AA/EE/ME 548: Linear Multivariable Control

Lecture 07

4/21/2025

Announcements

- Homework 1 due this week
  - No late days. Graded based on on-time submission + reasonable attempt
- Homework 2 out
- Project proposal due next week
  - Encouraged to attend OH
- Release form

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Sequential Decision Making

Control/decision-making over a horizon: choosing a sequence of actions where each one may have enduring consequences

Freshman

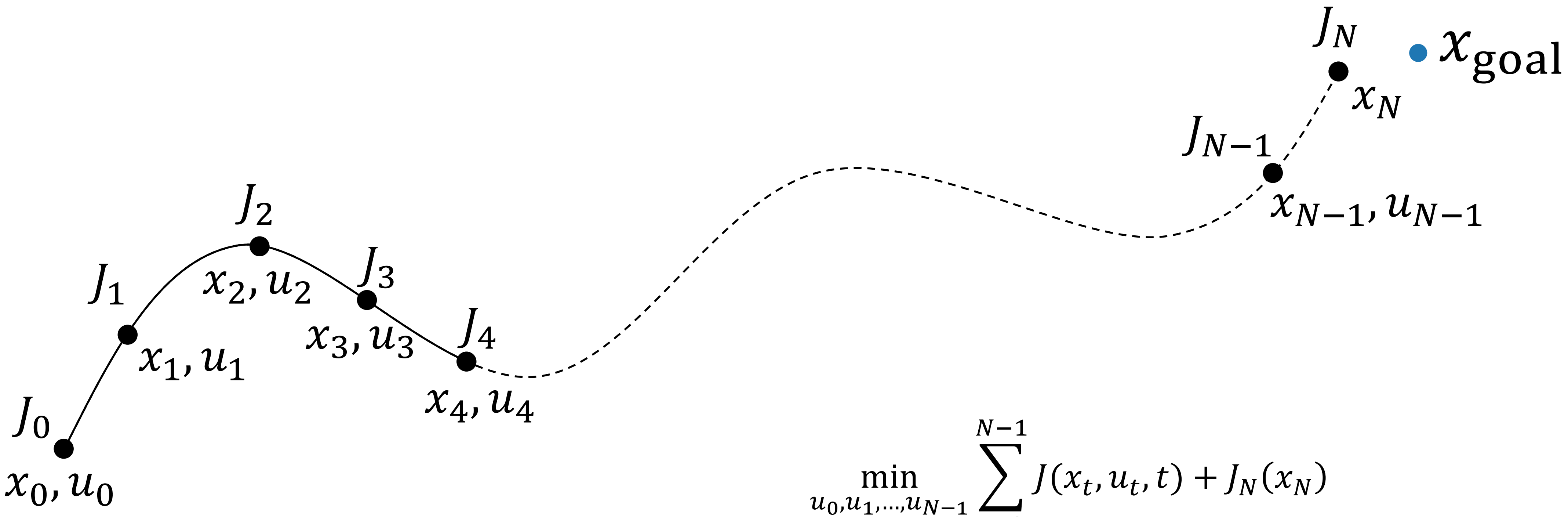
What should I spend my time on now so I can have a successful career in the future?

I have a midterm tomorrow morning. Should I eat a healthy meal, go to sleep early, or study through the night?

I have a homework due in 2 weeks, should I start now or go skiing with friends?

Should I attend that networking event or go home and just Netflix and chill?

Problem formulation (discrete time)



$$\min_{u_0, u_1, \ldots, u_{N-1}} \sum_{t=0}^{N-1} J(x_t, u_t, t) + J_N(x_N)$$

subject to $x_{t+1} = f(x_t, u_t, t)$
$x_0 = x_{\text{curr}}$
other constraints…

$$\min_{u_0,u_1,\ldots,u_{N-1}} \sum_{t=0}^{N-1} J(x_t,u_t,t) + J_N(x_N)$$
$$\text{subject to} \quad x_{t+1} = f(x_t,u_t,t)$$
$$x_0 = x_{\text{curr}}$$
other constraints…

Open-loop vs Closed-loop

Open-loop

- Solve the optimization problem directly.
  - Variables: controls (and states)
  - Constraints: dynamics, initial state, other constraints…
- Solution only for initial state x_0
  - Controls not valid if states changes
- Need to resolve for each new initial state
- Can leverage the suite of optimization techniques / solvers
- Generally, more tractable

(Will go over trajectory optimization and model predictive control later)

Closed-loop

- Find optimal policy $u^* = \pi^*(x)$ for all states
  - A formula to compute u for every x
- Solution for *all* initial states
- Solve for the policy *offline*
  - Rely on exploiting problem structure or offline data
- Generally, a very challenging problem.
  - Rarely closed-form unless for some simple settings (LQR)

Trajectory optimization

Sequential Convex (Quadratic) Programming

$(x_0^0, x_1^0, \dots, x_N^0)$ Initial
$(u_0^0, u_1^0, \dots, u_{N-1}^0)$ guess

$$\min_{u_0, u_1, \dots, u_{N-1}} \sum_{t=0}^{N-1} J(x_t, u_t, t) + J_N(x_N)$$

subject to $x_{t+1} = f(x_t, u_t, t)$

$x_0 = x_{\text{curr}}$

other constraints…

Nonlinear optimization

$$\min_{u_0, u_1, \dots, u_{N-1}} \sum_{t=0}^{N-1} \text{quadratic cost}$$

subject to linearized dynamics

$x_0 = x_{\text{curr}}$

linearized other constraints…

Turn into convex optimization
(quadratic program)

$(x_0^k, x_1^k, \dots, x_N^k)$
$(u_0^k, u_1^k, \dots, u_{N-1}^k)$

Module outline (homework 2)

- Principle of optimality (last week)
- Value function (Cost-to-go)
- Bellman equation
- Value iteration
- Stochastic case
- Continuous-time: Hamilton-Jacobi-Bellman equation
- HJ reachability
- Linear Quadratic Regulator
- Trajectory optimization

Principle of optimality

Let $\{\boldsymbol{u}_0^*, \boldsymbol{u}_1^*, \ldots, \boldsymbol{u}_{N-1}^*\}$ be an optimal control sequence, which together with $\boldsymbol{x}_0^*$ (initial state) determines the corresponding optimal state sequence $\{\boldsymbol{x}_0^*, \boldsymbol{x}_1^*, \ldots, \boldsymbol{x}_N^*\}$.

Consider the subproblem where we are at $\boldsymbol{x}_k^*$ at time k and want to minimize the cost-to-go from time k to N.

Then the truncated optimal sequence $\{\boldsymbol{u}_k^*, \boldsymbol{u}_{k+1}^*, \ldots, \boldsymbol{u}_{N-1}^*\}$ is optimal for the subproblem.

Tail of optimal sequences are optimal for tail subproblems