Neural Network and Deep Learning

McCulloch-Pitts Neuron

YANIKA KONGSOROT

Outline

- McCulloch-Pitts Neuron
- Linearly separable

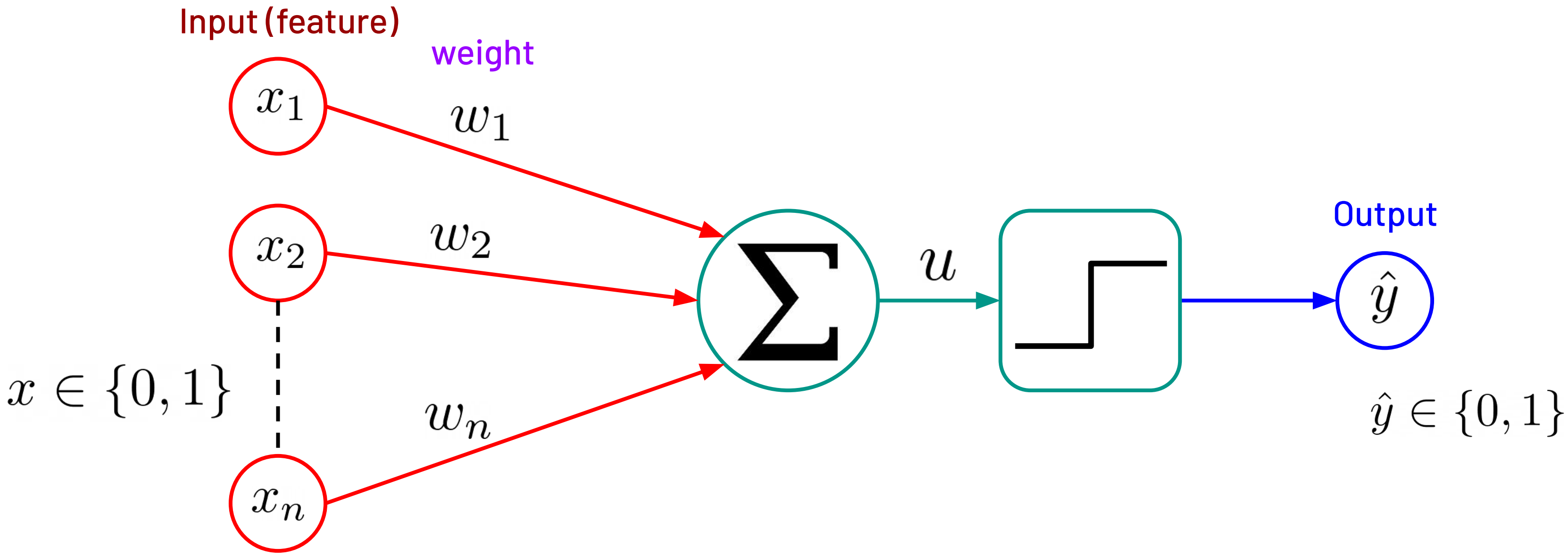
McCulloch-Pitts Neuron

The first computational model of a neuron

McCulloch-Pitts Neuron

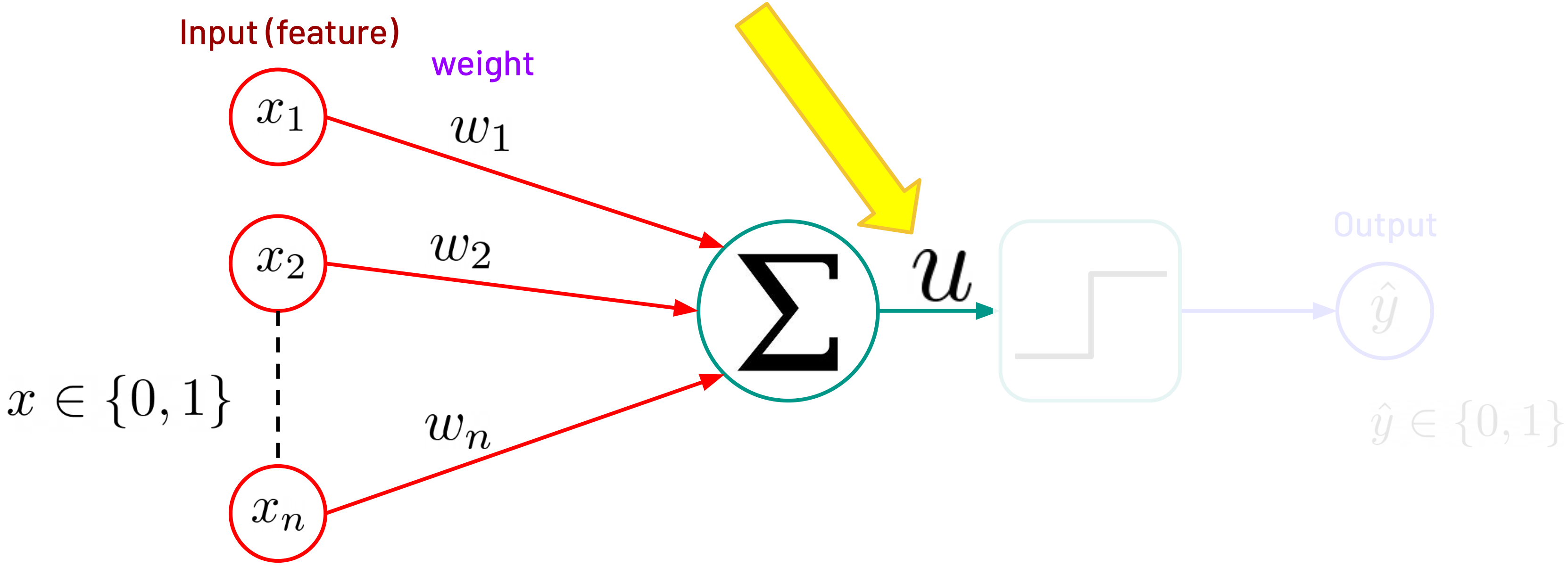
- The McCulloch-Pitts model was proposed by **Warren MuCulloch** (neuroscientist) and **Walter Pitts** (logician) in 1943.
- McCulloch and Pitts modeled computationally able to **emulate the behavior of a few boolean functions or *logical gates***, like the **AND gate** and the **OR gate**.
- **Neurons can be seen as biological computational devices**, in the sense that they can ***receive inputs***, ***apply calculations over those inputs algorithmically***, *and then* ***produce outputs***.
- The McCulloch-Pitts model is **the first computational model** of a neuron and it was an extremely simple artificial neuron. ***The inputs could be either a zero or a one. And the output was a zero or a one.***

