Below is the Mathematica code used for calculating values for the measurements PDF $f_Y(y)$, using said values to calculate approximate expectation values for bin counts $\{\nu_i\}$, using true PDF $f_X(x)$ to calculate exact expectation values for bin counts $\{\mu_j\}$, and exporting data to CSV files for use in R code.

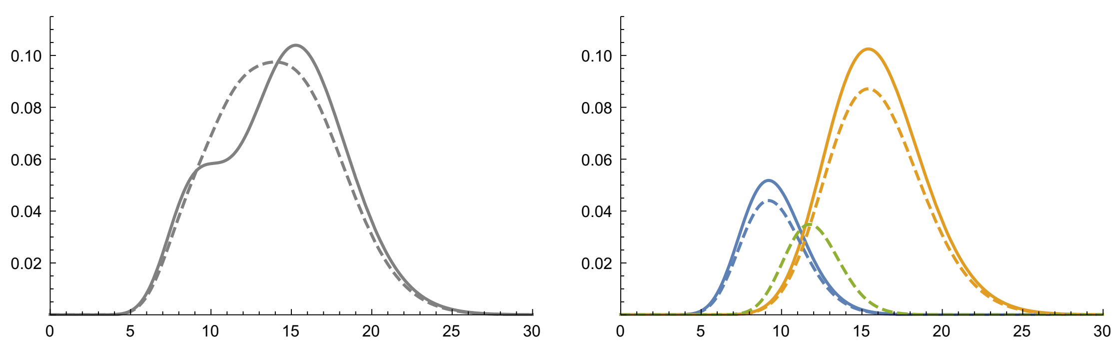
Defined below are:

- Each true PDF $f_X(x)$, **f1[x]** and **f2[x]**.
- The Kernel $g(x, y)$, **g[x,y]**.
- The efficiency $\epsilon(x)$, **ϵ[x]**.

In[1134]:=

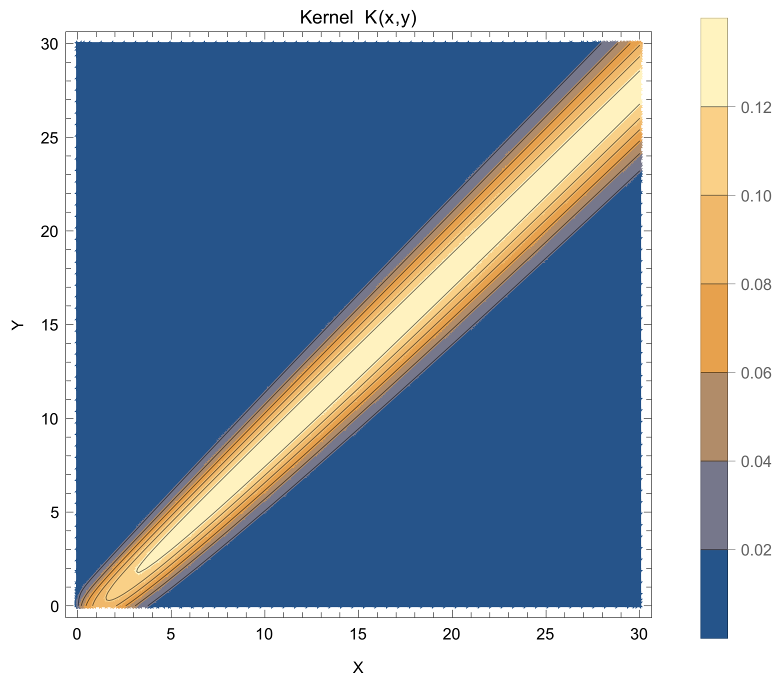
```
alpha1 = 24; alpha2 = 29; alpha3 = 48;
beta1 = 0.4; beta2 = 0.55; beta3 = 0.25;
p = 0.15;
p1 = {0.25, 0.75}; p2 = {(1 - p) p1[[1]], (1 - p) p1[[2]], p};
fp1[x_] := PDF[GammaDistribution[alpha1, beta1], x];
fp2[x_] := PDF[GammaDistribution[alpha2, beta2], x];
fp3[x_] := PDF[GammaDistribution[alpha3, beta3], x];
f1[x_] := p1[[1]] × fp1[x] + p1[[2]] × fp2[x];
f2[x_] := p2[[1]] × fp1[x] + p2[[2]] × fp2[x] + p2[[3]] × fp3[x];
g[x_, y_] := PDF[NormalDistribution[-x^(1/4), Log[(x + 10)/4]], y - x];
ϵ[x_] := 1 - Exp[-Sqrt[x] / 4];
GraphicsGrid[{{Plot[{f1[x], f2[x]}, {x, 0, 30}, PlotStyle → {Gray, {Dashed, Gray}},
    PlotRange → {{0, 30}, {0, 0.115}}, ImageSize → {250, 250}],
   Show[Plot[{p1[[1]] × fp1[x], p1[[2]] × fp2[x]}, {x, 0, 30},
     PlotRange → {{0, 30}, {0, 0.115}}, ImageSize → {250, 250}],
    Plot[{p2[[1]] × fp1[x], p2[[2]] × fp2[x], p2[[3]] × fp3[x]}, {x, 0, 30}, PlotStyle → {Dashed},
     PlotRange → {{0, 30}, {0, 0.115}}, ImageSize → {250, 250}]]}}, ImageSize → Large]
```

