STAT 621 HW 7 GARCH

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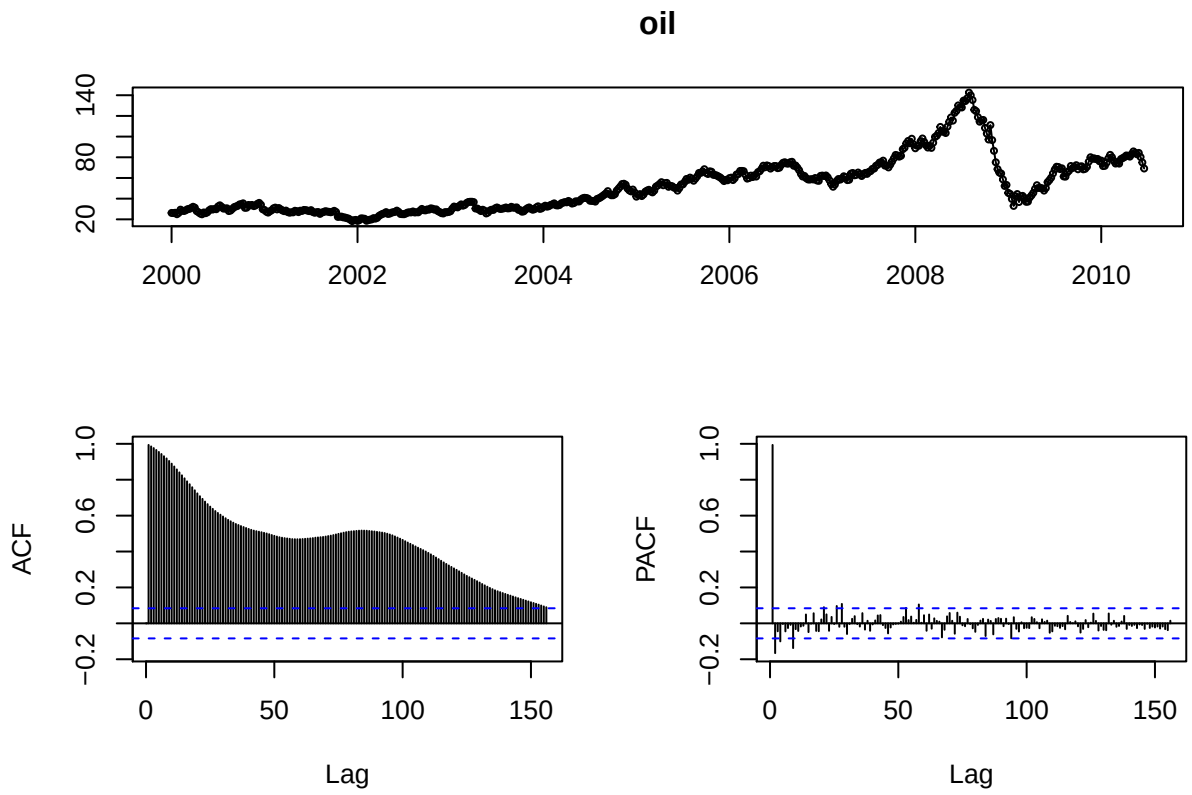
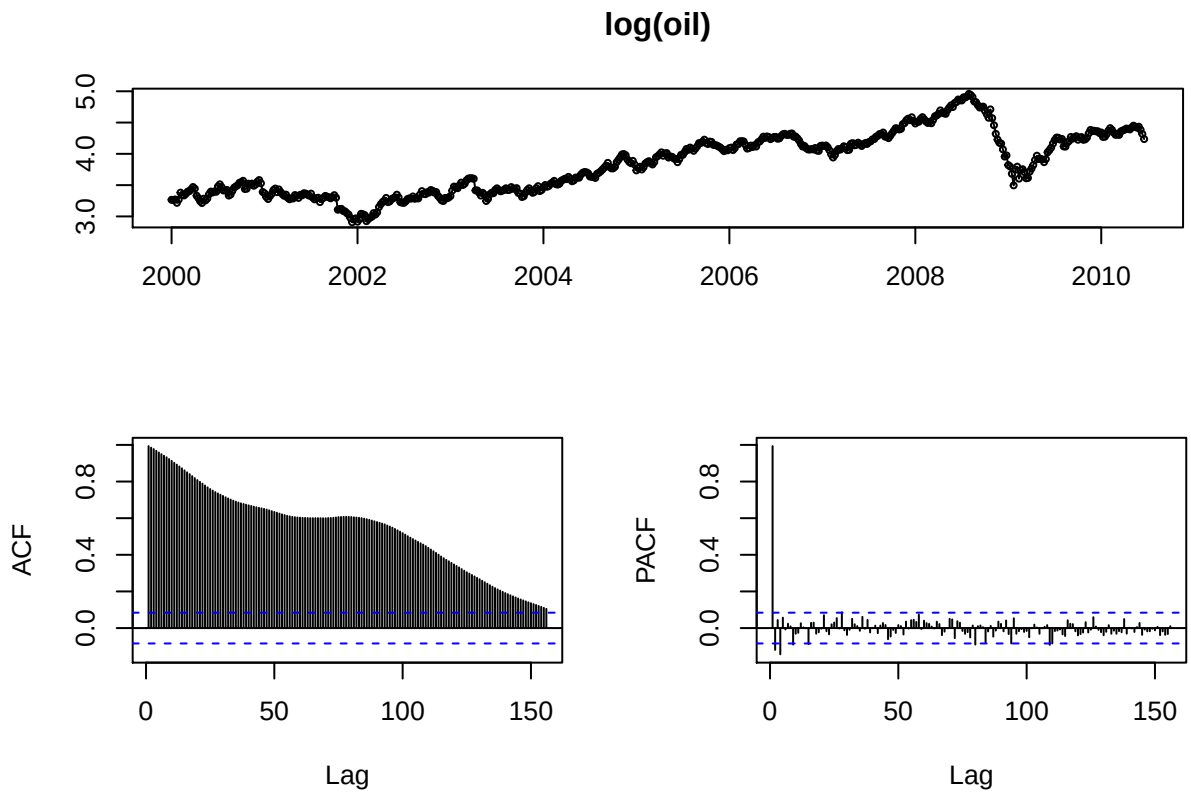
4/7/2018 (Due)

Q1

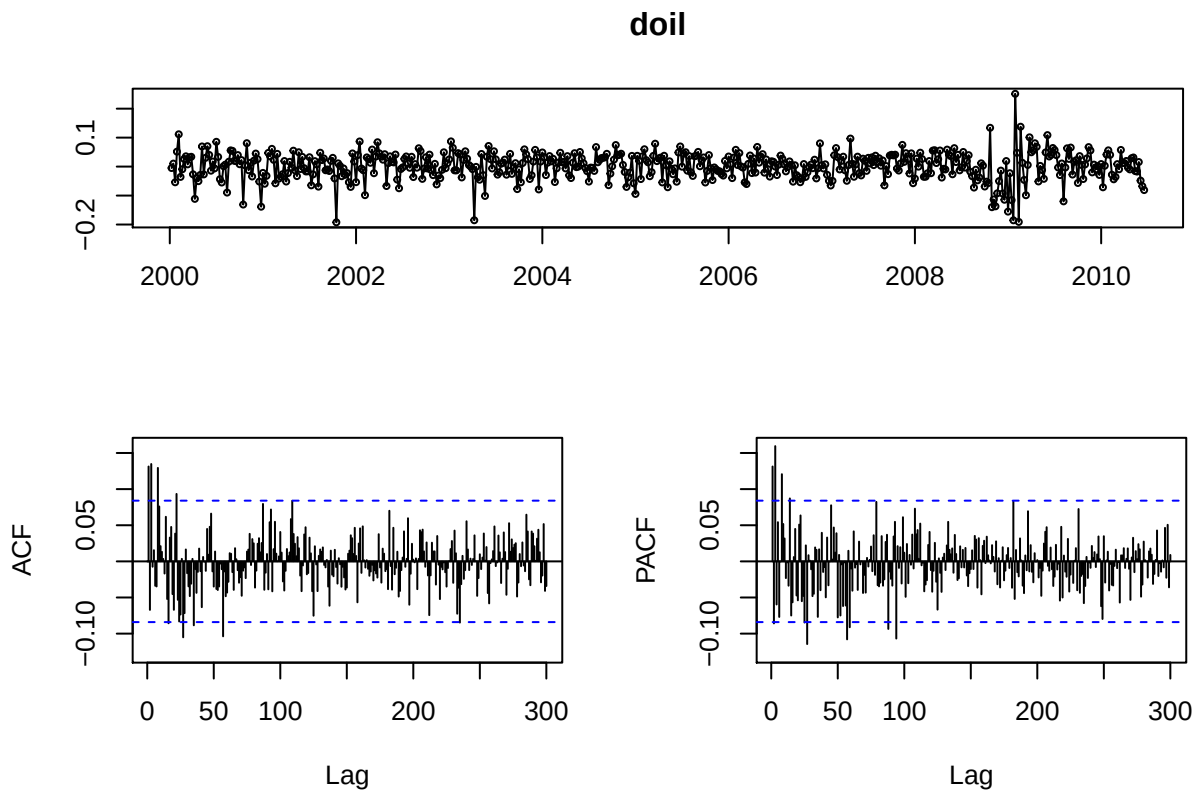
```
tsdisplay(oil)
```



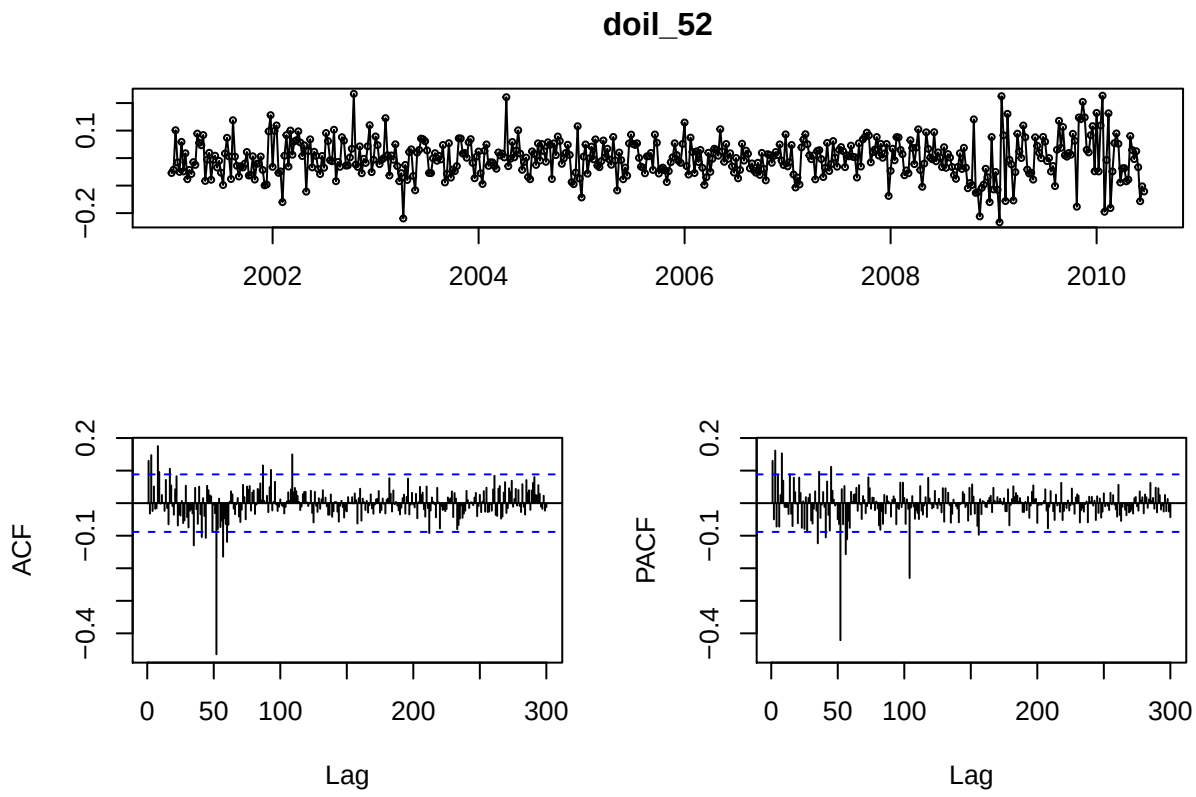

```
tsdisplay(log(oil))
```




```
doil=diff(log(oil))
tsdisplay(doil,lag.max = 300)
```




```
doil_52=diff(doil,52)    # Since it is weekly data
tsdisplay(doil_52,lag.max = 300)
```




```
auto.arima(log(oil))
```

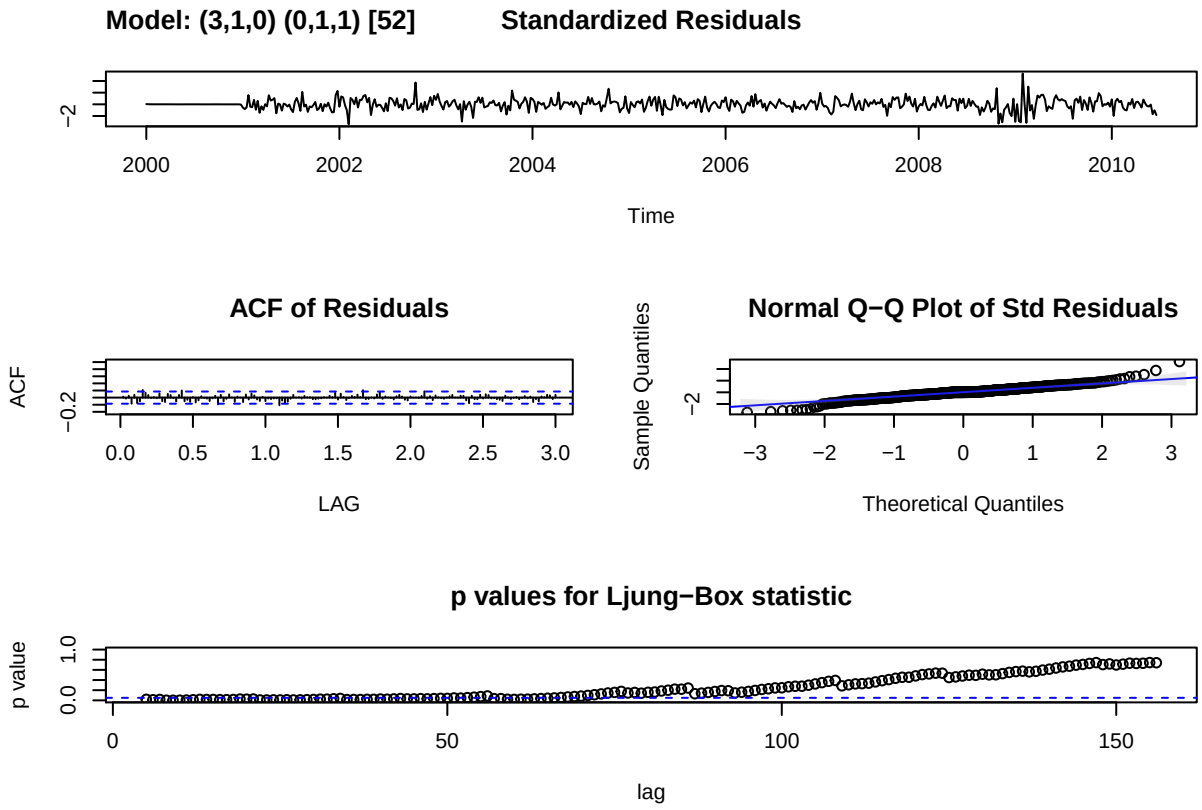
```
## Series: log(oil)
## ARIMA(3,1,0)(1,0,1)[52]
##
## Coefficients:
##          ar1      ar2     ar3    sar1     sma1
##       0.1586  -0.1105  0.1636  0.1446  -0.1952
## s.e.  0.0424   0.0428  0.0425  0.8856   0.8746
##
## sigma^2 estimated as 0.002111:  log likelihood=906.13
## AIC=-1800.27   AICc=-1800.11   BIC=-1774.48
```

Auto.Arima shows it might be a $[3,1,0][1,0,1][52]$ Sarima model.

```
sarima(log(oil),p=3,d=1,q=0,P=0,D=1,Q=1,S=52,xreg=1:545,no.constant=T, details=T)
```

```
## initial  value -2.690135
## iter   2 value -2.887661
## iter   3 value -2.920082
## iter   4 value -2.926528
## iter   5 value -2.930528
## iter   6 value -2.931395
## iter   7 value -2.931449
## iter   8 value -2.931449
## iter   8 value -2.931449
## iter   8 value -2.931449
## final  value -2.931449
```

```
## converged
## initial  value -2.946806
## iter   2 value -2.959379
## iter   3 value -2.960304
## iter   4 value -2.960508
## iter   5 value -2.960561
## iter   6 value -2.960575
## iter   7 value -2.960578
## iter   8 value -2.960578
## iter   8 value -2.960578
## iter   8 value -2.960578
## final  value -2.960578
## converged
```