McCulloch-Pitts Neuron



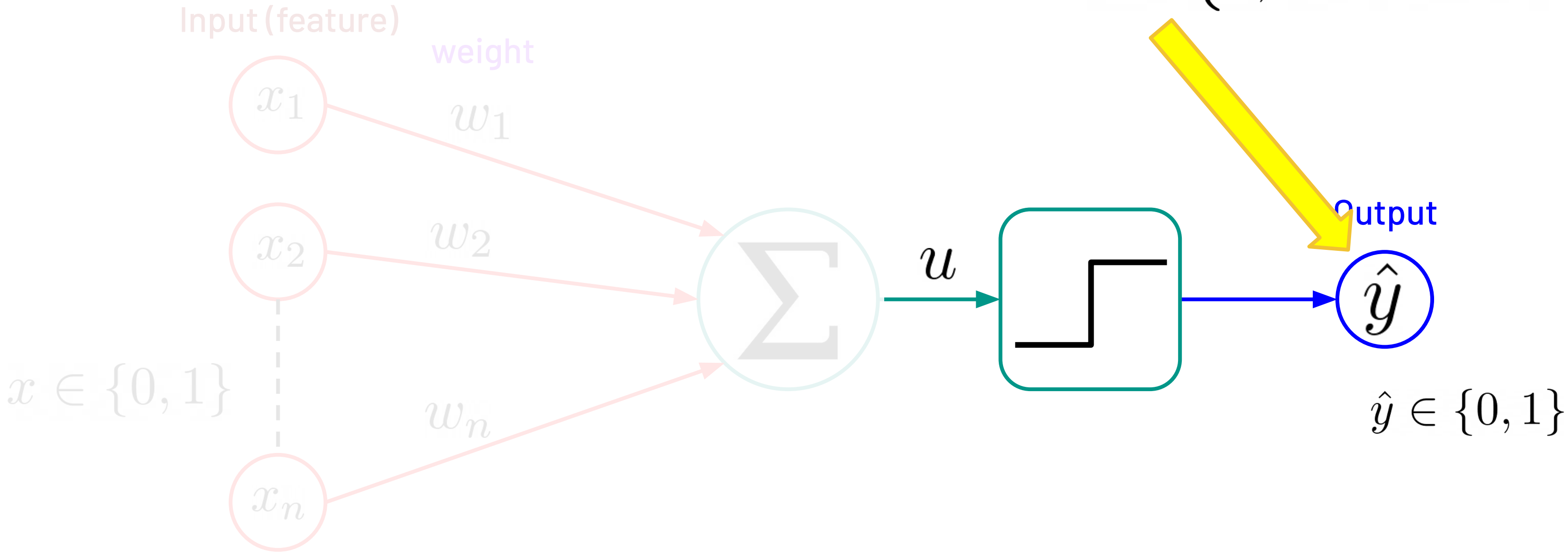
McCulloch-Pitts Neuron

$$u = w_1 x_1 + w_2 x_2 + \ldots + w_n x_n$$

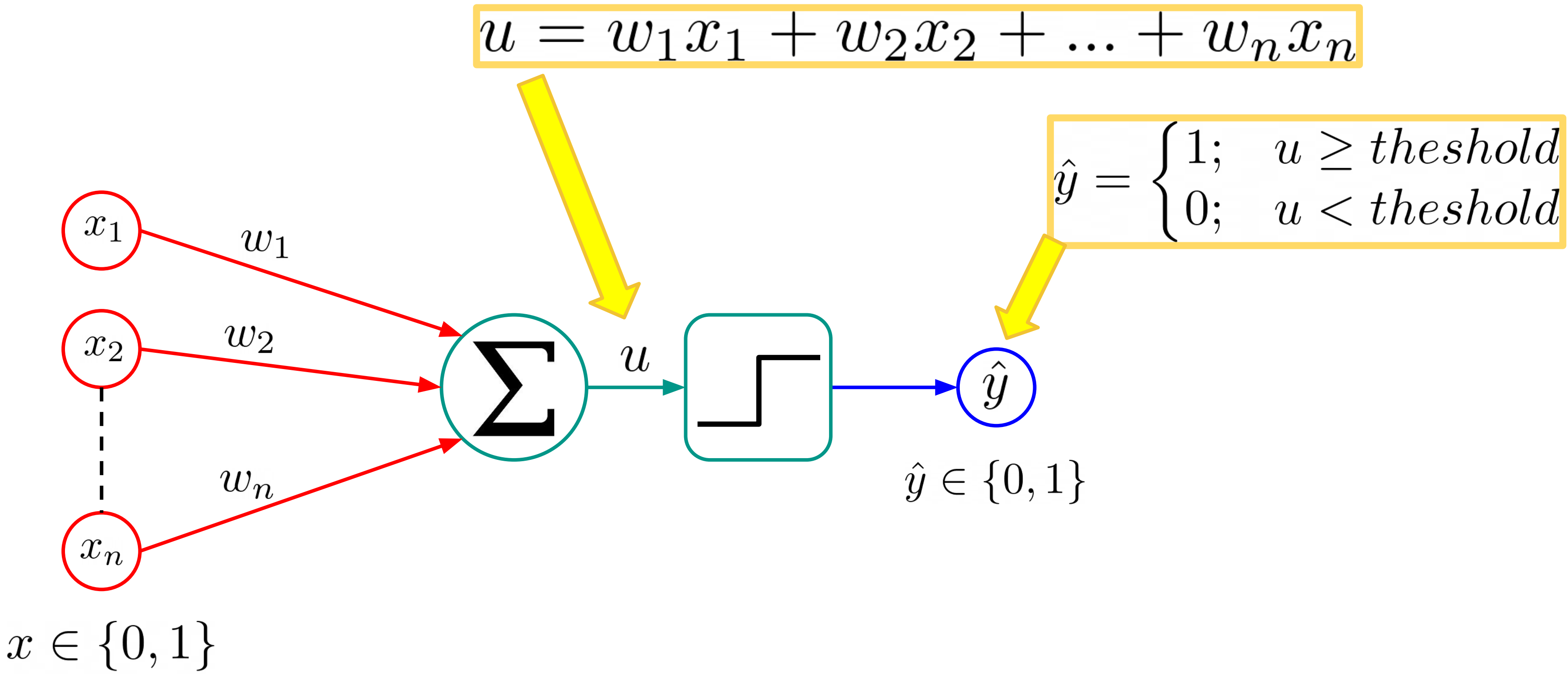


McCulloch-Pitts Neuron

$$\hat{y} = \begin{cases} 1; & u \geq theshold \\ 0; & u < theshold \end{cases}$$



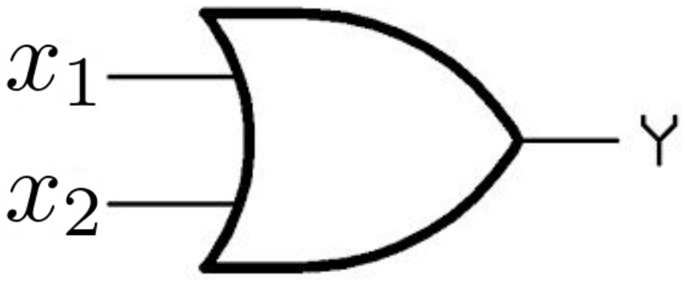
McCulloch-Pitts Neuron



McCulloch-Pitts Neuron
Computation

Emulate the behavior of ***simple logical gates***.

