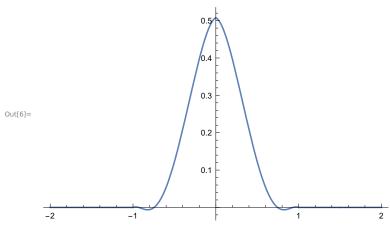
```
Out[3]= (1 + a2 x^2 + a4 x^4 + a6 x^6) (-UnitStep[-0.5 + x] + UnitStep[0.5 + x])
 ln[4]:= Plot[F[x, -4, -16/3, 1], \{x, -2, 2\}]
                                                                                      0.8
                                                                                      0.6
Out[4]=
                                                                                      0.2
                  -2
                                                      -1
                                                                                    -0.2
 In[5]:= FAC[\Delta_{,a2,a4,a6]} = Integrate[F[x,a2,a4,a6]*F[x-\Delta,a2,a4,a6], \{x,-1,1\}]
Out[5]= 0.0000832500832501 \times
                    (0. + (-1.00019992288 \times 10^{-12} \text{ a4 a6 } \Delta^6 + (-12012. -53.625 \text{ a6} - 0.2255859375 \text{ a6}^2 - 12012. \Delta - 12012. \Delta^2 + (-12012. \Delta^2 + 12012. 
                                                 375.375 a6 \Delta - 2.9326171875 a6<sup>2</sup> \Delta - 2252.25 a6 \Delta<sup>2</sup> - 15.99609375 a6<sup>2</sup> \Delta<sup>2</sup> -
                                                 7507.5 a6 \Delta^3 - 46.921875 a6<sup>2</sup> \Delta^3 - 15015. a6 \Delta^4 - 78.203125 a6<sup>2</sup> \Delta^4 -
                                                 18 018. a6 \Delta^5 - 70.3828125 a6^2 \Delta^5 - 12 012. a6 \Delta^6 - 26.8125 a6^2 \Delta^6 - 3432. a6 \Delta^7 +
                                                 2.13375983549 \times 10^{-11} \text{ a6}^2 \Delta^8 + 4.26751967098 \times 10^{-11} \text{ a6}^2 \Delta^{10} - 1. \text{ a6}^2 \Delta^{13} +
                                                 a2^{2} (-150.15 - 750.75 \Delta - 1001. \Delta^{2} - 400.4 \Delta^{5}) + a4^{2} (-5.21354166667 -
                                                            46.921875 \Delta - 160.875 \Delta^2 - 250.25 \Delta^3 - 150.15 \Delta^4 - 19.0666666667 \Delta^9) +
                                                 a2 (-2002. -6006. \Delta -12012. \Delta^2 -8008. \Delta^3 + a4 (-53.625 -375.375 \Delta -1051.05 \Delta^2 -
                                                                       1501.5 \Delta^3 - 1001. \Delta^4 - 228.8 \Delta^7) + a6 (-10.4270833333 - 93.84375 \Delta - 429.
