Homework 4 (LTL): COMS/SE/CPRE 412, ComS 512

Due-date: March 25 at 4:10PM (via Blackboard)
Late submissions are not allowed for this homework

Homework must be individual's original work. Collaborations and discussions of any form with any students (except our TA) or other faculty members are not allowed. If you have any questions/doubts/concerns, post your questions/doubts/concerns on Piazza and/or ask our TA or me.

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1. Consider a Kripke structure (S, T, L) such that

   - $S = \{s_1, s_2, s_3, s_4, s_5\}$;
   - $T = \{s_1, s_3), (s_1, s_4), (s_2, s_4), (s_3, s_4), (s_4, s_2), (s_4, s_3), (s_4, s_5), (s_5, s_4), (s_5, s_5)\})$ where $(s, s') \in T$ implies s has a transition to s';
   - $L(s_1) = \{a\}, L(s_2) = \{c\}, L(s_3) = \{b, c\}, L(s_4) = \{a, b, c\}$.

   Decide for each of the LTL formulas φ_i below whether $s_1 \models \varphi_i$ holds. If $s_1 \not\models \varphi_i$, present a path π such that $\pi[0] = s_1$ and $\pi \not\models \varphi_i$.

   (a) $\varphi_1 = \mathtt{FG}c$

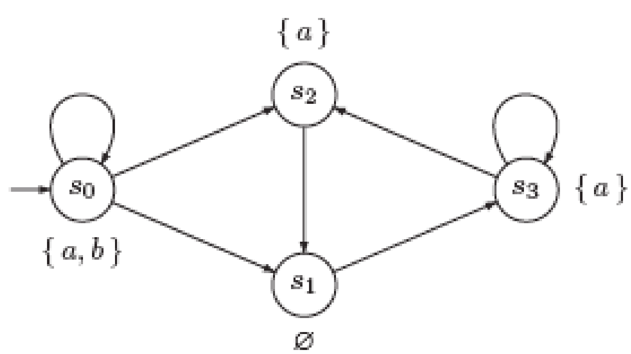
   (b) $\varphi_2 = \mathtt{GF}c$

   (c) $\varphi_3 = \mathtt{X}\neg c \Rightarrow \mathtt{XX}c$

   (d) $\varphi_4 = a\ \mathtt{U}\ \mathtt{G}(b\ \vee\ c)$

   (6)

2. Consider the following Kripke structure:



   Identify the states that satisfy the LTL formula $\mathtt{F}(a \wedge \mathtt{X}a)$. Justify your answer. (4)

3. Prove or disprove the following statement

   For any Kripke structure, a state satisfies the CTL formula $\mathtt{AG}(a \vee b)$ if it satisfies the LTL formula $\mathtt{G}(a) \vee \mathtt{G}(b)$, where a and b are atomic propositions.

   (5)

4. Can you use LTL to verify the following the properties for a state in a Kripke structure? Justify.

(a) There exists no path where p holds until q.

(b) There exists a path where p holds in a state, which is followed by a state where q holds.

(5)

5. Consider a new temporal operator in LTL $\mathtt{B}$ such that for any path π, $\pi \models \varphi \mathtt{B} \psi$ if and only if whenever there exists $i \geq 1.\pi^i \models \psi$, then there exists a $j < i.\pi^j \models \varphi$. Can you represent the LTL formula $\varphi \mathtt{B} \psi$ in terms of the LTL operators we have covered in class? Explain. (5)

6. Consider the following Büchi automata $\mathcal{B} = (Q, Q_0, \Gamma, \Delta, F)$ such that

   (a) $Q = \{q_0, q_1, q_2\}$

   (b) $Q_0 = \{q_0\}$

   (c) $\Gamma = \{a, b, c\}$

   (d) $\Delta = \{q_0 \xrightarrow{a} q_0,\ q_0 \xrightarrow{b} q_1,\ q_0 \xrightarrow{c} q_2,\ q_1 \xrightarrow{b} q_1,\ q_2 \xrightarrow{c} q_2\}$

   (e) $F = \{q_1, q_2\}$

   Does this automata accept all sequences that satisfy the LTL property $a\, \mathtt{U}(\mathtt{G}(b \vee c))$. Justify your answer. (5)