

## Problem I

# Intersecting Lines

Time limit: 3 seconds

Memory limit: 1024 megabytes

### Problem Description

We all know that a pair of distinct points on a plane defines a line and that a pair of lines on a plane will intersect in one of three ways:

1. no intersection because they are parallel;
2. intersect in a line because they are on top of one another (i.e. they are the same line);
3. intersect in a point.

In this problem, you will use your algebraic knowledge to create a program that determines how and where is the intersect point of these two lines.

Your program will repeatedly read in four points that define two lines in the  $x$ - $y$  plane and determine how and where the lines intersect ( $-1000 \leq x, y \leq 1000$ ).

### Hint 1: How to Derive the General Form of a Line from Two Points?

Given two points  $(x_1, y_1)$  and  $(x_2, y_2)$ , a line can be expressed in general form:

$$Ax + By = C$$

We can determine  $A$ ,  $B$  and  $C$  as:

$$A = y_2 - y_1$$

$$B = x_1 - x_2$$

$$C = A \times x_1 + B \times y_1$$

### Hint 2: How to Use Cramer's Rule to Find the Intersection?

For two lines:

$$L_1 : A_1 \times x + B_1 \times y = C_1$$

$$L_2 : A_2 \times x + B_2 \times y = C_2$$

**Step 1:** Determine the  $\Delta$ ,  $\Delta x$  and  $\Delta y$

$$\Delta = A_1 \times B_2 - A_2 \times B_1$$

$$\Delta x = C_1 \times B_2 - C_2 \times B_1$$

$$\Delta y = A_1 \times C_2 - A_2 \times C_1$$

**Step 2:** Determine relationship between the lines

If  $\Delta = 0$ :

- If also  $\Delta x = 0$  and  $\Delta y = 0$ , the lines are coincident (“LINE”).
- Otherwise, the lines are parallel (“NONE”).

If  $\Delta \neq 0$ :

- the lines intersect in one point.

**Step 3:** Determine the intersection point (if  $\Delta \neq 0$ )

$$x = \Delta x / \Delta$$

$$y = \Delta y / \Delta$$

This gives the coordinates of the intersection point.

## Input Format

The first line contains an integer  $N$  between 1 and 100 describing how many pairs of lines are represented.

The next  $N$  lines will each contain eight integers. These integers represent the coordinates of four points on the plane in the order  $x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$ . Thus each of these input lines represents two lines on the plane: one line through  $(x_1, y_1)$  and  $(x_2, y_2)$  and the other one line through  $(x_3, y_3)$  and  $(x_4, y_4)$ . The point  $(x_1, y_1)$  is always distinct from  $(x_2, y_2)$ . Likewise with  $(x_3, y_3)$  and  $(x_4, y_4)$ .

**Hint:**

- Use double for all calculations and storage of coordinates and results (**Do not use float**).
- When printing the  $x$  and  $y$  coordinates of the point in two decimal places, please use:  
`“System.out.printf(“POINT %.2f %.2f\n”, x, y);”`
- If the calculation result is “-0.00”, replace it with “0.00” before printing to pass the judge.

## Output Format

There should be  $N$  lines of output. There will then be one line of output for each pair of planar lines represented by a line of input, describing how the lines intersect: “NONE”, “LINE”, or “POINT”. If the intersection is a point then your program should output the  $x$  and  $y$  coordinates of the point, correct to **two decimal places**.

## Technical Specification

- $1 \leq N \leq 100$ .
- $-1,000 \leq x_i, y_i \leq 1,000$ , where  $1 \leq i \leq 4$

## Sample Input 1

## Sample Output 1

5	POINT 2.00 2.00
0 0 4 4 0 4 4 0	NONE
5 0 7 6 1 0 2 3	LINE
5 0 7 6 3 -6 4 -3	POINT 2.00 5.00
2 0 2 27 1 5 18 5	POINT 1.07 2.20
0 3 4 0 1 2 2 5	

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