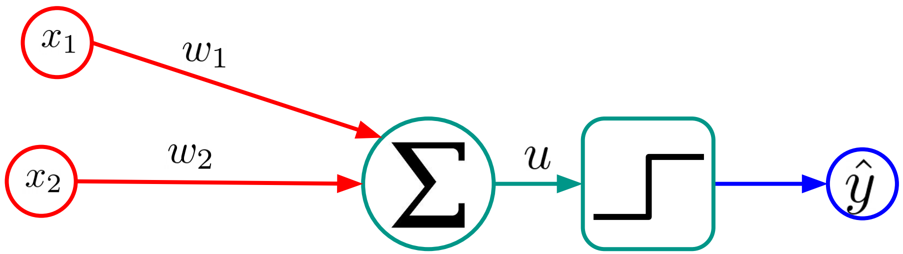
OR Gate via McCulloch-Pitts



x_1	x_2	Target
1	1	1
1	0	1
0	1	1
0	0	0

OR Gate via McCulloch-Pitts

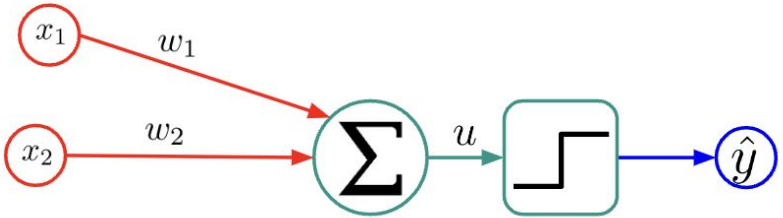
x_1	x_2	Target
1	1	1
1	0	1
0	1	1
0	0	0



assume: $w_1 = 1.5$

$w_2 = 0.5$

OR Gate via McCulloch-Pitts

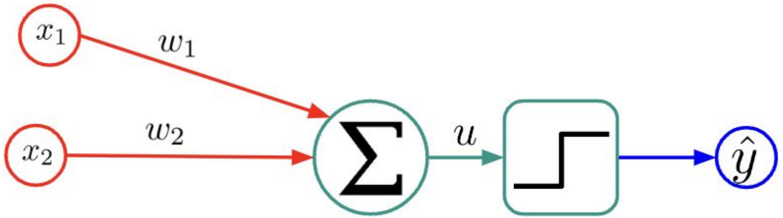


x_1	x_2
1	1

$u = w_1x_1 + w_2x_2$	$\hat{y}$

$$w_1 = 1.5$$
$$w_2 = 0.5$$

OR Gate via McCulloch-Pitts

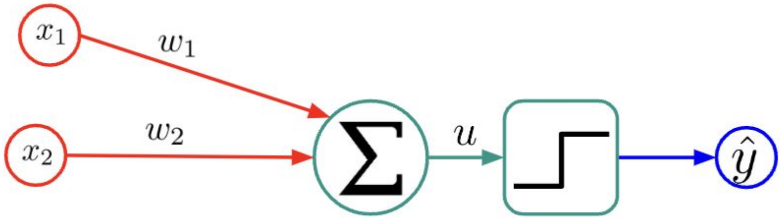


x_1	x_2
1	1

$u = w_1x_1 + w_2x_2$	$\hat{y}$
$(1.5^* \ ) + (0.5^* \ ) = \ u$	

$$w_1 = 1.5$$
$$w_2 = 0.5$$

OR Gate via McCulloch-Pitts

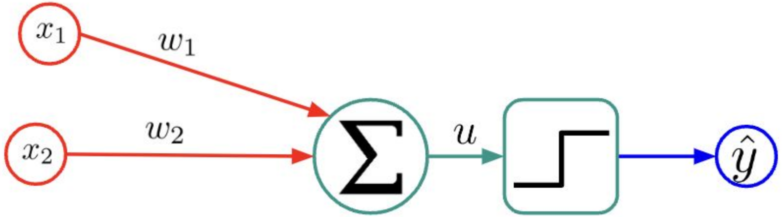


x_1	x_2
1	1

$u = w_1x_1 + w_2x_2$	$\hat{y}$
(1.5*1) + (0.5*1) = 2.0	

$$w_1 = 1.5$$
$$w_2 = 0.5$$

OR Gate via McCulloch-Pitts



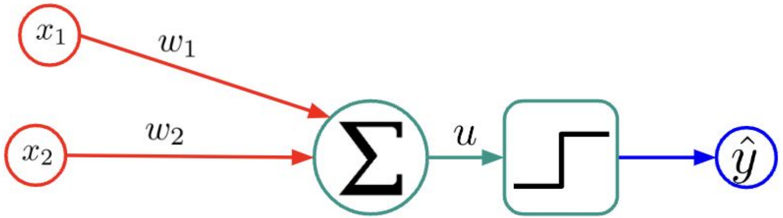
x_1	x_2
1	**1**
1	**0**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
$(1.5*\mathbf{1}) + (0.5*\mathbf{1}) = \mathbf{2.0}$	

$$w_1 = 1.5$$

$$w_2 = 0.5$$

OR Gate via McCulloch-Pitts

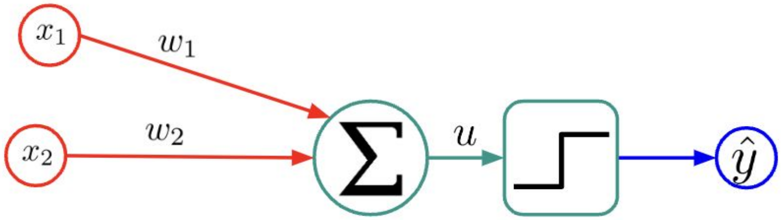


x_1	x_2
1	**1**
1	**0**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
(1.5***1**) + (0.5***1**) = **2.0**	
(1.5*) + (0.5*) = u	