                                                                         \Delta^2 - 1251.25 \Delta^3 - 2252.25 \Delta^4 - 2252.25 \Delta^5 - 1001. \Delta^6 - 95.3333333333 \Delta^9)) +
                                                 a4 (-300.3 - 1501.5 \Delta - 6006. \Delta^2 - 12012. \Delta^3 - 12012. \Delta^4 - 4804.8 \Delta^5 + a6
                                                               (-2.1328125 - 23.4609375 \Delta - 109.484375 \Delta^2 - 281.53125 \Delta^3 - 429. \Delta^4 - 375.375
                                                                          \Delta^5 - 150.15 \Delta^6 + 4.26751967098 \times 10^{-11} \Delta^8 - 10.4 \Delta^{11})) UnitStep [-1. -1. \Delta] +
                                      \Delta (12 012. + 375.375 a6 + 2.9326171875 a6<sup>2</sup> + 7507.5 a6 \Delta<sup>2</sup> + 46.921875 a6<sup>2</sup> \Delta<sup>2</sup> +
                                                 18 018. a6 \Delta^4 + 70.3828125 a6 \Delta^4 + 3432. a6 \Delta^6 - 5.33439958872 × 10<sup>-12</sup> a6 \Delta^6 -
                                                 2.13375983549 \times 10^{-11} \text{ a6}^2 \Delta^7 - 4.26751967098 \times 10^{-11} \text{ a6}^2 \Delta^9 + \text{a6}^2 \Delta^{12} +
                                                 a2^{2} (750.75 + 400.4 \Delta^{4}) + a4^{2} (46.921875 + 250.25 \Delta^{2} + 19.0666666667 \Delta^{8}) +
                                                 a2 (6006. +8008. \Delta^2 + a4 (375.375 +1501.5 \Delta^2 +228.8 \Delta^6) +a6 (93.84375 +1251.25
                                                                         \Delta^2 + 2252.25 \Delta^4 + 1.06687991774 \times 10^{-11} \Delta^5 + 95.33333333333 \Delta^8)) +
                                                 a4 (1501.5 + 12012. \Delta^2 + 4804.8 \Delta^4 + a6 (23.4609375 + 281.53125 \Delta^2 + 375.375
                                                                         \Delta^4 - 1.06687991774 \times 10^{-11} \Delta^5 - 4.26751967098 \times 10^{-11} \Delta^7 + 10.4 \Delta^{10}))
                                          UnitStep [0. - 1. \Delta]) UnitStep [0.5 - 1. \Delta] + (5.33439958872 \times 10^{-12} \text{ a6}^2 \Delta^7 +
                                      \Delta (12012. + 375.375 a6 + 2.9326171875 a6<sup>2</sup> + 7507.5 a6 \Delta<sup>2</sup> + 46.921875 a6<sup>2</sup> \Delta<sup>2</sup> +
                                                 18 018. a6 \Delta^4 + 70.3828125 a6<sup>2</sup> \Delta^4 + 3432. a6 \Delta^6 - 5.33439958872 × 10<sup>-12</sup> a6<sup>2</sup> \Delta^6 +
```

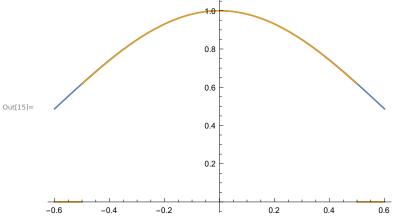
 $ln[3]:= F[x_, a2_, a4_, a6_] = (UnitStep[x + 0.5] - UnitStep[x - 0.5]) * (1 + a2 * x^2 + a4 * x^4 + a6 * x^6)$

UnitStep[1. – 1. ∆]) UnitStep[1.5 – 1. ∆])

In[6]:= Plot[FAC[t, -4, -16/3, 1], {t, -2, 2}]



```
Integrate [Evaluate @ FAC[x, a2, a4, a6]^2, {x, -1, 1}]
 In[7]:= \epsilon[a2_, a4_, a6_] =
                                                                                                                FAC[0, a2, a4, a6]^2
out_{7} = (1. \times (9.6192096 \times 10^7 + 5530.525 \text{ a}2^4 + 4.05171566621 \text{ a}4^4 + 715715. \text{ a}6 + 2300.51250017 \text{ a}6^2 + 2300.51250017 \text{ a}6^
                                 3.65673952643 a6^3 + 0.00543617066614 a6^4 + a4^3 (645.740333327 + 3.04719540558 a6) +
                                 a2^{3}(194038.288889 + 3495.9925 a4 + 628.961666667 a6) +