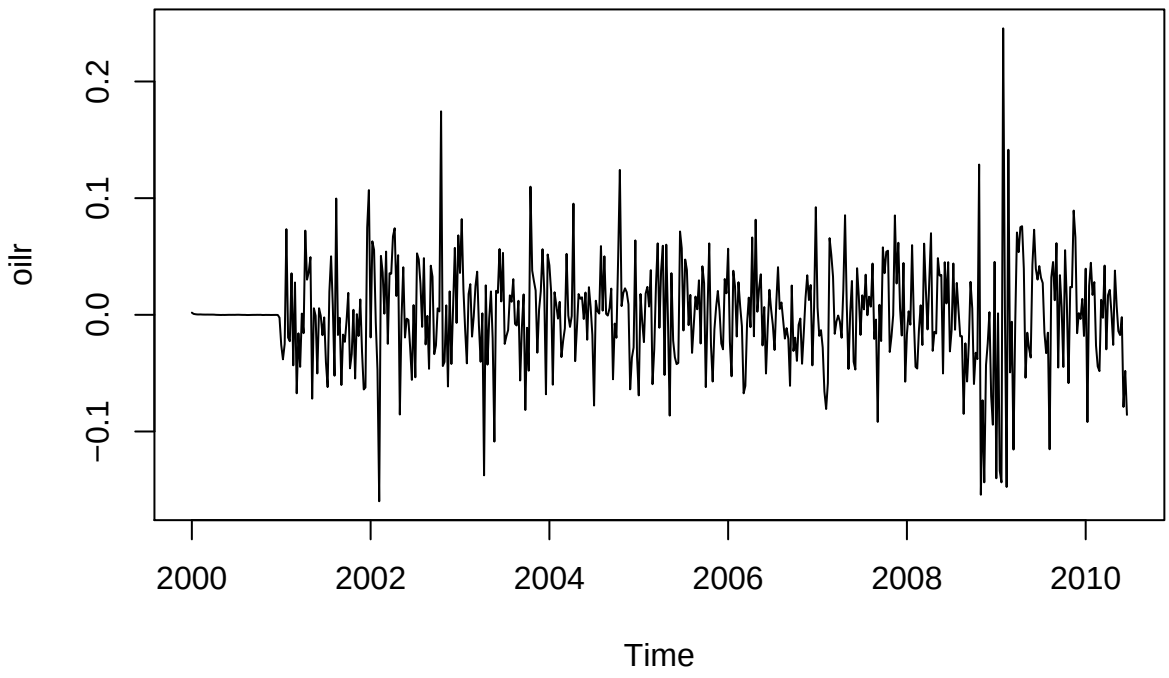



```
## $fit
##
## Call:
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
##     Q), period = S), xreg = xreg, optim.control = list(trace = trc, REPORT = 1,
##     reltol = tol))
##
## Coefficients:
##          ar1      ar2     ar3     sma1    xreg
##       0.1506  -0.0993  0.1535  -0.9604  0.0112
## s.e.  0.0447   0.0451  0.0448   0.2199  5.4448
##
```

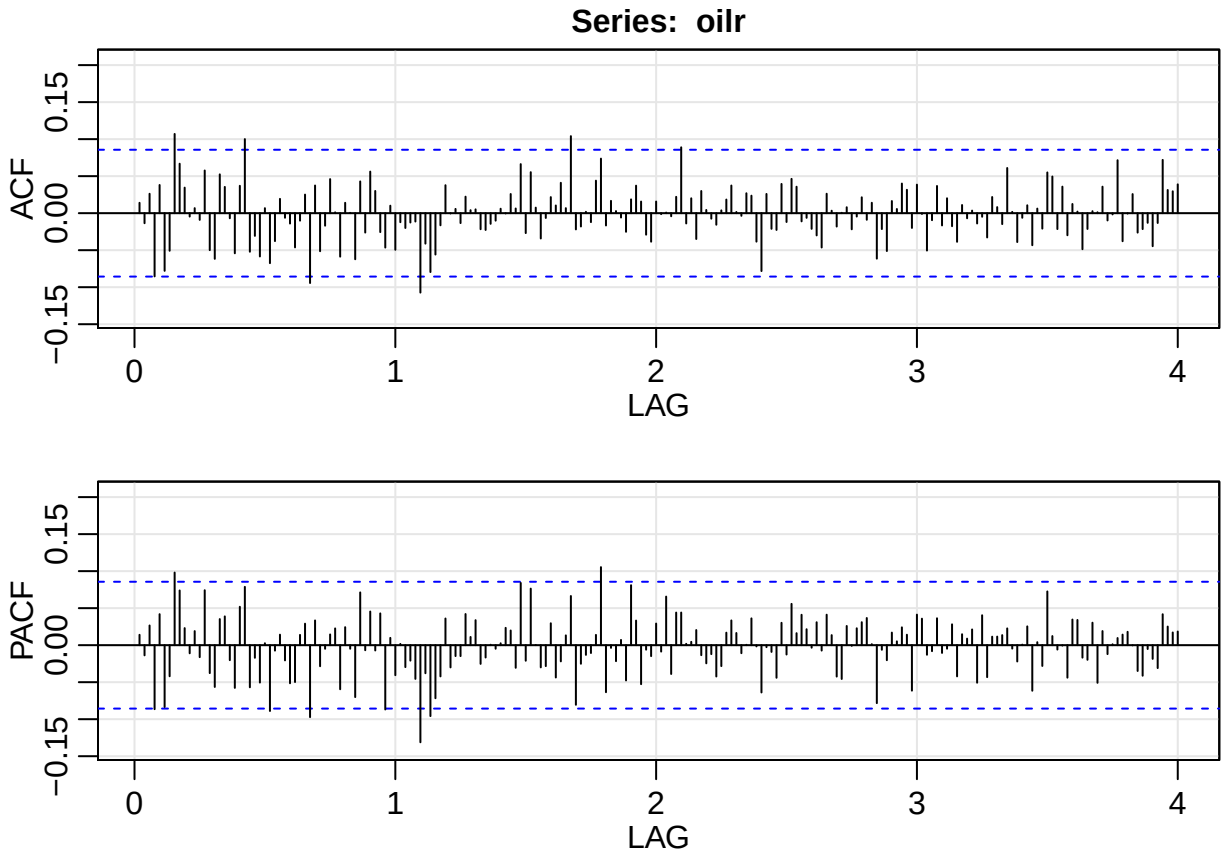
```
## sigma^2 estimated as 0.002172:  log likelihood = 758.49,  aic = -1504.97
##
## $degrees_of_freedom
## [1] 487
##
## $ttable
##      Estimate     SE t.value p.value
## ar1    0.1506 0.0447  3.3727  0.0008
## ar2   -0.0993 0.0451 -2.2022  0.0281
## ar3    0.1535 0.0448  3.4244  0.0007
## sma1  -0.9604 0.2199 -4.3674  0.0000
## xreg   0.0112 5.4448  0.0021  0.9984
##
## $AIC
## [1] -5.113762
##
## $AICc
## [1] -5.109806
##
## $BIC
## [1] -6.074305
```

The normal Q-Q Plot of Std Residuals suggests it might fit a (G)arch model.

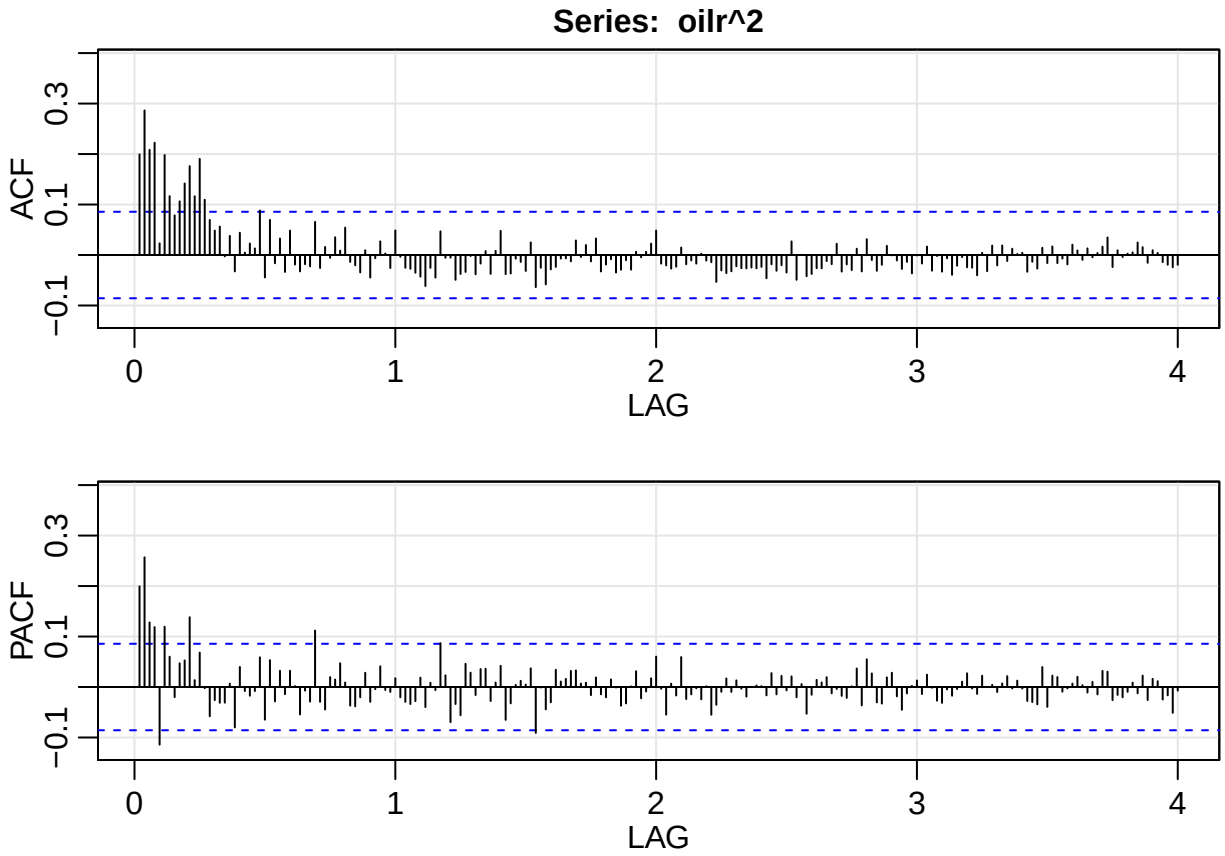
```
oilr <- resid(sarima(log(oil),p=3,d=1,q=0,P=0,D=1,Q=1,S=52,xreg=1:545,no.constant=T, details=F)$fit )
invisible(plot(oilr))              # plot residuals
```




```
invisible(acf2(oilr))             # P/ACF suggest noise
```




```
invisible(acf2(oilr^2))           # some correlation
```



The $residual^2$ ($oilr^2$) suggests it could be a GARCH(2,0) model.

```
summary(fit <- garchFit(~garch(2,0), data=oilr))  # alpha1 and alpha2 are significant
```

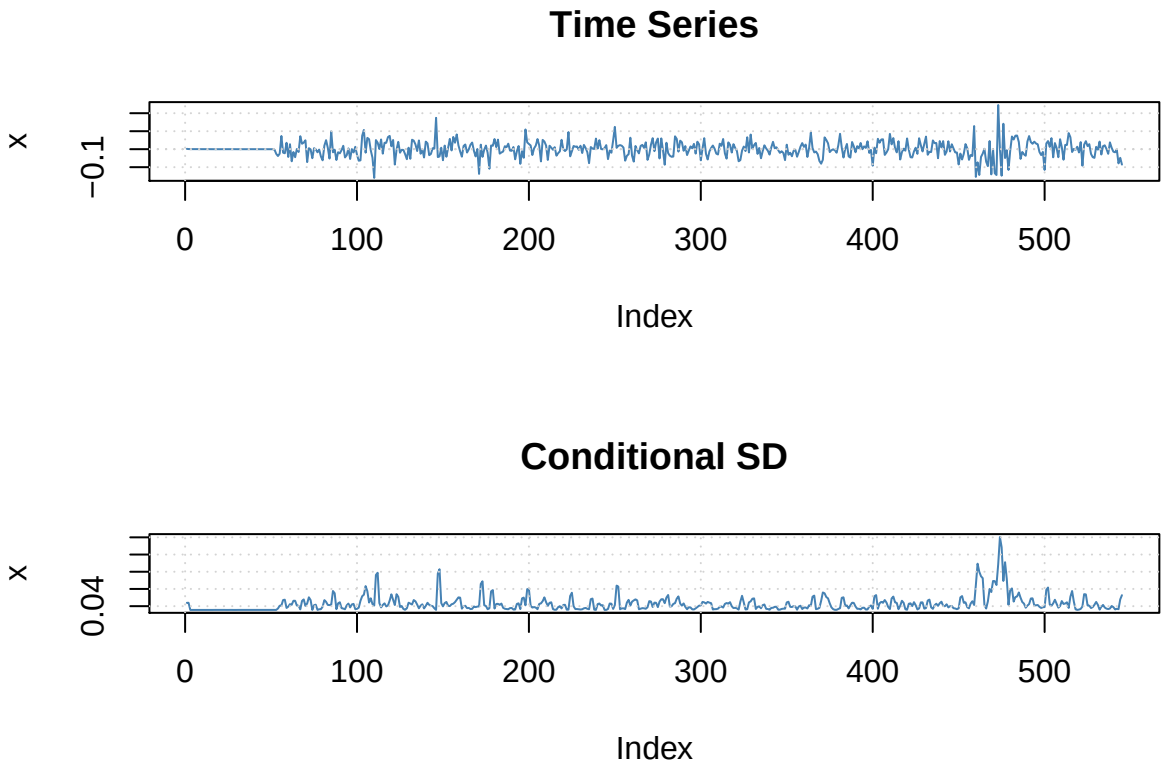
```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 0)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               2 0
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          545
##  Recursion Init:            mci
##  Series Scale:              0.04433526
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
```

```
##                     U            V      params includes
##     mu     -0.02603469   0.02603469 0.002603469     TRUE
##     omega   0.00000100 100.00000000 0.100000000     TRUE
##     alpha1  0.00000001   0.99999999 0.050000000     TRUE
##     alpha2  0.00000001   0.99999999 0.050000000     TRUE
##     gamma1 -0.99999999   0.99999999 0.100000000    FALSE
##     gamma2 -0.99999999   0.99999999 0.100000000    FALSE
##     delta   0.00000000   2.00000000 2.000000000    FALSE
##     skew    0.10000000  10.00000000 1.000000000    FALSE
##     shape   1.00000000  10.00000000 4.000000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1 alpha2
##      1      2      3      4
##  Persistence:                  0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     1482.7699: 0.00260347 0.100000 0.0500000 0.0500000
##   1:     779.90516: 0.00260355 0.945127 0.423843 0.432102
##   2:     771.88002: 0.00260648  1.20391 0.0910835 0.163212
##   3:     769.11388: 0.00260794  1.18734 0.119879 0.0733098
##   4:     762.74041: 0.00260800  1.09856 0.0867253 0.0589984
##   5:     747.87045: 0.00260830 0.725400 0.0620264 0.143409
##   6:     746.08512: 0.00260847 0.724503 0.154185 0.167333
##   7:     745.72588: 0.00261575 0.637501 0.158162 0.128948
##   8:     745.21441: 0.00273234 0.649692 0.150425 0.182793
##   9:     745.18880: 0.00277573 0.647807 0.165099 0.172259
##  10:     745.18374: 0.00283920 0.647803 0.161565 0.171343
##  11:     745.18082: 0.00299634 0.648850 0.159509 0.171564
##  12:     745.17489: 0.00341365 0.649680 0.157334 0.171856
##  13:     745.15506: 0.00508302 0.651496 0.152632 0.172594
##  14:     745.11271: 0.00961196 0.653843 0.145909 0.174083
##  15:     745.07001: 0.0141409 0.654312 0.143765 0.175124
##  16:     744.95071: 0.0260347 0.647161 0.157548 0.176288
##  17:     744.95002: 0.0260347 0.645883 0.159178 0.175968
##  18:     744.95000: 0.0260347 0.645819 0.159547 0.175910
##  19:     744.95000: 0.0260347 0.645801 0.159545 0.175923
##
## Final Estimate of the Negative LLH:
##  LLH:  -953.2563    norm LLH:  -1.749094
##          mu       omega      alpha1      alpha2
## 0.001154255 0.001269397 0.159544520 0.175923064
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu         omega      alpha1      alpha2
## mu     -335579.5811    -170739.51   -248.8301    908.3523
## omega  -170739.5058 -103232438.07 -92587.0529 -90981.7440
## alpha1    -248.8301     -92587.05   -404.7351   -119.1043
## alpha2     908.3523     -90981.74   -119.1043   -356.4917
## attr(,"time")
```