$$w_1 = 1.5$$
$$w_2 = 0.5$$

OR Gate via McCulloch-Pitts

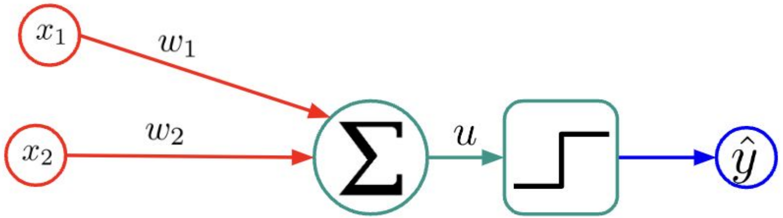


x_1	x_2
1	**1**
1	**0**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
(1.5***1**) + (0.5***1**) = **2.0**	
(1.5***1**) + (0.5***0**) = **1.5**	

$$w_1 = 1.5$$
$$w_2 = 0.5$$

OR Gate via McCulloch-Pitts

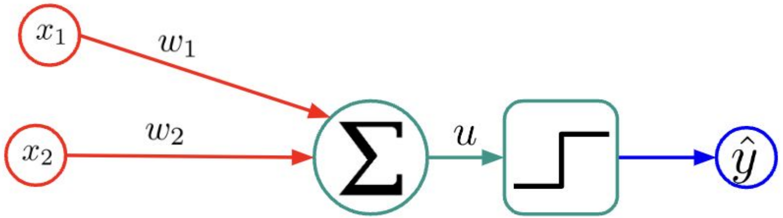


x_1	x_2
1	**1**
1	**0**
0	**1**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
$(1.5*\mathbf{1}) + (0.5*\mathbf{1}) = \mathbf{2.0}$	
$(1.5*\mathbf{1}) + (0.5*\mathbf{0}) = \mathbf{1.5}$	

$$w_1 = 1.5$$
$$w_2 = 0.5$$

OR Gate via McCulloch-Pitts

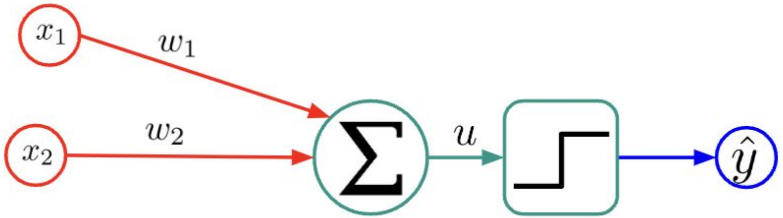


x_1	x_2
1	**1**
1	**0**
0	**1**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
$(1.5*\mathbf{1}) + (0.5*\mathbf{1}) =$ **2.0**	
$(1.5*\mathbf{1}) + (0.5*\mathbf{0}) =$ **1.5**	
$(1.5*\ \ ) + (0.5*\ \ ) = u$	

$$w_1 = 1.5$$
$$w_2 = 0.5$$

OR Gate via McCulloch-Pitts

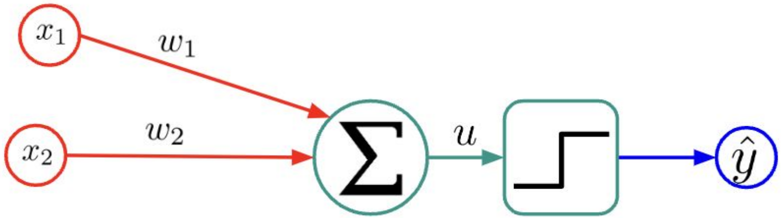


x_1	x_2
1	**1**
1	**0**
0	**1**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
$(1.5*\mathbf{1}) + (0.5*\mathbf{1}) = \mathbf{2.0}$	
$(1.5*\mathbf{1}) + (0.5*\mathbf{0}) = \mathbf{1.5}$	
$(1.5*\mathbf{0}) + (0.5*\mathbf{1}) = \mathbf{0.5}$	

$$w_1 = 1.5$$
$$w_2 = 0.5$$

OR Gate via McCulloch-Pitts

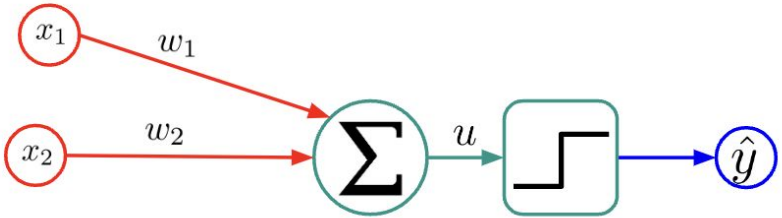


x_1	x_2
1	**1**
1	**0**
0	**1**
0	**0**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
$(1.5*\mathbf{1}) + (0.5*\mathbf{1}) = \mathbf{2.0}$	
$(1.5*\mathbf{1}) + (0.5*\mathbf{0}) = \mathbf{1.5}$	
$(1.5*\mathbf{0}) + (0.5*\mathbf{1}) = \mathbf{0.5}$	

$$w_1 = 1.5$$
$$w_2 = 0.5$$

OR Gate via McCulloch-Pitts

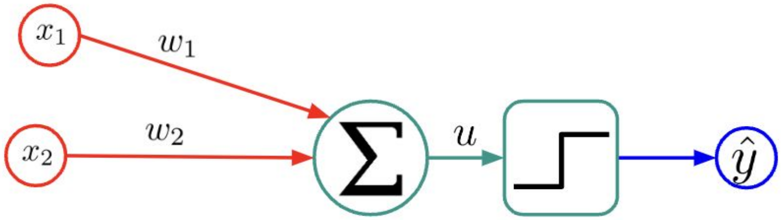


x_1	x_2
1	**1**
1	**0**
0	**1**
0	**0**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
$(1.5*\mathbf{1}) + (0.5*\mathbf{1}) =$ **2.0**	
$(1.5*\mathbf{1}) + (0.5*\mathbf{0}) =$ **1.5**	
$(1.5*\mathbf{0}) + (0.5*\mathbf{1}) =$ **0.5**	
$(1.5*\ \ ) + (0.5*\ \ ) = u$	

$$w_1 = 1.5$$
$$w_2 = 0.5$$

OR Gate via McCulloch-Pitts



x_1	x_2
1	**1**
1	**0**
0	**1**
0	**0**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
(1.5***1**) + (0.5***1**) = **2.0**	
(1.5***1**) + (0.5***0**) = **1.5**	
(1.5***0**) + (0.5***1**) = **0.5**	
(1.5***0**) + (0.5***0**) = **0.0**	

$$w_1 = 1.5$$
$$w_2 = 0.5$$

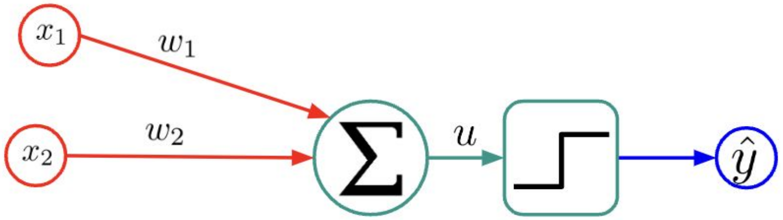
Now we have ***u*** of all data.

