科目 机率

長庚大學期中、期末考試答案用紙

____學年度 第____學期____考 資工 系 姓名 黃建銘 學號 B0729014

[1]

(1) $\sum_{x=0}^{10} (x, 10; \frac{1}{10}) = C^{10}_{x} (\frac{1}{10})^{x} (\frac{9}{10})^{10-x}$

$f_X(0) = 0.3487$ $f_X(5) = 0.0015$

$f_X(1) = 0.3874$ $f_X(6) = 0.0001$

$f_X(2) = 0.1937$ $f_X(7) = 8.748e-06$

$f_X(3) = 0.0574$ $f_X(8) = 3.645e-07$

$f_X(4) = 0.0112$ $f_X(9) = 9e-09$

$f_X(10) = 1e-10$

(2) $n \cdot p = \mu = 10 \cdot \frac{1}{10} = 1$

(3) $\sigma^2 = n \cdot p \cdot (1-p) = 10 \times \frac{1}{10} \times \frac{9}{10} = \frac{9}{10}$

$\sigma = \sqrt{\frac{9}{10}} = 0.9487$

(4) $f_Y(k) = \frac{C^{10}_{k} \times C^{90}_{10-k}}{C^{100}_{10}} \rightarrow$

$f_Y(0) = 0.3305$ $f_Y(5) = 6.398e-4$ $f_Y(10) = 5.177e-14$

$f_Y(1) = 0.4080$ $f_Y(6) = 3.1e-5$

$f_Y(2) = 0.2015$ $f_Y(7) = 8.144e-7$

$f_Y(3) = 0.0518$ $f_Y(8) = 1.0411e-8$

$f_Y(4) = 0.0076$ $f_Y(9) = 5.1992e-11$

(5)

$f_Y(3) = 0.0518$ $f_Y(8) = 1.0411e-8$

$f_Y(4) = 0.0076$ $f_Y(9) = 5.1992e-11$

(5)

期望值與標準差 與放不放回無關

$E[Y] + std[Y] = 1 + 0.8487 \doteq 1.8487$

(6)

[4]

$b(x, n, p) = \binom{n}{x} p^{x} q^{n-x} \xrightarrow[n \cdot p = \mu]{n \to \infty,\ p \to 0} p(x, \mu) = \frac{\mu^{x}}{x!} e^{-\mu}$

$\hookrightarrow \frac{n!}{} p^{x} (1-p)^{n-x} = \underline{n(n-1)(n-2) \dots (n-x+1)} (\frac{\mu}{n})^{x} (1 - \frac{\mu}{n})^{n-x}$

[4]

$$b(x;n,p) = \binom{n}{x} p^x q^{n-x} \xrightarrow[n \cdot p = \mu]{n \to \infty,\ p \to 0} p(x;\mu) = \frac{\mu^x}{x!} e^{-\mu}$$

$$\Rightarrow \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} = \frac{n(n-1)(n-2)\cdots(n-x+1)}{x!} \left(\frac{\mu}{n}\right)^x \left(1-\frac{\mu}{n}\right)^{n-x}$$

$$= \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right) \frac{1}{x!} \mu^x \left(\left(1-\frac{\mu}{n}\right)^{-\frac{n}{x}}\right)^{-x} \left(1-\frac{\mu}{n}\right)^{-x}$$

去士不變，$n \to \infty \Rightarrow \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right) \to 1$

$\left(1-\frac{\mu}{n}\right)^{-\frac{n}{x}} \to e$

$\left(1-\frac{\mu}{n}\right)^{-x} \to 1$

So $b(x;n,p) \to \frac{\mu^x e^{-\mu}}{x!}$

because $\sum_{x=0}^{\infty} p(x;\mu) = e^{-\mu} \sum_{x=0}^{\infty} \frac{\mu^x}{x!} = e^{-\mu} e^{\mu} = 1$

（請翻面繼續作答）

長庚大學期中、期末考試答案用紙

科目＿＿＿＿

＿＿學年度 第＿＿學期＿＿考＿＿＿＿系 姓名＿＿＿＿ 學號＿＿＿＿

[2]

(1) $f(w) = P(100;1) = \frac{e^{-1} 1^{100}}{100!} = 3.9417e-159$

(2) $E[W] = \lambda t = 100 \times 1 = 100$

$\mu = \sigma^2 = 100 \qquad \sigma^2 = 100 \Rightarrow \sigma = \sqrt{100} = 10$

$E[W] + std[W] = 100 + 10 = 110$

(4) $P(W > 120) = 0.9819$

(5) 接受，因為在現實中時常發生不代表每一天都會發生，因此就常態分布的機率而言，平均一天會有一次火災是合理的，火災可能在某一段期間很集中，也可能一段時間內都未發生，從長遠來看可看作平均一天發一次火災。

[3]

(1) no more than 5% defective

(5) 接受，因為在現實中時常發生不代表每一天都會發生，因此就常態分布的机率而言，平均一天會有一次火災是合理的。火災可能在某一段期間很集中，也可能一段時間内都不發生，從長遠來看可看作平均一天發一次火災。

[3]

(1.) no more than 5% detective

$p(X=10) = \binom{100}{10}(0.05)^{10}(0.95)^{90}$

$= 1.6715 \times 10^{-2}$

(2) A buyer would suspect the claim isn't correct because assuming a correct claim, probability of having 10 detective item in sample is 1.6715×10^{-2} and event would occur only 1.6715% of time.