IA006 – Exercícios de Fixação de Conceitos
EFC 1 – 1s2019

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Parte 1 – Atividades teóricas

Exercício 1

a) $P(A^C) = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$

b) $P(A^C \cup B) = P(A^C) + P(B) - P(A^C B) = P(A^C) + P(B) - (P(B) - P(AB)) = P(A^C) + P(AB) = \frac{2}{3} + \frac{1}{6} = \frac{5}{6}$

c) $P(A \cup B^C) = P(A) + P(B^C) - P(AB^C) = P(B^C) + P(AB) = (1 - P(B)) + P(AB) = (1 - \frac{1}{4}) + \frac{1}{6} = \frac{11}{12}$

d) $P(AB^C) = P(A) + P(B^C) - P(A \cup B^C) = \frac{1}{3} + \frac{3}{4} - \frac{11}{12} = \frac{2}{12} = \frac{1}{6}$

e) $P(A^C \cup B^C) = 1 - P(AB) = 1 - \frac{1}{6} = \frac{5}{6}$

Exercício 2

a)

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(\xi) d\xi = \int_{-\infty}^{x} \frac{1}{2} d\xi = \left[\frac{1}{2}\xi\right]_0^x = \frac{1}{2}x, \forall X \in [0, 2]$$

b)

$$E\{X\} = \int_{-\infty}^{\infty} x f_X(x) dx = \int_{0}^{2} \frac{1}{2} x dx = \left[\frac{x^2}{4}\right]_0^2 = 1$$

$$E\{X^2\} = \int_{-\infty}^{\infty} x^2 f_X(x) dx = \int_{0}^{2} \frac{1}{2} x^2 dx = \left[\frac{x^3}{6}\right]_0^2 = \frac{4}{3}$$

$$E\{X^3\} = \int_{-\infty}^{\infty} x^3 f_X(x) dx = \int_{0}^{2} \frac{1}{2} x^3 dx = \left[\frac{x^4}{8}\right]_0^2 = 2$$

Exercício 3

a) X^2 pois ela é a variável que transmite mais informações sobre o evento, sendo assim, a mais provável de resolver, com maior eficiência, a "confusão" quando não tivermos nenhum tipo de informação inicial sobre o problema, no pior dos cenários, sempre teremos 25% com X2.

b) $$H(X_1) = -\sum_x p(X_1) log_2[p(X_1)] = -[0,1(-3,32) + 0,2(-2,32) + 0,3(-1,74) + \ 0,4(-1,32)] = 1,85$$

c) $$H(X_2) = -\sum_x p(X_2) log_2[p(X_2)] = -[0,25(-2)+0,25(-2)+0,25(-2)+0,25(-2)] = 2$$

Exercício 4

a) $$\mathcal{L}(\mu) = p(x|\mu) = \frac{p(x\mu)}{p(\mu)} = p(x) = f_\mu(x)$$

b) $$\mathcal{L}(\mu) = p(\mathbf{x}|\mu) = \prod_{k=1}^{N} p(x_k|\mu) = \prod_{k=1}^{N} \frac{p(x_k|\mu)}{p(\mu)} = \prod_{k=1}^{N} f_\mu(x_k)$$

c) $$\mu_{ML} = argmax_\mu \mathcal{L}(\mu) = argmax_\mu ln\left[\prod_{k=1}^{N} f_\mu(x_k)\right] = argmax_\mu \sum_{k=1}^{N} ln[f_\mu(x_k)] = argmax_\mu \sum_{k=1}^{N} ln\left[\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\right] =$$
$$= argmax_\mu \sum_{k=1}^{N}\left[ln\left(\frac{1}{\sigma\sqrt{2\pi}}\right) - \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] = argmax_\mu \sum_{k=1}^{N} -\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 - Nln(\sigma\sqrt{2\pi}) = argmax_\mu A \iff \frac{dA}{d\mu} = 0 \Rightarrow \sum_{k=1}^{N} \frac{-2}{2}\left(\frac{x-\mu}{\sigma}\right)(-1) = 0 \Rightarrow$$
$$\sum_{k=1}^{N} x_k - N\mu = 0 \Rightarrow \mu_{ML} = \frac{1}{N}\sum_{k=1}^{N} x_k$$