

# NANOPHYSIQUE

## INTRODUCTION PHYSIQUE AUX NANOSCIENCES

### *3. AMAS OU AGREGATS ATOMIQUES*

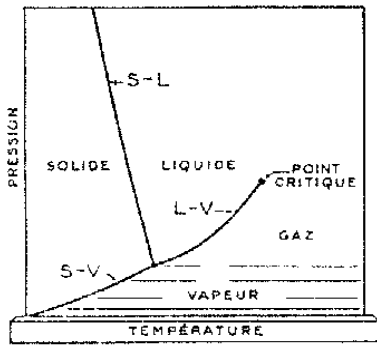
James Lutsko

2021-2022

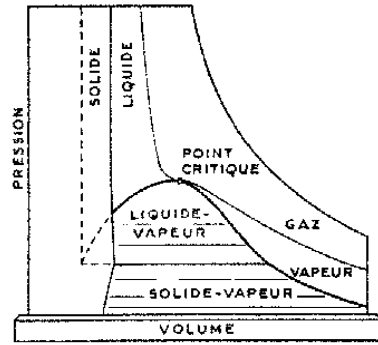
# *AMAS OU AGREGATS ATOMIQUES*

- L'auto-assemblage: Nucleation
  - Modele de l'amas: “capillary model”.
  - Thermodynamics
  - Becker-Doring model
  - Zeldovich equation
  - Taux de nucléation
- Nanoparticules cristalline
  - Structure cristalline
  - Indices de Miller
  - Tension de surface
  - Forme des Cristaux
  - Transitions de phase
- Propriétés électronique des agregats

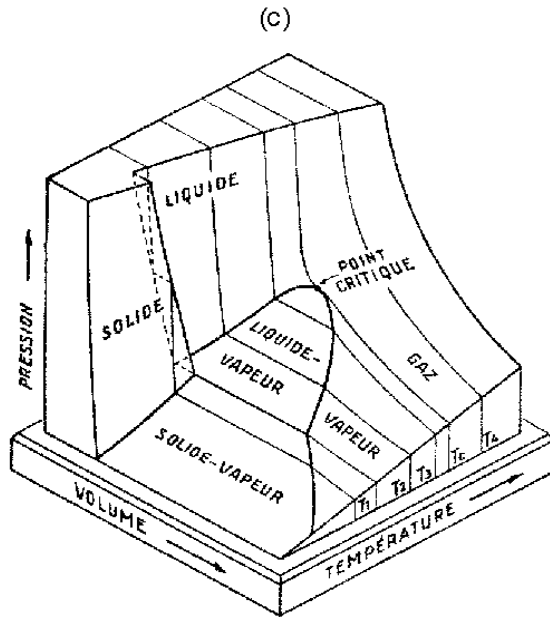
# A simple but realistic example: Liquid-vapor transition



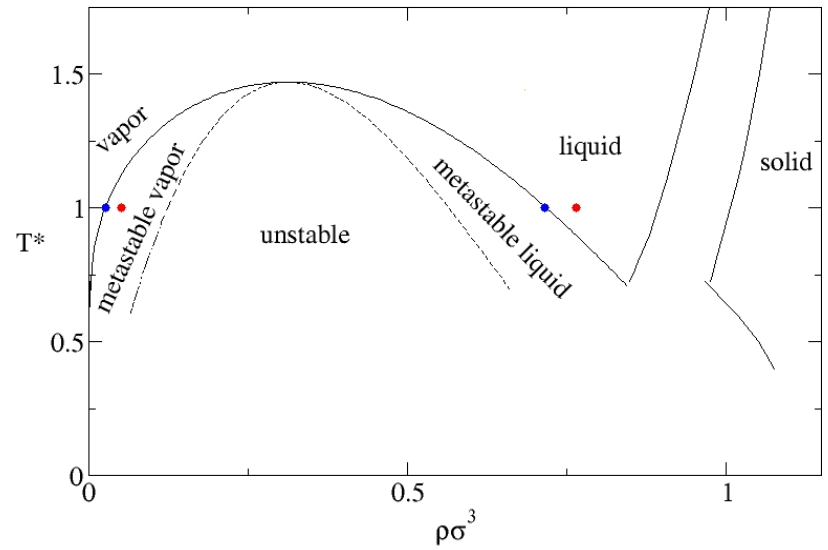
(a)



