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HOMEWORK #7

#1 $s_0 \in [\![P_{\geq 1}(F p_i)]\!]$ for some $i \in \{1,2,3\}$

I took this to mean

$\exists i \in \{1,2,3\} \; s_0 \in [\![P_{\geq 1}(F p_i)]\!]$

This is true. Let $i=2$

Let P_i : prob that $s_i \models F p_2$

$P_0 : (.3)P_1 + (.7)P_2 \quad = .3(1) + .7(1) = 1$

$P_1 : (.2)P_3 + (.4)P_7 + (.4)P_4 \quad = .2(1) + .4(1) + .4(1) = 1$

$P_2 : (.2)P_4 + (.8)P_5 \quad = .2(1) + .8(1) = 1$

$P_3 : (1)P_6 = 1$

$P_4 : (1)P_8 = 1$

$P_5 : (1)P_8 = 1$

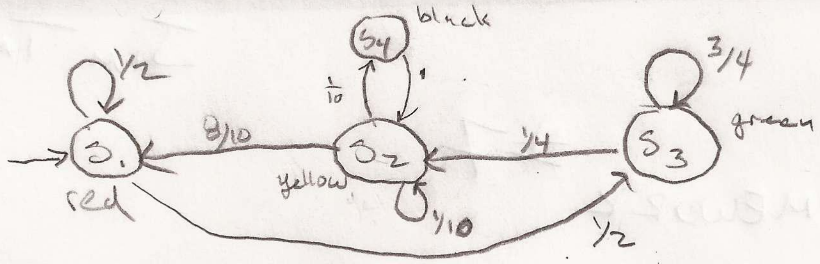
$P_6 : 1$

$P_7 : 1$

$P_8 : 1$

Thus $P_0 = 1$ and $s_0 \in [\![P_{\geq 1}(F p_2)]\!]$

$\square$



#2a Let P_i: Prob that $s_i \models X$ RED

P_1 : .5
P_3 : $\emptyset$
P_2 : .8
P_4 : $\emptyset$

$[\![P_{\geq 0.69} (X \langle red \rangle)]\!] = \{P_2\}$

#2b Let P_i: Prob that $s_i \models F$ black

$P_1 : .5(P_1) + .5(P_3)$ $\quad P_1 = P_3$
$P_3 : .25(P_2) + .75(P_3)$ $\quad P_3 = P_2$
$P_2 : .1(P_2) + .1(P_4) + .8(P_1)$
$P_4 : 1$

$P_2 = .1(P_2) + .1 + .8(P_1)$

$P_2 = .1(P_2) + .1 + .8(P_2)$

$P_2 = 1 \quad P_3 = 1 \quad P_1 = 1$

$[\![P_{>0.1} (F \langle black \rangle)]\!] = \{s_1, s_2, s_3, s_4\}$

This is because its the probability of finite paths,

#2C Let P_i: Prob. that $S_i \models F_{\leq 3}$ black

$$T = \begin{array}{c|cccc} & S_1 & S_2 & S_3 & S_4 \\ \hline S_1 & 1/2 & \varnothing & 1/2 & \varnothing \\ S_2 & 8/10 & 1/10 & \varnothing & 1/10 \\ S_3 & \varnothing & 1/4 & 3/4 & \varnothing \\ S_4 & \varnothing & 1 & \varnothing & \varnothing \end{array} \qquad V = \begin{bmatrix} \varnothing \\ \varnothing \\ \varnothing \\ 1 \end{bmatrix}$$

← S_4 only state that immediately satisfies $F\{\text{black}\}$

$$T' = \begin{bmatrix} 1/2 & 0 & 1/2 & 0 \\ 8/10 & 1/10 & \varnothing & 1/10 \\ 0 & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/10 \\ 0 \\ 1 \end{bmatrix}$$ ← $F_{\leq 1}\{\text{black}\}$

$$T' \times \begin{bmatrix} 0 \\ 1/10 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 11/100 \\ 1/40 \\ 1 \end{bmatrix}$$ ← $F_{\leq 2}\{\text{black}\}$

$$T' \times \begin{bmatrix} 0 \\ 1/100 \\ 1/40 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/80 \\ 101/1000 \\ 17/800 \\ 1 \end{bmatrix}$$ ← $F_{\leq 3}\{\text{black}\}$

Thus $[\![P_{>0.1}(F_{\leq 3}(\text{black}))]\!] = \{S_2, S_4\}$

#3 Thm:

$$s \models P_{>0}(Gp) \text{ iff } s \models EGp$$

Pf:

$s \models P_{>0}(Gp)$ iff

$s \models \neg P_{\leq 0}(\neg Gp)$ iff

$\neg [s \models P_{\leq 0}(Gp)]$ iff

$s \models P_{\leq 0}(Gp)$ says it is impossible for any path of s to satisfy Gp

$\neg [\forall \pi \in Path(s)\ \pi \not\models Gp]$ iff

$\exists \pi \in Path(s)\ \pi \models Gp$ iff

$s \models EGp$

#3 THM:

$$s \models P_{<1}(Fp) \quad \text{iff} \quad s \not\models AFp$$

i.e

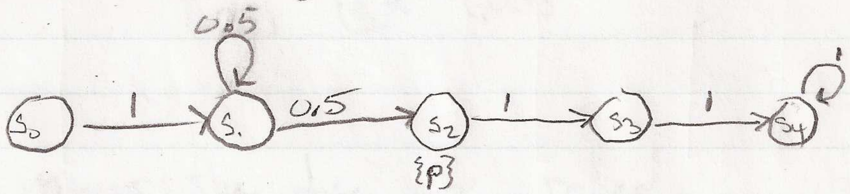
$$s \not\models P_{<1}(Fp) \quad \text{iff} \quad s \models AFp$$

i.e

$$s \models P_{\geq 1}(Fp), \quad \text{iff} \quad s \models AFp$$

DISPROOF;

