EE 131A | Homework Set 4
Probability | Wednesday, October 21, 2015
Instructor: Professor Roychowdhury | Due: Thursday, October 29, 2015

1. Two players, A and B, play a series of points in a game with player A winning each point with probability p and player B winning each point with probability $q = 1 - p$. The first player to win n points wins the game. What is the probability that the player A wins?

2. *Problem 2.125* Compare the binomial probability law and the hypergeometric law introduced in Problem 2.54 as follows.

   (a) Suppose a lot has 20 items of which five are defective. A batch of ten items is tested without replacement. Find the probability that k are found defective for k = 0,..., 10. Compare this to the binomial probabilities with $n = 10$ and $p = 5/20 = .25$.

   (b) Repeat but with a lot of 1000 items of which 250 are defective. A batch of ten items is tested without replacement. Find the probability that k are found defective for k = 0,..., 10. Compare this to the binomial probabilities with n = 10 and $p = 5/20 = .25$.

3. *Problem 2.126* Suppose that in Example 2.43, computer A sends each message to computer B simultaneously over two unreliable radio links. Computer B can detect when errors have occurred in either link. Let the probability of message transmission error in link 1 and link 2 be q_1 and q_2 respectively. Computer B requests retransmissions until it receives an error-free message on either link.

   (a) Find the probability that more than k transmissions are required.

   (b) Find the probability that in the last transmission, the message on link 2 is received free of errors.

4. A fair coin is thrown n times. Show that the conditional probability of a head on any specified trial, given a total of k heads over the n trials, is $\frac{k}{n}$.

5. *Problem 2.100* Each of n broadcasts a message in a given time slot with probability p.

   (a)Find the probability that exactly one terminal transmits so the message is received by all terminals without collision.

   (b) Find the value of p that maximizes the probability of successful transmission in the previous part.

   (c) Find the asymptotic value of the probability of successful transmission as n becomes large.

6. *Problem 3.13* Let X be a random variable with pmf $p_k = c/k^2$ for $k = 1, 2, ...$

   (a) Estimate the value of c numerically. Note that the series converges.

   (b) Find $P[X > 4]$.

   (c) Find $P[6 \leq X \leq 8]$.

7. *Problem 3.49* Let X be binomial random variable that results from performance of n Bernoulli trials with probability of success p.

   (a) Suppose that $X = 1$. Find the probability that the single event occurred in the kth Bernoulli trial.

   (b) Suppose that $X = 2$. Find the probability that the two events occurred in the jth and kth Bernoulli trials where $j < k$

   (c) In light of your answers to part a and part b in what sense are the successes distributed "completely at random" over the n Bernoulli trials?