                                 a4^{2} (73 783.71 + 344.055634189 a6 + 0.868747215165 a6^{2}) +
                                 a4(4.1225184 \times 10^6 + 26013.4874998 \ a6 + 61.324427966 \ a6^2 + 0.111432481455 \ a6^3) +
                                 a2^{2}(3.435432 \times 10^{6} + 848.013833332 a4^{2} + 15152.6375001 a6 +
                                            28.3015933307 \text{ a6}^2 + \text{a4} (86146.0600001 + 308.720965076 \text{ a6})) +
                                 a2(2.88576288 \times 10^7 + 94.0027849263 \ a4^3 + 175675.5 \ a6 + 403.153538615 \ a6^2 +
                                           0.606775186607 \text{ a6}^3 + \text{a4}^2 (12860.3475 + 52.0568215461 \text{ a6}) +
                                           a4 (1.002001 \times 10^6 + 4545.97 \ a6 + 9.69052583957 \ a6^2)))) /
                   (12.012. + 150.15 a2^2 + 5.21354166667 a4^2 + 53.625 a6 + 0.2255859375 a6^2 +
                              a4(300.3 + 2.1328125 \ a6) + a2(2002. + 53.625 \ a4 + 10.4270833333 \ a6))^{2}
               Maximize [\epsilon[a2, a4, a6], {a2, a4, a6}]
               \{0.686981293031, \{a2 \rightarrow -1.79005037628, a4 \rightarrow 1.1026880166, a6 \rightarrow -0.317743535132\}\}
Out[8]=
                ## Compare to the actual Taylor series.
 ln[9]:= Series [Cos[1.3359281199932025 * x]^2, {x, 0, 6}]
               1 - 1.78470394179 \ x^2 + 1.06172271995 \ x^4 - 0.25264809645 \ x^6 + 0[x]^7
Out[9]=
                ## Compare Cos[x]^2 to the Taylor polynomial found above.
               Plot[\{Cos[1.3309 * x]^2, (UnitStep[x + 0.5] - UnitStep[x - 0.5]) * (1 - 1.7900503762763724) x^2 + ...
                                 1.1026880165979676 x^4 - 0.31774353513175274 x^6, \{x, -0.6, 0.6\}
                                                                                  0.8
```



Show that both the $Cos[x]^2$ series as well as the solved series give the same ϵ

 $N[\epsilon[-1.7847039417885724], 1.0617227199452226], -0.25264809644969644]$ In[10]:=

Out[10]= 0.686981276785

```
N[\epsilon[-1.7900503762763724], 1.1026880165979676], -0.31774353513175274]
Out[11]= 0.686981293031
                       ## Calculate slope of \epsilon at solution.
  ln[12]:= DEp[a2_, a4_, a6_] = D[\epsilon[a2, a4, a6], a2, a4, a6]
out_{12} = (1. \times (4545.97 + 617.441930152 \ a2 + 104.113643092 \ a4 + 19.3810516791 \ a6)) / 
                               (12.012. + 150.15 a2^2 + 5.21354166667 a4^2 + 53.625 a6 + 0.2255859375 a6^2 +
                                              a4(300.3 + 2.1328125 \ a6) + a2(2002. + 53.625 \ a4 + 10.4270833333 \ a6))^{2} -
                           (2. × (2002. + 300.3 a2 + 53.625 a4 + 10.4270833333 a6) ×
                                        (26013.4874998 + 308.720965076 \ a2^2 + 9.14158621675 \ a4^2 + 122.648855932 \ a6 + 122.64885593 \ a6 + 122.64885593 \ a6 + 122.64885593 \ a6 + 122.64885593 \ a6 + 122.648859 \ a6 +
                                                 0.334297444366 \ a6^2 + 2 \ a4 \ (344.055634189 + 1.73749443033 \ a6) +
                                                 a2 (4545.97 + 104.113643092 a4 + 19.3810516791 a6))) /
                                (12012. + 150.15 a2^2 + 5.21354166667 a4^2 + 53.625 a6 + 0.2255859375 a6^2 +
                                              a4(300.3 + 2.1328125 \ a6) + a2(2002. + 53.625 \ a4 + 10.4270833333 \ a6))^3 -
