Bayesian Estimation of Daily Return and Volatility of XRP/USDT using MCMC

Project-1 — MA4740: Bayesian Statistics

Under the guidance of Prof. Arunabha Majumdar

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Motivation and Problem Statement

Challenge: Cryptocurrency markets exhibit high volatility and uncertainty

Limitations of Traditional Approaches:

- Point estimates fail to capture uncertainty
- Frequentist confidence intervals lack intuitive interpretation
- No principled way to incorporate prior knowledge

Our Solution: Bayesian inference with Rejection Sampling

- Complete probability distributions for parameters
- Credible intervals with direct probabilistic interpretation
- Posterior predictive distributions for forecasting

Bayesian Model Specification

Likelihood: Log returns follow a Normal distribution

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) \sim \mathcal{N}(\mu, \sigma^2)$$

Prior Distributions: Non-informative conjugate priors

$$\mu \sim \mathcal{N}(\mu_0 = 0, \sigma_0^2 = 1000)$$
$$\sigma^2 \sim \text{Inv-Gamma}(\alpha_0 = 0.001, \beta_0 = 0.001)$$

Posterior: Normal-Inverse-Gamma (conjugate)

$$p(\mu, \sigma^2|\mathbf{r}) \propto p(\mathbf{r}|\mu, \sigma^2) \cdot p(\mu) \cdot p(\sigma^2)$$

MCMC Implementation: Rejection Sampling

Algorithm:

1. Define proposal distributions centered on data:

$$q(\mu) = \mathcal{N}\left(\bar{r}, \sqrt{\sigma_0^2 + \frac{s^2}{n}}\right)$$

$$q(\sigma^2) = \text{Inv-Gamma}\left(\frac{n}{2}, \frac{1}{2}\sum_{i=1}^{n}(r_i - \bar{r})^2\right)$$

2. Draw candidate (μ^*, σ^{2*}) from proposal
3. Accept with probability: $\alpha = \min\left(1, \frac{p^*(\mu^*, \sigma^{2*}|\mathbf{r})}{M \cdot q(\mu^*, \sigma^{2*})}\right)$

Implementation Details:

- Target samples: 2,000 — Max attempts: 1,000,000
- Envelope constant: $M = 10$
- Typical acceptance rate: 20-25%

Results: Posterior Distributions

Key Findings:

- **Mean Return (μ):**
  -0.0012 (-0.12% daily)
  95% CI: $[-0.088, 0.090]$
- **Volatility (σ):**
  0.0545 (5.45% daily)
  95% CI: $[0.053, 0.056]$
- **Prob(Positive Return):**
  49% (nearly neutral)