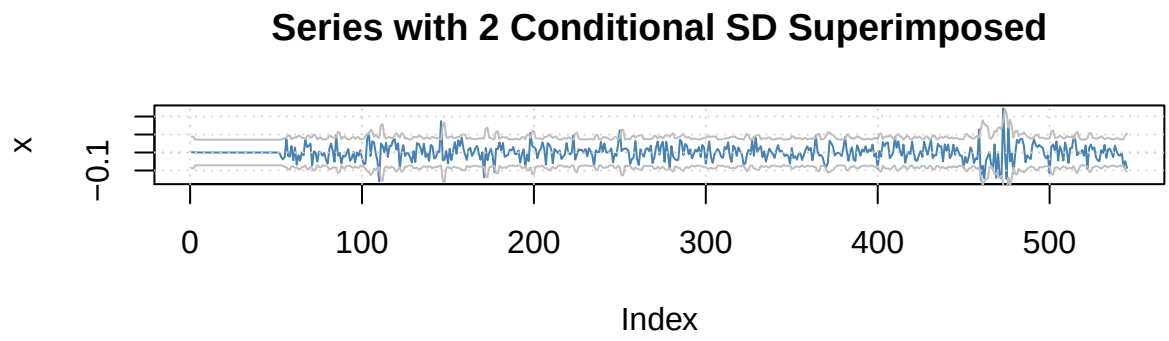
```
## Time difference of 0.02005816 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.06942415 secs
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 0), data = oilr)
##
## Mean and Variance Equation:
##  data ~ garch(2, 0)
## <environment: 0x000000001e48ce30>
##  [data = oilr]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##        mu      omega     alpha1     alpha2
## 0.0011543  0.0012694  0.1595445  0.1759231
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0011543   0.0017380    0.664  0.50661
## omega  0.0012694   0.0001203   10.554  < 2e-16 ***
## alpha1 0.1595445   0.0562160    2.838  0.00454 **
## alpha2 0.1759231   0.0610256    2.883  0.00394 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  953.2563    normalized:  1.749094
##
## Description:
##  Sat Apr 07 10:34:35 2018 by user: yydab
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  66.44128  3.774758e-15
##  Shapiro-Wilk Test  R    W      0.9814722 2.081421e-06
##  Ljung-Box Test     R    Q(10)  14.81414  0.1389862
##  Ljung-Box Test     R    Q(15)  19.72929  0.1825653
##  Ljung-Box Test     R    Q(20)  23.57797  0.2613134
##  Ljung-Box Test     R^2  Q(10)  19.19412  0.03786519
##  Ljung-Box Test     R^2  Q(15)  22.86526  0.0870429
```

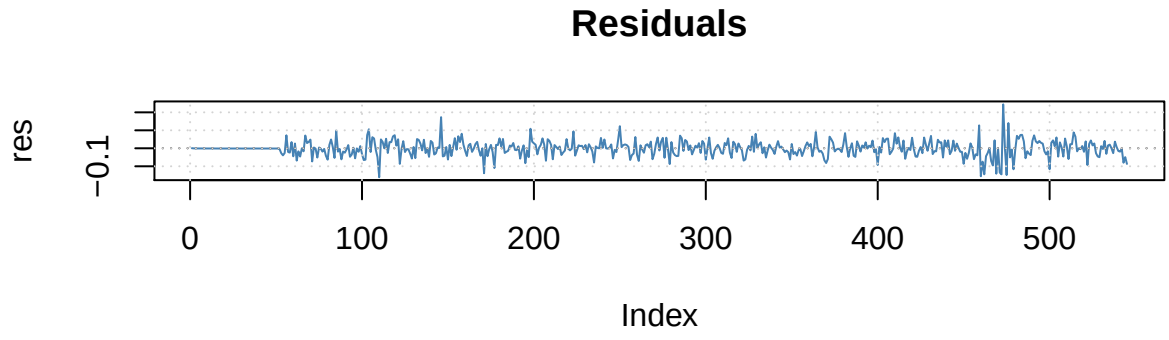
```
##  Ljung-Box Test       R^2  Q(20)  25.29213  0.1904827
##  LM Arch Test         R    TR^2   18.70875  0.0958038
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.483510 -3.451944 -3.483616 -3.471169
```

```
par(mfrow=2:1)
plot(fit, which=1:2)    # plot data and root volatility
```




```
plot(fit, which=c(3,7)) # Series with 2 Conditonal SD; Standardized Residuals
```



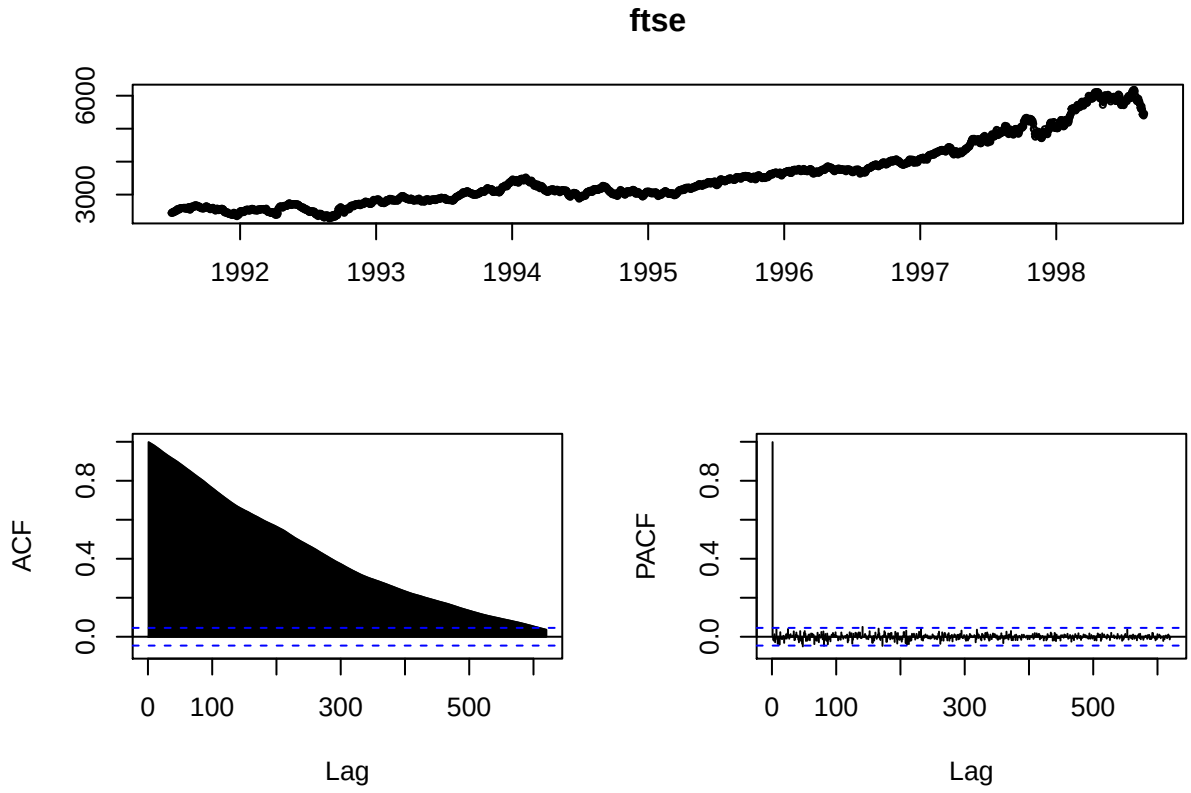


Q1 Conclusion:

First I use auto.arima to detect the model and then fit the sarima model. The normal Q-Q Plot of Std Residuals suggests it might fit a (G)arch model. After trying different garch combination, Garch(2,0) fit well. Both alpha1 and alpha2 are significant. I suggest it is a $SARIMA[3,1,0] \times [0,1,1]_{52} + GARCH(2,0)$ model.

Q2 Eu Stock Markets

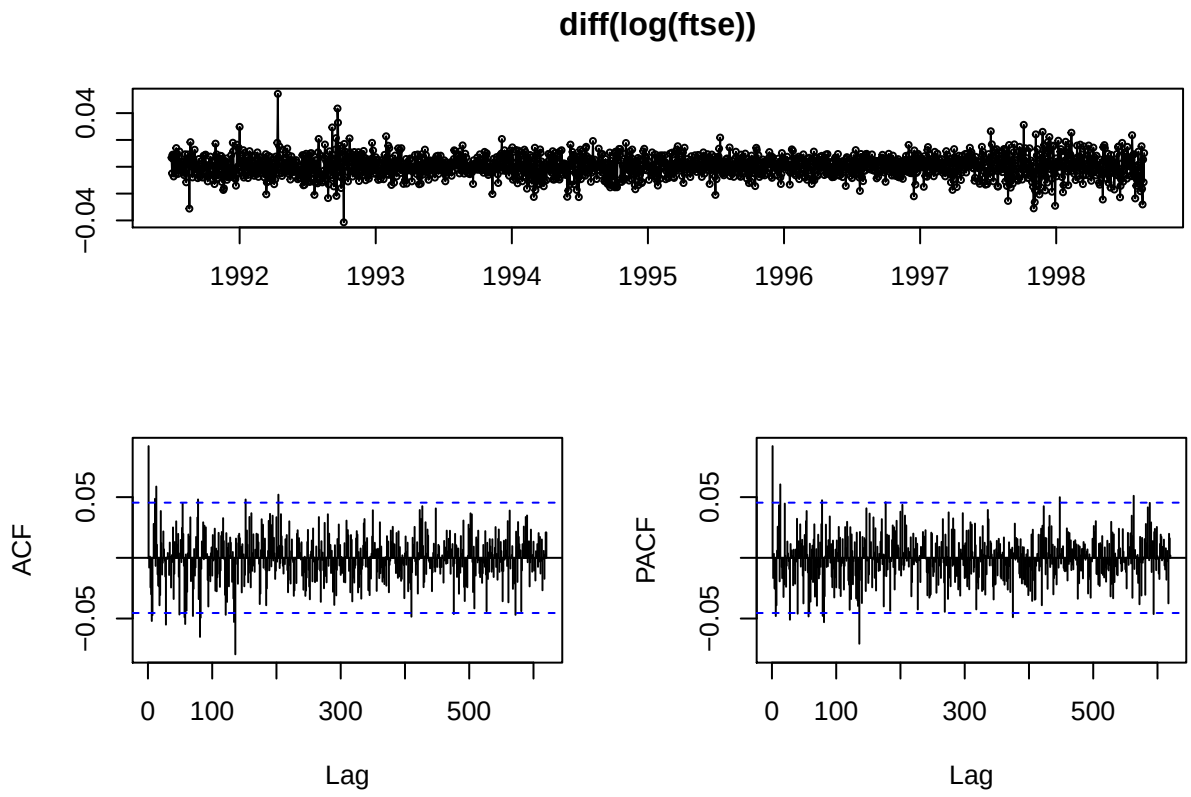
```
ftse<-EuStockMarkets[, "FTSE"]
tsdisplay(ftse)
```



Transfer the data as return

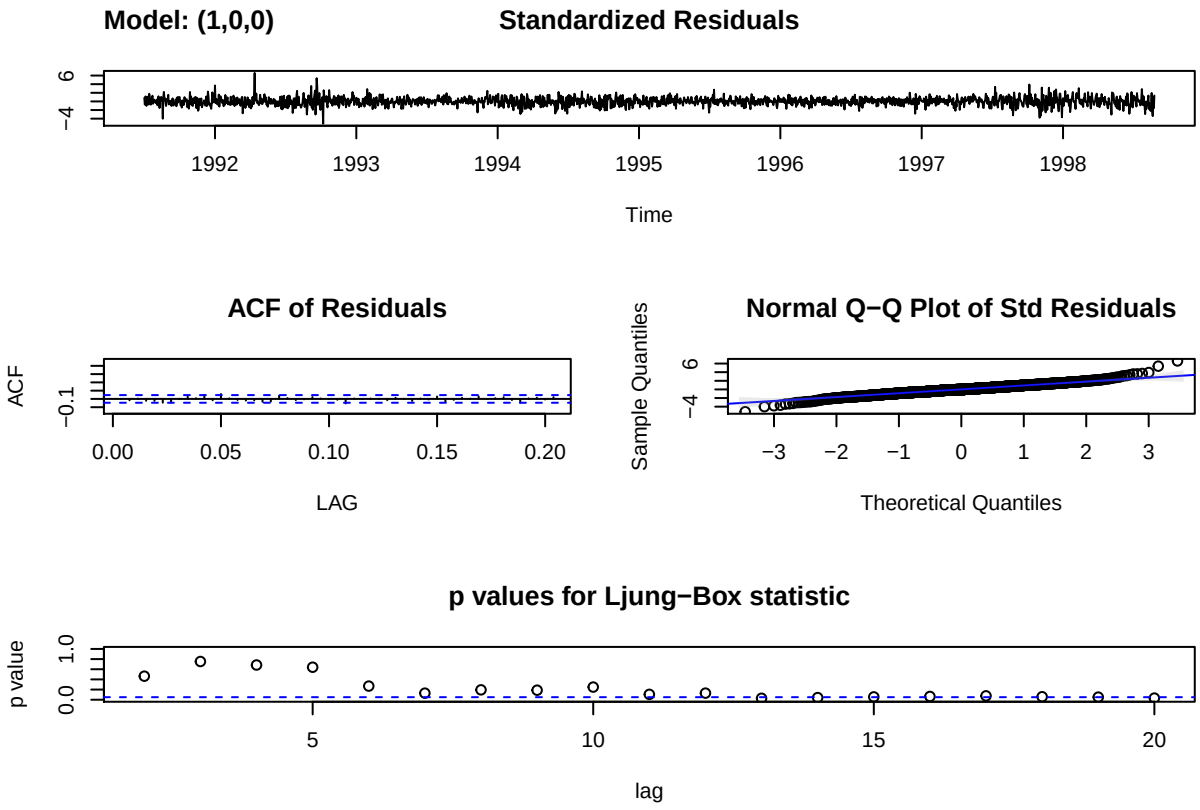
$r_t = \frac{x_t - x_{t-1}}{x_{t-1}} \approx log(r_t) = log(\frac{x_t}{x_{t-1}})$, so we gan use $diff(log(data))$ as a proxy of growth rate.

```
tsdisplay(diff(log(ftse)))
```




```
sarima(diff(log(ftse)),1,0,0)
```

```
## initial  value -4.833783
## iter   2 value -4.838040
## iter   2 value -4.838040
## iter   2 value -4.838040
## final  value -4.838040
## converged
## initial  value -4.838136
## iter   1 value -4.838136
## final  value -4.838136
## converged
```

