EE 131A | Homework Set 1
Probability | Wednesday, September 30, 2015
Instructor: Professor Roychowdhury | Due: Thursday October 8, 2015

1. *Problem 2.2* A die is tossed twice and the number of dots facing up in each toss is counted and noted in the order of occurrence.

   (a) Find the sample space.

   (b) Find the set A corresponding to the event "the number of dots in first toss is not less than number of dots in second toss"

   (c) Find the set B corresponding to the event "number of dots in first toss is 6"

   (d) Does A imply B or does B imply A?

   (e) Find $A \cap B^c$ and describe this event in words.

   (f) Let C correspond to the event "number of dots in dice differs by 2". Find $A \cap C$

2. *Problem 2.8* A number U is selected uniformly at random from the unit interval. Let the events A and B be

$$A = \text{"}U \text{ differs from } \tfrac{1}{2} \text{ by more than } \tfrac{1}{4}\text{"},$$
$$\text{and } \ B = \text{"}1 - U \text{ is less than } \tfrac{1}{2}\text{"}.$$

   Find the events $A \cap B$, $A^c \cap B$, and $A \cup B$.

3. *Problem 2.10* Use Venn diagrams to verify the set identities given in
   Eqs.(2.2) $A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$
   Eqs.(2.3) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   You will need to use different colors or difference shadings to denote the various regions clearly.

4. *Problem 2.12* Show that if $A \cup B = A$ and $A \cap B = A$ then $A = B$

5. *Problem 2.19* A random experiment has sample space $S = \{-1, 0, +1\}$.

   (a) Find all the subsets of S.

   (b) The outcome of a random experiment consists of pairs of outcomes from S where the elements of the pair cannot be equal. Find the sample space S' of this experiment. How many subsets does S' have?

6. *Problem 2.34* A number x is selected at random in the interval $[-1, 2]$. Let the events $A = \{x < 0\}$,$B = \{|x - 0.5| < 0.5\}$, and $C = \{x > 0.75\}$

   (a) Find the probability of A,B, $A \cap B$, $A \cap C$.

   (b) Find the probability of $A \cup B$,$A \cup C$,$A \cup B \cup C$, first, by directly evaluating the sets and then their probabilities, and second, by using the appropriate axioms or corollaries.

7. *Problem 2.36* The lifetime of a device behaves according to the probability law $P[(t, \infty)] = 1/t$ for $t > 1$. Let A be the event " lifetime is greater than 4, " and B the event "lifetime is greater than 8."

   (a) Find the probability of $A \cap B$ and $A \cup B$

   (b) Find the probability of the event "lifetime is greater than 6 but less than or equal to 12."