Then set the *"**threshold**"*

$$\boxed{theshold = 0.25}$$

$$\hat{y} = \begin{cases} 1; & u \geq theshold \\ 0; & u < theshold \end{cases}$$

OR Gate via McCulloch-Pitts



x_1	x_2
1	**1**
1	**0**
0	**1**
0	**0**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
(1.5***1**) + (0.5***1**) = 2.0	
(1.5***1**) + (0.5***0**) = 1.5	
(1.5***0**) + (0.5***1**) = 0.5	
(1.5***0**) + (0.5***0**) = 0.0	

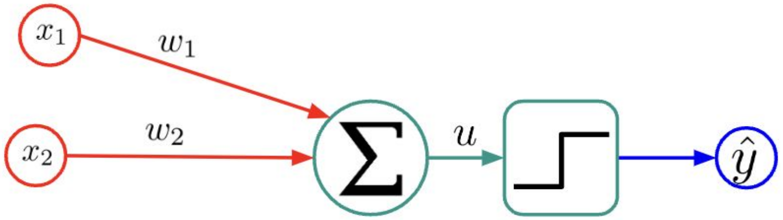
$$w_1 = 1.5$$

$$w_2 = 0.5$$

$$\boxed{theshold = 0.25}$$

$$\hat{y} = \begin{cases} 1; & u \geq theshold \\ 0; & u < theshold \end{cases}$$

OR Gate via McCulloch-Pitts



x_1	x_2
1	**1**
1	**0**
0	**1**
0	**0**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
(1.5***1**) + (0.5***1**) = **2.0**	**1**
(1.5***1**) + (0.5***0**) = **1.5**	
(1.5***0**) + (0.5***1**) = **0.5**	
(1.5***0**) + (0.5***0**) = **0.0**	

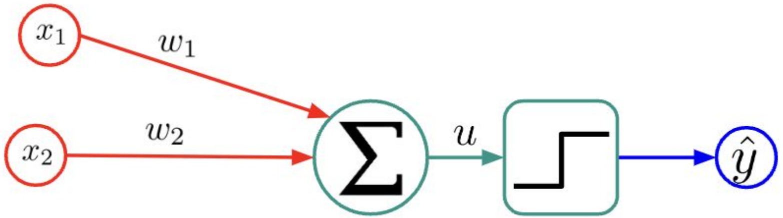
$$w_1 = 1.5$$

$$w_2 = 0.5$$

$$\boxed{theshold = 0.25}$$

$$\hat{y} = \begin{cases} 1; & u \geq theshold \\ 0; & u < theshold \end{cases}$$

OR Gate via McCulloch-Pitts



x_1	x_2
1	**1**
1	**0**
0	**1**
0	**0**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
(1.5***1**) + (0.5***1**) = **2.0**	**1**
(1.5***1**) + (0.5***0**) = **1.5**	**1**
(1.5***0**) + (0.5***1**) = **0.5**	
(1.5***0**) + (0.5***0**) = **0.0**	

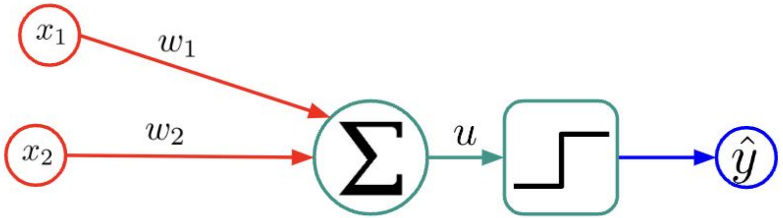
$$w_1 = 1.5$$

$$w_2 = 0.5$$

$$\boxed{theshold = 0.25}$$

$$\hat{y} = \begin{cases} 1; & u \geq theshold \\ 0; & u < theshold \end{cases}$$

OR Gate via McCulloch-Pitts



x_1	x_2
1	**1**
1	**0**
0	**1**
0	**0**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
(1.5***1**) + (0.5***1**) = 2.0	1
(1.5***1**) + (0.5***0**) = 1.5	1
(1.5***0**) + (0.5***1**) = 0.5	1
(1.5***0**) + (0.5***0**) = 0.0	

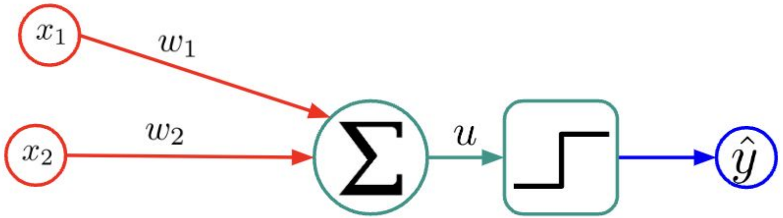
$$w_1 = 1.5$$

$$w_2 = 0.5$$

$$\boxed{theshold = 0.25}$$

$$\hat{y} = \begin{cases} 1; & u \geq theshold \\ 0; & u < theshold \end{cases}$$

OR Gate via McCulloch-Pitts



x_1	x_2
1	**1**
1	**0**
0	**1**
0	**0**

$u = w_1x_1 + w_2x_2$	$\hat{y}$
(1.5***1**) + (0.5***1**) = **2.0**	**1**
(1.5***1**) + (0.5***0**) = **1.5**	**1**
(1.5***0**) + (0.5***1**) = **0.5**	**1**
(1.5***0**) + (0.5***0**) = **0.0**	**0**

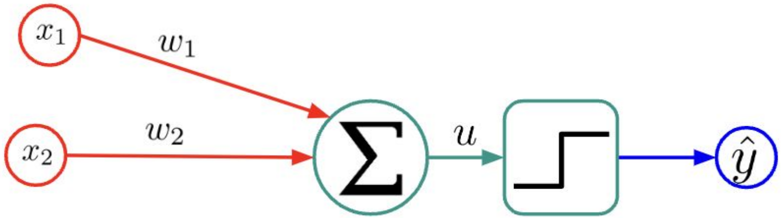
$$w_1 = 1.5$$

$$w_2 = 0.5$$

$$\boxed{theshold = 0.25}$$

$$\hat{y} = \begin{cases} 1; & u \geq theshold \\ 0; & u < theshold \end{cases}$$

