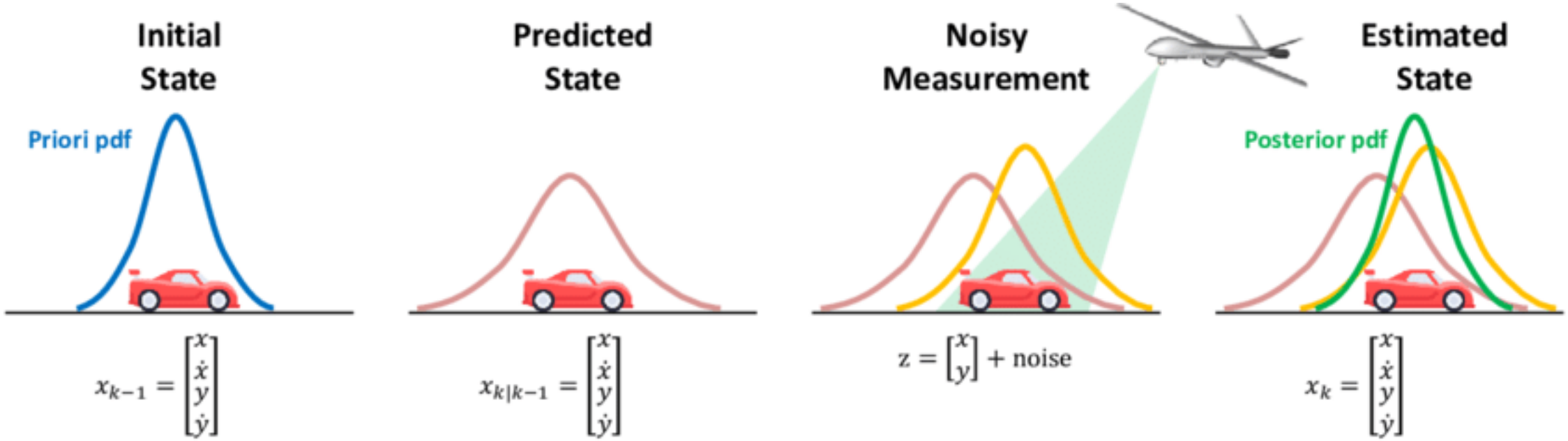
AA/EE/ME 548: Linear Multivariable Control

Lecture 18

5/28/2025

Announcements

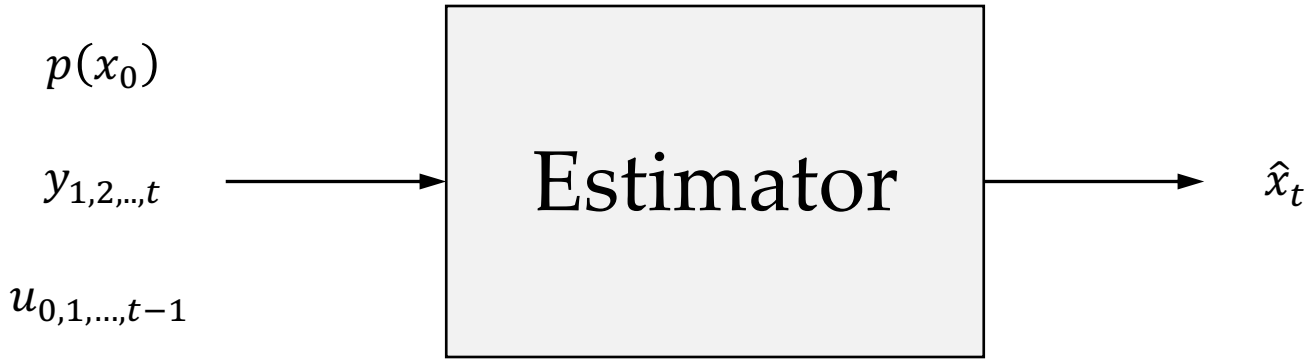
- Homework 3 due Thursday
- More notes updated. Let us know of any typos/issues
- Project details on website
  - **Week 10 Monday Lightning Pitch June 2**:
    - 1.5 minutes pp, +1 min extra for each person in group
    - Add slide to google slide deck.
  - **Week 10 Wednesday Poster Session June 4**:
    - GUG 305! Bring poster to session (Limited time to set-up)
    - Poster session schedule. 30 minutes each session. Peer review assignments.
  - **Finals Week Wednesday June 11**: Report/website submission
    - Submit to Canvas



Kalman Filter (State estimation)

Problem set up

- Given initial state estimate, measurements, and control inputs, estimate the state.