(b)



(c)



# Modele de l'Amas : “Capillary model”

L'energie libre (ensemble généralisé)

$$\Omega = V(R) \omega_1 + S(R) \gamma_{12} + (V - V(R)) \omega_2$$

$\omega$  est l'energie volumique par unite de volume,  $\gamma$  est l'energie surface

Minimisez par rapport à R:

$$0 = \frac{\partial \Omega}{\partial R} \rightarrow S(R) \omega_1 + S'(R) \gamma_{12} = S(R) \omega_2$$

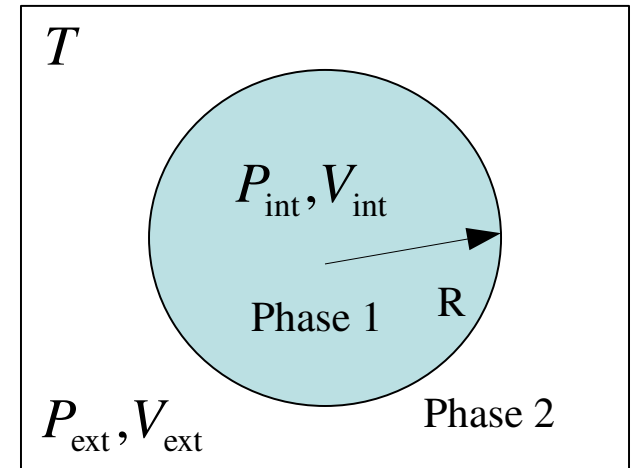
Si  $\omega_i = \omega(\rho_i)$  on doit minimiser par rapport aux densities:

$$0 = \frac{\partial \Omega}{\partial \rho_i} \rightarrow \frac{\partial \omega(\rho_i)}{\partial \rho_i} = 0$$

Rappelez-vous que  $\omega(\rho) = f(\rho) - \mu \rho \rightarrow \omega'(\rho) = f'(\rho) - \mu$

Donc,  $f'(\rho_i) = \mu \rightarrow \omega(\rho_i) = f(\rho_i) - f'(\rho_i) \rho_i = -P(\rho_i)$

$$0 = \frac{\partial \Omega}{\partial R} \rightarrow -P(\rho_1) + \frac{S'(R)}{S(R)} \gamma_{12} = -P(\rho_2)$$



Equation de Laplace: 
$$P_{\text{int}} = P_{\text{ext}} + \frac{2\gamma}{R}$$

# Modele de l'Amas : “Capillary model”

L'energie libre (ensemble canonique)

$$F = V(R) f_1 + S(R) \gamma_{12} + (V - V(R)) f_2$$

$f$  est l'energie volumique par unite de volume,  
est c'est  $N$  qui est constante:  $\rho_1 V(R) + \rho_2 (V - V(R)) = N$

Minimiser par rapport à  $R$ :

$$0 = \frac{\partial F}{\partial R} = S(R) f_1 + S'(R) \gamma_{12} - S(R) f_2 + (V - V(R)) f'_2 \frac{\partial \rho_2}{\partial R}$$

$$S(R) f_1 + S'(R) \gamma_{12} = S(R) f_2 + S(R) f'_2 (\rho_1 - \rho_2)$$

$$S(R) (f_1 - f'_2 \rho_1) + S'(R) \gamma_{12} = S(R) (f_2 - f'_2 \rho_2)$$

Minimize par rapport à la premiere densite:

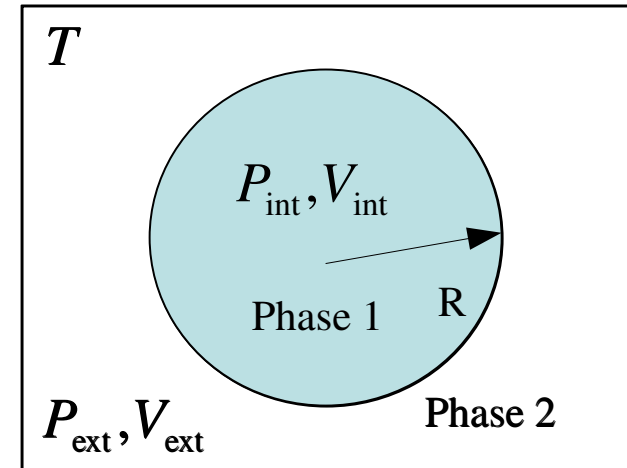
$$0 = V(R) f'_1 + (V - V(R)) f'_2 \frac{\partial \rho_2}{\partial \rho_1} = V(R) (f'_1 - f'_2)$$