                           (2. × (300.3 + 53.625 a2 + 10.4270833333 a4 + 2.1328125 a6) ×
                                        (175675.5 + 1886.885 a2^2 + 52.0568215461 a4^2 + 806.307077229 a6 +
                                                  1.82032555982 \ a6^2 + a4 (4545.97 + 19.3810516791 \ a6) +
                                                 2 a2 (15152.6375001 + 308.720965076 a4 + 56.6031866614 a6))) /
                                (12.012. + 150.15 a2^2 + 5.21354166667 a4^2 + 53.625 a6 + 0.2255859375 a6^2 +
                                              a4(300.3 + 2.1328125 \ a6) + a2(2002. + 53.625 \ a4 + 10.4270833333 \ a6))^3 -
                           (2. × (53.625 + 10.4270833333 a2 + 2.1328125 a4 + 0.451171875 a6) ×
                                        (1.002001 \times 10^6 + 10487.9775 \text{ a}2^2 + 282.008354779 \text{ a}4^2 + 4545.97 \text{ a}6 +
                                                 9.69052583957 \ a6^2 + 2 \ a4 \ (12860.3475 + 52.0568215461 \ a6) +
                                                  2 a2 (86 146.0600001 + 1696.02766666 a4 + 308.720965076 a6))) /
                                (12.012. + 150.15 \text{ a}^2 + 5.21354166667 \text{ a}^4 + 53.625 \text{ a}^6 + 0.2255859375 \text{ a}^6 + 0.225585975 \text{ a}^6 + 0.225585975 \text{ a}^6 + 0.225585975 \text{ a}^6 + 0.225585975 \text{ a}^6 + 0
                                              a4(300.3 + 2.1328125 \ a6) + a2(2002. + 53.625 \ a4 + 10.4270833333 \ a6))^3 +
                           (6. \times (300.3 + 53.625 \text{ a2} + 10.4270833333 \text{ a4} + 2.1328125 \text{ a6}) \times
                                        (2002. + 300.3 a2 + 53.625 a4 + 10.4270833333 a6) \times
                                        (715715. + 628.961666667 a2^3 + 3.04719540558 a4^3 + 4601.02500033 a6 +
                                                 a2^{2}(15152.6375001 + 308.720965076 a4 + 56.6031866614 a6) +
                                                  a4(26013.4874998 + 122.648855932 \ a6 + 0.334297444366 \ a6^2) +
                                                 a2(175675.5 + 52.0568215461 a4^2 + 806.307077229 a6 +
                                                               1.82032555982 a6^2 + a4 (4545.97 + 19.3810516791 a6)))) /
                                (12.012. + 150.15 a2^2 + 5.21354166667 a4^2 + 53.625 a6 + 0.2255859375 a6^2 +
                                              a4(300.3 + 2.1328125 \ a6) + a2(2002. + 53.625 \ a4 + 10.4270833333 \ a6))^4 -
                           (107.25 \times (715715. + 628.961666667 \ a2^3 + 3.04719540558 \ a4^3 + 4601.02500033 \ a6 + 628.961666667 \ a2^3 + 3.04719540558 \ a4^3 + 4601.02500033 \ a6 + 628.961666667 \ a2^3 + 3.04719540558 \ a4^3 + 4601.02500033 \ a6 + 628.961666667 \ a2^3 + 3.04719540558 \ a4^3 + 4601.02500033 \ a6 + 628.961666667 \ a2^3 + 3.04719540558 \ a4^3 + 4601.02500033 \ a6 + 628.961666667 \ a2^3 + 3.04719540558 \ a4^3 + 4601.02500033 \ a6 + 628.961666667 \ a2^3 + 3.04719540558 \ a4^3 + 4601.02500033 \ a6 + 628.961666667 \ a2^3 + 3.04719540558 \ a4^3 + 4601.02500033 \ a6 + 628.961666667 \ a2^3 + 3.04719540558 \ a4^3 + 4601.02500033 \ a6 + 628.961666667 \ a2^3 + 3.04719540558 \ a4^3 + 4601.02500033 \ a6 + 628.96166667 \ a2^3 + 628.9616667 \ a2^3 + 628.96166667 \ a2^3 + 628.9616667 \ a2^3 + 628.9616667 \ a2^3 + 628.9616667 \ a2^3 + 628.961667 \ a2^3 + 628.96167 \ a2^3 + 628.9617 \ a2^3 + 628.96170 \ a2^3 + 628.
