑
7	↓	↑
8	↑	↓
9	↓	↑
10	↓	↑

* A SEES ↑ ⟺ B SEES ↓
* A " ↓ ⟺ B " ↑

PERFECTLY CORRELATED!

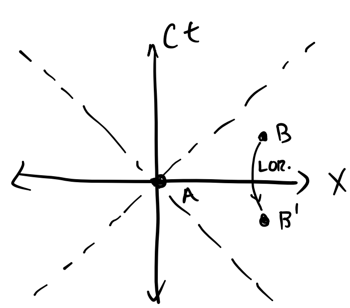
* 50/50.

WHAT THE "PARADOX" IS NOT:

* SUPPOSE A & B SPACELIKE SEP.

* A MEASURES, DET'S WHAT B MUST SEE? ⟹ $V_{SIGNAL} > c$?

NO



- NO "BEFORE" & "AFTER" FOR SPC-LIKE SEP PTS!
- CORRELATION IS NOT CAUSATION
- NOT SREL. VIOLATION

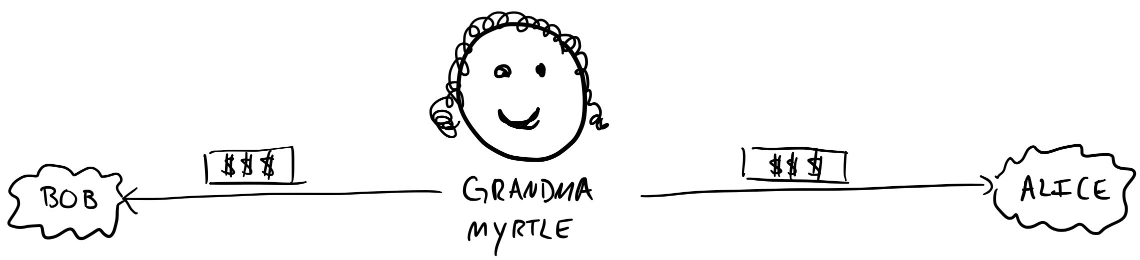
CLAIMED PARADOX

"ANY REASONABLE DEFINITION OF THE NATURE OF REALITY" – EINSTEIN
REQS. THAT "THE REAL FACTUAL SITUATION OF THE SYSTEM S_2 IS INDEP. OF ... S_1."

CLAIM: THIS $\Rightarrow \exists$ SOME HIDDEN VARIABLE DET $|\downarrow\uparrow\rangle$ VS $|\uparrow\downarrow\rangle$ AT THE SOURCE.
(IE. $|\uparrow\downarrow\rangle$ OR $|\downarrow\uparrow\rangle$ AT SOURCE. NOT $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$.)

* 50/50 ENSEMBLE OF $|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle \Rightarrow$ SAME MEASUREMENT PREDICTIONS!
  ↳ CLASSICAL

TOY H.V. & DEF. STATE EX. HOLIDAY CHECKS



- \$70 TO SPEND EVERY YEAR \$50 + \$20, BILLS
  (loves A & B equally, ⇒ equally likely as $t \to \infty$ dist. of bills)

YEAR	ALICE	BOB
1	50	20
2	50	20
3	20	50
4	50	20
5	20	50
6	20	50

* A SEES 50 $\Longleftrightarrow$ B SEES 20 } PERFECTLY
* A " 20 $\Longleftrightarrow$ B " 50 } CORRELATED!
* 50/50 CHANCE OF SEEING \$50 VS \$20

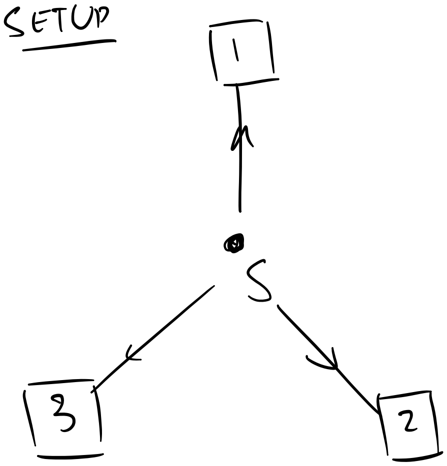
HIDDEN VARIABLE = MYRTLE. SHE DET. DEF STATE.