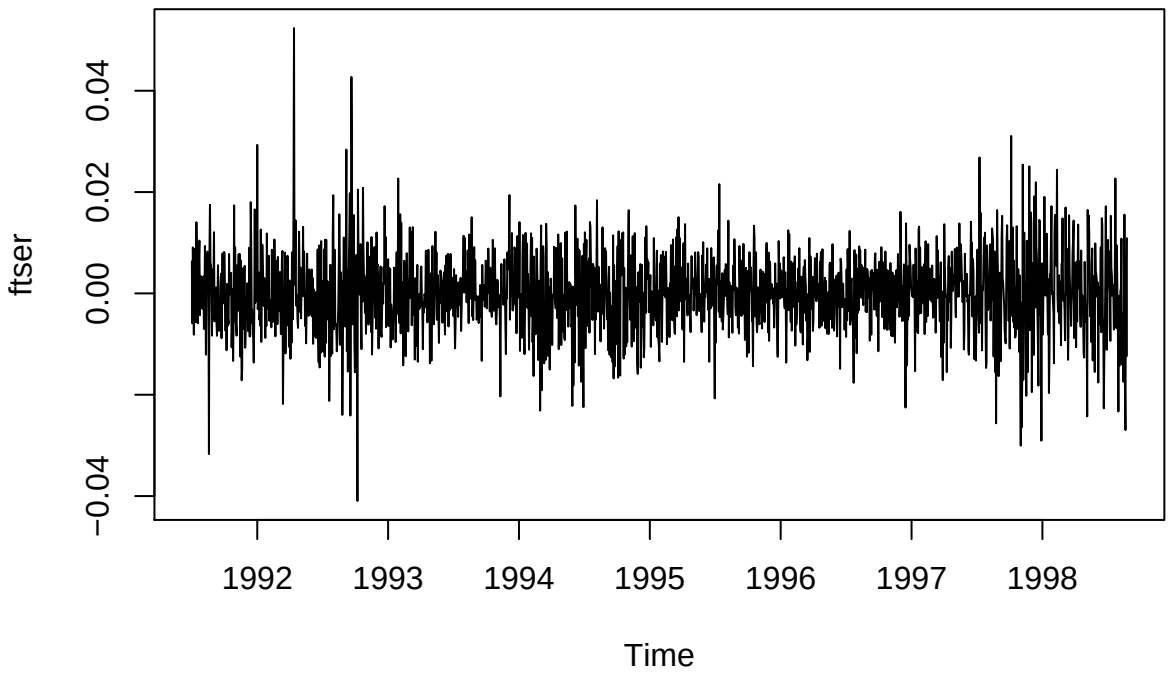



```
## $fit
##
## Call:
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
##     Q), period = S), xreg = xmean, include.mean = FALSE, optim.control = list(trace = trc,
##     REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1  xmean
##       0.0921  4e-04
## s.e.  0.0231  2e-04
##
## sigma^2 estimated as 6.275e-05:  log likelihood = 6356.29,  aic = -12706.58
##
## $degrees_of_freedom
## [1] 1857
##
## $ttable
##       Estimate     SE t.value p.value
## ar1     0.0921 0.0231  3.9865  0.0001
## xmean   0.0004 0.0002  2.1190  0.0342
##
## $AIC
## [1] -8.674125
##
## $AICc
```

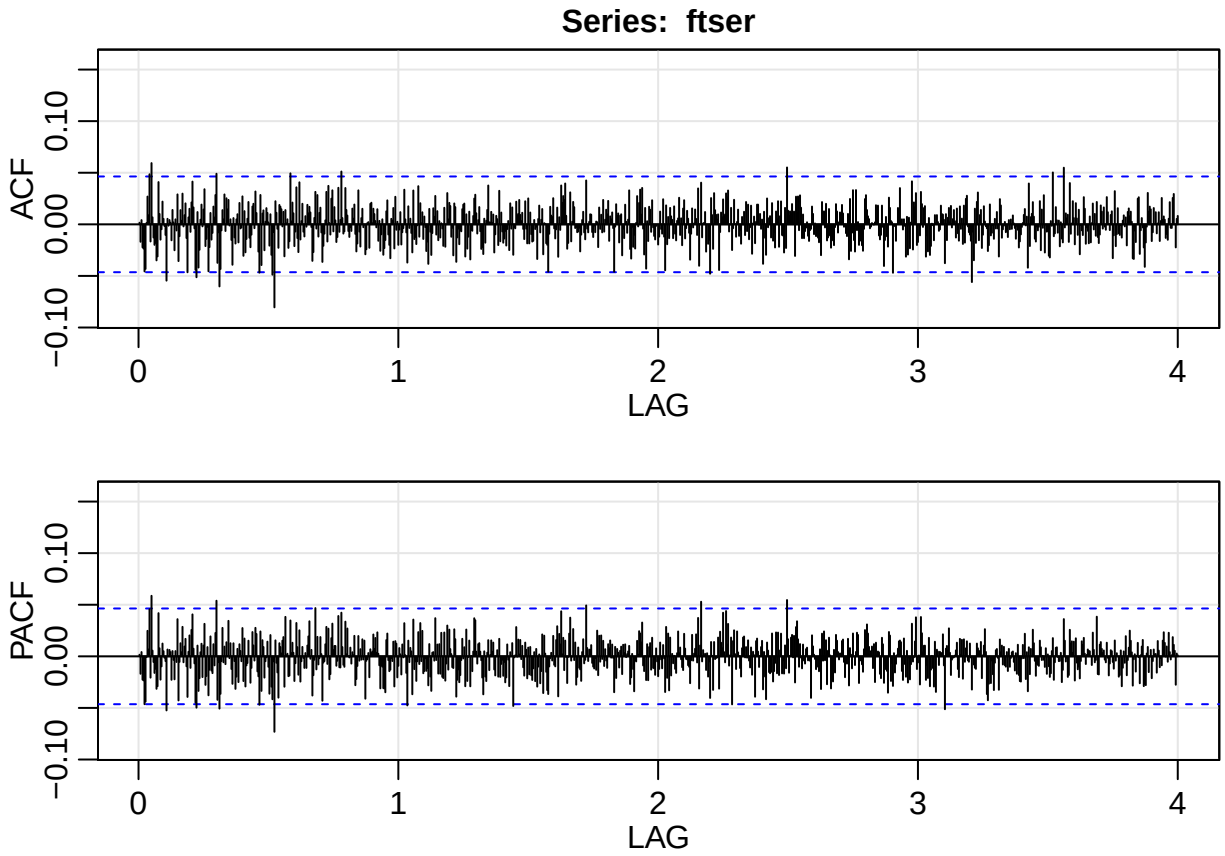
```
## [1] -8.673042
##
## $BIC
## [1] -9.668178
```

Get the residuals from an AR(1) fit

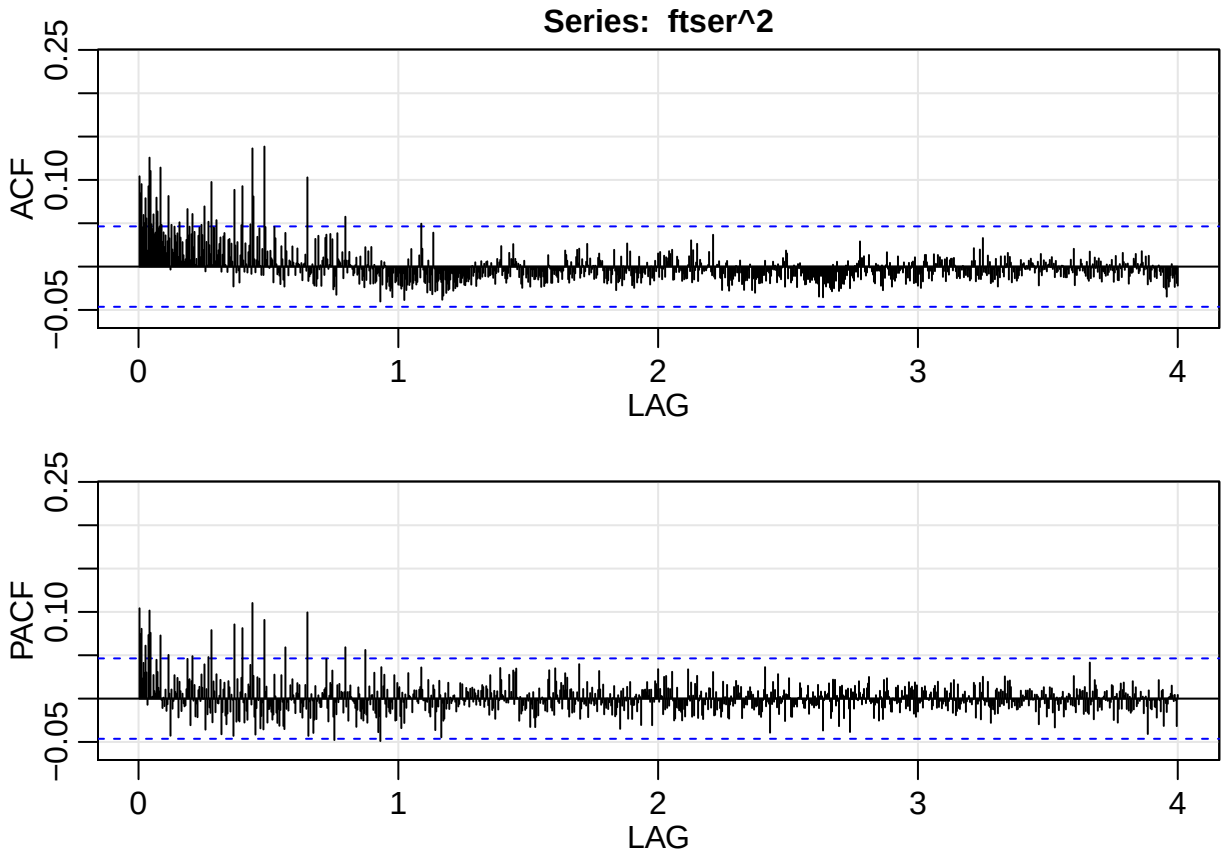
```
ftser <- resid(sarima(diff(log(ftse)), 1,0,0, details=FALSE)$fit )

# Looking at residuals to growth rate
invisible(plot(ftser))              # plot residuals
```




```
invisible(acf2(ftser))              # P/ACF suggest noise
```




```
invisible(acf2(ftser^2))           # some correlation
```




```
# Let's try fitting an ARCH model
summary(fit <- garchFit(~garch(1,1), data=ftser))
```

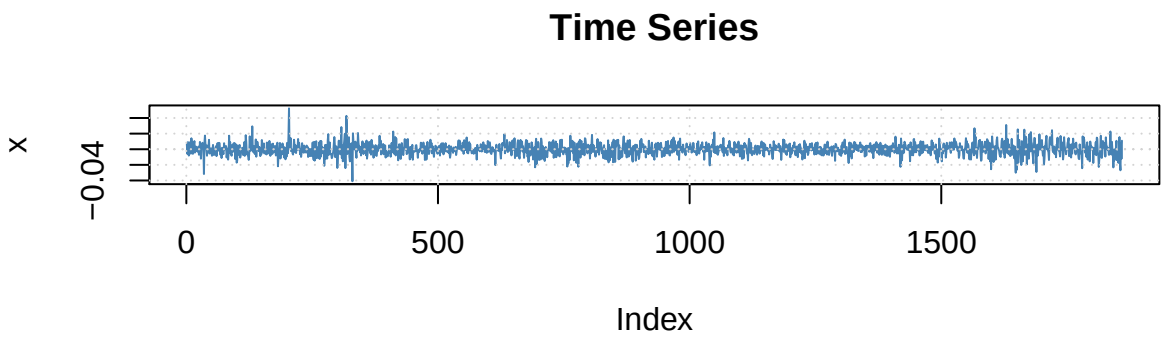
```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(0, 0)
##  GARCH Model:                garch
##  Formula Variance:           ~ garch(1, 1)
##  ARMA Order:                 0 0
##  Max ARMA Order:             0
##  GARCH Order:                1 1
##  Max GARCH Order:            1
##  Maximum Order:              1
##  Conditional Dist:           norm
##  h.start:                    2
##  llh.start:                  1
##  Length of Series:           1859
##  Recursion Init:             mci
##  Series Scale:               0.007923918
##
## Parameter Initialization:
##  Initial Parameters:           $params
##  Limits of Transformations:    $U, $V
##  Which Parameters are Fixed?   $includes
##  Parameter Matrix:
```

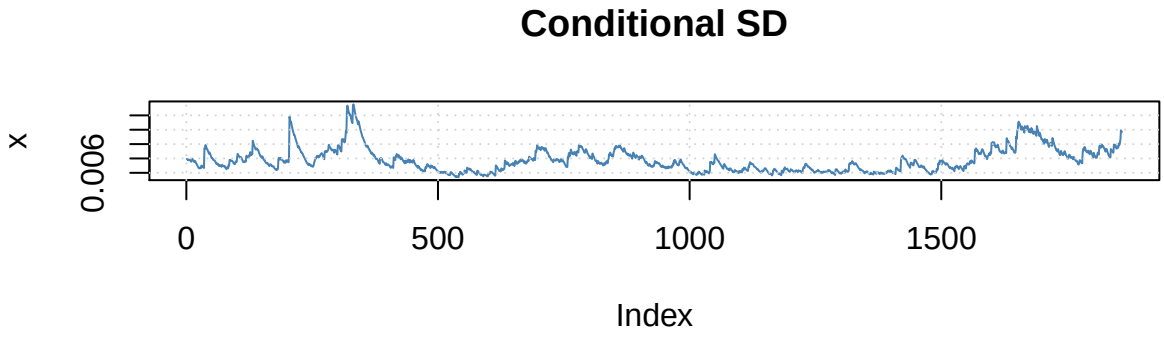
```
##                       U          V     params includes
##     mu     -0.00104750 1.0475e-03 0.00010475     TRUE
##     omega   0.00000100 1.0000e+02 0.10000000     TRUE
##     alpha1  0.00000001 1.0000e+00 0.10000000     TRUE
##     gamma1 -0.99999999 1.0000e+00 0.10000000    FALSE
##     beta1   0.00000001 1.0000e+00 0.80000000     TRUE
##     delta   0.00000000 2.0000e+00 2.00000000    FALSE
##     skew    0.10000000 1.0000e+01 1.00000000    FALSE
##     shape   1.00000000 1.0000e+01 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1
##      1      2      3      5
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     2574.3908: 0.000104750 0.100000 0.100000 0.800000
##   1:     2573.6072: 0.000104750 0.0944825 0.104466 0.807772
##   2:     2572.5218: 0.000104750 0.0844104 0.104407 0.810830
##   3:     2571.5614: 0.000104750 0.0791065 0.107162 0.819495
##   4:     2569.9584: 0.000104750 0.0630579 0.105506 0.833019
##   5:     2565.1180: 0.000104750 0.0452323 0.0814825 0.875602
##   6:     2565.0303: 0.000104750 0.0439424 0.0808264 0.876184
##   7:     2564.9267: 0.000104751 0.0443359 0.0803625 0.877620
##   8:     2564.8015: 0.000104754 0.0436324 0.0776741 0.879037
##   9:     2564.7338: 0.000104757 0.0446117 0.0763941 0.881014
##  10:     2564.6639: 0.000104752 0.0417001 0.0804014 0.879793
##  11:     2564.4843: 0.000104752 0.0403824 0.0793960 0.884618
##  12:     2563.9799: 0.000104757 0.0387602 0.0752936 0.887181
##  13:     2563.5540: 0.000104766 0.0377754 0.0690680 0.895204
##  14:     2563.5508: 0.000104770 0.0360319 0.0667081 0.895864
##  15:     2563.3094: 0.000104778 0.0364159 0.0671538 0.897245
##  16:     2563.1482: 0.000104793 0.0336686 0.0674558 0.898423
##  17:     2562.8796: 0.000104816 0.0335808 0.0637604 0.903162
##  18:     2562.4372: 0.000104902 0.0263033 0.0602135 0.912023
##  19:     2562.2042: 0.000104926 0.0256757 0.0652918 0.911840
##  20:     2561.9950: 0.000104934 0.0264412 0.0608002 0.914187
##  21:     2561.9726: 0.000104934 0.0259773 0.0606038 0.914092
##  22:     2561.9455: 0.000104939 0.0259180 0.0606823 0.914593
##  23:     2561.8998: 0.000104954 0.0252346 0.0602931 0.915235
##  24:     2561.1735: 0.000105488 0.0155976 0.0471259 0.937782
##  25:     2561.1712: 0.000105550 0.0147333 0.0481634 0.938394
##  26:     2561.1567: 0.000105585 0.0146077 0.0475630 0.938782
##  27:     2561.1502: 0.000105615 0.0147452 0.0469817 0.939222
##  28:     2561.1472: 0.000105795 0.0142323 0.0460085 0.940641
##  29:     2561.1472: 0.000105969 0.0142799 0.0460050 0.940580
##  30:     2561.1472: 0.000106108 0.0142711 0.0460065 0.940593
##  31:     2561.1472: 0.000106367 0.0142681 0.0460077 0.940597
##  32:     2561.1472: 0.000108803 0.0142542 0.0460129 0.940613
##  33:     2561.1471: 0.000114120 0.0142385 0.0460189 0.940632
```

```
##  34:     2561.1470: 0.000129948 0.0142117 0.0460296 0.940664
##  35:     2561.1466: 0.000174644 0.0141665 0.0460488 0.940716
##  36:     2561.1457: 0.000289302 0.0140982 0.0460808 0.940793
##  37:     2561.1434: 0.000575009 0.0140010 0.0461342 0.940897
##  38:     2561.1366: 0.00104750 0.0140138 0.0461571 0.940861
##  39:     2561.1334: 0.00104750 0.0142079 0.0460331 0.940665
##  40:     2561.1332: 0.00104750 0.0142621 0.0459968 0.940612
##  41:     2561.1332: 0.00104750 0.0142655 0.0460024 0.940603
##
## Final Estimate of the Negative LLH:
##  LLH:  -6432.466    norm LLH:  -3.460175
##           mu        omega       alpha1        beta1
## 8.300307e-06 8.957092e-07 4.600240e-02 9.406031e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         omega        alpha1         beta1
## mu     -3.589849e+07 -1.360249e+08 -4.894673e+02  5.080255e+01
## omega  -1.360249e+08 -1.012940e+14 -4.264539e+09 -5.089170e+09
## alpha1 -4.894673e+02 -4.264539e+09 -2.292469e+05 -2.457651e+05
## beta1   5.080255e+01 -5.089170e+09 -2.457651e+05 -2.789727e+05
## attr(,"time")
## Time difference of 0.03125501 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.211838 secs
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = ftser)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001f930818>
##  [data = ftser]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 8.3003e-06  8.9571e-07  4.6002e-02  9.4060e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     8.300e-06   1.669e-04    0.050  0.96034
## omega  8.957e-07   4.616e-07    1.941  0.05231 .
```