                                                 a2^{2}(15152.6375001 + 308.720965076 a4 + 56.6031866614 a6) +
                                                  a4(26013.4874998 + 122.648855932 \ a6 + 0.334297444366 \ a6^2) +
```

```
a2(175675.5 + 52.0568215461 a4^2 + 806.307077229 a6 +
                           1.82032555982 a6^2 + a4 (4545.97 + 19.3810516791 a6)))) /
   (12012. + 150.15 a2^2 + 5.21354166667 a4^2 + 53.625 a6 + 0.2255859375 a6^2 +
              a4 (300.3 + 2.1328125 \ a6) + a2 (2002. + 53.625 \ a4 + 10.4270833333 \ a6))^3 +
(6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times
          (2002. + 300.3 a2 + 53.625 a4 + 10.4270833333 a6) ×
          (4.1225184 \times 10^6 + 3495.9925 \text{ a}2^3 + 16.2068626648 \text{ a}4^3 + 26013.4874998 \text{ a}6 +
                 61.324427966 \ a6^2 + 0.111432481455 \ a6^3 + 3 \ a4^2 \ (645.740333327 + 3.04719540558 \ a6) +
                 a2^{2} (86 146.0600001 + 1696.02766666 a4 + 308.720965076 a6) +
                 2 \text{ a4} (73783.71 + 344.055634189 a6 + 0.868747215165 a6^2) +
                 a2 (1.002001 \times 10^6 + 282.008354779 \text{ a4}^2 + 4545.97 \text{ a6} +
                           9.69052583957 \text{ a6}^2 + 2 \text{ a4} (12860.3475 + 52.0568215461 \text{ a6})))) /
   (12012. + 150.15 a2^2 + 5.21354166667 a4^2 + 53.625 a6 + 0.2255859375 a6^2 +
              a4(300.3 + 2.1328125 \ a6) + a2(2002. + 53.625 \ a4 + 10.4270833333 \ a6))^4 -
(20.8541666667 \times (4.1225184 \times 10^6 + 3495.9925 \text{ a}2^3 + 16.2068626648 \text{ a}4^3 + 26.013.4874998 \text{ a}6 +
                 61.324427966 \ a6^2 + 0.111432481455 \ a6^3 + 3.44^2 (645.740333327 + 3.04719540558 \ a6) +
                 a2^{2} (86146.0600001 + 1696.02766666 a4 + 308.720965076 a6) +
                 2 \text{ a4} (73783.71 + 344.055634189 a6 + 0.868747215165 a6^2) +
                 a2 (1.002001 \times 10^6 + 282.008354779 \text{ a4}^2 + 4545.97 \text{ a6} +
                           9.69052583957 \ a6^2 + 2 \ a4 (12860.3475 + 52.0568215461 \ a6)))) /
   (12012. + 150.15 a2^2 + 5.21354166667 a4^2 + 53.625 a6 + 0.2255859375 a6^2 +
              a4(300.3 + 2.1328125 \ a6) + a2(2002. + 53.625 \ a4 + 10.4270833333 \ a6))^3 +
(6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.4270833333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a6) \times (6. \times (53.625 + 10.42708333333 \ a2 + 2.1328125 \ a4 + 0.451171875 \ a4 + 0.451171875 \ a4 + 0.451171875 \ a4 + 0.451171
          (300.3 + 53.625 a2 + 10.4270833333 a4 + 2.1328125 a6) \times
          (2.88576288 \times 10^7 + 22122.1 \text{ a}2^3 + 94.0027849263 \text{ a}4^3 + 175675.5 \text{ a}6 +
