EE 131A
Probability
Instructor: Professor Roychowdhury

Homework Set 7
Friday, November 13, 2015
Due: Thursday, November 19, 2015

1. *Problem 4.61* Let X be an exponential random variable with parameter λ.

   (a) For $d > 0$ and k a nonnegative integer, find $P[kd < X < (k+1)d]$.

   (b) Segment the positive real line into four equiprobable disjoint intervals.

2. *Problem 4.62* The rth percentile, $\pi(r)$, of a random variable X is defined by $P[X \leq \pi(r)] = r/100$.

   (a) Find the 90%, 95% and 99% percentiles of the exponential random variable with parameter λ.

   (b) Repeat part a for the Gaussian random variable with parameters $m = 0$ and σ^2.

3. *Problem 4.67* A binary transmission system transmits a signal X (-1 to send a "0" bit; +1 to send a "1" bit). The received signal is $Y = X + N$ where noise N has a zero-mean Gaussian distribution with variance σ^2. Assume that "0" bits are three times as likely as "1" bits.

   (a) Find the conditional pdf of Y given the input value: $f_Y(y|X = +1)$ and $f_Y(y|X = -1)$.

   (b) The receiver decides a "0" was transmitted if the observed value of y satisfies

   $$f_Y(y|X = -1)P[X = -1] > f_Y(y|X = +1)P[X = +1]$$

   and it decides a "1" was transmitted otherwise. Use the results from part (a) to show that this decision rule is equivalent to: If $Y < T$ decide "0"; if $Y \geq T$ decide "1".

   (c) What is the probability that the receiver makes an error given that a +1 was transmitted? a -1 was transmitted? Assume $\sigma^2 = 1/16$.

   (d) What is the overall probability of error?

4. *Problem 4.69* Passengers arrive at a taxi stand at an airport at a rate of one passenger per minute. The taxi driver will not leave until seven passengers arrive to fill his van. Suppose that passenger interarrival times are exponential random variable, and let X be the time to fill a van. Find the probability that more than 10 minutes elapse until the van is full.

5. *Problem 4.74*

   (a) Find the cdf of the m-Erlang random variable by integration of the pdf. Hint: Use integration by parts.

   (b) Show that the derivative of the cdf given by

   $$F_{S_m}(t) = 1 - \sum_{k=0}^{m-1} \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

gives the pdf of an m-Erlang random variable.

6. *Problem 4.90* A voltage X is a Gaussian random variable with mean 1 and variance 2. Find the pdf of the power dissipated by an R-ohm resistor $P = RX^2$

7. Let $Y = e^X$, find the pdf of Y when X is a Gaussian random variable with mean m and variance σ. In this case Y is said to be a lognormal random variable.