OR Gate via McCulloch-Pitts



x_1	x_2	Target	$u = w_1x_1 + w_2x_2$	$\hat{y}$
1	1	1	(1.5*1) + (0.5*1) = 2.0	1
1	0	1	(1.5*1) + (0.5*0) = 1.5	1
0	1	1	(1.5*0) + (0.5*1) = 0.5	1
0	0	0	(1.5*0) + (0.5*0) = 0.0	0

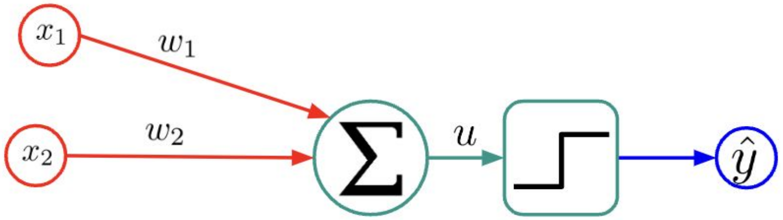
$$w_1 = 1.5$$
$$w_2 = 0.5$$
$$theshold = 0.25$$

$$\boxed{\hat{y} = \begin{cases} 1; & u \geq theshold \\ 0; & u < theshold \end{cases}}$$

Discuss !!

If *threshold = 1* ?

OR Gate via McCulloch-Pitts



x_1	x_2
1	1
1	0
0	1
0	0

$u = w_1x_1 + w_2x_2$	$\hat{y}$
(1.5*1) + (0.5*1) = 2.0	
(1.5*1) + (0.5*0) = 1.5	
(1.5*0) + (0.5*1) = 0.5	
(1.5*0) + (0.5*0) = 0.0	

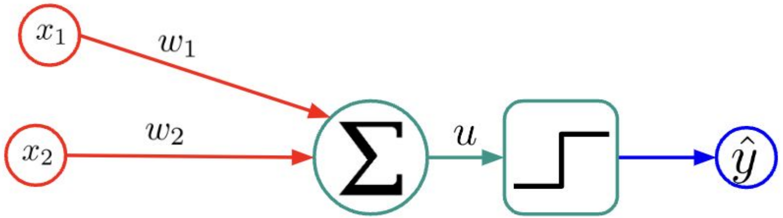
$$w_1 = 1.5$$

$$w_2 = 0.5$$

$$theshold = \boxed{1}$$

$$\hat{y} = \begin{cases} 1; & u \geq theshold \\ 0; & u < theshold \end{cases}$$

OR Gate via McCulloch-Pitts



x_1	x_2
1	1
1	0
0	1
0	0

$u = w_1x_1 + w_2x_2$	$\hat{y}$
$(1.5*1) + (0.5*1) = 2.0$	1
$(1.5*1) + (0.5*0) = 1.5$	1
$(1.5*0) + (0.5*1) = 0.5$	0
$(1.5*0) + (0.5*0) = 0.0$	0

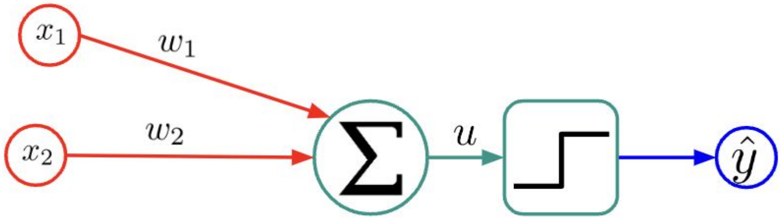
$$w_1 = 1.5$$

$$w_2 = 0.5$$

$$theshold = \boxed{1}$$

$$\hat{y} = \begin{cases} 1; & u \geq theshold \\ 0; & u < theshold \end{cases}$$

OR Gate via McCulloch-Pitts



x_1	x_2	Target	$u = w_1x_1 + w_2x_2$	$\hat{y}$
1	1	1	(1.5*1) + (0.5*1) = 2.0	1
1	0	1	(1.5*1) + (0.5*0) = 1.5	1
0	1	1	(1.5*0) + (0.5*1) = 0.5	0
0	0	0	(1.5*0) + (0.5*0) = 0.0	0

$$w_1 = 1.5$$

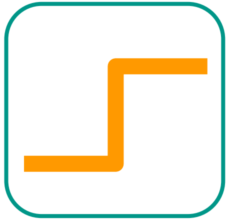
$$w_2 = 0.5$$

$$theshold = \boxed{1}$$

$$\hat{y} = \begin{cases} 1; & u \geq theshold \\ 0; & u < theshold \end{cases}$$

Limitation of McCulloch-Pitts Neuron

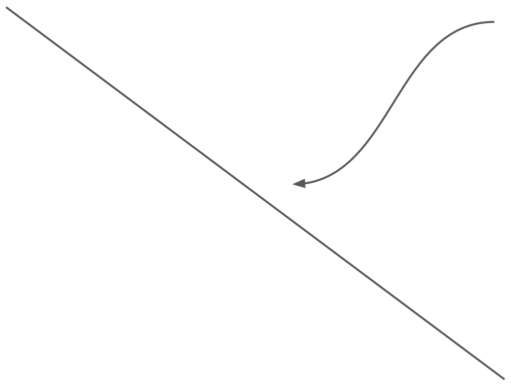
*Do we always need to hand set the **weights** & **threshold** ?*