Want to make sure $|x_t - \hat{x}_t|$ is small as possible.

Set up

- Gaussian linear dynamics

White noise

$$x_{t+1} = Ax_t + Bu_t + w_t, \qquad \mathbb{E}[w_t w_t^T] = Q, \qquad w_t \sim \mathcal{N}(0, Q)$$
$$y_t = Cx_t + v_t, \qquad \mathbb{E}[v_t v_t^T] = R, \qquad v_t \sim \mathcal{N}(0, R)$$

- Estimate of state

$$\hat{x}_t \sim p(\hat{x}_t \mid u_{0:t-1}, y_{1:t}, x_0) \qquad \hat{x}_t \sim \mathcal{N}(\mu_t, P_t)$$

- Covariance of error $P_t = \mathbb{E}[e_t e_t^T] = \mathbb{E}[(x_t - \hat{x}_t)(x_t - \hat{x}_t)^T]$
- What is μ_t, P_t ? Depend on previous values.

Predict step

- Given μ_{t-1}, P_{t-1}, estimate at previous time step
- Predict next state estimate using noise-free dynamics only

$$\mu_t^p = A\mu_{t-1} + Bu_{t-1} \qquad P_t^p = AP_{t-1}A^T + Q$$

- Receive measurement & compare with predicted measurement

$$h_t = y_t - C\hat{x}_t^p$$

Update step

- Update mean give prediction and measurement

$$\mu_t = \mu_t^p + K_t(y_t - C\mu_t^p)$$

- Update error covariance $P_t = (I - K_t C)P_t^p(I - K_t C)^T + K_t R K_t^T$
- How to choose K_t ?
  - The one that minimizes $\mathrm{Tr}(P_t)$ (mean square error)

… some algebra… $K_t = P_t^p C^T (C P_t^p C^T + R)^{-1}$

- Final error covariance update

$$P_t = (I - K_t C)P_t^p$$

Kalman Filter algorithm

Start with initial state estimate $\hat{x}_0 \sim \mathcal{N}(\mu_0, P_0)$

For t = 1, 2, …

1. Predict step $\mu_{t-1}, P_{t-1} \rightarrow \mu_t^p, P_t^p$
2. Update step $\mu_t^p, P_t^p \rightarrow \mu_t, P_t$

- Iterate *forward* in time

Do these equations look familiar?

$$P_t^p = AP_{t-1}A^T + Q \qquad K_t = P_t^p C^T (CP_t^p C^T + R)^{-1}$$

- Kind of…but not quite
- Some more algebra.
- Sub $P_{t-1} = (I - K_{t-1}C)P_{t-1}^p$ into $P_t^p = AP_{t-1}A^T + Q$
- We get…

$$P_{t+1}^p = Q + AP_t^p A^T - AP_t^p C^T (CP_t^p C^T + R)^{-1} CP_t^p A^T$$

From LQR

$$P_{t-1} = Q + A^T P_t A - A^T P_t B(B^T P_t B + R)^{-1} B^T P_t A$$

$$K_t = (B^T P_{t+1} B + R)^{-1} B^T P_{t+1} A$$

Duality!