Consider the following



Let $P_i : S_i \models Fp$

$P_0 = P_1$
$P_1 = 0.5(P_1) + 0.5(P_2)$
$P_2 = \underline{1}$

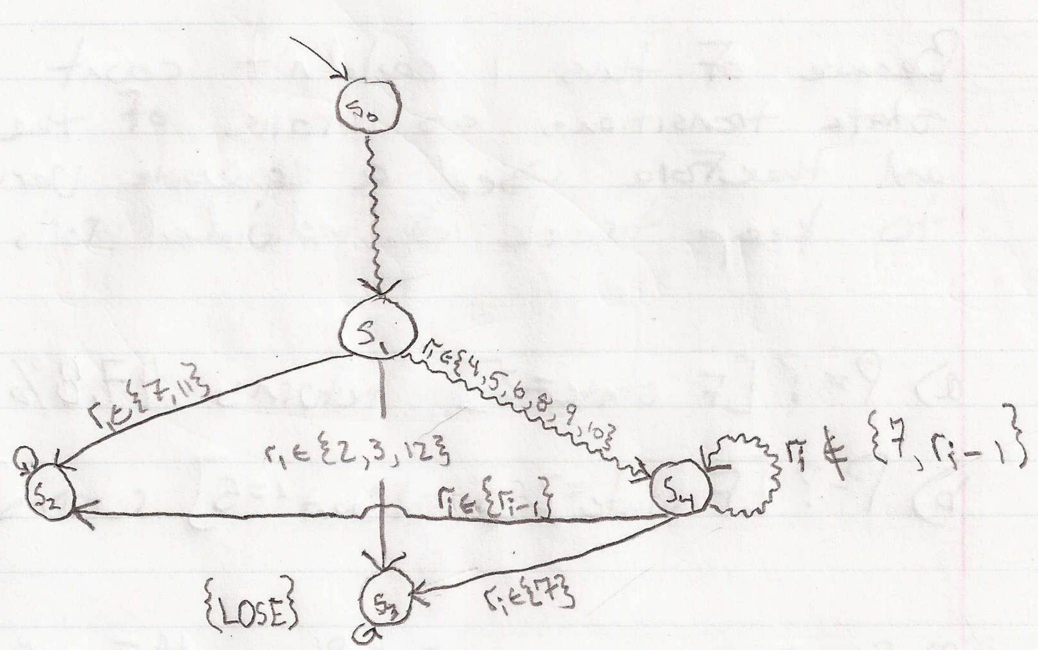
$P_1 = .5 + .5P_1 = \underline{1}$

Here we see $s_1 \models P_{\geq 1}(Fp)$ however $s_1 \not\models AFp$ because of the path $s_1 \to s_1 \to s_1 \dashrightarrow s_1$
Hence it NOT the case that

$s \models P_{\geq 1}(Fp)$ iff $s \models AFp$ and it is Not true that

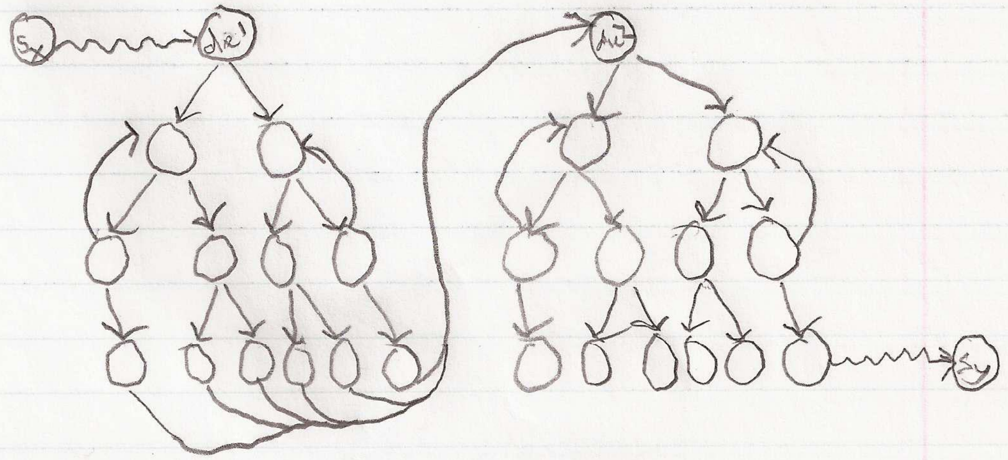
$s \models P_{<1}(Fp)$ iff $s \not\models AFp$

#4



$\rightsquigarrow$ means the dice are rolled.

In my model this was represented by "state = 5" and used the dice.pm example to roll two dice. This in turn meant using multiple states in between two rolls. I.E. between any two states S_x and S_y $S_x \rightsquigarrow S_y$ is represented by



Because of this I couldn't count state transitions as rolls of the dice and therefore used a seperate variable to keep track of individual rolls.

a) P=? [F state=2] returns 47.8%

b) P=? [F state=2 & rollCount <=5] returns 40.7%

a) Prob of winning a game > 50% is NOT verified

b) Prob of winning in at most 5 rolls >30% IS verified