- Mathematics
- Informatics
- Communication

Vulgarize

github.com

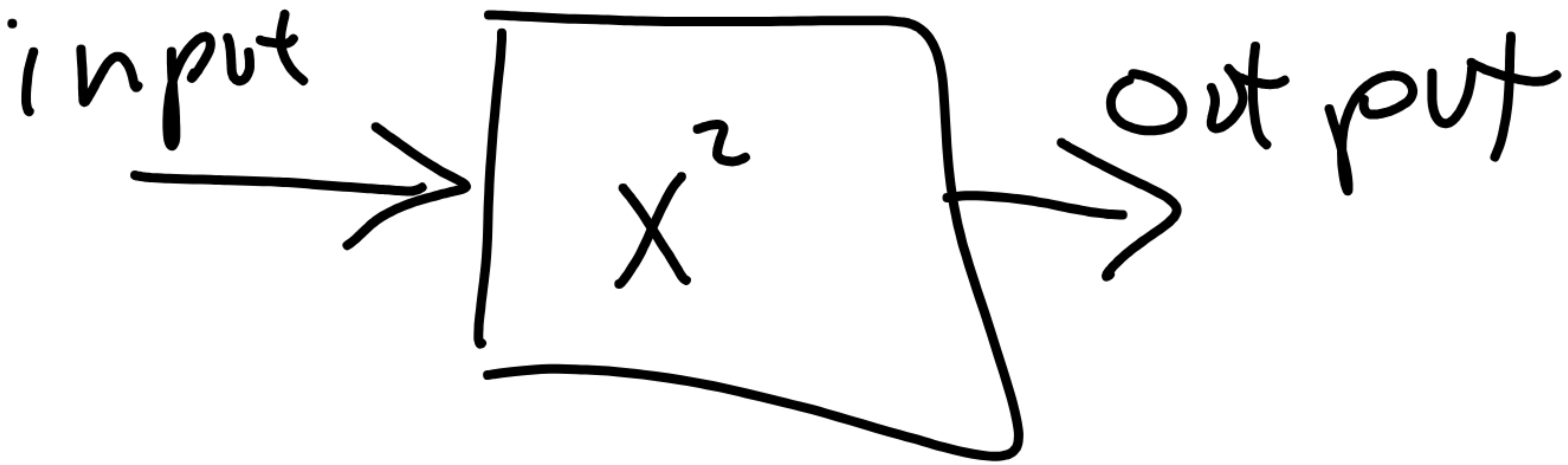
associates output
with an input.

$$f : S \to T$$

Python

functions

Types



$(-1, 1)$

$(3, 9)$ $(-3, 9)$

int

float

string "hello"

$x \in S$

$\mathbb{Z} \subseteq \mathbb{R}$

$3 \in \mathbb{N}$ $\pi \in \mathbb{R}$ $-3 \in \mathbb{Z}$

$\frac{3}{5} \in \mathbb{Q}$ $\mathbb{C}$

$\mathbb{R} \subseteq \mathbb{C}$ true

$\mathbb{Q} \subseteq \mathbb{Z}$ false

proposition

$f(x)$ $3x^2+2x-1$ $\log$ $\ln$

$y = mx + b$

$\sin$ $\cos$

e^x $\exp$ $\tan$

Combinatorics

$$n! = n \cdot (n-1) \cdot (n-2)$$

$$\cdot \cdots\cdots \quad 1$$

$$3! = 3 \cdot 2 \cdot 1 = 6$$

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$4! = 4 \cdot 3!$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n(n-1)! & \text{if } n > 0 \end{cases}$$

$1 \times 2 \times 3 \cdots n$

```
def fac(n):
    if n == 0:
        return 1
    else:
        return n * fac(n-1)
```

termination

recursion

reduction

combine

$$\binom{n}{k} \quad \binom{10}{4} = \frac{n!}{k!\,(n-k)!}$$

$$\binom{n}{k} \quad k \le n \qquad = \frac{n!}{k!\,(n-k)!}$$

$$\binom{7}{3} = \frac{7!}{3!\,(7-3)!} = \frac{7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2}}{\cancel{3 \cdot 2} \cdot 4 \cdot \cancel{3} \cdot \cancel{2}}$$

$$= 35.0$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{n\,(n-1)!}{k\,(k-1)!\,(n-k)!} = \frac{n}{k}\binom{n-1}{k-1}$$