Description	LQR	Kalman filter	Description
Dynamics	A	A^T	Dynamics transposed
Control matrix	B	C^T	Observation transposed
State cost	Q	Q	Process noise covariance
Control cost	R	R	Measurement noise covariance
Terminal cost	Q_t	P_0	Initial state covariance
Value function	P_t	P_t	Error covariance

Continuous time Kalman Filter

- $\dot{x}(t) = Ax(t) + Bu(t) + w(t), \;\; y(t) = Cx(t) + v(t)$
  - $w(t) \sim N\big(0, Q(t)\big), v(t) \sim N(0, R(t))$
- $x_{t+1} = (I + A\Delta t)x_t + \mathrm{B}\Delta t u_t + \tilde{w}_t, \;\; y_t = Cx_t + \tilde{v}_t$
- $\tilde{w}_t \sim N(0, Q\Delta t), \tilde{v}_t \sim N(0, \frac{R}{\Delta t})$

- Same steps as discrete time…

More details…

- $\frac{1}{\Delta t} K_t = P_t^p C^T \left(C P_t^p C \, \Delta t + R \right)^{-1}$
  - If $\Delta t \to 0$, then $K_t \to 0$
  - But $\lim_{\Delta t \to 0} \frac{K_t}{\Delta t} = P_t^p C^T R^{-1}$ (CT Kalman gain)
  - Don't just use discrete time KF for very small timesteps
- $\lim_{\Delta t \to 0} \frac{1}{\Delta t} \left(P_t^p - P_{t-1}^p \right)$
  - $\dot{P} = AP + PA^T - PC^T R^{-1} CP + Q$

Giving an effective pitch

Elevator pitches

- **What**: A short persuasive summary of an idea/product/project.
- **Why**: Capture interest and persuade listener to want to learn more
- **When**:
  - Research (symposium, conference, paper abstract)
  - Professional networking events (mingling, introducing yourself)
  - Raising money for your start-up
  - Convincing your manager to let you work on a project

- Similar to writing a proposal. But first, you need to convince someone want to read your proposal!

(Discussion) What makes a pitch a good pitch?

- Charisma / Enthusiasm. “RIZZ”
- Knowing your audience
- Concise
- Motivation / Impact
- Uniqueness (stand out from the crowd)
- Actionable

RISE framework

- **R**elevance: Quickly establish the relevance.
  - Why should the audience care? Is it something they can relate to?
  - A "hook" to reel the audience in
- **I**ssue: Describe the problem.
  - What is the gap? Or missing piece? Or challenge
  - After **R**, the audience should feel compelled to help solve the problem
- **S**olution: State your solution to the problem.
  - Explain your innovative idea to solving the issue
  - Convey the contribution/impact of your solution
- **E**xpertise (or **E**vidence): Provide support or proof that your solution is legitimate
  - Demonstrate that you have the (unique) expertise to execute the solution
  - Describe the evidence (e.g., experimental results) that validate your solution

Activity: Identify RISE & reduce length

Run For Your Life

Fear is a remarkable thing. The fear of missing out can make you go out and do something you initially lacked the energy to do. The fear of missing an important work deadline can be more effective than taking Adderall. Imagine what fear can do when it comes to your cardiovascular health. Cardiovascular disease is the leading cause of death in the US, accounting for approximate one in every five deaths nationwide. Well, if you have ever lacked motivation to go out for a run and keep that heart rate up, then Run For Your Life is all you need. Just download the Run For Your Life app right before you start your run, and within 5 minutes, we will release your greatest fear on you. You can choose from our list of fears such as clowns, axe murderers, and a werewolf, and with a premium account, we can customize the fear experience just for you and even send in an actual convict to chase you. I am Karen, a Co-Founder of Run For Your Life, and I have over 10 years of experience in procrastinating and pulling all-nighters for work deadlines, so I personally understand what our customers are looking for. My other co-founder Penny Wise, has over 40 years of clowning experience scaring young children and adults and Penny has several feature-length films made about her career. Our CTO Fenrir Greyback has over 30 years of experience as a werewolf at the haunted mansion ride at Universal Studios and has a PhD degree in costume design. Together, we are committed to making Run For Your Life solve the country's cardiovascular health crisis one fright at a time.

Activity: “Sharktank”

Category	What to Look For
Structure	Was the RISE framework followed?
Clarity	Was the pitch easy to understand? Was the idea clearly explained?
Confidence & Delivery	Did the speaker sound confident, enthusiastic, and engaging?
Memorability	Was it memorable or did something stand out (tagline, phrasing, etc.)?