                 403.153538615 \ a6^2 + 0.606775186607 \ a6^3 + a4^2 (12860.3475 + 52.0568215461 \ a6) +
                 3 a2^{2} (194038.288889 + 3495.9925 a4 + 628.961666667 a6) +
                 a4 (1.002001 \times 10^6 + 4545.97 \ a6 + 9.69052583957 \ a6^2) +
                 2 a2 (3.435432 \times 10^{6} + 848.013833332 a4^{2} + 15152.6375001 a6 +
                           28.3015933307 \text{ a6}^2 + \text{a4} (86146.0600001 + 308.720965076 \text{ a6})))) /
   (12\,012. + 150.15 \text{ a2}^2 + 5.21354166667 \text{ a4}^2 + 53.625 \text{ a6} + 0.2255859375 \text{ a6}^2 +
              a4 (300.3 + 2.1328125 \ a6) + a2 (2002. + 53.625 \ a4 + 10.4270833333 \ a6))^4 -
(4.265625 \times (2.88576288 \times 10^7 + 22122.1 \text{ a}2^3 + 94.0027849263 \text{ a}4^3 + 175675.5 \text{ a}6 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^
                 403.153538615 \text{ a6}^2 + 0.606775186607 \text{ a6}^3 + \text{a4}^2 (12860.3475 + 52.0568215461 \text{ a6}) +
                 3 a2^{2} (194038.288889 + 3495.9925 a4 + 628.961666667 a6) +
                 a4(1.002001 \times 10^6 + 4545.97 \ a6 + 9.69052583957 \ a6^2) +
                 2 a2 (3.435432 \times 10^6 + 848.013833332 a4^2 + 15152.6375001 a6 +
                           28.3015933307 \text{ a6}^2 + \text{a4} (86146.0600001 + 308.720965076 \text{ a6})))) /
   (12\,012. + 150.15 \text{ a2}^2 + 5.21354166667 \text{ a4}^2 + 53.625 \text{ a6} + 0.2255859375 \text{ a6}^2 +
              a4 (300.3 + 2.1328125 \ a6) + a2 (2002. + 53.625 \ a4 + 10.4270833333 \ a6))^3 -
(24. × (53.625 + 10.4270833333 a2 + 2.1328125 a4 + 0.451171875 a6) ×
```

```
(300.3 + 53.625 a2 + 10.4270833333 a4 + 2.1328125 a6) ×
          (2002. + 300.3 a2 + 53.625 a4 + 10.4270833333 a6) ×
          (9.6192096 \times 10^7 + 5530.525 \text{ a2}^4 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6} + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6} + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6} + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6} + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6}^4 + 2300.51250017 \text{ a6}^2 + 4.05171566621 \text{ a4}^4 + 71571566621 \text{ a4}^4 + 71571566621 \text{ a6}^4 + 4.05171566621 \text{ a4}^4 + 71571566621 \text{ a6}^4 + 715715666621 \text{ a6}^4 + 71571566621 \text{ a6}^4 + 715715666161 \text{ a6}^4 + 7
                  3.65673952643 a6^3 + 0.00543617066614 a6^4 + a4^3 (645.740333327 + 3.04719540558 a6) +
                  a2^{3}(194038.288889 + 3495.9925 a4 + 628.961666667 a6) +
                  a4^{2}(73783.71 + 344.055634189 \ a6 + 0.868747215165 \ a6^{2}) +
                  a4(4.1225184 \times 10^6 + 26013.4874998 \ a6 + 61.324427966 \ a6^2 + 0.111432481455 \ a6^3) +