Hands On

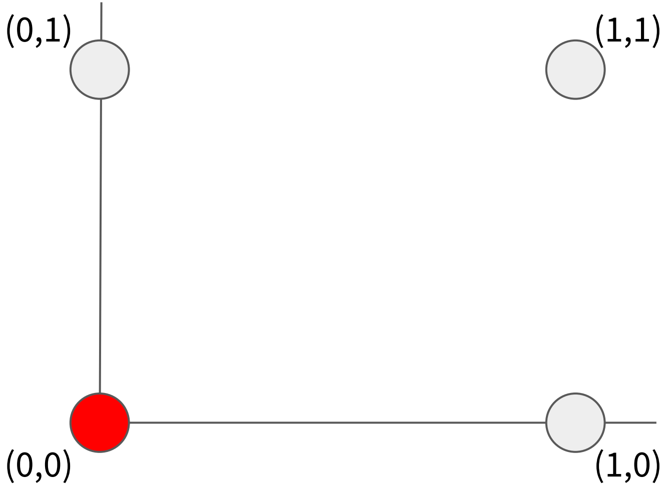
Linearly separable

Linear equation



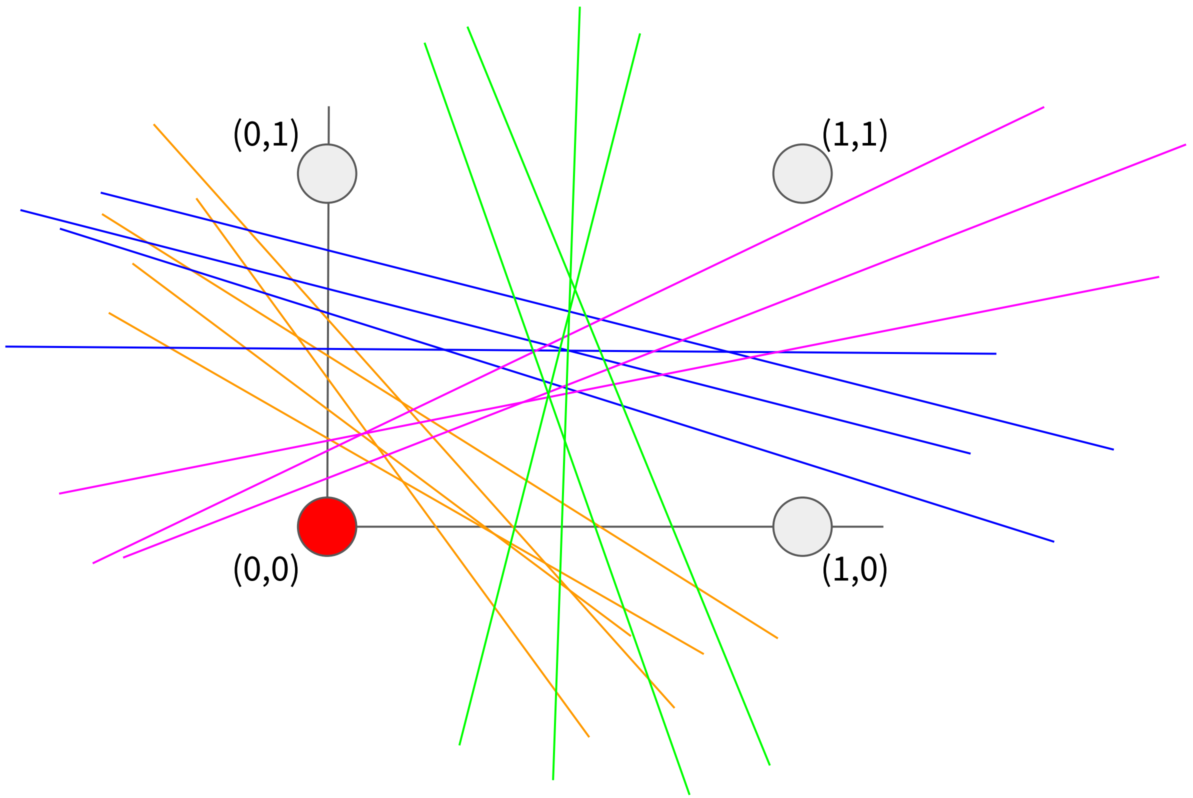
$$y = mx + c$$

Linearly separable

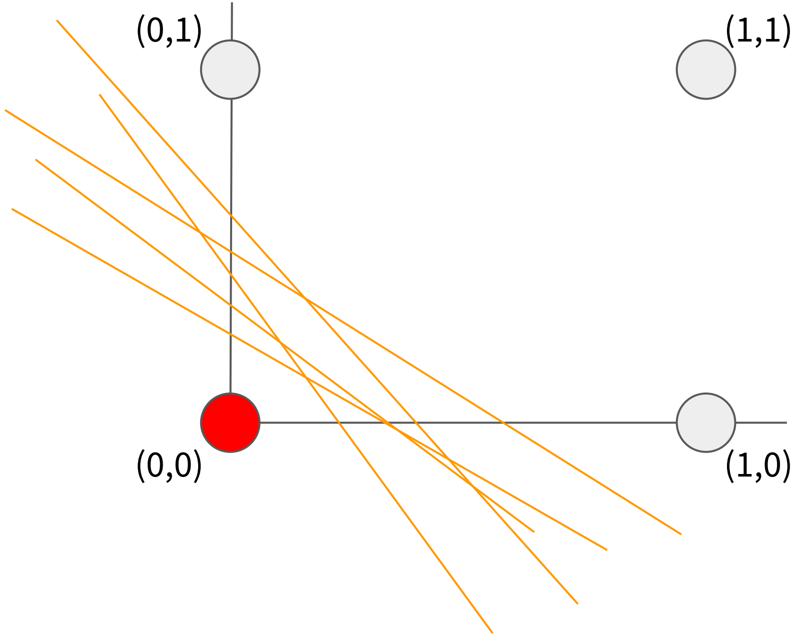


x	y	class
0	0	
0	1	
1	0	
1	1	

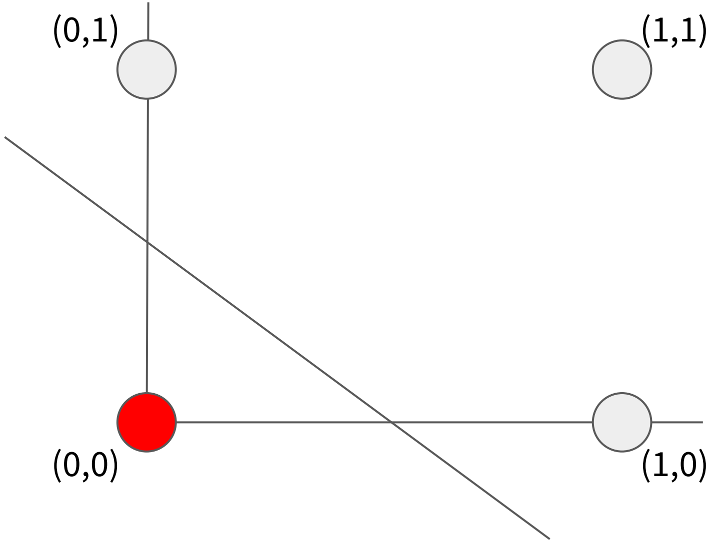
Linearly separable



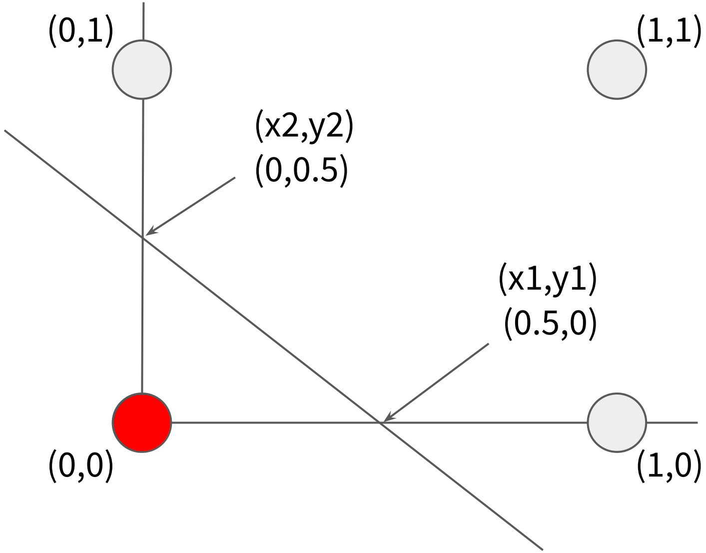
Linearly separable



Linearly separable



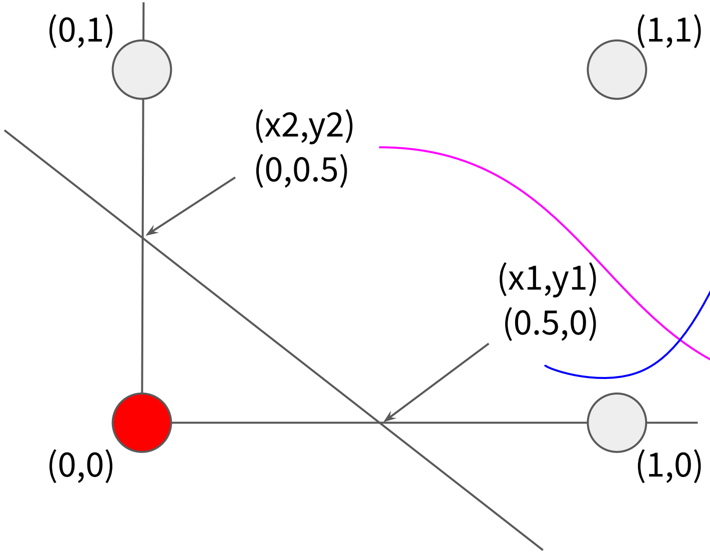
Linearly separable



$$y = mx + c$$

$$\boxed{\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0.5 - 0}{0 - 0.5} \\ &= -1 \end{aligned}}$$

Linearly separable