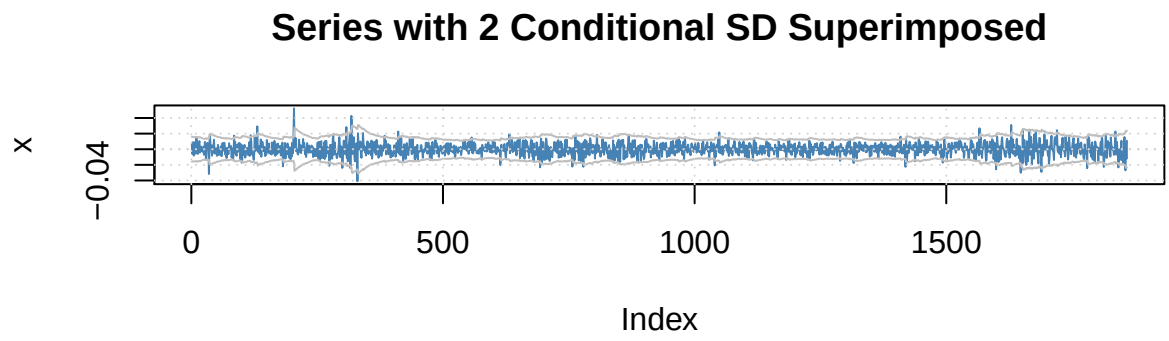
```
## alpha1 4.600e-02   1.189e-02    3.868  0.00011 ***
## beta1  9.406e-01   1.737e-02   54.135  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  6432.466    normalized:  3.460175
##
## Description:
##  Sat Apr 07 10:34:36 2018 by user: yydab
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  175.0399  0
##  Shapiro-Wilk Test  R    W      0.9901225 6.229921e-10
##  Ljung-Box Test     R    Q(10)  7.392143  0.6879777
##  Ljung-Box Test     R    Q(15)  16.34177  0.359708
##  Ljung-Box Test     R    Q(20)  23.31753  0.2734735
##  Ljung-Box Test     R^2  Q(10)  4.415861  0.9266432
##  Ljung-Box Test     R^2  Q(15)  8.447815  0.904578
##  Ljung-Box Test     R^2  Q(20)  11.53827  0.9310524
##  LM Arch Test       R    TR^2   6.353908  0.8972016
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -6.916047 -6.904153 -6.916057 -6.911664
```

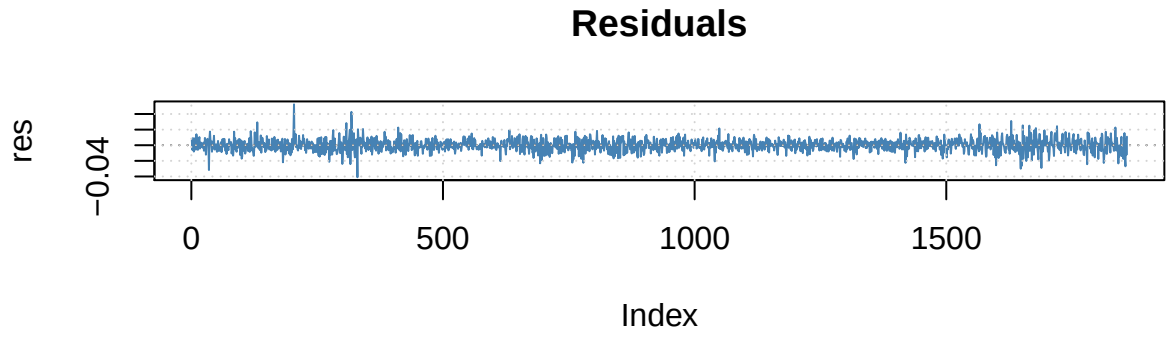
```
# alpha1 and beta1 are significant. I also tried other different combinations but Garch(1,1) is the bes

par(mfrow=2:1)
plot(fit, which=1:2)    # plot data and root volatility
```






```
plot(fit, which=c(3,7)) # Series with 2 Conditonal SD; Standardized Residuals
```






```
# ARMA with ARCH errors
ftsedl <- diff(log(ftse))
fit2<- garchFit(~arma(1,0)+garch(1,1), data=ftsedl)
```

```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(1, 0)
##  GARCH Model:                garch
##  Formula Variance:           ~ garch(1, 1)
##  ARMA Order:                 1 0
##  Max ARMA Order:             1
##  GARCH Order:                1 1
##  Max GARCH Order:            1
##  Maximum Order:              1
##  Conditional Dist:           norm
##  h.start:                    2
##  llh.start:                  1
##  Length of Series:           1859
##  Recursion Init:             mci
##  Series Scale:               0.007957728
##
## Parameter Initialization:
##  Initial Parameters:           $params
##  Limits of Transformations:    $U, $V
##  Which Parameters are Fixed?   $includes
```

```
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.54284978   0.5428498 0.05439580     TRUE
##     ar1    -0.99999999   1.0000000 0.09208644     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1   0.00000001   1.0000000 0.80000000     TRUE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1  omega alpha1  beta1
##      1      2      3      4      6
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     2566.1950: 0.0543958 0.0920864 0.100000 0.100000 0.800000
##   1:     2566.0010: 0.0543899 0.0959064 0.0318122 0.125884 0.850195
##   2:     2564.3364: 0.0543899 0.0958215 0.0365554 0.126030 0.853242
##   3:     2563.4066: 0.0543900 0.0956233 0.0364293 0.120600 0.851734
##   4:     2561.7230: 0.0543904 0.0950928 0.0461914 0.114993 0.851255
##   5:     2560.3734: 0.0543910 0.0938305 0.0490588 0.0927912 0.853762
##   6:     2557.5021: 0.0543902 0.0934531 0.0439034 0.0875225 0.875082
##   7:     2555.4819: 0.0543896 0.0930316 0.0317217 0.0785271 0.891801
##   8:     2554.4767: 0.0543900 0.0919706 0.0231445 0.0703703 0.910979
##   9:     2554.1159: 0.0543978 0.0876211 0.0193378 0.0534546 0.924743
##  10:     2553.9889: 0.0544040 0.0853049 0.0232704 0.0517306 0.927731
##  11:     2553.0974: 0.0544101 0.0852014 0.0204800 0.0523369 0.927826
##  12:     2552.9963: 0.0544161 0.0865829 0.0185735 0.0527493 0.929394
##  13:     2552.9935: 0.0544161 0.0865826 0.0186155 0.0527925 0.929499
##  14:     2552.9916: 0.0544162 0.0865817 0.0185137 0.0527372 0.929533
##  15:     2552.9864: 0.0544172 0.0865811 0.0184903 0.0526643 0.929761
##  16:     2552.9035: 0.0544482 0.0865807 0.0171050 0.0495520 0.933884
##  17:     2552.9000: 0.0544482 0.0865796 0.0171353 0.0496185 0.933967
##  18:     2552.8977: 0.0544482 0.0865771 0.0170297 0.0496299 0.933996
##  19:     2552.8957: 0.0544490 0.0864216 0.0169992 0.0497016 0.934130
##  20:     2552.8925: 0.0544513 0.0860048 0.0168762 0.0496536 0.934164
##  21:     2552.8899: 0.0544595 0.0851692 0.0167956 0.0496669 0.934385
##  22:     2552.8810: 0.0545216 0.0864312 0.0164811 0.0495887 0.934702
##  23:     2552.8763: 0.0545668 0.0849468 0.0163000 0.0494720 0.935104
##  24:     2552.8758: 0.0545668 0.0849518 0.0162466 0.0494340 0.935096
##  25:     2552.8752: 0.0545683 0.0849917 0.0162586 0.0494441 0.935139
##  26:     2552.8745: 0.0545717 0.0850821 0.0162045 0.0494059 0.935170
##  27:     2552.8735: 0.0545811 0.0852748 0.0162106 0.0494125 0.935226
##  28:     2552.8722: 0.0546055 0.0855368 0.0161450 0.0493602 0.935281
##  29:     2552.8553: 0.0557378 0.0821543 0.0150985 0.0479871 0.937683
##  30:     2552.8383: 0.0568791 0.0847432 0.0142464 0.0469280 0.939855
##  31:     2552.8306: 0.0569453 0.0853137 0.0141008 0.0458495 0.940710
```

```
##  32:     2552.8295: 0.0568405 0.0853181 0.0141379 0.0458615 0.940783
##  33:     2552.8292: 0.0567357 0.0853385 0.0141025 0.0458499 0.940803
##  34:     2552.8290: 0.0565261 0.0853918 0.0140620 0.0458481 0.940857
##  35:     2552.8289: 0.0563637 0.0856447 0.0140951 0.0458991 0.940763
##  36:     2552.8289: 0.0563906 0.0856440 0.0140908 0.0459091 0.940761
##  37:     2552.8289: 0.0563930 0.0855965 0.0140854 0.0458900 0.940788
##  38:     2552.8289: 0.0563935 0.0856158 0.0140874 0.0458984 0.940776
##  39:     2552.8289: 0.0563934 0.0856160 0.0140874 0.0458984 0.940776
##
## Final Estimate of the Negative LLH:
##  LLH:  -6432.855    norm LLH:  -3.460385
##           mu          ar1        omega       alpha1        beta1
## 4.487632e-04 8.561600e-02 8.920910e-07 4.589840e-02 9.407760e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu           ar1         omega        alpha1
## mu     -35886704.462   -15873.9144  4.286244e+07  5.265352e+03
## ar1       -15873.914    -1744.9895 -2.793655e+06 -2.026047e+02
## omega   42862440.966 -2793654.9294 -1.019452e+14 -4.287758e+09
## alpha1      5265.352     -202.6047 -4.287758e+09 -2.304051e+05
## beta1       8187.296     -306.7414 -5.118538e+09 -2.470751e+05
##                beta1
## mu      8.187296e+03
## ar1    -3.067414e+02
## omega  -5.118538e+09
## alpha1 -2.470751e+05
## beta1  -2.805122e+05
## attr(,"time")
## Time difference of 0.04688382 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.346226 secs
```

```
summary(fit2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = ftsedl)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000001a213a90>
##  [data = ftsedl]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu         ar1       omega      alpha1       beta1
```

```
## 4.4876e-04  8.5616e-02  8.9209e-07  4.5898e-02  9.4078e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     4.488e-04   1.673e-04    2.683 0.007304 **
## ar1    8.562e-02   2.402e-02    3.565 0.000364 ***
## omega  8.921e-07   4.599e-07    1.940 0.052421 .
## alpha1 4.590e-02   1.189e-02    3.860 0.000113 ***
## beta1  9.408e-01   1.736e-02   54.207  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  6432.855    normalized:  3.460385
##
## Description:
##  Sat Apr 07 10:34:37 2018 by user: yydab
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  178.8086  0
##  Shapiro-Wilk Test  R    W      0.9900033 5.151592e-10
##  Ljung-Box Test     R    Q(10)  7.513918  0.6761988
##  Ljung-Box Test     R    Q(15)  16.53416  0.3474659
##  Ljung-Box Test     R    Q(20)  23.55409  0.2624131
##  Ljung-Box Test     R^2  Q(10)  4.468907  0.9237251
##  Ljung-Box Test     R^2  Q(15)  8.535124  0.9005445
##  Ljung-Box Test     R^2  Q(20)  11.61467  0.9287007
##  LM Arch Test       R    TR^2   6.422273  0.8933186
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -6.915390 -6.900523 -6.915405 -6.909911
```

Skew Normal fit

Note the null hypothesis of normality was rejected. Try a skew-normal. Skew parameter is significant under the model.

```
fit3<- garchFit(~arma(1,0)+garch(1,1), cond.dist="snorm", data=ftsedl)
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                1 0
##  Max ARMA Order:            1
##  GARCH Order:               1 1
```

```
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          snorm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          1859
##  Recursion Init:            mci
##  Series Scale:              0.007957728
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U           V     params includes
##     mu     -0.54284978   0.5428498 0.05439580     TRUE
##     ar1    -0.99999999   1.0000000 0.09208644     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1   0.00000001   1.0000000 0.80000000     TRUE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000     TRUE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1  omega alpha1  beta1   skew
##      1      2      3      4      6      8
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     2566.1950: 0.0543958 0.0920864 0.100000 0.100000 0.800000  1.00000
##   1:     2565.7404: 0.0543899 0.0958992 0.0319416 0.125835 0.850100 0.989762
##   2:     2564.1258: 0.0543899 0.0958074 0.0366237 0.125963 0.853119 0.989630
##   3:     2563.2169: 0.0543899 0.0955971 0.0364674 0.120599 0.851638 0.989414
##   4:     2561.5649: 0.0543902 0.0950504 0.0460765 0.114995 0.851265 0.989800
##   5:     2560.1512: 0.0543901 0.0937224 0.0485003 0.0930829 0.854306 0.989218
##   6:     2557.3570: 0.0543888 0.0932693 0.0433019 0.0879689 0.875222 0.986669
##   7:     2555.4134: 0.0543874 0.0927596 0.0312823 0.0792111 0.891660 0.984285
##   8:     2554.4471: 0.0543868 0.0915242 0.0229554 0.0711154 0.910634 0.983317
##   9:     2553.8989: 0.0543903 0.0870521 0.0196704 0.0547145 0.923465 0.989053
##  10:     2553.8187: 0.0543936 0.0849822 0.0234363 0.0529829 0.926290 0.990449
##  11:     2553.0495: 0.0543975 0.0850825 0.0209323 0.0530952 0.926559 0.989233
##  12:     2552.9468: 0.0544011 0.0859585 0.0189423 0.0535273 0.928098 0.988483
##  13:     2552.9370: 0.0544012 0.0859587 0.0190349 0.0536110 0.928237 0.988481
##  14:     2552.9295: 0.0544013 0.0859596 0.0186169 0.0534743 0.928583 0.988455
##  15:     2552.9033: 0.0544050 0.0863034 0.0184835 0.0529983 0.929451 0.988090
##  16:     2552.8194: 0.0544331 0.0852157 0.0168305 0.0493247 0.934259 0.988112
##  17:     2552.8133: 0.0544332 0.0852175 0.0169451 0.0494799 0.934434 0.988112
##  18:     2552.8070: 0.0544350 0.0851515 0.0167918 0.0494264 0.934451 0.988301
```

```
##  19:     2552.8023: 0.0544392 0.0851156 0.0165937 0.0495597 0.934771 0.988625
##  20:     2552.7936: 0.0544592 0.0858372 0.0162378 0.0493846 0.935061 0.989059
##  21:     2552.7897: 0.0544722 0.0851912 0.0161750 0.0494481 0.935206 0.988295
##  22:     2552.7853: 0.0545191 0.0862955 0.0159006 0.0492291 0.935691 0.989718
##  23:     2552.7850: 0.0545192 0.0862879 0.0158973 0.0492219 0.935757 0.989706
##  24:     2552.7844: 0.0545207 0.0862492 0.0158594 0.0491954 0.935768 0.989697
##  25:     2552.7836: 0.0545238 0.0861724 0.0158404 0.0491802 0.935858 0.989674
##  26:     2552.7822: 0.0545326 0.0860039 0.0157587 0.0491377 0.935931 0.989602
##  27:     2552.7789: 0.0545704 0.0858637 0.0155922 0.0491011 0.936229 0.988794
##  28:     2552.7783: 0.0545957 0.0850777 0.0154825 0.0489775 0.936353 0.989421
##  29:     2552.7763: 0.0546524 0.0853367 0.0155060 0.0489444 0.936436 0.989097
##  30:     2552.7760: 0.0546685 0.0857294 0.0154983 0.0489012 0.936411 0.988701
##  31:     2552.7742: 0.0547309 0.0853284 0.0154658 0.0488091 0.936588 0.988978
##  32:     2552.7729: 0.0548601 0.0846993 0.0153020 0.0484937 0.936928 0.989305
##  33:     2552.7680: 0.0549863 0.0852679 0.0152988 0.0483860 0.937149 0.988517
##  34:     2552.7678: 0.0550758 0.0856528 0.0151257 0.0480377 0.937502 0.987956
##  35:     2552.7634: 0.0551752 0.0855248 0.0150756 0.0479896 0.937767 0.987817
##  36:     2552.7620: 0.0551878 0.0843222 0.0148750 0.0477338 0.938209 0.989105
##  37:     2552.7562: 0.0564549 0.0847928 0.0142522 0.0465926 0.939880 0.988665
##  38:     2552.7552: 0.0560997 0.0850601 0.0142849 0.0463994 0.940078 0.988969
##  39:     2552.7552: 0.0560997 0.0850642 0.0142424 0.0464500 0.940100 0.988961
##  40:     2552.7551: 0.0560959 0.0850638 0.0142345 0.0464447 0.940097 0.988957
##  41:     2552.7550: 0.0559766 0.0850424 0.0142120 0.0464184 0.940149 0.988807
##  42:     2552.7550: 0.0558905 0.0851870 0.0142027 0.0464280 0.940152 0.988672
##  43:     2552.7550: 0.0559027 0.0851619 0.0142067 0.0464318 0.940143 0.988637
##  44:     2552.7550: 0.0559037 0.0851506 0.0142059 0.0464290 0.940146 0.988650
##
## Final Estimate of the Negative LLH:
##  LLH:  -6432.929    norm LLH:  -3.460425
##           mu          ar1        omega       alpha1        beta1
## 4.448661e-04 8.515062e-02 8.995943e-07 4.642901e-02 9.401463e-01
##         skew
## 9.886496e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu           ar1         omega        alpha1
## mu      -35916127.30 -1.556910e+04 -6.694879e+08 -2.719471e+04
## ar1        -15569.10 -1.744539e+03 -2.667938e+06 -2.050571e+02
## omega  -669487907.87 -2.667938e+06 -9.998224e+13 -4.199581e+09
## alpha1     -27194.71 -2.050571e+02 -4.199581e+09 -2.255025e+05
## beta1      -28718.83 -3.037074e+02 -5.016185e+09 -2.419136e+05
## skew        12576.34  8.018133e+01  1.338307e+07 -3.630815e+01
##                beta1          skew
## mu     -2.871883e+04  1.257634e+04
## ar1    -3.037074e+02  8.018133e+01
## omega  -5.016185e+09  1.338307e+07
## alpha1 -2.419136e+05 -3.630815e+01
## beta1  -2.748331e+05  4.510115e+02
## skew    4.510115e+02 -1.183642e+03
## attr(,"time")
## Time difference of 0.08070016 secs
##
## --- END OF TRACE ---
##
```

```
##
## Time to Estimate Parameters:
##  Time difference of 0.3591251 secs
```

```
summary(fit3)
```

