

**ENSEA**

Beyond Engineering

## TP1 DIGITAL COMMUNICATION

RTS TP1

### Communication of the M-QAM

PAN Jimmy-Antoine - LI Guillaume

## Question 1

To display the constellation of the M-QAM, we use two intermediate functions : `const_M_QAM` and `Gray_M_QAM`. The function `const_M_QAM` gives the the matrix of coordinates of our symbols in 2D. The function `Gray_M_QAM` associates the value of the coordinate in Gray code.

So we can display the constellation of any M-QAM and its Gray Code. We can test it for several values of M :

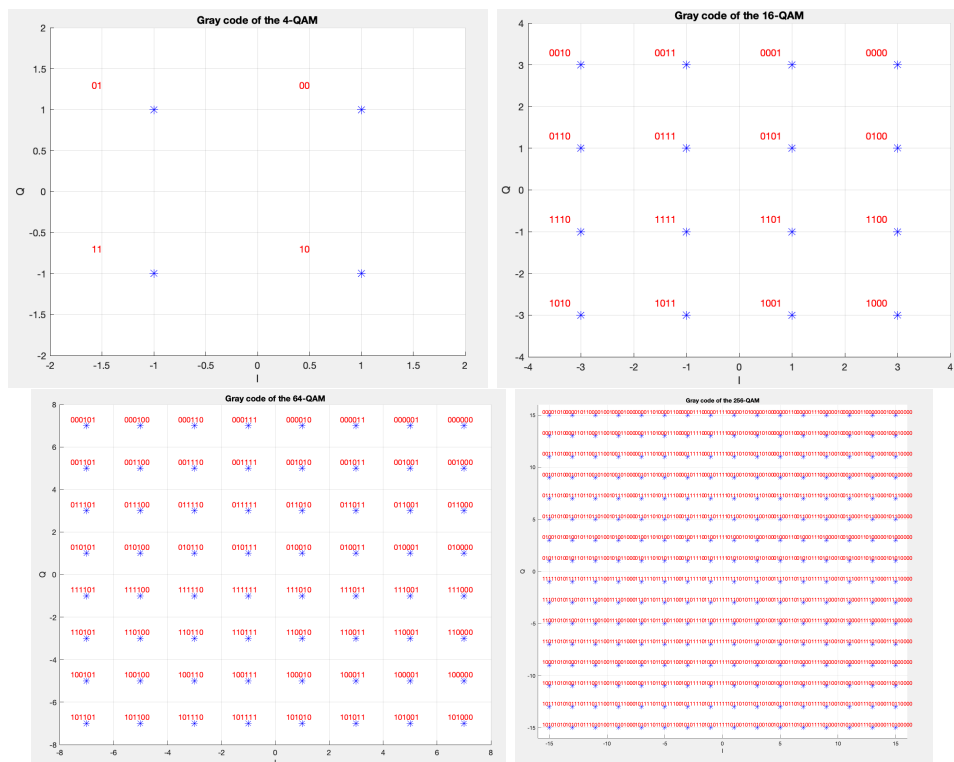


FIGURE 1 – Gray Code of the M-QAM for  $M = \{4, 16, 64, 256\}$

We can observe that all symbole are equally separated : the probability to get one symbol is perfectly equiprobable. Between two neighboring symbols, we only modify 1 bit according to code Gray mapping.

## Question 2

To simulate the chain of communication of the system, we generate a sequence of  $N$  bits which go through a AWGN. The goal is to test our chain and see if we can get back the beginning sequence.

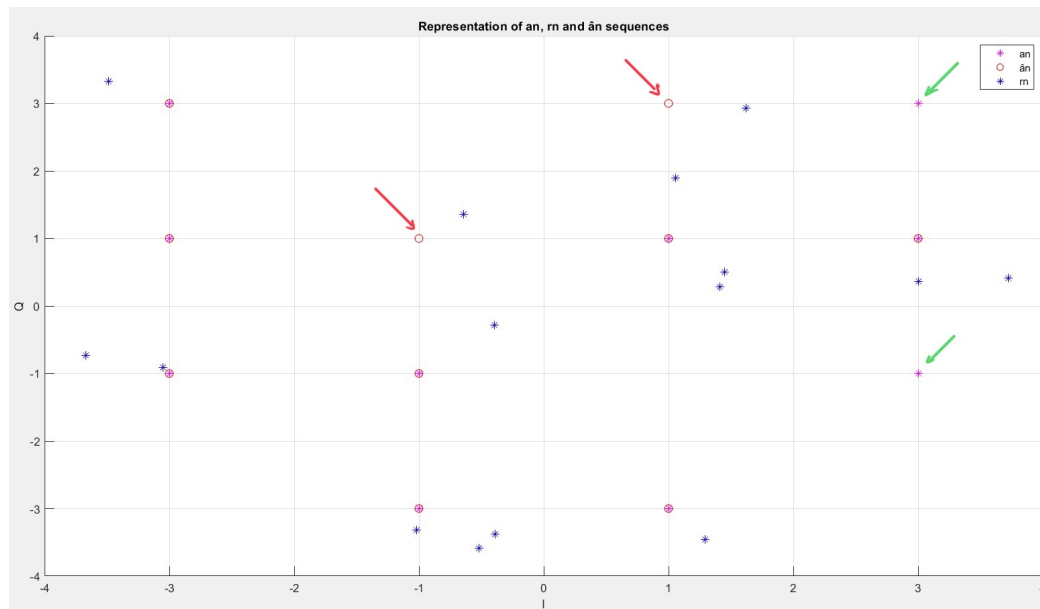


FIGURE 2 – Simulation of the chain of communication for  $M=16$

On this figure :

- $a_n$  represent the symbols of the initial sequence.
- $m$  represent the symbols of the sequence after the AWGN.
- $\hat{a}_n$  represent the symbols of the sequence after the AWGN after the decision.