                  a2^{2} (3.435432 \times 10^{6} + 848.013833332 \ a4^{2} + 15152.6375001 \ a6 +
                             28.3015933307 \text{ } a6^2 + a4 (86146.0600001 + 308.720965076 \text{ } a6)) +
                  a2 (2.88576288 \times 10^7 + 94.0027849263 \text{ a4}^3 + 175675.5 \text{ a6} + 403.153538615 \text{ a6}^2 +
                            0.606775186607 \text{ a6}^3 + \text{a4}^2 (12860.3475 + 52.0568215461 \text{ a6}) +
                            a4 (1.002001 \times 10^6 + 4545.97 \ a6 + 9.69052583957 \ a6^2)))) /
   (12\,012. + 150.15 \text{ a2}^2 + 5.21354166667 \text{ a4}^2 + 53.625 \text{ a6} + 0.2255859375 \text{ a6}^2 +
              a4 (300.3 + 2.1328125 \ a6) + a2 (2002. + 53.625 \ a4 + 10.4270833333 \ a6))^5 +
(321.75 × (53.625 + 10.4270833333 a2 + 2.1328125 a4 + 0.451171875 a6) ×
          (9.6192096 \times 10^7 + 5530.525 \text{ a}2^4 + 4.05171566621 \text{ a}4^4 + 715715. \text{ a}6 + 2300.51250017 \text{ a}6^2 + 4.05171566621 \text{ a}4^4 + 715715. \text{ a}6 + 2300.51250017 \text{ a}6^2 + 4.05171566621 \text{ a}4^4 + 715715. \text{ a}6 + 2300.51250017 \text{ a}6^2 + 4.05171566621 \text{ a}4^4 + 715715. \text{ a}6 + 2300.51250017 \text{ a}6^2 + 4.05171566621 \text{ a}4^4 + 715715. \text{ a}6 + 2300.51250017 \text{ a}6^2 + 4.05171566621 \text{ a}4^4 + 715715. \text{ a}6^2 + 4.05171566621 \text{ a}4^4 + 71571566621 \text{ a}4^4 + 7157156661 \text{ a}4^4 + 71571566610 \text{ a}4^4 + 7157156661 \text{ a}4^4 + 71571566610 \text{ a}4^4 + 71571
                 3.65673952643 \text{ } a6^3 + 0.00543617066614 \text{ } a6^4 + a4^3 (645.740333327 + 3.04719540558 \text{ } a6) +
                  a2^{3}(194038.288889 + 3495.9925 a4 + 628.961666667 a6) +
                  a4^{2}(73783.71 + 344.055634189 \ a6 + 0.868747215165 \ a6^{2}) +
                  a4(4.1225184 \times 10^6 + 26013.4874998 \ a6 + 61.324427966 \ a6^2 + 0.111432481455 \ a6^3) +
                  a2^{2}(3.435432 \times 10^{6} + 848.013833332 \ a4^{2} + 15152.6375001 \ a6 +
                            28.3015933307 \text{ } a6^2 + a4 (86146.0600001 + 308.720965076 \text{ } a6)) +
                 a2 (2.88576288 \times 10^7 + 94.0027849263 \text{ a4}^3 + 175675.5 \text{ a6} + 403.153538615 \text{ a6}^2 +
                            0.606775186607 \ a6^3 + a4^2 (12860.3475 + 52.0568215461 \ a6) +
                            a4 (1.002001 \times 10^6 + 4545.97 \ a6 + 9.69052583957 \ a6^2)))) /
   (12\,012. + 150.15 \text{ a2}^2 + 5.21354166667 \text{ a4}^2 + 53.625 \text{ a6} + 0.2255859375 \text{ a6}^2 +
              a4 (300.3 + 2.1328125 \ a6) + a2 (2002. + 53.625 \ a4 + 10.4270833333 \ a6))^4 +
