Two-sample tests of proportion

- The implementation of two-sample tests in R is similar to one-sample test but there are differences to be aware of.
- As before, we use the command prop.test to handle tests of proportion problems. We just need to learn when to use it and how.

Example: Two surveys

- A survey is taken two times over the course of two weeks. The pollsters wish to see if there is a difference in the results as there has been a new advertising campaign run.
- Here is the data:

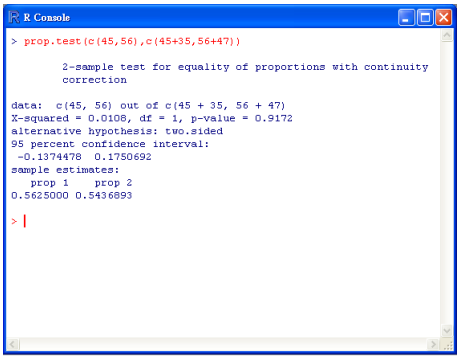
	Week 1	Week 2
Favorable	45	56
Unfavorable	35	47

假設設定與prop.test函數

- The standard hypothesis test is
  $H_0 : P_1 = P_2$ (*or* $H_0 : P_1 - P_2 = 0$)
  against the alternative (two-sided)
  $H_1 : P_1 \neq P_2$ (*or* $H_1 : P_1 - P_2 \neq 0$)
- The function prop.test is used to being called as prop.test(x,n) where x is the number favorable and n is the total. Here it is no different, but since there are two x's it looks slightly different.

R 指令

- We observe that the p-value is 0.9172, so we accept the null hypothesis that $P_1 = P_2$.



Two-sample t-test

- one-sample t-test 的統計公式：

$$t = \frac{\bar{X} - \mu}{s / \sqrt{n}}$$

1. 使用在資料近似於常態分配時
2. σ未知

- two-sample t-test 的統計公式：

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

假設 X_i 是常態或趨近於常態分配

Equal Variances

- 當我們假設兩樣本之變異數相同，則兩組資料可以合併，估計共同的變異數。
- 參見下頁 t.test 線上說明檔。

Equal Variances

- t.test(x,
  - y = NULL, ← 另一筆數值向量物件
  - alternative = c("two.sided", "less", "greater"), ← 對立假設 ← 預設為two.sided
  - mu = 0, ← 兩個樣本mean的差距
  - paired = FALSE, ← 是否為成對樣本
  - var.equal = FALSE, ← 變異數是否相等
  - conf.level = 0.95) ← 信賴區間的信心水準

Example: Recovery time for new drug

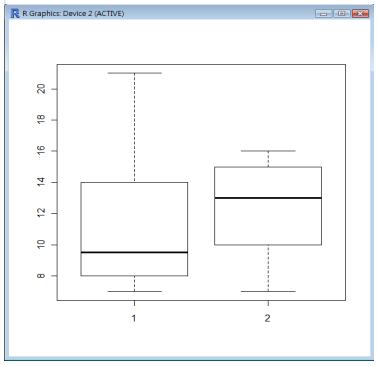
- Suppose the recovery time for patients taking a new drug is measured (in days). A placebo group is also used to avoid the placebo effect. The data are as follows

  with drug: 15 10 13 7 9 8 21 9 14 8

  placebo: 15 14 12 8 14 7 16 10 15 12

Example: Recovery time for new drug

```
> x = c(15, 10, 13, 7, 9, 8, 21, 9, 14, 8)
> y = c(15, 14, 12, 8, 14, 7, 16, 10, 15, 12)
> boxplot(x,y)
>
```



- 利用盒鬚圖決定
  - 變異數是否相同 (建議用var.test函數)
  - 資料是否服從常態 (呈偏態分佈)

Example: Recovery time for new drug

$$H_0: \mu_1 - \mu_2 \geq 0$$
$$H_1: \mu_1 - \mu_2 < 0$$

```
> x = c(15, 10, 13, 7, 9, 8, 21, 9, 14, 8)
> y = c(15, 14, 12, 8, 14, 7, 16, 10, 15, 12)
> boxplot(x,y)
> t.test(x,y,alt="less",var.equal=TRUE)

        Two Sample t-test

data:  x and y
t = -0.5331, df = 18, p-value = 0.3002
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
     -Inf 2.027436
sample estimates:
mean of x mean of y
     11.4      12.3
```

- 根據此結果，我們無法拒絕虛無假設。

Unequal Variances

```
> t.test(x,y,alt="less")

        Welch Two Sample t-test

data:  x and y
t = -0.5331, df = 16.245, p-value = 0.3006
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
     -Inf 2.044664
sample estimates:
mean of x mean of y
     11.4      12.3
```

- 根據此結果，我們無法拒絕虛無假設。

Comparisons

Equal variances

```
> t.test(x,y,alt="less",var.equal=TRUE)

        Two Sample t-test

data:  x and y
t = -0.5331, df = 18, p-value = 0.3002
```

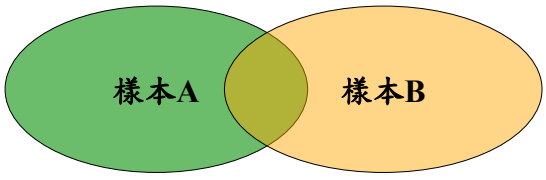
Unequal variances

```
> t.test(x,y,alt="less")

        Welch Two Sample t-test

data:  x and y
t = -0.5331, df = 16.245, p-value = 0.3006
```

- 變異數相等與不相等的結果有些不同，但此例中結論是相同的。(無法拒絕H_0)
- 當變異數相等時，抽樣分配之t分配自由度較大，愈接近標準常態，兩尾面積較小，p值因此較小，虛無假設較容易被拒絕。