BELL 1960's: CLASS. SYS. W. HIDDEN VARS MAKE TESTABLE PRED, DIFF FROM QM.

ALAIN ASPECT: EXPT $\Rightarrow$ TEST'S BELL, QM RIGHT, HV WRONG

② GREENBERGER-HORNE-ZEILINGER (1990) (same idea as Bell, more direct)

SETUP



- S SENDS BOXES TO 1,2,3
- LABS 1,2,3 IDENTICAL MACHINES
- MACHINE: TWO SETTINGS, X OR Y
  TWO POSS. MEAS. FOR EACH, ± 1

- PROCEDURE: (@ EACH LAB)
  1. CHOOSE X VS Y
  2. MEASURE
  3. RECORD RESULT, ± 1
  4. REPEAT

- EX RESULTS

X	X	Y	Y	X	Y	X
+	−	+	−	−	−	+

⋮

10^{100} MEASUREMENTS

⋮

- 1,2,3 COMPARE RESULTS: FACT IF ONE MEAS. X, OTHERS Y $\Rightarrow$ 3 RESULTS MULT. TO 1.

($\Rightarrow$ + + + OR + − −)

IE $X_i Y_j Y_k = 1$, $i \neq j \neq k$

CALL FACT ⊛

Q: COULD THIS BE CLASSICAL?

IE. DET IN DEFINITE STATE AT S?

A: YES! (vote!)

FACT ⊛ $\Rightarrow$ 8 POSS. DEF STATES @ S (consistent with the fact)

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} + & + \\ + & + \\ + & + \end{bmatrix}, \begin{bmatrix} - & - \\ - & - \\ + & + \end{bmatrix}, \begin{bmatrix} - & + \\ - & + \\ + & - \end{bmatrix}, \begin{bmatrix} - & - \\ + & + \\ - & - \end{bmatrix}$$

$$\begin{bmatrix} - & + \\ + & - \\ - & + \end{bmatrix}, \begin{bmatrix} + & - \\ + & - \\ + & - \end{bmatrix}, \begin{bmatrix} + & + \\ - & - \\ - & - \end{bmatrix}, \begin{bmatrix} + & - \\ - & + \\ - & + \end{bmatrix}$$

(combinatorics problem)

(all satisfy fact)

$\Longrightarrow$ CLASSICAL PREDICTION:

ALL 3 MEASURE X $\Rightarrow$ PROD OF RES. IS 1. $X_1 X_2 X_3 = 1$

NATURE GHZ HAS BEEN DONE. $X_1 X_2 X_3 = -1$

CONCRETE QUANTUM GHZ SYSTEM

$$X = \frac{\sigma_x}{\hbar} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \frac{\sigma_y}{\hbar} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$X|\uparrow\rangle = |\downarrow\rangle \quad X|\downarrow\rangle = |\uparrow\rangle \qquad Y|\uparrow\rangle = i|\downarrow\rangle \quad Y|\downarrow\rangle = -i|\uparrow\rangle$$

SOURCE SENDS $\quad |\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle\right)$

$X_i, Y_i \quad i = 1,2,3$

$$X_1 Y_2 Y_3 |\psi\rangle = X_1 Y_2 Y_3 \left[|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle\right] = (1)(i)(i)|\downarrow\downarrow\downarrow\rangle - (1)(-i)(-i)|\uparrow\uparrow\uparrow\rangle$$
$$= +1\,|\psi\rangle$$

SIMILARLY $\quad Y_1 X_2 Y_3 |\psi\rangle = Y_1 Y_2 X_3 |\psi\rangle \Rightarrow$ FACT (*)

BUT NOW $\quad X_1 X_2 X_3 |\psi\rangle = X_1 X_2 X_3 \left[|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle\right]$

$$= (1)(1)(1)|\downarrow\downarrow\downarrow\rangle - (1)(1)(1)|\uparrow\uparrow\uparrow\rangle$$
$$= -1\,|\psi\rangle$$

SUMMARY $\quad$ QM PRED ✓

CL PRED ✗

(the putative classical system that explains fact (*) makes another prediction that has been falsified in real experiments.

$\Rightarrow \exists$ experiments where the initial state doesn't have definite properties!)