Puis, c'est la même qu'avant sauf

$$\rho_2 = \frac{N - \rho_1 V(R)}{(V - V(R))} = \frac{N}{V} + \left( \frac{N}{V} - \rho_1 \right) \frac{V(R)}{V} + \dots$$

Donc, la deux calcul (dans l'ensemble canonique est dans l'ensemble généralisé) sont équivalent dans la limit  $V \rightarrow \infty$

En fait et en general, les ensembles sont equivalent dans ce cas.



# Modele de l'Amas : “Capillary model”

Équivalence d'ensemble (d'après Oxtoby et Evans, J. Chem. Phys. 89, 7521 (1988)):

$$\Omega = -PV + \gamma S$$

$$F = -PV + \gamma S + \mu N = \Omega + \mu N$$

Donc, pour la difference:

$$\Omega_1 - \Omega_2 = -(P_1 - P_2)V + \gamma S$$

$$F_1 - F_2 = -(P_1 - P_2)V + \gamma S + (\mu_1 - \mu_2)N$$

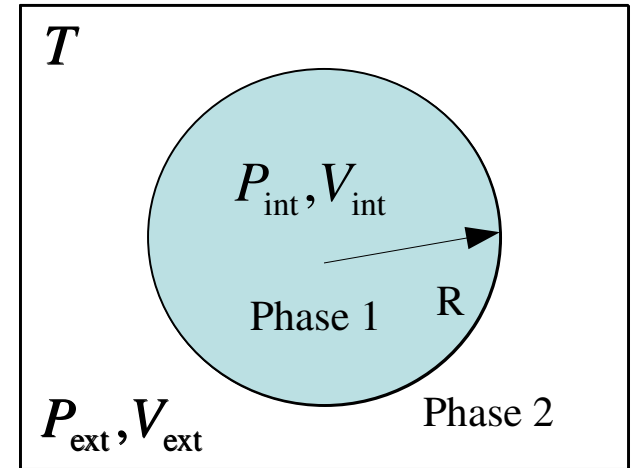
$$\Delta \Omega = \Delta F \Leftrightarrow \mu_1 = \mu_2 \Leftrightarrow \text{thermodynamic limit}$$

# Modele de l'Amas : “Capillary model”

Par ailleurs, une méthode plus simple  
dans l'ensemble canonique:

$$dF = -SdT - PdV + \gamma dA$$

$$dF = -SdT - P_{\text{int}} dV_{\text{int}} - P_{\text{ext}} dV_{\text{ext}} + \gamma dA$$



$$V_{\text{int}} = \frac{4\pi}{3} R^3 \quad dV_{\text{int}} = 4\pi R^2 dR$$

$$V_{\text{ext}} = V - V_{\text{int}} \quad dV_{\text{ext}} = -dV_{\text{int}}$$

$$A = 4\pi R^2 \quad dA = 8\pi R dR$$

$$0 = dF = (-P_{\text{int}} + P_{\text{ext}}) 4\pi R^2 dR + \gamma 8\pi R$$

Equation de Laplace:

$$P_{\text{int}} = P_{\text{ext}} + \frac{2\gamma}{R}$$

# Energie libre

Energie libre de Helmholtz (ensemble canonique):

$$F(\rho; T, V) = f(\rho; T) V$$

Gaz parfait:

$$\beta f(\rho; T) \equiv \beta f_{id}(\rho; T) = \rho \ln \rho - \rho$$

Gaz hard-sphere avec diametre  $d$  (Carnahan-Starling approximation):