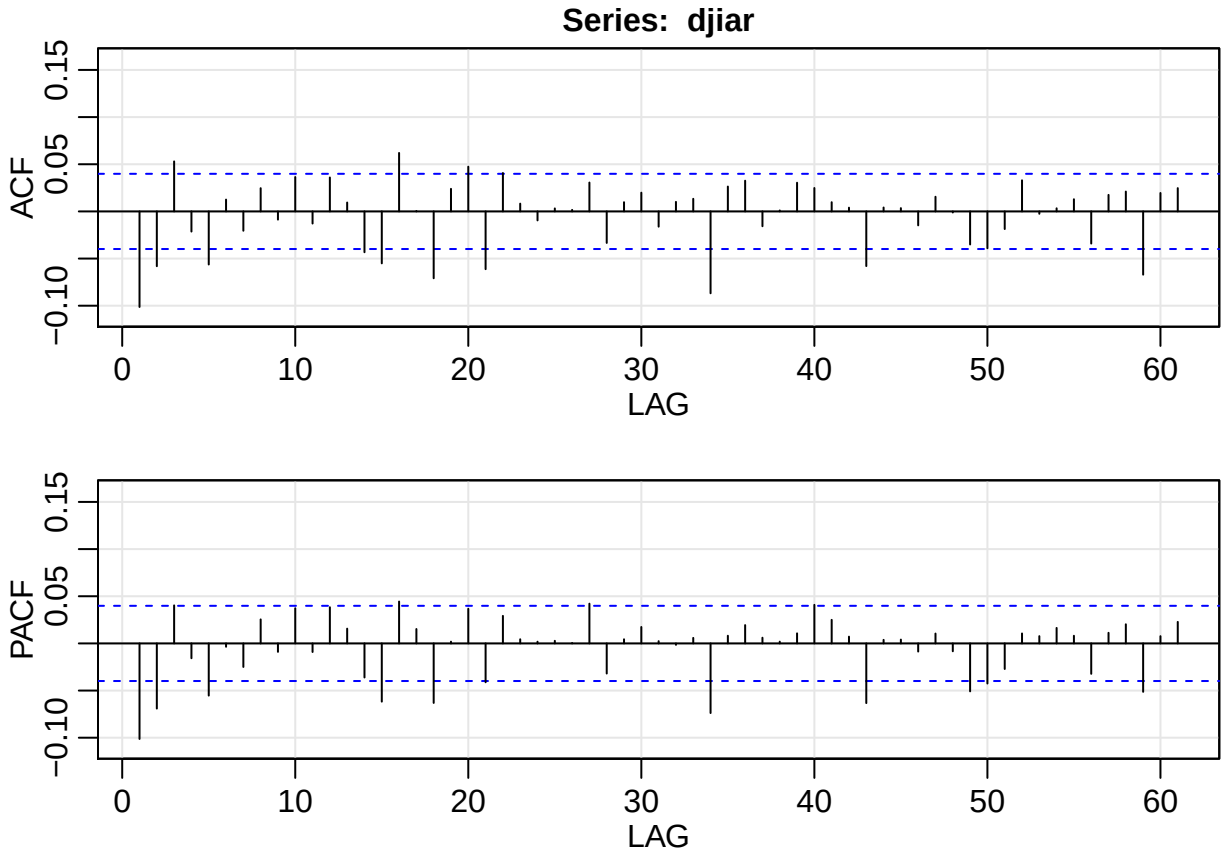
```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = ftsedl,
##     cond.dist = "snorm")
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000001d359448>
##  [data = ftsedl]
##
## Conditional Distribution:
##  snorm
##
## Coefficient(s):
##         mu         ar1       omega      alpha1       beta1        skew
## 4.4487e-04  8.5151e-02  8.9959e-07  4.6429e-02  9.4015e-01  9.8865e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     4.449e-04   1.675e-04    2.656 0.007911 **
## ar1    8.515e-02   2.405e-02    3.541 0.000398 ***
## omega  8.996e-07   4.608e-07    1.952 0.050929 .
## alpha1 4.643e-02   1.199e-02    3.872 0.000108 ***
## beta1  9.401e-01   1.743e-02   53.951  < 2e-16 ***
## skew   9.886e-01   2.937e-02   33.665  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  6432.929    normalized:  3.460425
##
## Description:
##  Sat Apr 07 10:34:37 2018 by user: yydab
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  178.3195  0
##  Shapiro-Wilk Test  R    W      0.9900119 5.22203e-10
##  Ljung-Box Test     R    Q(10)  7.512191  0.6763662
##  Ljung-Box Test     R    Q(15)  16.51273  0.348817
##  Ljung-Box Test     R    Q(20)  23.53262  0.2634044
##  Ljung-Box Test     R^2  Q(10)  4.473355  0.9234776
```

```
##  Ljung-Box Test     R^2  Q(15)  8.566279  0.8990826
##  Ljung-Box Test     R^2  Q(20)  11.64354  0.9277988
##  LM Arch Test       R    TR^2   6.419559  0.8934741
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -6.914394 -6.896553 -6.914415 -6.907819
```

Conclusion: the model $ARMA(1,0)+GARCH(1,1)$ with skew normal distribution fit the data.

Q3

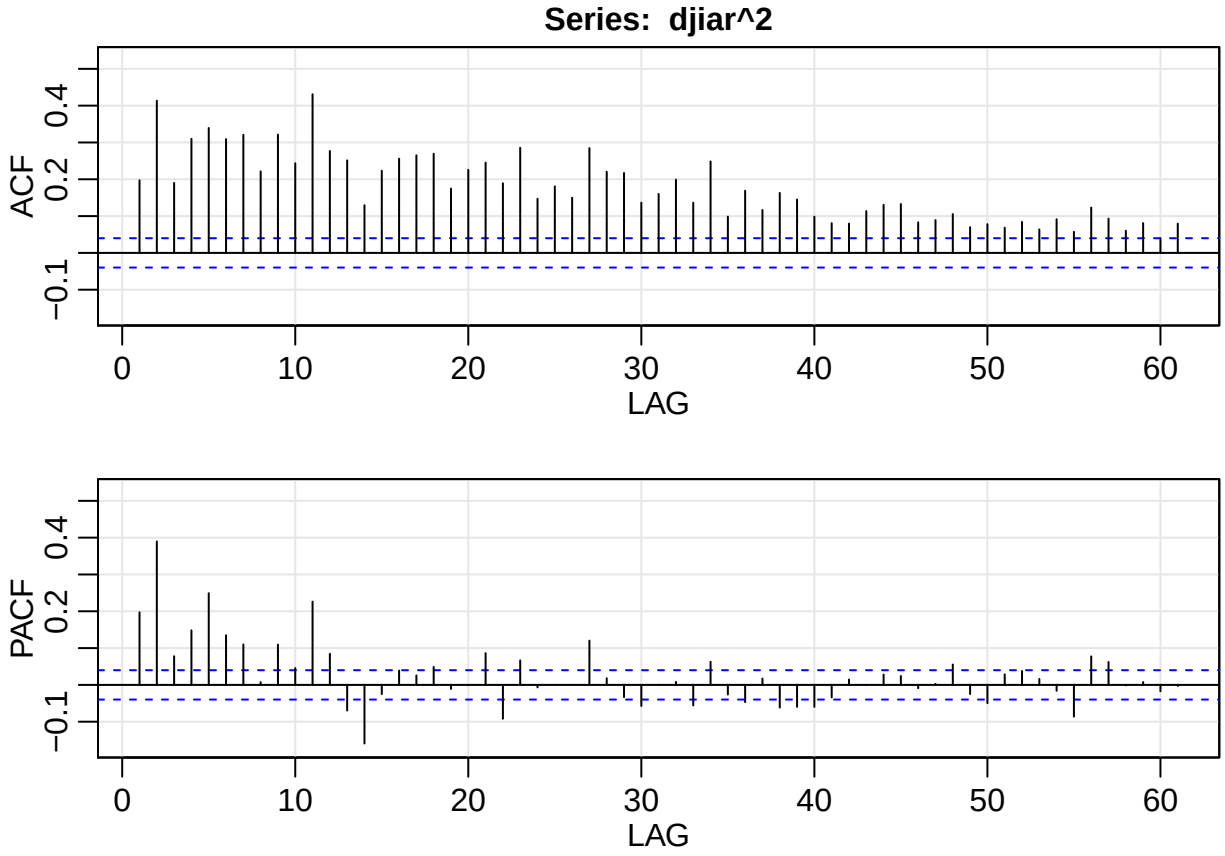
```
djiar = diff(log(djia$Close))[-1]
acf2(djiar)    # exhibits some autocorrelation
```




```
##          ACF  PACF
##  [1,] -0.10 -0.10
##  [2,] -0.06 -0.07
##  [3,]  0.05  0.04
##  [4,] -0.02 -0.02
##  [5,] -0.06 -0.06
##  [6,]  0.01  0.00
##  [7,] -0.02 -0.02
##  [8,]  0.02  0.03
```

```
##  [9,] -0.01 -0.01
## [10,]  0.04  0.04
## [11,] -0.01 -0.01
## [12,]  0.04  0.04
## [13,]  0.01  0.02
## [14,] -0.04 -0.04
## [15,] -0.06 -0.06
## [16,]  0.06  0.04
## [17,]  0.00  0.02
## [18,] -0.07 -0.06
## [19,]  0.02  0.00
## [20,]  0.05  0.04
## [21,] -0.06 -0.04
## [22,]  0.04  0.03
## [23,]  0.01  0.00
## [24,] -0.01  0.00
## [25,]  0.00  0.00
## [26,]  0.00  0.00
## [27,]  0.03  0.04
## [28,] -0.03 -0.03
## [29,]  0.01  0.00
## [30,]  0.02  0.02
## [31,] -0.02  0.00
## [32,]  0.01  0.00
## [33,]  0.01  0.01
## [34,] -0.09 -0.07
## [35,]  0.03  0.01
## [36,]  0.03  0.02
## [37,] -0.02  0.01
## [38,]  0.00  0.00
## [39,]  0.03  0.01
## [40,]  0.02  0.04
## [41,]  0.01  0.02
## [42,]  0.00  0.01
## [43,] -0.06 -0.06
## [44,]  0.00  0.00
## [45,]  0.00  0.00
## [46,] -0.01 -0.01
## [47,]  0.02  0.01
## [48,]  0.00 -0.01
## [49,] -0.04 -0.05
## [50,] -0.04 -0.04
## [51,] -0.02 -0.03
## [52,]  0.03  0.01
## [53,]  0.00  0.01
## [54,]  0.00  0.02
## [55,]  0.01  0.01
## [56,] -0.03 -0.03
## [57,]  0.02  0.01
## [58,]  0.02  0.02
## [59,] -0.07 -0.05
## [60,]  0.02  0.01
## [61,]  0.02  0.02
```

```r
acf2(djiar^2)  # oozes autocorrelation
```




```
##         ACF  PACF
##  [1,] 0.20  0.20
##  [2,] 0.41  0.39
##  [3,] 0.19  0.08
##  [4,] 0.31  0.15
##  [5,] 0.34  0.25
##  [6,] 0.31  0.13
##  [7,] 0.32  0.11
##  [8,] 0.22  0.01
##  [9,] 0.32  0.11
## [10,] 0.24  0.05
## [11,] 0.43  0.23
## [12,] 0.28  0.08
## [13,] 0.25 -0.07
## [14,] 0.13 -0.16
## [15,] 0.22 -0.03
## [16,] 0.26  0.04
## [17,] 0.27  0.03
## [18,] 0.27  0.05
## [19,] 0.17 -0.01
## [20,] 0.23  0.00
## [21,] 0.25  0.09
## [22,] 0.19 -0.09
## [23,] 0.29  0.07
```

```
## [24,] 0.15 -0.01
## [25,] 0.18  0.00
## [26,] 0.15  0.00
## [27,] 0.28  0.12
## [28,] 0.22  0.02
## [29,] 0.22 -0.03
## [30,] 0.14 -0.06
## [31,] 0.16  0.00
## [32,] 0.20  0.01
## [33,] 0.14 -0.06
## [34,] 0.25  0.06
## [35,] 0.10 -0.03
## [36,] 0.17 -0.05
## [37,] 0.12  0.02
## [38,] 0.16 -0.06
## [39,] 0.15 -0.06
## [40,] 0.10 -0.06
## [41,] 0.08 -0.03
## [42,] 0.08  0.01
## [43,] 0.11  0.00
## [44,] 0.13  0.03
## [45,] 0.13  0.02
## [46,] 0.08 -0.01
## [47,] 0.09  0.00
## [48,] 0.11  0.06
## [49,] 0.07 -0.02
## [50,] 0.08 -0.05
## [51,] 0.07  0.03
## [52,] 0.08  0.04
## [53,] 0.06  0.02
## [54,] 0.09 -0.02
## [55,] 0.06 -0.09
## [56,] 0.12  0.08
## [57,] 0.09  0.06
## [58,] 0.06  0.00
## [59,] 0.08  0.01
## [60,] 0.04 -0.02
## [61,] 0.08  0.00
```

```
# GARCH fit
summary(fit_31<- garchFit(~arma(1,0)+garch(1,1), data=djiar, cond.dist='std'))
```