(62.5625 × (300.3 + 53.625 a2 + 10.4270833333 a4 + 2.1328125 a6) ×
          (9.6192096 \times 10^7 + 5530.525 \text{ a2}^4 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6} + 2300.51250017 \text{ a6}^2 +
                 3.65673952643 a6^3 + 0.00543617066614 a6^4 + a4^3 (645.740333327 + 3.04719540558 a6) +
                  a2^{3}(194038.288889 + 3495.9925 a4 + 628.961666667 a6) +
                  a4^{2}(73783.71 + 344.055634189 \ a6 + 0.868747215165 \ a6^{2}) +
                  a4(4.1225184 \times 10^6 + 26013.4874998 \ a6 + 61.324427966 \ a6^2 + 0.111432481455 \ a6^3) +
                  a2^{2}(3.435432 \times 10^{6} + 848.013833332 \ a4^{2} + 15152.6375001 \ a6 +
                             28.3015933307 \text{ } a6^2 + a4 (86146.0600001 + 308.720965076 \text{ } a6)) +
                 a2 (2.88576288 \times 10^7 + 94.0027849263 \text{ a4}^3 + 175675.5 \text{ a6} + 403.153538615 \text{ a6}^2 +
                            0.606775186607 \ a6^3 + a4^2 (12860.3475 + 52.0568215461 \ a6) +
                            a4 (1.002001 \times 10^6 + 4545.97 \ a6 + 9.69052583957 \ a6^2)))) /
   (12\,012. + 150.15 \ a2^2 + 5.21354166667 \ a4^2 + 53.625 \ a6 + 0.2255859375 \ a6^2 +
              a4(300.3 + 2.1328125 \ a6) + a2(2002. + 53.625 \ a4 + 10.4270833333 \ a6))^4 +
(12.796875 × (2002. + 300.3 a2 + 53.625 a4 + 10.4270833333 a6) ×
          (9.6192096 \times 10^7 + 5530.525 \text{ a2}^4 + 4.05171566621 \text{ a4}^4 + 715715. \text{ a6} + 2300.51250017 \text{ a6}^2 +
```

```
3.65673952643 \text{ a6}^3 + 0.00543617066614 \text{ a6}^4 + \text{a4}^3 (645.740333327 + 3.04719540558 \text{ a6}) +
      a2<sup>3</sup> (194 038.288889 + 3495.9925 a4 + 628.961666667 a6) +
      a4^{2}(73783.71 + 344.055634189 \ a6 + 0.868747215165 \ a6^{2}) +
      a4(4.1225184 \times 10^6 + 26013.4874998 \ a6 + 61.324427966 \ a6^2 + 0.111432481455 \ a6^3) +
      a2^{2}(3.435432 \times 10^{6} + 848.013833332 \ a4^{2} + 15152.6375001 \ a6 +
          28.3015933307 \text{ } a6^2 + a4 (86146.0600001 + 308.720965076 \text{ } a6)) +
      a2 (2.88576288 \times 10^7 + 94.0027849263 \text{ a4}^3 + 175675.5 \text{ a6} + 403.153538615 \text{ a6}^2 +
          0.606775186607 \ a6^3 + a4^2 (12860.3475 + 52.0568215461 \ a6) +
          a4 (1.002001 \times 10^6 + 4545.97 \ a6 + 9.69052583957 \ a6^2)))) /
(12.012. + 150.15 a2^2 + 5.21354166667 a4^2 + 53.625 a6 + 0.2255859375 a6^2 +
    a4(300.3 + 2.1328125 \ a6) + a2(2002. + 53.625 \ a4 + 10.4270833333 \ a6))^4
```

DEp[-1.7847039417885724`, 1.0617227199452226`, -0.25264809644969644`]

0.0000727862428794 Out[13]=

The value of ϵ is relatively insensitive to the parameters:

ln[14]:= Plot3D[ϵ [-1.8, a4, a6], {a4, 0, 2}, {a6, -1, 2}]