We can observe that most of the symbols perturbed by the AWGN canal can be right identified after the decision block (cf green arrows). However, the decision block can make mistakes (cf red arrows). The code Gray is useful in this situation : even if the decision block make mistakes, we need to correct only 1 bit to get back the right symbol.

### Question 3

The goal of this question is to determine the bit and symbol error rate in function of the normalised SNR,  $SNR_0$ , in dB. We must determine  $\sigma$  from the value of SNR in dB,  $SNR_{0dB}$ .  $\sigma$  will allow us to build our different AWGP channels.

#### Determination of $\sigma$

We want to display the graphic with the normalised SNR in dB from 0 to 12. However, to determine the BER/SER we must convert the value of  $SNR_{0dB}$  to standard SNR then express  $SNR_0$  in function of  $\sigma$  by using its definition.

We know that :

$$SNR_{dB} = 10\log(SNR) \text{ and } SNR_0 = \frac{E_b}{N_0} = \frac{3\log_2(M)}{M-1} \frac{E_b}{N_0} = \frac{g(0)^2}{\sigma_b^2}$$

We have :

$$SNR = 10^{\frac{SNR_{dB}}{10}} \text{ then } \sigma^2 = \frac{g(0)^2(M-1)}{3\log_2(M)*10. \frac{SNR_{0dB}}{10}}$$

We got the expression of  $\sigma^2$  depending of  $SNR_{0dB}$

We can now repeat what we did last question and create our AWGP channels and determine BER and SER :

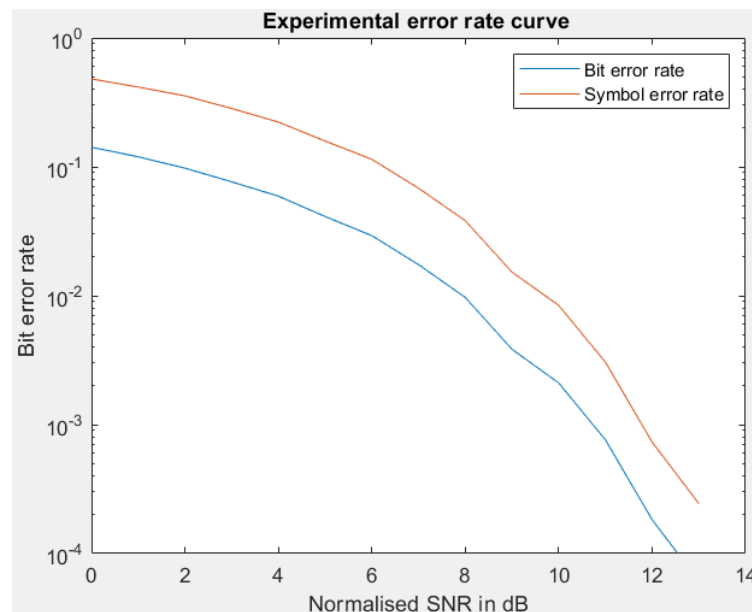


FIGURE 3 – Estimated BER/SER in function of  $SNR_{0dB}$

Let's compare this graphic in next question.

## Question 4

We now want to compare the estimated curves to the theoretical one. In the course we have the following formula to calculate the symbol error probability :

$$P_s(e) = 4 \frac{\sqrt{M} - 1}{\sqrt{M} \log_2(M)} Q \left( \sqrt{\frac{3 \log_2(M)}{M - 1} \frac{E_b}{N_0}} \right)$$

And for the binary error probability we have :

$$P_b(e) = \frac{P_s(e)}{\log_2(M)}$$

Moreover the subject says that :

$$Q = \frac{1}{2} \operatorname{erfc} \left( \frac{x}{\sqrt{2}} \right)$$

Which gives us the following plot with the 2 curves :

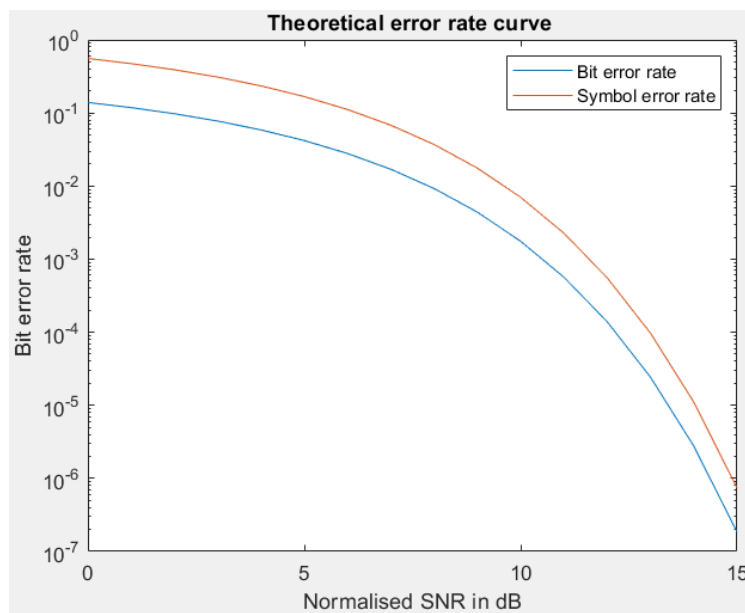


FIGURE 4 – Theoretical error rate curve of the 16-QAM

We observe that both curves are really close to the lecture's curves. However, we can see that estimated error rate is bigger than theoretical error rate : we have more error in labs than in the theory.