$$\therefore \binom{n-1}{k-1} = \frac{(n-1)!}{(k-1)!\,(n-k)!}$$

$$\binom{n}{1} = n$$

$$\binom{n}{0} = 1$$

$$\frac{n!}{1!\,(n-1)!} = \frac{n\,\cancel{(n-1)!}}{1!\,\cancel{(n-1)!}} = n$$

$$\binom{n-1}{k-1} = \frac{n-1}{k-1}\binom{n-2}{k-2}$$

$$\binom{n}{k} \qquad k \le n \qquad = \frac{n!}{k!\,(n-k)!}$$

$$\binom{7}{3} = \frac{7!}{3!\,(7-3)!} = \frac{7\cdot\cancel{6}\cdot 5\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}}{\cancel{3\cdot 2}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}}$$

$$= 35.0$$

```
def choose(n, k):   #arity
    if k==1:
        return n
    elif k==0:
        return 1
    else:
        return n * choose(n-1, k-1) // k
```

termination

$\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$

$\underbrace{n \ *}_{\text{combin}}$ $\underbrace{\text{choose(n-1, k-1) // k}}_{\text{reduction}}$

$F_0 = 1$

$F_1 = 1$

$F_2 = F_1 + F_0$

$F_3 = F_2 + F_1$

0 1 1 2 3 5

8 13 21 34 ....

$F_n = F_{n-1} + F_{n-2}$

$$F_n = F_{n-1} + F_{n-2}$$

$$F_{12} = F_{11} + F_{10}$$

$$F_{11} = F_{10} + F_9$$

$$F_{10} = F_9 + F_8$$

```
def fib(n):
    if n == 0:
        return 1
    elif n == 1:
        return 1
    else:
        return fib(n-1)+fin(n-2)
```

termination

combine

reduction

reduction

$$3/2 = 1.5$$

$$4/2 = 2.0$$

$$1 \quad \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}}} = a$$

closed form

z

Xeno's paradox

$$a = \frac{1}{1+a}$$

$$a(1+a) = 1$$

$$a + a^2 = 1$$

$$a^2 + a - 1 = 0$$

$$a = \frac{-1 \pm \sqrt{1^2 - 4(-1)}}{2}$$

$$a = \frac{-1 \pm \sqrt{5}}{2}$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}} \longrightarrow$$

$$a_0 = x$$

$$a_1 = \frac{1}{1+x} = \frac{1}{1+a_0}$$

$$a_2 = \cfrac{1}{1+\cfrac{1}{1+x}} = \frac{1}{1+a_1}$$

$$a_3 = \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+x}}} = \frac{1}{1+a_2}$$

$$\vdots$$

Helger

$$a_{n-1} = \frac{1}{1+a_{n-2}}$$

$$a_n = \frac{1}{1+a_{n-1}}$$

```
def frac(n,x):
    if n==0:
        return x

1/3
                                1.0
    else:
        return 1.0/(1+frac(n-1,x))
```

~~fre~~ fac(10,3.5)

$$x^n = \overbrace{x \cdot x \cdot x \cdot \cdots x}^{n \text{ times}}$$

$$x^2$$

$$\cdot \; x \cdot x \cdot x$$

$$x^n = \begin{cases} 1 & \text{if } n=0 \\ x \cdot x^{n-1} & \text{if } n>0 \end{cases}$$

$n \in \mathbb{N}$

recursive dpf

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \end{cases}$$

relation

reflexive $a \sim a$

transitive $a \sim b \wedge b \sim c \Rightarrow a \sim c$

symmetric $a \sim b \Rightarrow b \sim a$

predicate equivalence

$A \sim B$ if they have a element in common

$A \sim B \wedge B \sim C \implies A \sim C$

$\underbrace{\{1, 2\}}_{A} \quad \underbrace{\{2, 3\}}_{B} \quad \underbrace{\{3, 4\}}_{C}$

