225 Notes

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March 28, 2018

Contents

1 Chapter 1

$|\psi\rangle = |+\rangle + |-\rangle$ This is a Quantum State ψ it needs the money signs to be in the math mode in order to use it.

1.1 Stern-Gerlach Experiments

Stern-Gerlach Experiments are where a particle (or many particles) is passed through a magnetic field in a certain direction and then they are observed going in a specific direction.

After a particle goes through an SG it will align itself with that SG's direction. So if a particle is 100% positive Z then it goes through an X SG, it will then be split in to 50/50 positive and negative Z.

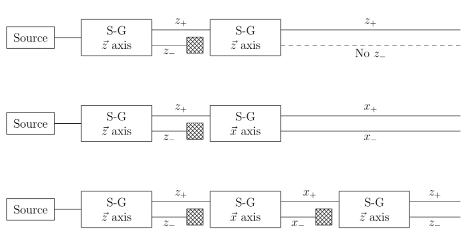

Figure 1: An example of a stern Gerlach Experiment

1.2 Postulate One

The state of a quantum Mechanical system, including all the information you can know about it, is represented mathematically by a normalized Ket.

1.3 Quantum State Vectors

These act like vectors as a basis state for a Hilbert Space.[1] They are normally in the Z basis or the S_z basis. The general Quantum state vector ψ is the equation that represents a basis state. The $|+\rangle$ and $|-\rangle$ represent orthogonal vectors. The Ket is how they are normally represented and pictured.

Ket:

$$|\psi\rangle = a|+\rangle + b|-\rangle \tag{1}$$

Conversely there is also a Bra that is like the opposite of a Ket in that it is the complex conjugate of the Ket. All that this means is that the imaginary numbers all go from positive to negative.

Bra:

$$\langle\psi| = a^*\langle+| + b^*\langle-| \tag{2}$$

Because the vectors $|+\rangle$ and $|-\rangle$ are orthonormal to each other when the inner product of a bra and a ket are taken only the respecting parts are multiplied.

$$\begin{aligned} \langle+|\psi\rangle &= \langle+|(a|+\rangle + b|-\rangle) \\ &= \langle+|a|+\rangle + \langle+|b|-\rangle \\ &= a\langle+|+\rangle + b\langle+|-\rangle \\ &= a \end{aligned} \tag{3}$$

[1] Hilbert Space is the space where quantum state vectors live and work.

When they are flipped they become the complex conjugate of the inner product calculated $\langle +|\psi\rangle \rightarrow \langle\psi|+\rangle^*$. For two Quantum States $\langle\phi|\psi\rangle \rightarrow \langle\psi|\phi\rangle^*$

1.3.1 Example 1.1

Normalize the vector $|\psi\rangle = C(1|+\rangle + 2i|-\rangle)$. The complex constant C is often reffered to as the **Normalization Constant**.

To normalize $|\psi\rangle$, we set the inner product of the vector with itself equal to unity and then solve for C, note the requisite complex conjugation.

$$\begin{aligned}
1 &= \langle\psi|\psi\rangle \\
&= C^*(1|\langle +| - 2i\langle -|)C(1|+\rangle + 2i|-\rangle) \\
&= C^*C(1\langle +|+\rangle + 2i\langle +|-\rangle - 2i\langle -|+\rangle + 4\langle -|-\rangle) \\
&= |C^2|(1\langle +|+\rangle + 4\langle -|-\rangle) \\
&= 5|C^2| \\
\frac{1}{5} &= C^2 \\
\sqrt{\frac{1}{5}} &= C
\end{aligned} \tag{4}$$

The overall phase of the normalization constant is not physically meaningful, so we follow the standard convention and choose it to be real and positive. This yields $C = \frac{1}{\sqrt{5}}$. The normalized quantum state is

$$|\psi\rangle = \frac{1}{\sqrt{5}}(1|+\rangle + 2i|-\rangle) \tag{5}$$

1.4 Postulate 4 (Spin-one half System)

The probability of obtaining the value $\pm\hbar/2$ in a measurement of the observable S_z on a system in the state $|\psi\rangle$ is

$$\mathcal{P}_{\pm} = |\langle\pm|\psi\rangle|^2, \tag{6}$$

where $|\pm\rangle$ is the basis ket of S_z corresponding to the result $\pm\hbar/2$