成對樣本(Matched Samples)

- 雙樣本t檢定(two sample t-test)之成對t檢定(paired t-test)，使用不同於獨立性t檢定的統計模型。
- 成對樣本，假設兩樣本中存在有共同的特點。



Basic model

- Yi = Xi + εi ，其中εi 是隨機變數。

檢驗：H0: 平均數=0，H1: 平均數≠0。

將Yi減去Xi，進行單一樣本t檢定。

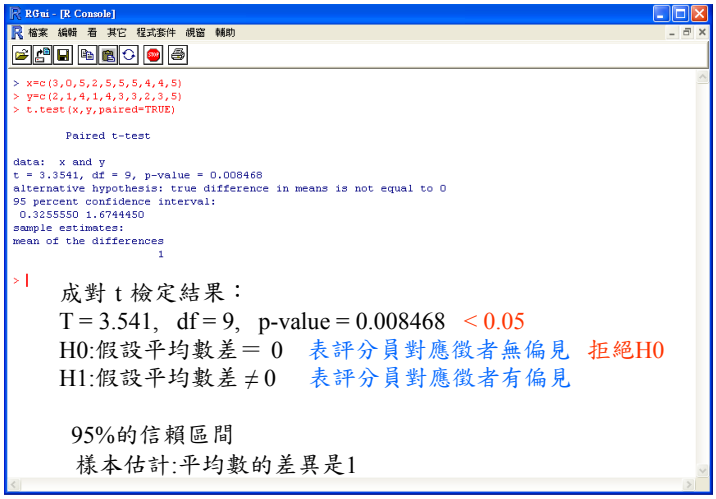
指令：t.test(x,y,paired=TRUE)

Example: Dilemma of two graders

- 為了公平起見，每位應徵者都接受兩位評分員評分。資料如下(閱同一份試卷，所以非獨立樣本)：

Appl. :	1	2	3	4	5	6	7	8	9	10
Grader1:	3	0	5	2	5	5	5	4	4	5
Grader2:	2	1	4	1	4	3	3	2	3	5

Example: Dilemma of two graders



成對 t 檢定結果：

T = 3.541, df = 9, p-value = 0.008468 < 0.05

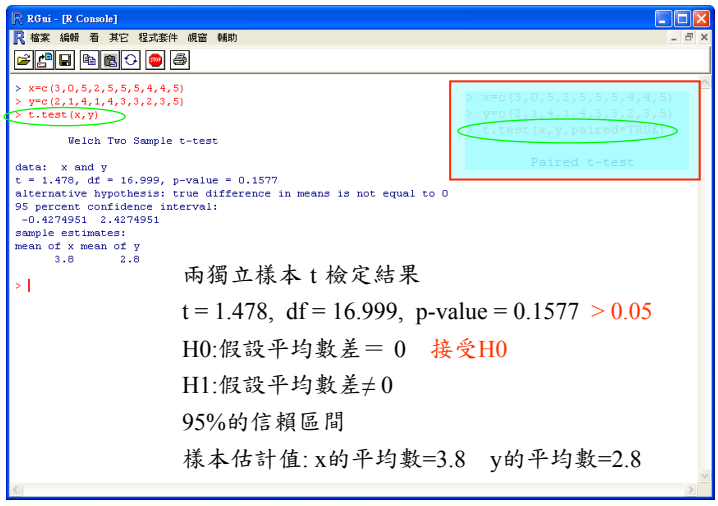
H0:假設平均數差＝ 0 表評分員對應徵者無偏見 拒絕H0

H1:假設平均數差 ≠ 0 表評分員對應徵者有偏見

95%的信賴區間

樣本估計:平均數的差異是1

Example: Dilemma of two graders



兩獨立樣本 t 檢定結果

t = 1.478, df = 16.999, p-value = 0.1577 > 0.05

H0:假設平均數差＝ 0 接受H0

H1:假設平均數差≠ 0

95%的信賴區間

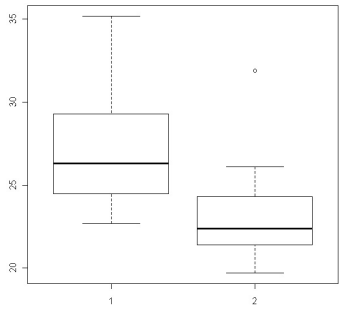
樣本估計值: x的平均數=3.8 y的平均數=2.8

Resistant two-sample tests

- 抗雜訊雙樣本檢定使用Wilcox的檢定函數(wilcox.test())，用法與單一樣本中位數檢定相似。
- Example : Taxi out times
  - 比較美國和西北航空公司在Newark機場的降落滑行時間，資料集為ewr。

先觀察資料分佈

```
> data(ewr)
> attach(ewr)
> tmp=subset(ewr,inorout == "out",select=c("AA","NW"))
> x=tmp[['AA']]
> y=tmp[['NW']]
> boxplot(x,y)
>
```



- 呈偏態分配
- 使用Wilcox中位數檢定

Wilcoxon Test 檢定結果

```
> wilcox.test(x,y)

        Wilcoxon rank sum test with continuity correction

data:  x and y
W = 460.5, p-value = 1.736e-05
alternative hypothesis: true location shift is not equal to 0

Warning message:
In wilcox.test.default(x, y) : cannot compute exact p-value with ties
>
```

無法很準確計算出p值

- 由wilcox.test得知，有強烈的證據可拒絕虛無假設，即接受中位數不相等之對立假設。

The End