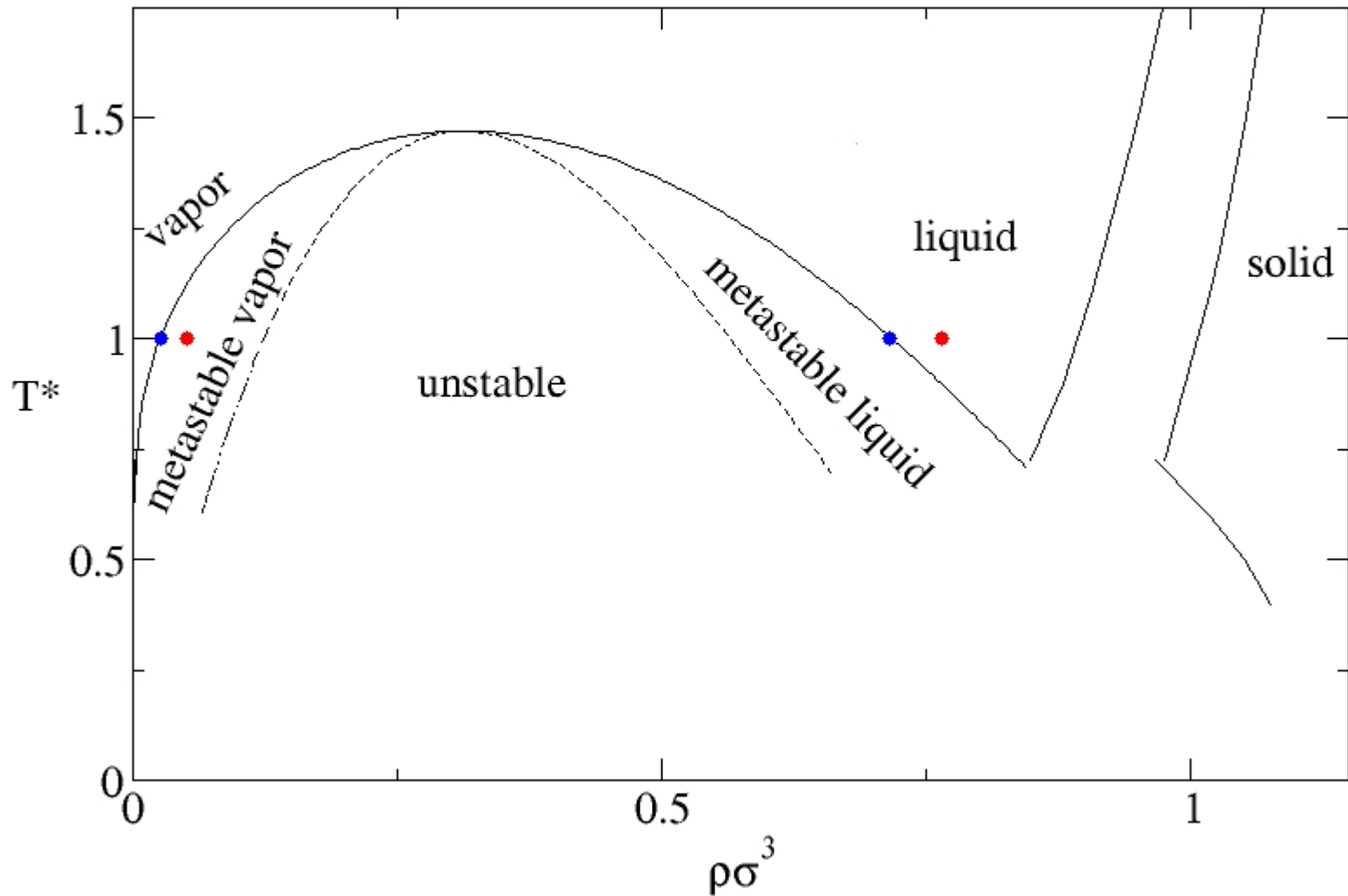
$$\beta f_{hs}(\rho; T) = \beta f_{id}(\rho; T) + \rho \eta \frac{4 - 3\eta}{(1 - 2\eta)^2}, \quad \eta = \frac{4\pi}{3} (d/2)^3 \rho = \frac{\pi}{6} \rho d^3$$

