Master en Sciences Physiques

Nanophysique PHYS-F-475

CHAPITRE 10. MeNT

Exercices

1. The form of the Dean-Kawasaki model is

$$\frac{\partial}{\partial t}\rho_t\left(\mathbf{r}\right) = D\frac{\partial}{\partial \mathbf{r}}\cdot\rho_t\left(\mathbf{r}\right)\frac{\partial}{\partial \mathbf{r}}\frac{\delta F\left[\rho_t\right]}{\delta\rho_t\left(\mathbf{r}\right)} - \frac{\partial}{\partial \mathbf{r}}\cdot\sqrt{2D\rho_t\left(\mathbf{r}\right)}\boldsymbol{\xi}_t\left(\mathbf{r}\right), \quad \left\langle \boldsymbol{\xi}_t\left(\mathbf{r}\right)\boldsymbol{\xi}_{t'}\left(\mathbf{r}'\right)\right\rangle = \mathbf{1}\delta\left(t-t'\right)\delta\left(\mathbf{r}-\mathbf{r}'\right)$$

   (a) Integrate this equation over a ball of radius R and determine the explicit expressions for each term.

   (b) What is the autocorrelation of the noise term (i.e. the equivalent of $\left\langle\left(\sqrt{2D\rho_t\left(\mathbf{r}\right)}\boldsymbol{\xi}_t\left(\mathbf{r}\right)\right)\left(\sqrt{2D\rho_{t'}\left(\mathbf{r}'\right)}\boldsymbol{\xi}_{t'}\left(\mathbf{r}'\right)\right)\right\rangle$)?

   (c) Are any special assumptions necessary in the previous calculation? If so, when are they true?

2. Show that if

$$\frac{\partial}{\partial t}m_t\left(r\right) = D4\pi r^2\rho_t\left(r\right)\frac{\partial}{\partial r}\frac{\delta F\left[\rho_t\right]}{\delta\rho_t\left(r\right)} + \sqrt{8\pi r^2 D\rho_t\left(r\right)}\xi_t\left(r\right)$$

   then it is also true that

$$\frac{\partial}{\partial t}m_t\left(r\right) = -D\frac{\partial m_t\left(r\right)}{\partial r}\frac{\delta F\left[\rho_t\right]}{\delta\rho_t\left(r\right)} + \sqrt{2D\frac{\partial m_t\left(r\right)}{\partial r}}\xi_t\left(r\right)$$

3. The distance between two densities is

$$d\left[\rho_1\left(r\right),\rho_2\left(r\right)\right] = \min_{\substack{m_t(r)\\ m_0(r)=\rho_1(r)\\ m_T(r)=\rho_2(r)}}\int_0^T\left(\sqrt{\int_0^\infty\frac{1}{4\pi r^2\rho_t\left(r\right)}\left(\frac{\partial m_t\left(r\right)}{\partial t}\right)^2 dr}\right)dt$$

   (a) Express this in terms of $r\left(m\right)\equiv m^{-1}\left(m\right)$.

   (b) For the capillary approximation, $\rho\left(r\right)=\rho_0+\rho_1\Theta\left(R-r\right)$, where ρ_0,ρ_1 are fixed constants, what is $m\left(r;R\right)$? What is $d\left[\rho\left(r;R_1\right),\rho\left(r;R_2\right)\right]$?