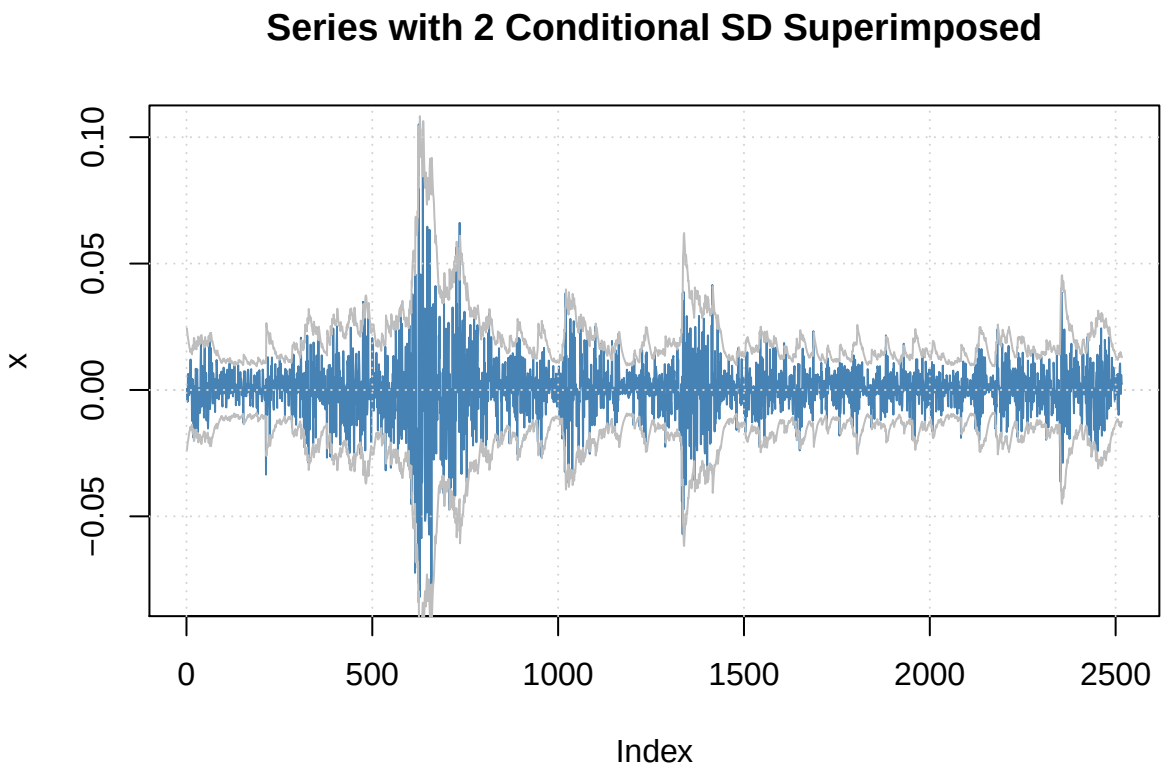
```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                1 0
##  Max ARMA Order:            1
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          std
##  h.start:                   2
```

```
##  llh.start:                1
##  Length of Series:         2517
##  Recursion Init:           mci
##  Series Scale:             0.01210097
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U           V      params includes
##     mu     -0.15336279   0.1533628  0.01533395     TRUE
##     ar1    -0.99999999   1.0000000 -0.10129752     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
##     alpha1  0.00000001   1.0000000  0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000    FALSE
##     beta1   0.00000001   1.0000000  0.80000000     TRUE
##     delta   0.00000000   2.0000000  2.00000000    FALSE
##     skew    0.10000000  10.0000000  1.00000000    FALSE
##     shape   1.00000000  10.0000000  4.00000000     TRUE
##  Index List of Parameters to be Optimized:
##     mu    ar1  omega alpha1  beta1  shape
##      1      2      3      4      6      9
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     2966.5649: 0.0153339 -0.101298 0.100000 0.100000 0.800000  4.00000
##   1:     2944.7773: 0.0153351 -0.0991717 0.0803851 0.105688 0.794226  3.99986
##   2:     2910.9444: 0.0153425 -0.0864391 0.0198839 0.162519 0.809277  4.00053
##   3:     2891.9987: 0.0153434 -0.0855698 0.0350001 0.168391 0.817706  4.00077
##   4:     2882.6364: 0.0153798 -0.0489486 0.0211395 0.164589 0.840541  4.00286
##   5:     2881.9304: 0.0153823 -0.0484851 0.0204244 0.166791 0.845392  4.00314
##   6:     2881.5678: 0.0153882 -0.0491681 0.0164060 0.164252 0.847791  4.00365
##   7:     2880.9562: 0.0153959 -0.0493829 0.0174447 0.161530 0.852258  4.00434
##   8:     2880.3922: 0.0154144 -0.0492493 0.0148742 0.153976 0.859252  4.00602
##   9:     2880.0788: 0.0154929 -0.0464297 0.0155698 0.139117 0.871707  4.01357
##  10:     2879.5719: 0.0156667 -0.0502107 0.0125331 0.137634 0.873099  4.03121
##  11:     2879.2754: 0.0158465 -0.0534691 0.0130134 0.140284 0.873078  4.04888
##  12:     2878.9979: 0.0160179 -0.0464745 0.0137975 0.139197 0.871470  4.06589
##  13:     2878.9707: 0.0160186 -0.0464112 0.0132405 0.139289 0.871633  4.06595
##  14:     2878.9510: 0.0160193 -0.0463401 0.0134262 0.139559 0.872118  4.06603
##  15:     2878.9240: 0.0160288 -0.0462943 0.0129595 0.139571 0.872158  4.06693
##  16:     2878.8886: 0.0160494 -0.0462517 0.0131197 0.139735 0.872456  4.06886
##  17:     2872.3653: 0.0239833 -0.0535089 0.0143767 0.137582 0.858370  4.81181
##  18:     2871.5453: 0.0252884 -0.0550775 0.0137416 0.122667 0.870984  4.83396
##  19:     2871.3687: 0.0252889 -0.0548939 0.0140394 0.123736 0.872173  4.83400
##  20:     2871.2367: 0.0253052 -0.0547681 0.0128189 0.123951 0.872237  4.83405
##  21:     2871.1340: 0.0253530 -0.0546310 0.0132252 0.124568 0.872883  4.83413
##  22:     2871.0366: 0.0254525 -0.0544735 0.0123989 0.124866 0.872942  4.83425
```

```
##  23:     2870.9225: 0.0256529 -0.0543024 0.0127160 0.125465 0.873271  4.83439
##  24:     2865.2785: 0.0596248 -0.0456160 0.0160045 0.157797 0.842171  4.84608
##  25:     2862.9397: 0.0677622 -0.0434674 0.0102271 0.136586 0.867384  5.04667
##  26:     2862.1051: 0.0743737 -0.0636105 0.0112956 0.123954 0.872267  5.35478
##  27:     2861.7101: 0.0719553 -0.0554743 0.0110194 0.124369 0.870600  5.66110
##  28:     2861.6164: 0.0709488 -0.0545210 0.0111056 0.125771 0.869190  5.85080
##  29:     2861.6013: 0.0708292 -0.0551561 0.0109174 0.124128 0.870274  5.97092
##  30:     2861.6001: 0.0709556 -0.0553034 0.0110181 0.124507 0.869952  5.97943
##  31:     2861.6000: 0.0709486 -0.0553165 0.0109951 0.124451 0.870011  5.97873
##  32:     2861.6000: 0.0709475 -0.0553146 0.0109959 0.124444 0.870013  5.97878
##
## Final Estimate of the Negative LLH:
##  LLH:  -8249.619    norm LLH:  -3.27756
##           mu           ar1         omega        alpha1         beta1
##  8.585338e-04 -5.531459e-02  1.610165e-06  1.244440e-01  8.700129e-01
##         shape
##  5.978777e+00
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu           ar1         omega        alpha1
## mu     -4.791648e+07 -4.661855e+04 -1.205649e+09 -3.466882e+04
## ar1    -4.661855e+04 -2.490883e+03 -1.234056e+06 -8.201170e+00
## omega  -1.205649e+09 -1.234056e+06 -1.703959e+13 -5.515908e+08
## alpha1 -3.466882e+04 -8.201170e+00 -5.515908e+08 -3.611267e+04
## beta1  -9.149142e+04 -9.743995e+01 -8.452232e+08 -4.469044e+04
## shape  -9.970359e+02  8.560505e-02 -3.050355e+06 -1.832751e+02
##                beta1         shape
## mu     -9.149142e+04 -9.970359e+02
## ar1    -9.743995e+01  8.560505e-02
## omega  -8.452232e+08 -3.050355e+06
## alpha1 -4.469044e+04 -1.832751e+02
## beta1  -6.270066e+04 -2.340750e+02
## shape  -2.340750e+02 -2.547444e+00
## attr(,"time")
## Time difference of 0.1431739 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.4600739 secs
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = djiar, cond.dist = "std")
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000001eea62a8>
##  [data = djiar]
##
## Conditional Distribution:
```

```
##  std
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
##  8.5853e-04  -5.5315e-02   1.6102e-06   1.2444e-01   8.7001e-01
##       shape
##  5.9788e+00
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      8.585e-04   1.470e-04    5.842 5.16e-09 ***
## ar1    -5.531e-02   2.023e-02   -2.735 0.006239 **
## omega   1.610e-06   4.459e-07    3.611 0.000305 ***
## alpha1  1.244e-01   1.660e-02    7.497 6.55e-14 ***
## beta1   8.700e-01   1.526e-02   57.022  < 2e-16 ***
## shape   5.979e+00   7.917e-01    7.552 4.31e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  8249.619    normalized:  3.27756
##
## Description:
##  Sat Apr 07 10:34:38 2018 by user: yydab
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  310.0055  0
##  Shapiro-Wilk Test  R    W      0.9820294 0
##  Ljung-Box Test     R    Q(10)  16.82241  0.07838716
##  Ljung-Box Test     R    Q(15)  26.44807  0.03356859
##  Ljung-Box Test     R    Q(20)  28.71095  0.09360887
##  Ljung-Box Test     R^2  Q(10)  15.36776  0.119218
##  Ljung-Box Test     R^2  Q(15)  19.13661  0.2076099
##  Ljung-Box Test     R^2  Q(20)  22.9289   0.2922993
##  LM Arch Test       R    TR^2   15.0399   0.2392619
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -6.550353 -6.536453 -6.550364 -6.545309
```

```
plot(fit_31,which=3)
```



APARCH fit

Suppose y_t is APARCH noise with conditional variance.

```
summary(fit_32<- garchFit(~arma(1,0)+aparch(1,1), data=djiar, cond.dist='std'))
```

```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(1, 0)
##  GARCH Model:                aparch
##  Formula Variance:           ~ aparch(1, 1)
##  ARMA Order:                 1 0
##  Max ARMA Order:             1
##  GARCH Order:                1 1
##  Max GARCH Order:            1
##  Maximum Order:              1
##  Conditional Dist:           std
##  h.start:                    2
##  llh.start:                  1
##  Length of Series:           2517
##  Recursion Init:             mci
##  Series Scale:               0.01210097
##
## Parameter Initialization:
```

