

Chun-Wei Chang, Takeshi Miki, Fuh-Kwo Shiah, Shuh-ji Kao, Jiunn-Tzong Wu, Akash R. Sastri and Chih-hao Hsieh. Year. Linking secondary structure of individual size distribution with nonlinear size-trophic level relationship in food webs.

APPENDIX B

Incorporating size-dependent transfer efficiency in the generalized MTE model

In Appendix A, the transfer efficiency (TE) was assumed as a constant, independent of body size in our model. However, it will be illuminating to relax this assumption and investigate how size-dependent TE (Barnes et al. 2010) affects the generalized MTE model. In the following theoretical argument, we aim to demonstrate that the prediction of generalized MTE model is not very sensitive to the size dependency of TE. Thus, our prediction, based on the assumption that TE is constant, should be robust.

To examine the effects of the size-dependent TE, we follow the suggestion of Barnes et al (2010) to firstly consider the behavior of log linear transfer efficiency (α). Barnes et al (2010) found that empirically $\log(\text{PPMR})$ is proportional to $\log M$, and suggested that \log transfer efficiency (α) to the predator with size M may be also proportional to $\log M$. This leads to $\log \alpha = v' + w' * \log M$ (v' and $w' < 0$), because $\log \alpha$ is proportional to $\log(\text{PPMR})$.

If α changes with trophic level and body size, the eqn A.2 shall be expressed by

$$\alpha(TL) = \alpha_2 * \alpha_3 * \dots * \alpha_{TL} = \prod_{i=2}^{TL} \alpha_i \text{ where } \alpha_i \text{ is the transfer efficiency from } TL=i-1 \text{ to}$$

$TL=i$. Here, we use a geometric mean of α over a range of TL between 1 and TL ,

$\alpha_m = \left(\prod_{i=2}^{TL} \alpha_i \right)^{\frac{1}{TL-1}}$. As $\log \alpha_i = v' + w' * \log M_i$, $\log \alpha_m$ can be formulated as

$$\begin{aligned} \log \alpha_m &= (TL - 1)^{-1} \sum_{i=2}^{TL} \log \alpha_i \\ &= (TL - 1)^{-1} [v'(TL - 1) + w'(\sum_{i=2}^{TL} \log M_i)] \\ &= v' + w' * E(\log M) \end{aligned}$$

, where $E(\log M)$ is the average body size over a range of TL between 1 and TL corresponding to a range of body size between $(\log M_2 \log M_{TL})$. We can approximate the $E(\log M)$ by a simple estimation $E(\log M) = 0.5 * (\log M + \log M_2)$, where the $\log M$ is the body size of predators at TL . This is based on an assumption that the average of the body size, $E(\log M) = E[\tau^{-1}(TL)]$ equals to $\tau^{-1}[E(TL)]$, where the τ^{-1} is the inverse function of size-TL relationship. For example, τ^{-1} is a linear function. This approximation is reasonable because our model is based on polynomial models. If the function of body size, $\tau^{-1}(TL)$, is a convex function (Barnes et al. 2010), we expect that the $E(\log M) = j * (\log M + \log M_2)$, where $j < 0.5$ according to Jensen's inequality that $E(\tau^{-1}(TL)) \leq \tau^{-1}(E(TL))$ (our approximation). We firstly show the result specifying $j=0.5$.

$$\begin{aligned} \log \alpha_m &= [v' + 0.5w' * (\log M + \log M_2)] = (v' + 0.5w' * \log M_2) + 0.5w' * \log M \\ &= v + w * \log M \end{aligned} \tag{B.1}$$

, where $v = (v' + 0.5w' * \log M_2)$ and $w = 0.5w'$.

Substitute constant α by α_m in eqn A.3,

$$N = aM^{-\lambda + \log \alpha_m \sum_{i=1}^m b_i \log^{i-1} M}$$

By approximating the α_m by eqn B.1, we obtain the following relationship:

$$\begin{aligned} N &= aM^{-\lambda + (v+w \log M) \sum_{i=1}^m b_i \log^{i-1} M} \\ &= aM^{-\lambda + (v \sum_{i=1}^m b_i \log^{i-1} M + w \sum_{i=1}^m b_i \log^i M)} \\ \log N &= \log a + (vb_1 - \lambda) \log M + (v \sum_{i=2}^m b_i \log^i M + w \sum_{i=1}^m b_i \log^{i+1} M) \end{aligned} \quad (\text{B.2})$$

Set B_i as the coefficient of the higher order terms ($\log^i M$) in eqn B.2. As such, coefficients of higher order terms are expressed as the following recursive form:

$$\begin{aligned} B_2 &= vb_2 + wb_1 \\ B_3 &= vb_3 + wb_2 \\ &\dots \\ B_n &= vb_n + wb_{n-1} \end{aligned} \quad (\text{B.3})$$

This result suggests the fact that size-dependent TE (i.e. $\log \alpha = v + w * \log M$) can potentially influence our prediction (eqn A.3) only if the size scaling exponent of TE, w , is large. Based on eqn B.3, when the size-dependent TE is included in our model (eqn B.2), the magnitude of secondary structure of ISD should be smaller than our original prediction (eqn B.3), because vb_n and wb_{n-1} have opposite signs (v and $w < 0$, but b_n and b_{n-1} have opposite signs) and thus the effects of size-dependence of TL and TE on secondary structure of ISD would be canceled out, qualitatively. If so, we expect that our predictions based on eqn A.3 should be systematically larger than the empirical observations. However, such systematic

overestimation is not observed in our empirical measurements (Fig. 3), implying that v is much larger than w . Therefore, we argue that the influence of size-dependent TE on ISD through the size scaling exponents, w , may be small based on both our empirical observations (Fig. 3) and the following explanation. The natural range of α (0.05~0.25) is much smaller than the range of body size (10^{-12} ~ 10^8 g). By assuming $\log_2 \alpha = v' + w' \log_2 M$ under the \log_2 scale (as we used in this study), the estimated $v' = -2.658$ and $w' = -0.073$ can be obtained according to the prediction from the previous study (Barnes et al. 2010) that α equals to 0.134 and 0.058 when M equals to 10^1 and 10^6 (g), respectively. The real slope, w' , might be even shallower than our estimation (-0.073) because the decrease of α with respect to body size may be very small for very tiny (or very large) predators since the real α is bounded in a narrow range. By assuming $M_2 = 10^1$, the body size of the smallest consumers (Barnes et al. 2010), we estimated $v = (v' + 0.5w' \log M_2)$ and $w = 0.5w'$. The estimated $v = -2.658 + 0.5 \times 0.073 \times \log_2 10^1 = -2.779$, $w = 0.5 \times -0.073 = -0.036$, and thus $v/w = 77.19$. Therefore, w is almost two orders smaller than v . Moreover, if the TL -size relationship, τ^{-1} , is a convex function (Barnes et al. 2010), $E(\log M)$ is equal to $j^*(\log M + \log M_0)$, where $j < 0.5$. When we try $j = 0.3$, w is even smaller than v ($v/w = 125.19$). Therefore, the eqn B.3 can be approximated by $B_n = vb_n$, which is similar with our original prediction. Our predictions are thus insensitive to inclusion of the assumption that TE is size-dependent.

LITERATURE CITED

Barnes, C., D. Maxwell, D. C. Reuman, and S. Jennings. 2010. Global patterns in predator–prey size relationships reveal size dependency of trophic transfer efficiency. *Ecology* 91:222–232.