$A \sim A$

$A \sim B \Rightarrow B \sim A$

$|A| > 1$ $A \neq \emptyset$

$$\neg\left(p(n) \wedge q(n)\right) \qquad p(n) \quad n>3$$

$$\neg\left(p(n) \vee q(n)\right) \qquad q(n) \quad n<10$$

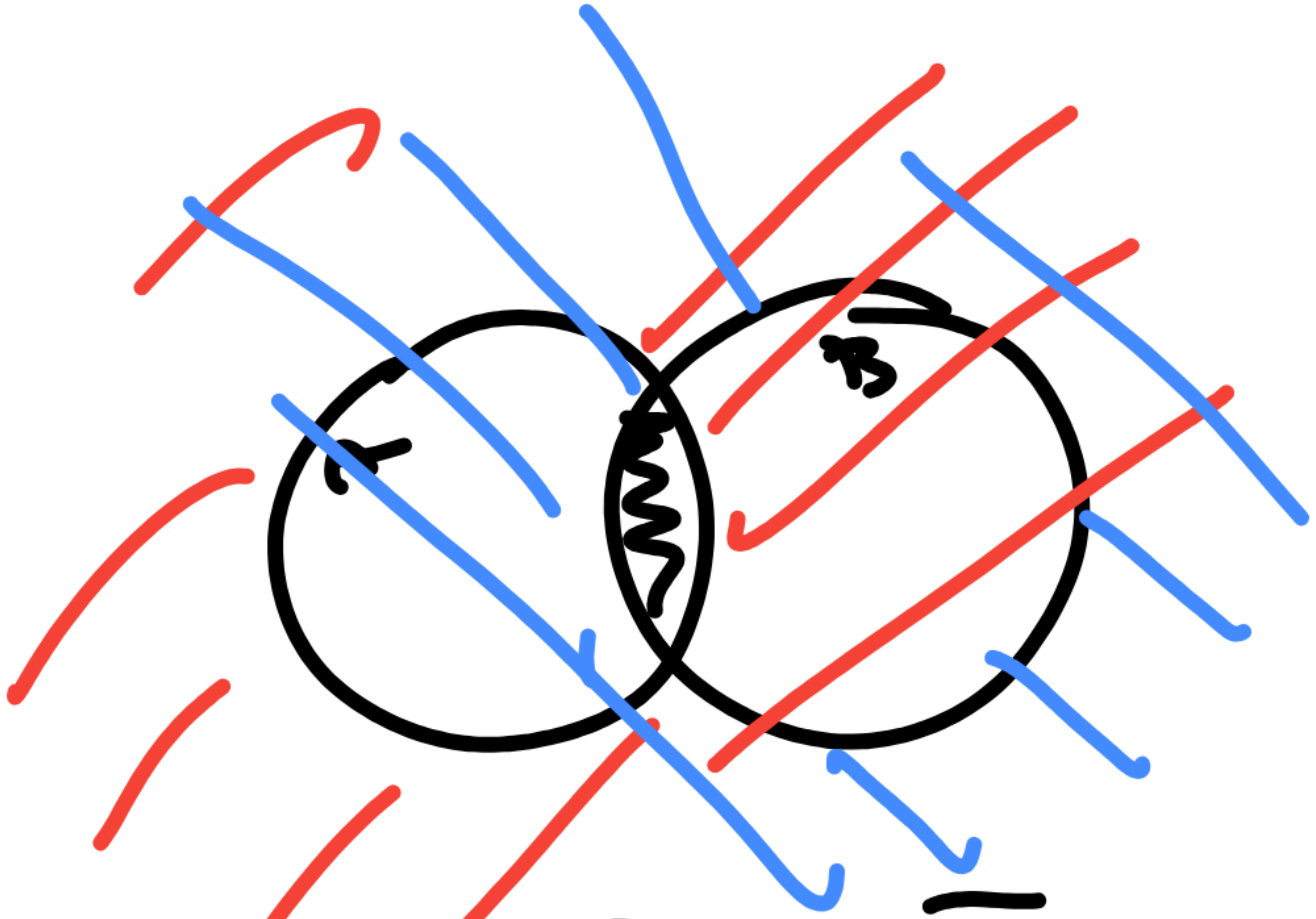
$$\neg \qquad \overline{n>3} \overset{\vee}{\wedge} \overline{n<10} \qquad p(n) \vee q(n)$$

$$\overline{n>3} \underset{\wedge}{\vee} \overline{n<10}$$

De Morgan's Theorem

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$



$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

if c_1 and c_2 :

{

else:

{

even(n) or composite(n)

```
if odd(n) and prime(n):

else:
```

quantifier

existential

universal

$\forall n > 10, \quad n > 0$

$\exists n > 10, \quad n > 100$

$$\neg(\forall x \in S,\ P(x))$$

$$\exists x \in S,\ \neg P(x)$$

$\forall x \in S \quad \exists n > X \quad$ n is even

$\exists x \in S \ \neg(\exists n > X \quad$ n is even $)$

$\exists x \in S \quad \forall n > X \quad$ n is odd

$\forall x \in S$ all(. . . . .)