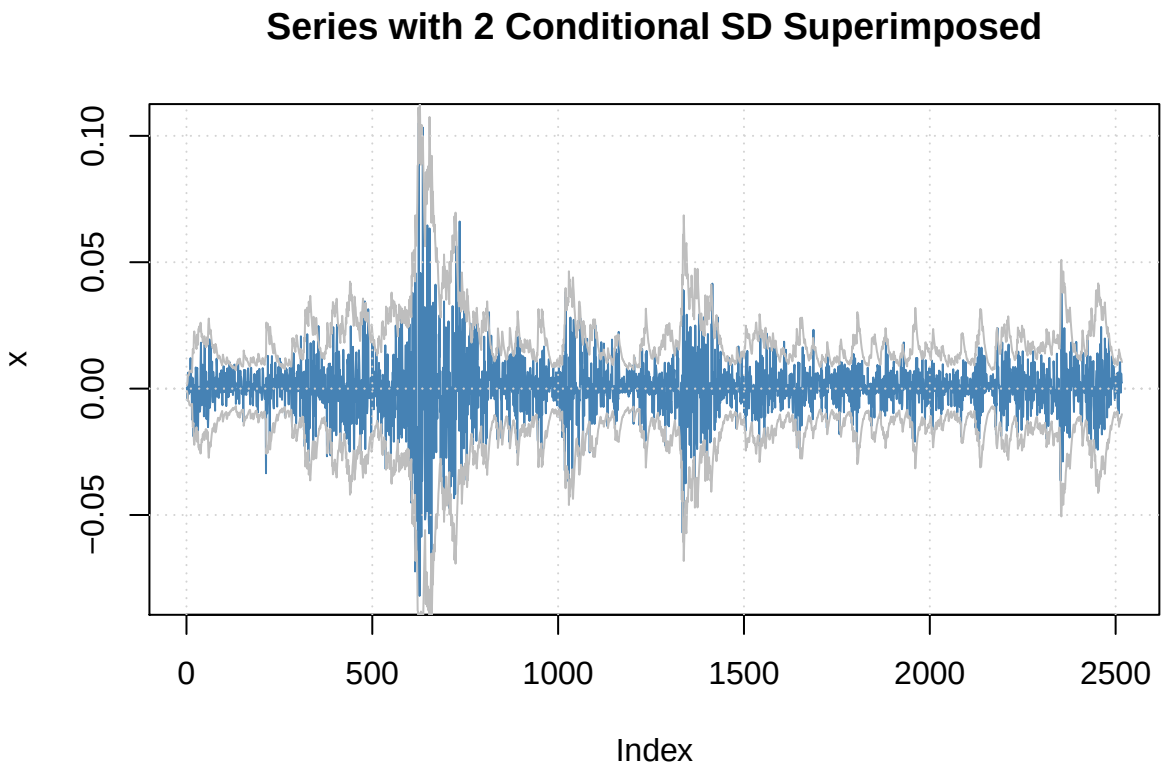
```
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U           V      params includes
##     mu     -0.15336279   0.1533628  0.01533395     TRUE
##     ar1    -0.99999999   1.0000000 -0.10129752     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
##     alpha1  0.00000001   1.0000000  0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000     TRUE
##     beta1   0.00000001   1.0000000  0.80000000     TRUE
##     delta   0.00000000   2.0000000  2.00000000     TRUE
##     skew    0.10000000  10.0000000  1.00000000    FALSE
##     shape   1.00000000  10.0000000  4.00000000     TRUE
##  Index List of Parameters to be Optimized:
##     mu    ar1  omega alpha1 gamma1  beta1  delta  shape
##      1      2      3      4      5      6      7      9
##  Persistence:                  0.901
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     2956.2216: 0.0153339 -0.101298 0.100000 0.100000 0.100000 0.800000  2.00000  4.00000
##   1:     2933.7102: 0.0153349 -0.0994951 0.0816702 0.105086 0.101441 0.794372  1.99989  3.99987
##   2:     2915.7665: 0.0153364 -0.0969033 0.0659019 0.116673 0.103734 0.795785  2.00000  3.99997
##   3:     2868.4754: 0.0153445 -0.0848309 0.0240064 0.174770 0.115713 0.826263  2.00000  4.00127
##   4:     2866.0092: 0.0153484 -0.0806185 0.0223796 0.179099 0.121370 0.835422  2.00000  4.00179
##   5:     2865.7443: 0.0153525 -0.0768099 0.0131353 0.176107 0.127199 0.838908  1.99992  4.00228
##   6:     2863.1450: 0.0153538 -0.0756692 0.0180404 0.176968 0.129004 0.841876  2.00000  4.00245
##   7:     2856.2636: 0.0153873 -0.0477472 0.00967343 0.170088 0.175541 0.860597  1.99995  4.00606
##   8:     2853.4395: 0.0154246 -0.0297309 0.0251948 0.165528 0.224263 0.839295  1.99939  4.00933
##   9:     2853.3035: 0.0154254 -0.0301564 0.0152622 0.163962 0.225414 0.836633  1.99923  4.00939
##  10:     2850.4729: 0.0154259 -0.0302849 0.0196607 0.165302 0.225960 0.839073  1.99934  4.00948
##  11:     2849.5507: 0.0154305 -0.0320034 0.0140574 0.167395 0.231771 0.845156  1.99944  4.01012
##  12:     2847.7408: 0.0154470 -0.0377746 0.0183930 0.169017 0.251279 0.844831  1.99918  4.01197
##  13:     2845.4604: 0.0154649 -0.0404007 0.0166508 0.170044 0.271510 0.841014  1.99879  4.01390
##  14:     2840.6097: 0.0155659 -0.0398960 0.0220312 0.179337 0.362238 0.827889  1.99564  4.02504
##  15:     2840.2159: 0.0155666 -0.0399220 0.0175565 0.177807 0.362739 0.826608  1.99550  4.02510
##  16:     2839.6368: 0.0155818 -0.0427577 0.0195156 0.177280 0.363341 0.828508  1.99333  4.02667
##  17:     2839.2986: 0.0156224 -0.0472413 0.0180222 0.174695 0.365336 0.830519  1.98742  4.03092
##  18:     2838.3755: 0.0158110 -0.0320240 0.0189810 0.174719 0.374079 0.830619  1.96003  4.05076
##  19:     2838.1806: 0.0158142 -0.0328761 0.0200414 0.172691 0.376493 0.833566  1.95990  4.05118
##  20:     2837.8264: 0.0158349 -0.0334367 0.0184927 0.171928 0.377234 0.833250  1.95678  4.05345
##  21:     2837.5310: 0.0158776 -0.0347303 0.0194491 0.170603 0.379522 0.834980  1.95050  4.05826
##  22:     2837.0525: 0.0159673 -0.0366418 0.0184466 0.169757 0.381820 0.835073  1.93697  4.06843
##  23:     2831.2211: 0.0181618 -0.0743039 0.0230404 0.163104 0.433944 0.843617  1.61079  4.32024
##  24:     2817.9701: 0.0238342 -0.0395263 0.0222446 0.116086 0.625292 0.887192 0.952834  4.96894
##  25:     2816.4431: 0.0238343 -0.0395438 0.0234501 0.116900 0.625343 0.888222 0.952877  4.96895
##  26:     2816.0968: 0.0238346 -0.0397073 0.0221922 0.118042 0.625724 0.888557 0.952974  4.96899
##  27:     2815.9413: 0.0238547 -0.0398291 0.0225848 0.118850 0.626312 0.888738 0.955815  4.97031
##  28:     2815.7521: 0.0238673 -0.0399496 0.0221607 0.119049 0.626607 0.888251 0.959122  4.97106
```

```
##  29:     2815.5147: 0.0238935 -0.0402344 0.0221744 0.120186 0.627325 0.888144 0.965762  4.97248
##  30:     2812.9395: 0.0247901 -0.0462755 0.0254998 0.131035 0.642407 0.870254  1.18463  5.01118
##  31:     2810.0292: 0.0275158 -0.0436698 0.0253200 0.137236 0.714729 0.865646  1.08442  5.09245
##  32:     2806.6352: 0.0307215 -0.0488468 0.0202554 0.123557 0.752027 0.880103  1.17165  5.11226
##  33:     2801.0388: 0.0434902 -0.0442804 0.0227921 0.122527 0.941344 0.881833  1.00718  5.42913
##  34:     2799.4186: 0.0454608 -0.0467090 0.0206287 0.110682 0.974697 0.890429  1.03440  5.52098
##  35:     2799.3567: 0.0454608 -0.0466941 0.0213348 0.110090 0.974750 0.890322  1.03440  5.52100
##  36:     2799.3092: 0.0454567 -0.0466982 0.0211179 0.109861 0.974937 0.890133  1.03430  5.52178
##  37:     2799.2864: 0.0454480 -0.0467008 0.0213211 0.109735 0.975290 0.890215  1.03417  5.52349
##  38:     2799.2562: 0.0454309 -0.0467111 0.0212701 0.109485 0.975963 0.890174  1.03390  5.52695
##  39:     2797.2264: 0.0440041 -0.0503513 0.0197231 0.0974177  1.00000 0.896930  1.04374  6.72117
##  40:     2796.9414: 0.0436647 -0.0468928 0.0199020 0.0999479  1.00000 0.893330  1.08177  6.93829
##  41:     2796.8934: 0.0439925 -0.0477799 0.0210184 0.0986150  1.00000 0.893827  1.06058  7.10174
##  42:     2796.8503: 0.0433479 -0.0484091 0.0202761 0.0980377  1.00000 0.894290  1.07211  7.22780
##  43:     2796.8478: 0.0433159 -0.0482204 0.0202479 0.0980887  1.00000 0.894301  1.07263  7.26082
##  44:     2796.8472: 0.0432654 -0.0481866 0.0202579 0.0980839  1.00000 0.894407  1.07101  7.28539
##  45:     2796.8472: 0.0432532 -0.0481827 0.0202595 0.0980866  1.00000 0.894447  1.07040  7.28659
##  46:     2796.8472: 0.0432526 -0.0481838 0.0202596 0.0980896  1.00000 0.894456  1.07023  7.28591
##  47:     2796.8472: 0.0432526 -0.0481839 0.0202594 0.0980893  1.00000 0.894457  1.07023  7.28576
##
## Final Estimate of the Negative LLH:
##  LLH:  -8311.583    norm LLH:  -3.302178
##            mu           ar1         omega        alpha1        gamma1
##  0.0005233986 -0.0481839398  0.0001798026  0.0980892614  0.9999999900
##         beta1         delta         shape
##  0.8944566891  1.0702336184  7.2857627317
##
## R-optimhess Difference Approximated Hessian Matrix:
##                    mu           ar1         omega        alpha1
## mu     -9.257268e+07 -1.304497e+05 -6.612990e+08 -2.435875e+06
## ar1    -1.304497e+05 -2.960736e+03 -7.708000e+05 -1.492695e+03
## omega  -6.612990e+08 -7.708000e+05 -9.933032e+09 -3.284959e+07
## alpha1 -2.435875e+06 -1.492695e+03 -3.284959e+07 -1.662177e+05
## gamma1 -1.277035e+05 -7.809040e+01 -1.721819e+06 -8.715942e+03
## beta1  -3.642711e+06 -3.791572e+03 -5.191596e+07 -2.185971e+05
## delta  -2.481012e+05 -2.643131e+02 -3.568315e+06 -1.497209e+04
## shape  -4.448811e+03 -2.850991e+00 -4.321053e+04 -2.197851e+02
##                gamma1         beta1         delta         shape
## mu     -1.277035e+05 -3.642711e+06  -248101.2220  -4448.811095
## ar1    -7.809040e+01 -3.791572e+03     -264.3131     -2.850991
## omega  -1.721819e+06 -5.191596e+07 -3568314.8658 -43210.531832
## alpha1 -8.715942e+03 -2.185971e+05   -14972.0922   -219.785097
## gamma1 -9.621456e+03 -1.146450e+04     -783.1003    -11.538915
## beta1  -1.146450e+04 -3.201325e+05   -21535.8361   -285.458078
## delta  -7.831003e+02 -2.153584e+04    -1520.6731    -19.344100
## shape  -1.153892e+01 -2.854581e+02      -19.3441     -1.118540
## attr(,"time")
## Time difference of 0.5037329 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 1.38564 secs
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + aparch(1, 1), data = djiar,
##     cond.dist = "std")
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + aparch(1, 1)
## <environment: 0x000000001c77b008>
##  [data = djiar]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu         ar1       omega      alpha1      gamma1       beta1
##  0.0005234  -0.0481839   0.0001798   0.0980893   1.0000000   0.8944567
##      delta       shape
##  1.0702336   7.2857627
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      5.234e-04   1.525e-04    3.432 0.000598 ***
## ar1    -4.818e-02   1.934e-02   -2.491 0.012727 *
## omega   1.798e-04   3.443e-05    5.222 1.77e-07 ***
## alpha1  9.809e-02   1.030e-02    9.525  < 2e-16 ***
## gamma1  1.000e+00   1.045e-02   95.731  < 2e-16 ***
## beta1   8.945e-01   1.049e-02   85.280  < 2e-16 ***
## delta   1.070e+00   1.350e-01    7.928 2.22e-15 ***
## shape   7.286e+00   1.123e+00    6.489 8.61e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  8311.583    normalized:  3.302178
##
## Description:
##  Sat Apr 07 10:34:39 2018 by user: yydab
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  245.1569  0
##  Shapiro-Wilk Test  R    W      0.9830579 0
##  Ljung-Box Test     R    Q(10)  15.59584  0.1118014
##  Ljung-Box Test     R    Q(15)  26.45095  0.03354158
##  Ljung-Box Test     R    Q(20)  30.17074  0.06713391
##  Ljung-Box Test     R^2  Q(10)  19.17687  0.0380728
##  Ljung-Box Test     R^2  Q(15)  30.46672  0.01034547
```

```
##  Ljung-Box Test     R^2  Q(20)  35.36462  0.01824615
##  LM Arch Test       R    TR^2   29.57741  0.003231586
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -6.598000 -6.579468 -6.598020 -6.591274
```

```
plot(fit_32,which=3)
```



Conclusion: The distribution of the noise is not normal. But in the fGarch package there are various distributions to fit the data. May use skew distribution or some other distributions. Compare model fits – essentially the same but skew coefficient is significant.I would choose the model with skew distribution.

```
fit_33<- garchFit(~arma(1,0)+garch(1,1), data=djiar, cond.dist="snorm")
```

```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(1, 0)
##  GARCH Model:                garch
##  Formula Variance:           ~ garch(1, 1)
##  ARMA Order:                 1 0
##  Max ARMA Order:             1
##  GARCH Order:                1 1
##  Max GARCH Order:            1
```

```
##  Maximum Order:             1
##  Conditional Dist:          snorm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          2517
##  Recursion Init:            mci
##  Series Scale:              0.01210097
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U           V      params includes
##     mu     -0.15336279   0.1533628  0.01533395     TRUE
##     ar1    -0.99999999   1.0000000 -0.10129752     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
##     alpha1  0.00000001   1.0000000  0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000    FALSE
##     beta1   0.00000001   1.0000000  0.80000000     TRUE
##     delta   0.00000000   2.0000000  2.00000000    FALSE
##     skew    0.10000000  10.0000000  1.00000000     TRUE
##     shape   1.00000000  10.0000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1  omega alpha1  beta1   skew
##      1      2      3      4      6      8
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3064.7864: 0.0153339 -0.101298 0.100000 0.100000 0.800000  1.00000
##   1:     2998.2884: 0.0153343 -0.100527 0.0719804 0.0985390 0.786107 0.997935
##   2:     2958.0939: 0.0153351 -0.0987608 0.0475087 0.117064 0.790039 0.992986
##   3:     2941.5371: 0.0153363 -0.0967019 0.0453166 0.138342 0.806616 0.987375
##   4:     2935.0495: 0.0153391 -0.0924070 0.0213701 0.132931 0.812846 0.976980
##   5:     2914.5879: 0.0153441 -0.0851296 0.0291722 0.133128 0.830661 0.958631
##   6:     2895.7673: 0.0153589 -0.0672051 0.0123449 0.116131 0.868972 0.909676
##   7:     2895.0876: 0.0153708 -0.0619546 0.0186594 0.119789 0.858316 0.881580
##   8:     2894.4638: 0.0153729 -0.0621146 0.0147645 0.119450 0.856359 0.879298
##   9:     2893.1783: 0.0153819 -0.0653354 0.0158836 0.119196 0.859716 0.878265
##  10:     2892.8715: 0.0153955 -0.0678638 0.0141583 0.118022 0.863048 0.876952
##  11:     2892.4419: 0.0154354 -0.0649186 0.0149319 0.117933 0.863625 0.867946
##  12:     2892.4191: 0.0154938 -0.0591800 0.0141979 0.118016 0.862258 0.861047
##  13:     2892.1396: 0.0155486 -0.0590475 0.0143136 0.116923 0.865268 0.862195
##  14:     2892.0677: 0.0155834 -0.0630643 0.0137028 0.116263 0.866292 0.861082
##  15:     2892.0315: 0.0156557 -0.0636925 0.0138591 0.115788 0.867453 0.860665
##  16:     2891.9929: 0.0157152 -0.0610225 0.0133482 0.115314 0.868229 0.861692
##  17:     2891.9493: 0.0158913 -0.0645879 0.0136067 0.115605 0.867053 0.856624
##  18:     2891.9359: 0.0158914 -0.0645752 0.0137544 0.115711 0.867205 0.856614
##  19:     2891.9280: 0.0158916 -0.0645492 0.0135606 0.115780 0.867316 0.856592
```

```
##  20:     2891.9221: 0.0158979 -0.0644900 0.0136587 0.115877 0.867497 0.856533
##  21:     2891.9129: 0.0159121 -0.0643880 0.0135018 0.115850 0.867527 0.856431
##  22:     2891.9019: 0.0159407 -0.0641853 0.0135465 0.115876 0.867707 0.856234
##  23:     2891.7289: 0.0170988 -0.0575362 0.0134040 0.113732 0.869100 0.849695
##  24:     2890.9588: 0.0244146 -0.0765944 0.0120269 0.105468 0.877786 0.845200
##  25:     2890.0556: 0.0317358 -0.0811964 0.0134596 0.114641 0.869856 0.840726
##  26:     2889.4834: 0.0390587 -0.0808913 0.0128869 0.117402 0.863991 0.847025
##  27:     2889.0193: 0.0418075 -0.0760610 0.0146025 0.113896 0.865574 0.859131
##  28:     2888.7351: 0.0445616 -0.0744274 0.0137870 0.119007 0.865300 0.853164
##  29:     2888.4421: 0.0473166 -0.0700594 0.0127260 0.118278 0.866010 0.857754
##  30:     2888.3918: 0.0500732 -0.0685051 0.0130424 0.118626 0.865062 0.857219
##  31:     2888.3834: 0.0499739 -0.0667871 0.0129414 0.117479 0.866280 0.857178
##  32:     2888.3828: 0.0500362 -0.0667716 0.0130134 0.117249 0.866158 0.855841
##  33:     2888.3820: 0.0501018 -0.0667571 0.0129964 0.117371 0.866167 0.856486
##  34:     2888.3820: 0.0500955 -0.0667593 0.0129952 0.117369 0.866171 0.856481
##
## Final Estimate of the Negative LLH:
##  LLH:  -8222.837    norm LLH:  -3.26692
##           mu           ar1         omega        alpha1         beta1
##  6.062042e-04 -6.675929e-02  1.902939e-06  1.173689e-01  8.661709e-01
##          skew
##  8.564812e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                    mu           ar1         omega        alpha1
## mu     -4.350147e+07 -2.897963e+04 -5.208178e+09 -1.921042e+05
## ar1    -2.897963e+04 -2.215986e+03 -1.394565e+06  1.160171e+02
## omega  -5.208178e+09 -1.394565e+06 -2.566138e+13 -8.569594e+08
## alpha1 -1.921042e+05  1.160171e+02 -8.569594e+08 -5.853531e+04
## beta1  -3.277218e+05 -1.760735e-01 -1.250712e+09 -6.810566e+04
## skew    1.578738e+04  2.235437e+02  7.745196e+06 -5.306309e+02
##                beta1         skew
## mu     -3.277218e+05   15787.3836
## ar1    -1.760735e-01     223.5437
## omega  -1.250712e+09 7745196.3326
## alpha1 -6.810566e+04    -530.6309
## beta1  -9.028858e+04    -470.9147
## skew   -4.709147e+02   -2274.4003
## attr(,"time")
## Time difference of 0.1546512 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.495188 secs
```

```
summary(fit_33)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = djiar, cond.dist = "snorm")
```

```
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x0000000021efe940>
##  [data = djiar]
##
## Conditional Distribution:
##  snorm
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
##  6.0620e-04  -6.6759e-02   1.9029e-06   1.1737e-01   8.6617e-01
##        skew
##  8.5648e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      6.062e-04   1.553e-04    3.903 9.50e-05 ***
## ar1    -6.676e-02   2.144e-02   -3.114  0.00185 **
## omega   1.903e-06   3.720e-07    5.115 3.13e-07 ***
## alpha1  1.174e-01   1.269e-02    9.248  < 2e-16 ***
## beta1   8.662e-01   1.286e-02   67.367  < 2e-16 ***
## skew    8.565e-01   2.122e-02   40.365  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  8222.837    normalized:  3.26692
##
## Description:
##  Sat Apr 07 10:34:40 2018 by user: yydab
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  282.4382  0
##  Shapiro-Wilk Test  R    W      0.9823857 0
##  Ljung-Box Test     R    Q(10)  16.69802  0.08131888
##  Ljung-Box Test     R    Q(15)  26.25585  0.03541792
##  Ljung-Box Test     R    Q(20)  28.5981   0.09597972
##  Ljung-Box Test     R^2  Q(10)  17.12103  0.0717285
##  Ljung-Box Test     R^2  Q(15)  20.22023  0.1636462
##  Ljung-Box Test     R^2  Q(20)  23.47313  0.2661642
##  LM Arch Test       R    TR^2   16.67805  0.1621144
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -6.529072 -6.515173 -6.529083 -6.524028
```

```
fit_31@fit$ics
```

```
##       AIC       BIC       SIC      HQIC
```

```
## -6.550353 -6.536453 -6.550364 -6.545309
```

```
fit_32@fit$ics
```

```
##       AIC       BIC       SIC      HQIC
## -6.598000 -6.579468 -6.598020 -6.591274
```

```
fit_33@fit$ics
```

```
##       AIC       BIC       SIC      HQIC
## -6.529072 -6.515173 -6.529083 -6.524028
```

```
# Forecasting from ARMA+GARCH models
#invisible(predict(fit_31,n.head=5,nx=200,plot=TRUE))
#invisible(predict(fit_32,n.head=5,nx=200,plot=TRUE))
invisible(predict(fit_33,n.head=5,nx=200,plot=TRUE))
```