EE 131A | Homework Set 3
Probability | Thursday, October 15, 2015
Instructor: Professor Roychowdhury | Due: Thursday, October 22, 2015

1. A deck of 52 playing cards is shuffled and the cards turned up one at a time until the first ace appears. Is the next card -that is, the card following the first ace- more likely to be the ace of spades or the two of clubs? What is the probability of the ace of spades showing up right after the first ace? What is the probability of the two of clubs showing up right after the first ace?

2. A plane is missing, and it is presumed that it was equally likely to have gone down in any of 3 possible regions. Let $1 - \beta_i$ denote the probability that the plane will be found upon a search of the $i$th region when the plane is, in fact, in that region, $i =$ 1,2,3. (The constants $\beta_i$ are called overlook probabilities because they represent the probability of overlooking the plane; they are generally attributable to the geographical and environmental conditions of the regions.) What is the conditional probability that the plane is in the $i$th region, given that a search of region 1 is unsuccessful, $i = 1,2,3$?

3. Consider two boxes, one containing 1 black and 1 white marble, the other 2 black and 1 white marble. A box is selected at random, and a marble is drawn from it at random. What is the probability that the marble is black? What is the probability that the first box was the one selected given that the marble is white?

4. There is a 50-50 chance that the queen carries the gene for hemophilia. If she is a carrier, then each prince has a 50-50 chance of having hemophilia. If the queen has had three princes without the disease, what is the probability that the queen is a carrier? If there is a fourth prince, what is the probability that he will have hemophilia?

5. A and B are involved in a duel. The rules of the duel are that they are to pick up their guns and shoot at each other simultaneously. If one or both are hit, then the duel is over. If both shots miss, then they repeat the process. Suppose that the results of the shots are independent and that each shot of A will hit B with probability $p_A$, and each shot of B will hit A with probability $p_B$. What is

   (a) the probability that A is not hit?

   (b) the probability that both duelists are hit?

   (c) the probability that the duel ends after the nth round of shots?

   (d) the conditional probability that the duel ends after the nth round of shots given that A is not hit?

   (e) the conditional probability that the duel ends after the nth round of shots given that both duelists are hit?