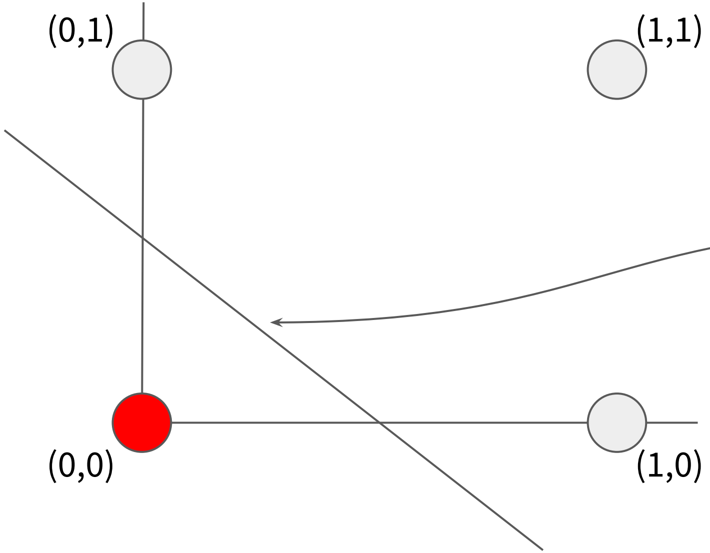


$$y = mx + c$$

$$\begin{aligned} c &= y_1 - mx_1 \\ &= 0 - (-1) \times 0.5 \\ &= 0.5 \end{aligned}$$

$$\begin{aligned} c &= y_2 - mx_2 \\ &= 0.5 - (-1) \times 0 \\ &= 0.5 \end{aligned}$$

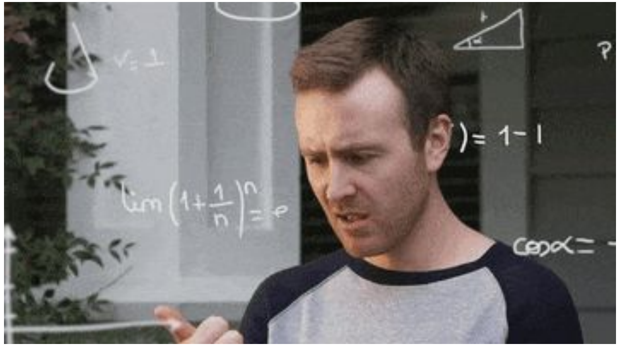
Linearly separable



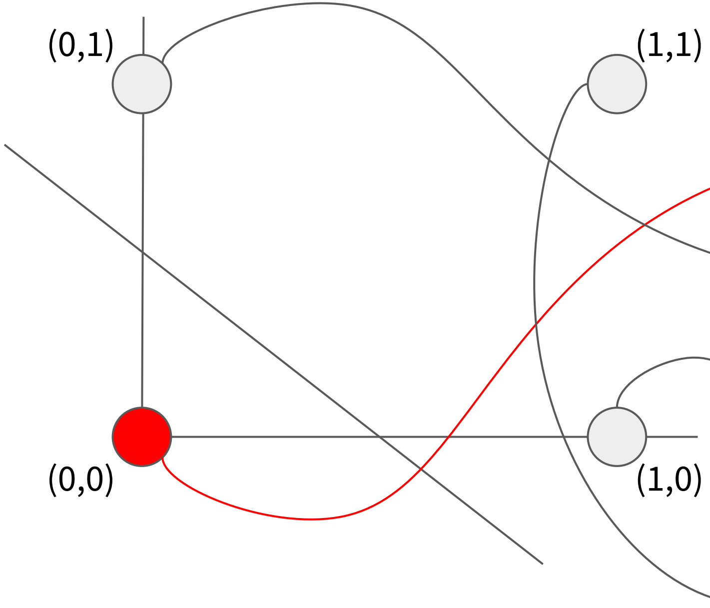
$$y = mx + c$$

$$y = (-1)x + 0.5$$

$$\boxed{f(x, y) = -x - y + 0.5}$$



Linearly separable



$$\boxed{f(x,y) = -x - y + 0.5}$$

$$f(0,0) = +0.5$$

POSITIVE

$$f(0,1) = -0.5$$

$$f(1,0) = -0.5$$

$$f(1,1) = -1.5$$

NEGATIVE

Linearly separable

$$f(x, y) = -x - y + 0.5$$