Champ moyenne (van der Waals):

$$\beta f(\rho; T) = \beta f_{hs}(\rho; T) - \frac{1}{2} a \rho^2 \quad a = \beta \int v_{att}(\mathbf{r}) d\mathbf{r}$$



# Les transitions de phase



Coexistence de deux phase:  $p_1 = p_2$ ,  $f_1 = f_2$



# Crystallization is a hot topic ...

**nature**  
International journal of science

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Letter | Published: 26 June 2019

## Observing crystal nucleation in four dimensions using atomic electron tomography

Jihan Zhou, Yongsoo Yang, Yao Yang, Dennis S. Kim, Andrew Yuan, Xuezheng Tian, Colin Ophus, I Sun, Andreas K. Schmid, Michael Nathanson, Hendrik Heinz, Qi An, Hao Zeng, Peter Ercius & Jianwei Miao ✉

DOI: 10.1038/s41586-019-1000-1

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## Controlling Zeolite Nucleation

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Science 06 Jan 2012:  
Vol. 335, Issue 6064, pp. 11  
DOI: 10.1126/science.335.6064.11-f

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## Crystallization by particle attachment in synthetic, biogenic, and geologic environments

James J. De Yoreo<sup>1,2</sup>, Pupa U. P. A. Gilbert<sup>3,4,\*</sup>, Nico A. J. M. Sommerdijk<sup>5,6</sup>, R. Lee Penn<sup>7</sup>, Stephen Whitelam<sup>8</sup>, Derk Joeste...  
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Science 31 Jul 2015:  
Vol. 349, Issue 6247, aaa6760  
DOI: 10.1126/science.aaa6760

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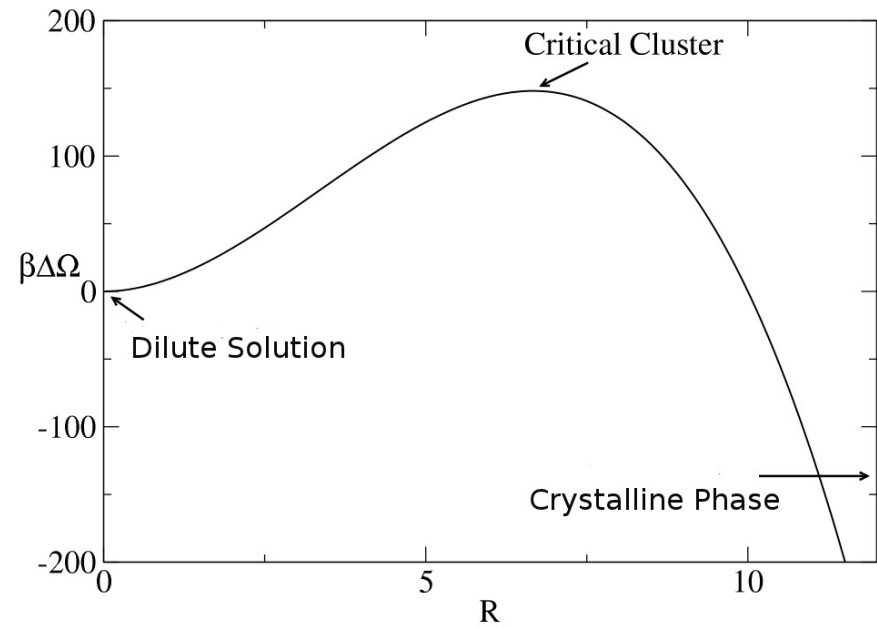
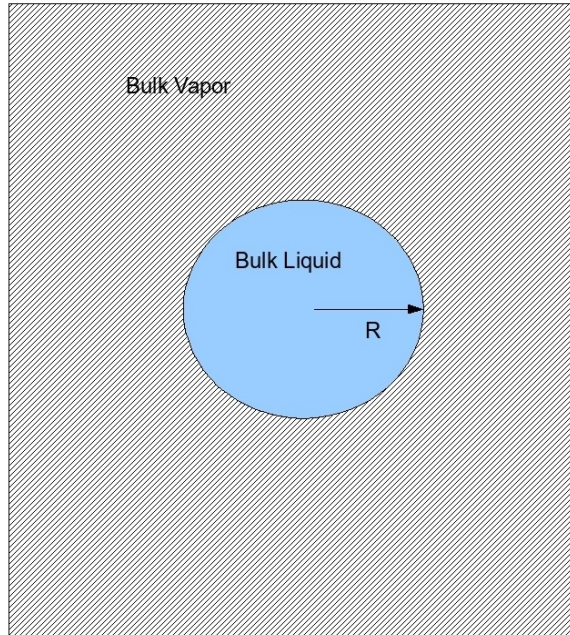
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## Observing classical nucleation theory at work by monitoring phase transitions with molecular precision

