

Objective

- In this project we will further analyze random variables and their various properties.
- We will use MATLAB to generate pseudo random number and analyze its properties.
- We will use pseudo random number to simulate practical problems.
- We will verify empirical result to analytical ones we have studied.

Basic random variable

Mathlab's pseudo random number generator

- Uniform random variable $U[0, 1]$: `rand(m,n)`
- Gaussian random variable $N(0,1)$: `randn(m,n)`

Note: the final outcome is determined by the initial state of rand pseudo random number generator, to set the initial state you can do the following: `rand('seed',1234);`

Other random variables I

Continuous uniform r.v. $[a,b]$

- $\text{uniform} = a + (b-a) \cdot \text{rand}(1,t);$

Discrete uniform r.v. $[a b]$

- $\text{uniform} = \text{ceil}(a + (b-a) \cdot \text{rand}(1,t));$

Gaussian r.v with mean m and variance of a^2

- $\text{normal} = m + a \cdot \text{randn}(1,t)$

Poisson r.v. with paramter 1: $\text{poissrnd}(1,1,t)$

Binomial r.v. with $n=100$, $p=0.05$: $\text{binornd}(100,0.05,1,t)$

Geometric r.v. with $p=0.1$: $\text{geornd}(0.1, 1,t)$

Other random variables II

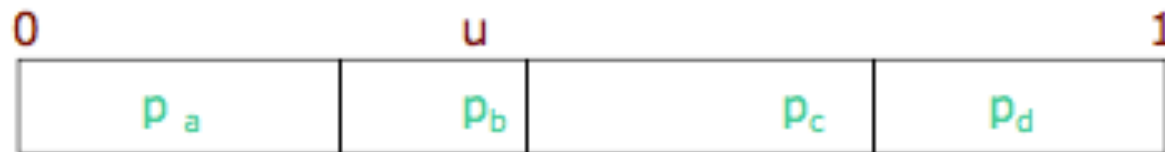
Now, consider the discrete random variable with given pmf. Any discrete distribution p_k can be sampled by constructing the cumulant, now say discrete random variable with pmf as

$$p(X=1)=p_a$$

$$p(X=2)=p_b$$

$$p(X=3)=p_c$$

$$p(X=4)=p_d$$



Other random variables III

Now, consider the **PR generation with an arbitrary cdf $F(x)$** using the transformation method. (Refer to Selection 4.9 in textbook)

Consider a sequence of independent identically distributed (iid) uniformly distributed PR rv's denoted by $\{U_i, i = 1, 2, \dots\}$ defined on $[0, 1]$. Then the desired $\{X_i, i = 1, 2, \dots\}$ defined by

$$X_i = F^{-1}(U_i), i=1,2,\dots, (1)$$

is a sequence of iid rv with a cdf of $F(x)$.

Other random variables III

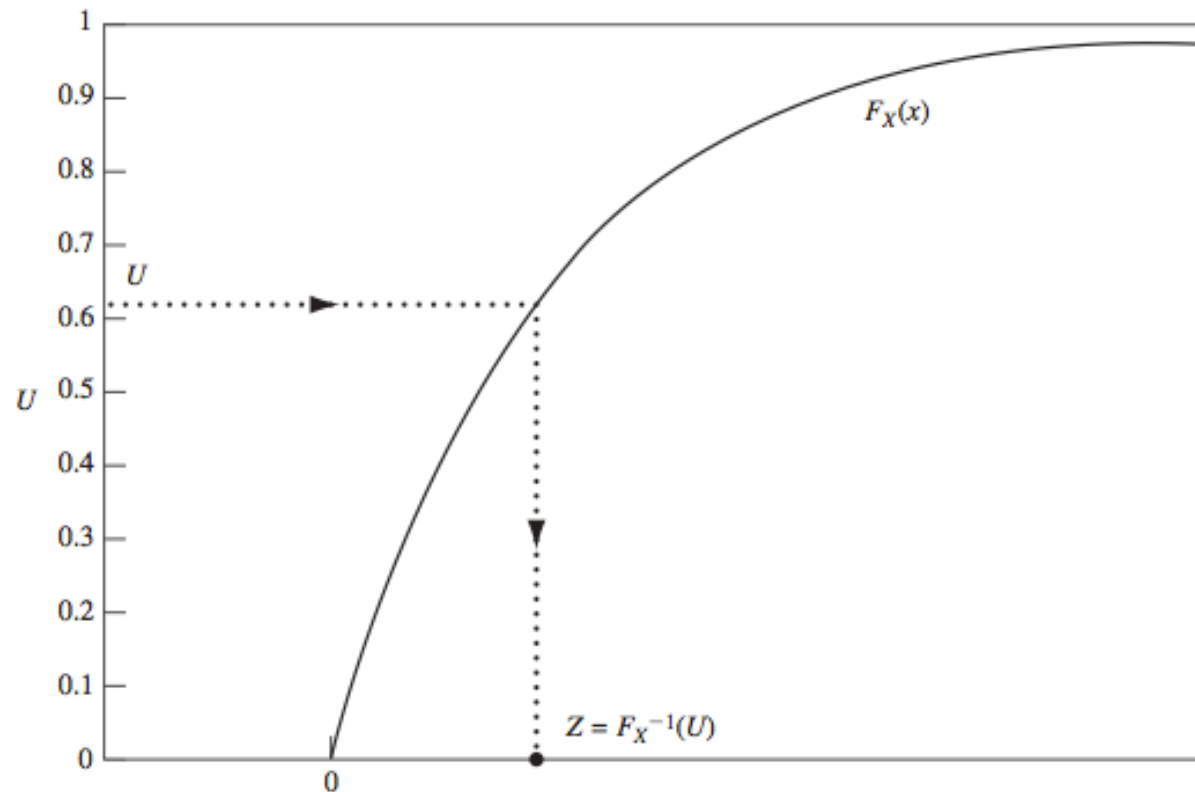


FIGURE 4.19

Transformation method for generating a random variable with cdf $F_X(x)$.

Evaluation

Let us evaluate its sample mean and sample variance to verify whether they are close to the theoretical mean and variance.

- sample mean: $\text{mean}(X)$
- sample variance: $\text{var}(X)$

Let us evaluate its pdf/pmf to verify whether they are close to the theoretical one.

- histogram: $\text{hist}(x)$ (needs to normalize)

Demo