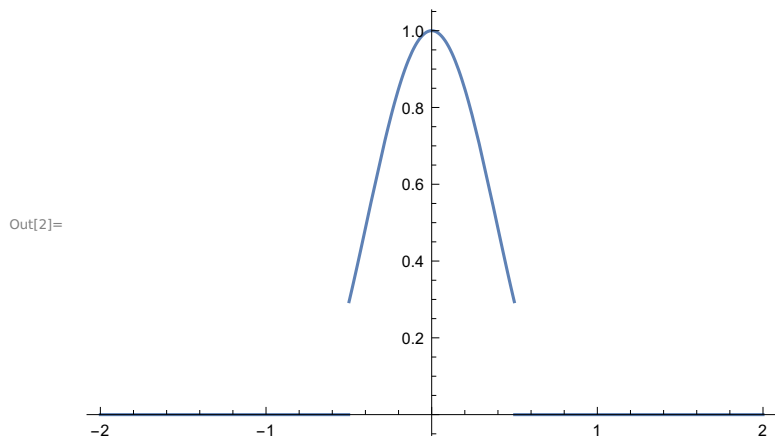


In[1]:= **F[x\_, α\_] = (UnitStep[x + 0.5] - UnitStep[x - 0.5]) \* Cos[α \* x]^2**

Out[1]=  $\text{Cos}[x \alpha]^2 (-\text{UnitStep}[-0.5 + x] + \text{UnitStep}[0.5 + x])$

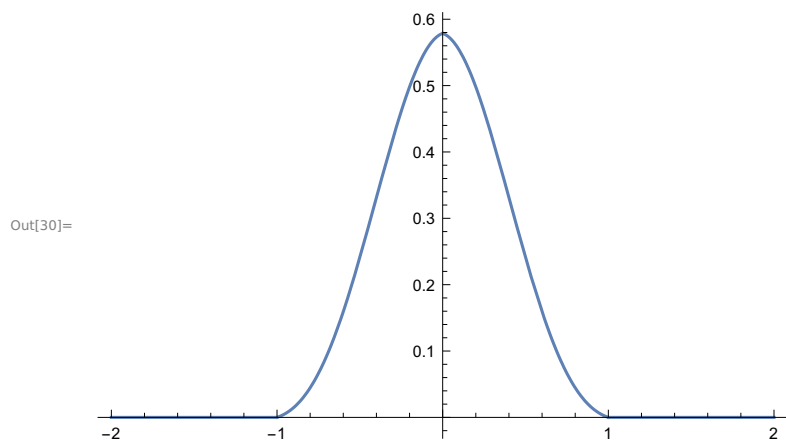
In[2]:= **Plot[F[x, 2], {x, -2, 2}]**



In[29]:= **FAC[Δ\_, α\_] = Rationalize[Convolve[F[x, α], F[x, α], x, Δ]]**

Out[29]= 
$$\frac{1}{\alpha} \left( \left( -\frac{\alpha}{4} + \frac{\alpha \Delta}{4} + \alpha \left( -\frac{1}{8} + \frac{\Delta}{8} \right) \cos[2 \alpha \Delta] - \frac{\sin[\alpha]}{4} - \frac{1}{4} \sin[\alpha (1 - 2 \Delta)] - \frac{1}{16} \sin[\alpha (2 - 2 \Delta)] \right) \text{UnitStep}[-1 + \Delta] + \right. \\ \left( -\frac{\alpha \Delta}{2} - \frac{1}{4} \alpha \Delta \cos[2 \alpha \Delta] - \frac{1}{2} \cos[\alpha] \sin[2 \alpha \Delta] - \frac{1}{8} \cos[2 \alpha] \sin[2 \alpha \Delta] \right) \text{UnitStep}[\Delta] + \\ \left. \left( \frac{\alpha}{4} + \frac{\alpha \Delta}{4} + \alpha \left( \frac{1}{8} + \frac{\Delta}{8} \right) \cos[2 \alpha \Delta] + \frac{\sin[\alpha]}{4} + \frac{1}{4} \sin[\alpha (1 + 2 \Delta)] + \frac{1}{16} \sin[\alpha (2 + 2 \Delta)] \right) \text{UnitStep}[1 + \Delta] \right)$$

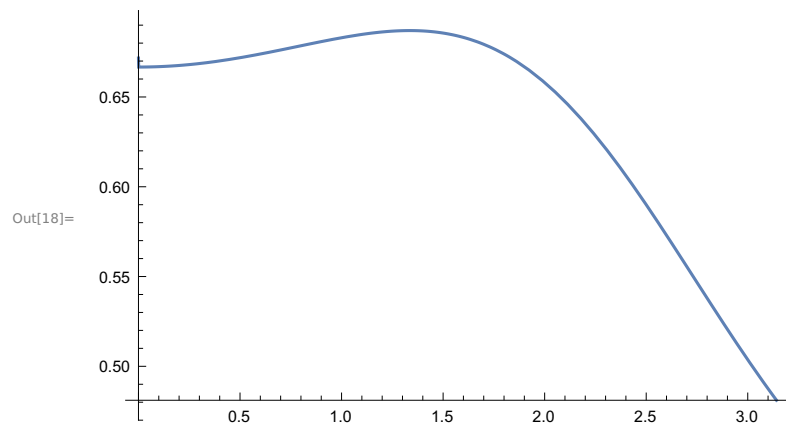
In[30]:= **Plot[FAC[t, 2], {t, -2, 2}]**



In[22]:=  $\epsilon[\alpha] = \frac{\text{Integrate}[\text{Evaluate}@\text{FAC}[x, \alpha]^2, \{x, -1, 1\}]}{\text{FAC}[0, \alpha]^2}$

Out[22]= 
$$\frac{(396 \alpha + 96 \alpha^3 + 440 \sin[\alpha] + 8 \alpha (-28 \cos[\alpha] - 25 \cos[2 \alpha] + 4 \alpha (10 + \cos[\alpha]) \sin[\alpha]) - 92 \sin[2 \alpha] - 72 \sin[3 \alpha] - 3 \sin[4 \alpha])}{\left(2048 \alpha \left(\frac{3 \alpha}{8} + \frac{\sin[\alpha]}{2} + \frac{1}{16} \sin[2 \alpha]\right)^2\right)}$$

In[18]:= **Plot**[ $\epsilon[\alpha]$ , { $\alpha$ , 0,  $\pi$ }]



In[20]:= **FindMaximum**[ $\epsilon[\alpha]$ , { $\alpha$ , 1.5}]

Out[20]= {0.686981281869 , { $\alpha \rightarrow 1.33592803895$ }}