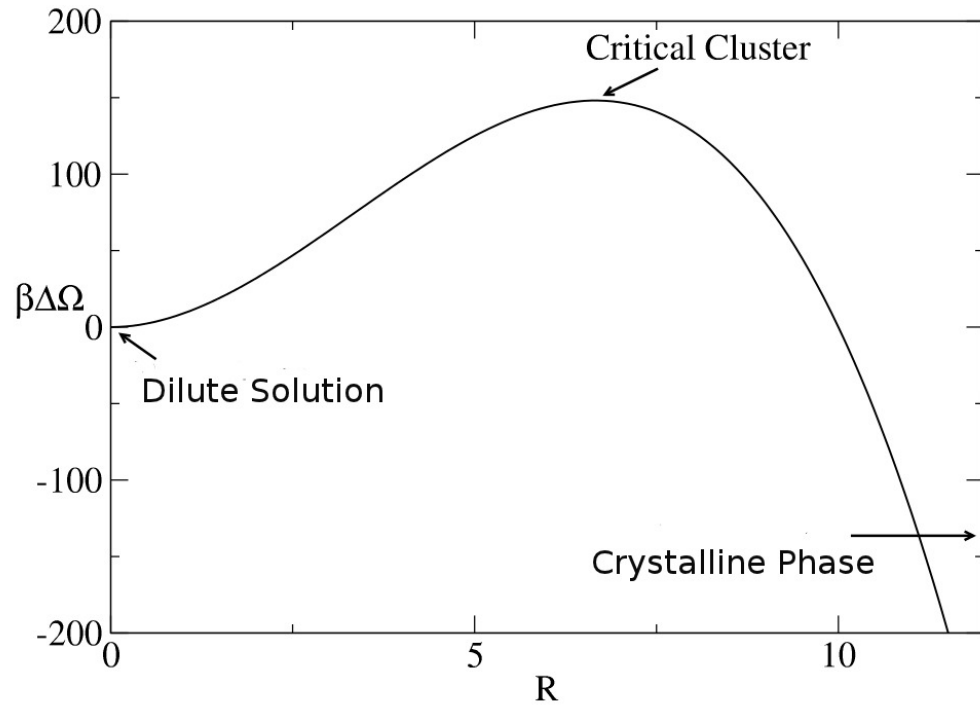
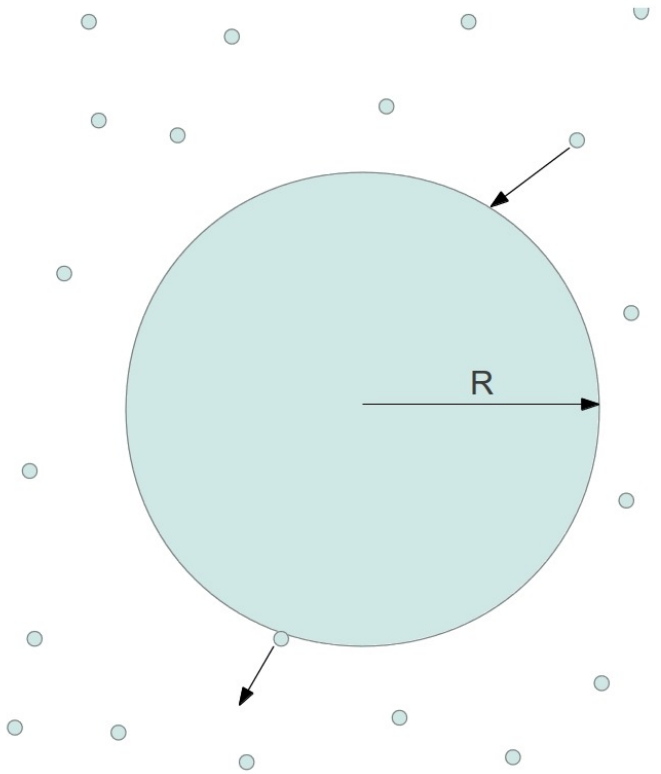
Mike Sleutel ✉, Jim Lutsko, Alexander E.S. Van Driessche, Miguel A. Durán-Olivencia & Dominique Maes