$\exists$ any(. . . . .)

False ← all(odd(n) for n in range(100))

all(n > -1 for n in range(100))

~~not n > -1~~

any ~~all~~ (n ≤ -1 for n in range(100))

```
def frac(n, x):
    if n == 0:
        return x

1/3
    else:                1.0
        return 1.0/( 1 + frac(n-1, x))

fac(10, 3.5)
```

$$\pi = 3.141592653589$$

$$\pi = 3.14 = \frac{314}{100} = \frac{157}{50} \rightarrow \frac{a}{b}$$

$$\frac{a}{51} \qquad \frac{17}{\cancel{8}51} = \frac{1}{3} \qquad \frac{22}{7}$$

North

Gordel P.

South

Monoid $(M, \circ)$

is a set M with a binary operator, $\circ$ which obeys

1) closure $\quad a, b \in M \Rightarrow a \circ b \in M$

2) associativity: $\quad a, b, c \in M \Rightarrow$

$$a \circ (b \circ c) = (a \circ b) \circ c$$

3) identity: $\quad \exists e \in M, \quad x \in M$

$$\Rightarrow e \circ x = x \circ e = x$$

$\mathbb{N}$ $+$

a, b, c

$(a+b)+c = a+(b+c)$

$0+x$

$x+0$

x

$(1+3)+7 \qquad 1+(3+7)$

$4+7 \qquad 1+10$

$(1-3)-7 \qquad 1-(3-7)$

$-2-7 \qquad 1--4$

$-9 \qquad 5$

3×3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$u(x) = odd(x)$

$w(x) = prime(x)$

$v(x) = x > 12$

$odd(x) \wedge prime(x)$

$odd(x) \vee prime(x)$

$u(x) \wedge (w(x) \wedge v(x))$

$(u(x \wedge w(x)) \wedge v(x)$

The set of all Python strings

closure ( )

assoc "I" · "am" "happy"

( )

ident

$$[1, 2, 3] + [10, 11, 12]$$

a	b	c
F	F	F
F	F	T
F	T	F
F	T	T
T	F	F
T	F	T
	⋮	

$(a \wedge b) \wedge c$

F

$a \wedge (b \wedge c)$

F

$$\Big(\underbrace{[1, 12, -10 \cdots\cdots}\Big) + \Big(\underbrace{\quad\quad]}\Big)$$