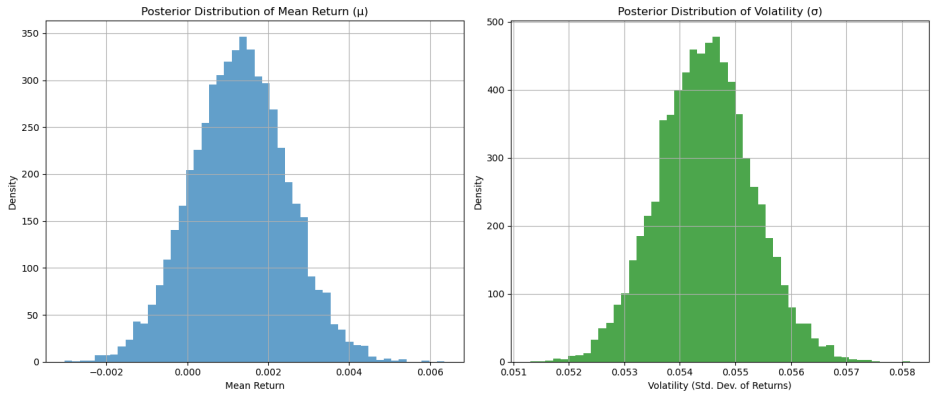

Figure: Posterior distributions from 2,000 MCMC samples

Posterior Predictive Distribution

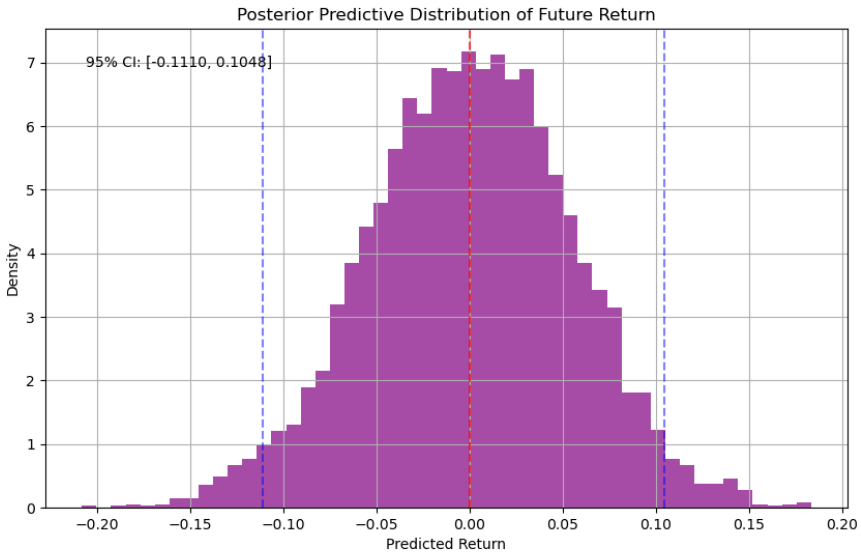

Figure: Next-day return prediction

Interpretation:

- 95% Predictive Interval: $[-14.3\%, +15.0\%]$
- Wide interval reflects:
  - Parameter uncertainty
  - Market randomness
  - Crypto volatility
- Honest quantification of uncertainty

Multi-Horizon Price Forecasts

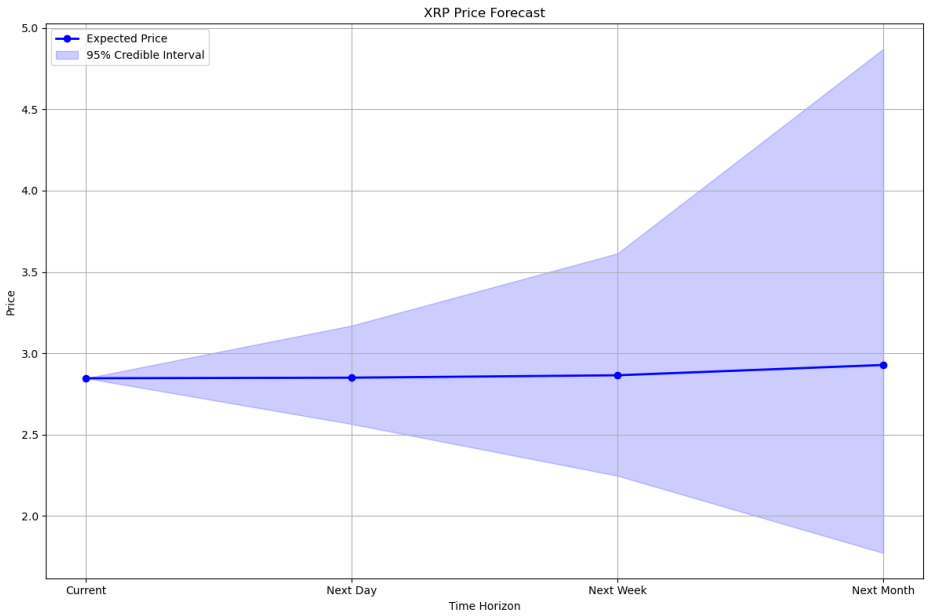

Figure: Widening uncertainty over time

Current Price: $2.85

Horizon	Expected	95% CI
Next Day	$2.84	[$2.46, $3.31]
Next Week	$2.83	[$2.03, $3.96]
Next Month	$2.77	[$1.36, $5.61]

Key Insight:

Uncertainty compounds over time, illustrating the difficulty of long-term prediction in volatile markets

Trading Implications and Risk Assessment

Risk Classification: MEDIUM (volatility = 5.45%)

Trading Signal: NEUTRAL (Low Confidence)

- Nearly 50-50 probability of positive return
- Wide credible interval containing zero
- Suggests market-neutral or wait-and-see approach

Position Sizing Recommendation:

- Suggested allocation: 22.5% (split long/short or reduced exposure)
- Based on volatility-adjusted Kelly criterion
- Incorporates directional uncertainty

Risk Management:

- Stop-loss: $\pm$ 8% (1.5 $\times$ daily volatility)
- Take-profit: $\pm$ 11% (2 $\times$ daily volatility)

Conclusions and Future Work

Key Achievements:

- Successfully implemented Bayesian inference with Rejection Sampling
- Generated 2,000 independent posterior samples (1-20% acceptance rate)
- Complete uncertainty quantification for crypto returns
- Automated analysis pipeline with comprehensive reporting

Advantages of Bayesian Approach:

- Intuitive probability statements about parameters
- Natural incorporation of uncertainty in predictions
- Flexible framework for model extensions

Future Extensions:

- Time-varying volatility models (GARCH, Stochastic Volatility)
- Alternative MCMC methods (Metropolis-Hastings, HMC)
- Multi-asset portfolio optimization
- Incorporation of external covariates and regime-switching

Thank You!

Questions?

Project Repository:
github.com/jimmy2683/Bayesian_Analysis_on_XRP_USDT
Course: MA4740 - Bayesian Statistics
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