Nature Communications 5, Article number: 5598 (2014) | Download Citation ⏏

# Les transitions de phase : le processus

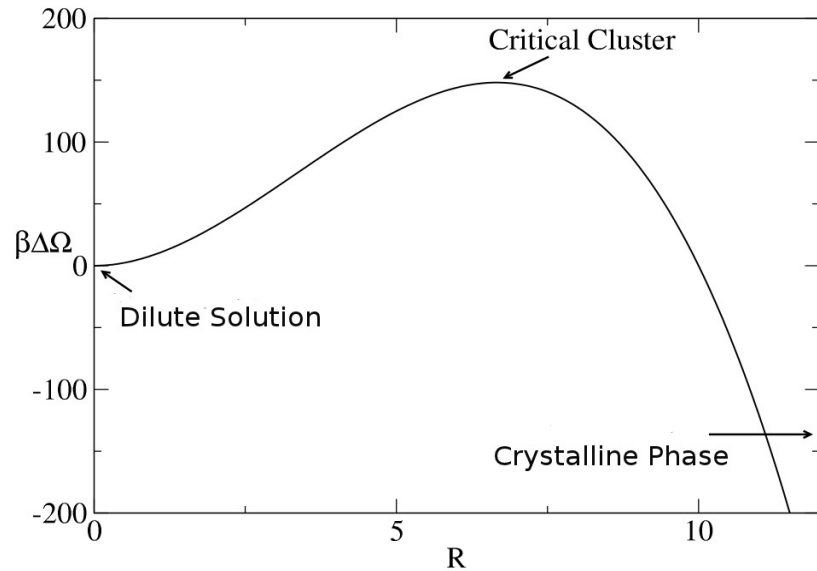
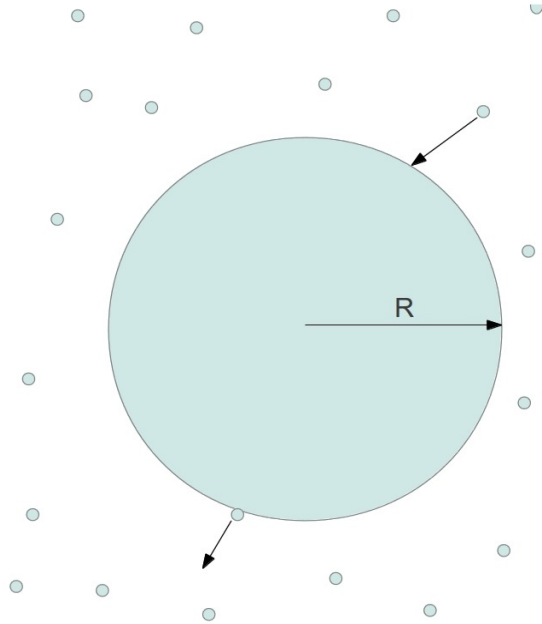


# Classical Nucleation Theory (CNT) : Thermodynamics



$$\begin{aligned}\Omega &= V(R)\omega_2 + S(R)\gamma + (V - V(R))\omega_1 \\ &= V(R)\Delta\omega + S(R)\gamma + V\omega_1 \\ \Delta\Omega &= V(R)\Delta\omega + S(R)\gamma\end{aligned}$$

# Classical Nucleation Theory (CNT) : Dynamics



Monomer attachment/detachment (Becker-Doring c. 1930):

$$\frac{dc_n}{dt} = (f_{n-1}c_{n-1}c_1 - g_nc_n) - (f_nc_nc_1 - g_{n+1}c_{n+1})$$

$f_n, g_n$  sont les taux de fixation et de détachement des monomères