10,000

$$(((1 + 12) + -10) +$$

10000 n

5000 n

2500 n

$a_0 \quad n$

$a_1 \quad n/2$

$a_2 \quad n/4$

$a_3 \quad n/8$

$\vdots$

$a_k = n/2^k = 1$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

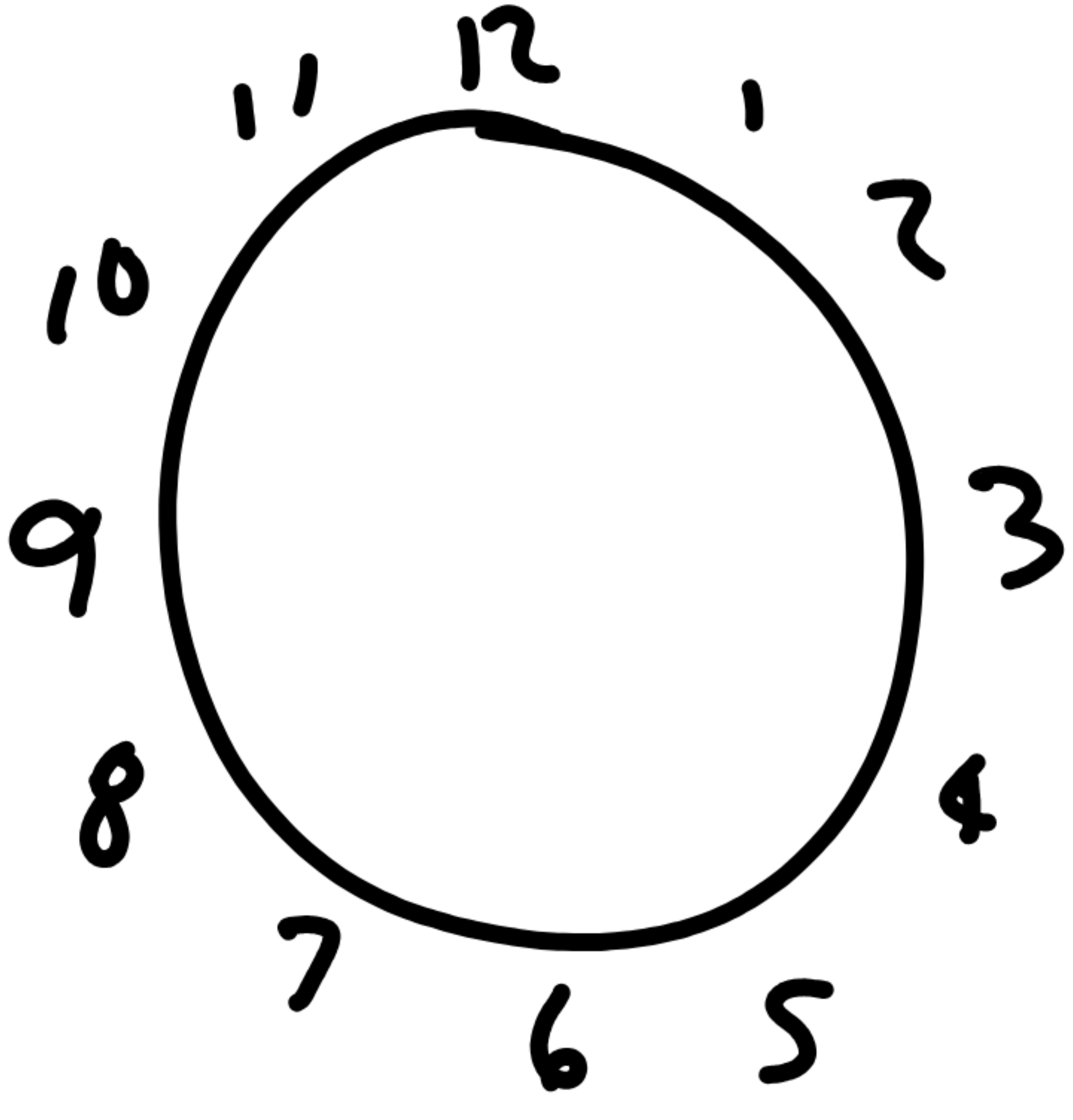
$$\log_2 n = \log_2 2^k = k$$

$$\log_b n = \frac{\log_{10} n}{\log_{10} b}$$

$\cdot$	a	b	c	d
a	a	b	c	d
b	b	b	d	d
c	c	d	c	d
d	d	d	d	d

$\cup$	$\emptyset$	$\{1\}$	$\{2\}$	$\{1,2\}$
$\emptyset$	$\emptyset$	1	2	1, 2
$\{1\}$	1	1	1, 2	1, 2
$\{2\}$	2	1, 2	2	1, 2
$\{1,2\}$	1, 2	1, 2	1, 2	1, 2

duals



9×7

7×9

```
def mult_12(a, b):

    p = (a * b) % 12
    if p == 0
    els return 12
```

$$
\begin{aligned}
7^{51} &= 7 \cdot 7^{50} \\
&= 7 \cdot 7 \cdot 7^{49} \\
&= 7 \cdot (7^{2})^{25} \\
&= 7 \cdot 1 = 7
\end{aligned}
$$

7^0

$$7^{12} = 7^{12} \cdot 7^0$$
$$= 7^{12+0}$$
$$= 7^{12}$$

Someone wrote this backwards

$$7\cdot\left(7^2\right)^{25}$$

$$7\cdot\left(7^2\right)\left(7^2\right)^{24}$$

$$7\cdot\left(7^2\right)\left(\left(7^2\right)^2\right)^{12}$$

$$7\cdot\left(7^2\right)\left(\left(\left(7^2\right)^2\right)^2\right)^{6}$$

```
def power(b, p, op, id):
    if p == 0:
        return id
    elif p % 2 == 0:
        return power(op(b, b), p//2)
    else
        return op(b, power(b, p-1))
```

$$\pi = 3.141592653589$$

$$\pi = 3.14 = \frac{314}{100} = \frac{157}{50} \to \frac{a}{b}$$

$$\frac{a}{51} \qquad \frac{17}{51} = \frac{1}{3} \qquad \frac{22}{7}$$