Out[1145]=



In[1146]:=
```
ContourPlot[ϵ[x] × g[x, y], {x, 0, 30},
 {y, 0, 30}, PlotRange → All, PlotLegends → Automatic,
 PlotPoints → 50, MaxRecursion → 3, ImageSize → {350, 350},
 PlotLabel → "Kernel  K(x,y)", FrameLabel → {"X", "Y"}]
```

Out[1146]=



Performed below:

- The sequence of values for x and y from 0 to 30 with step sizes of 0.01 are generated for plotting, **xys**.
- The values of each true PDF $f_x(x)$ are calculated for plotting, **f1xs** and **f2xs**.
- Each PDF $f_x(x)$ is integrated across bins of width $\Delta x = 1$ to produce its corresponding histogram, **hist1xs** and **hist2xs**.

In[1178]:=
```
xys = Table[N[x / 100], {x, 0, 3000}];
f1xs = N[f1[xys]];
f2xs = N[f2[xys]];
hist1xs = N[Table[∫_{i-1}^{i} f1[x] ⅆx, {i, 1, 30}]];
hist2xs = N[Table[∫_{i-1}^{i} f2[x] ⅆx, {i, 1, 30}]];
```

Performed below:

- Point-by-point calculations of $\int_{-\infty}^{\infty} f_X(x)\, \epsilon(x)\, g(x, y)\, dx$ to get values for each $f_Y(y)$ for each bin separately, **f1ysd[[i]]** and **f2ysd[[i]]**.
- The mean for each bin is then found get **hist1ys** and **hist2ys**.
- The bins are combined into the values of each $f_y(y)$ to be plotted, **f1ys** and **f2ys**.

The first item takes several minutes to perform.

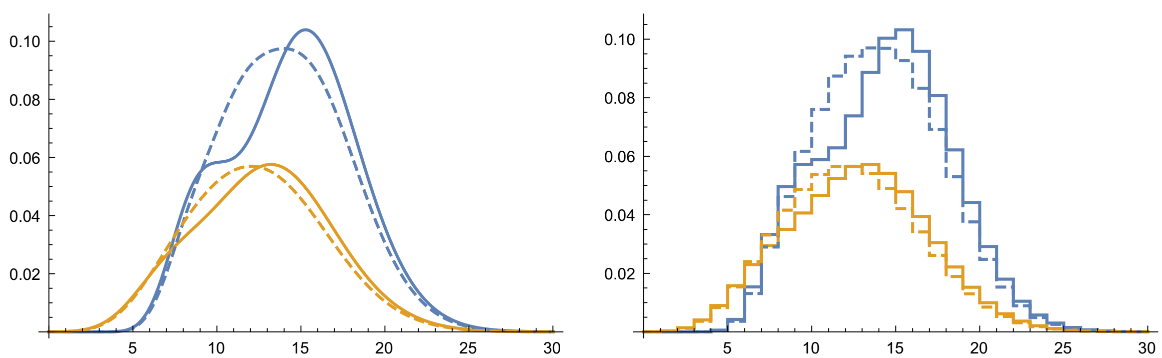
In[1152]:=
```
f1ysd = Table[Null, {x, 1, 30}];
For[i = 1, i < 31, i++,
  f1ysd[[i]] = NIntegrate[
    ExpandAll[f1[x] × ϵ[x] × g[x, N[Table[y / 100, {y, (i - 1) * 100, i * 100}]]]], {x, -∞, ∞}]];
f2ysd = Table[Null, {x, 1, 30}];
For[i = 1, i < 31, i++,
  f2ysd[[i]] =
   NIntegrate[ExpandAll[f2[x] × ϵ[x] × g[x, N[Table[y / 100, {y, (i - 1) * 100, i * 100}]]]], {x, -∞, ∞}]];
```

In[1183]:=
```
hist1ys = Table[Mean[f1ysd[[i]]], {i, 1, 30}];
hist2ys = Table[Mean[f2ysd[[i]]], {i, 1, 30}];
seq = Table[x, {x, 2, 101}];
f1ys = Flatten[Join[{f1ysd[[1]]}, Table[f1ysd[[i]][[seq]], {i, 2, 30}]]];
f2ys = Flatten[Join[{f2ysd[[1]]}, Table[f2ysd[[i]][[seq]], {i, 2, 30}]]];
```

Plotting **f1xs**, **f1ys**, **f2xs**, **f2ys**, **hist1xs**, **hist1ys**, **hist2xs**, and **hist2ys**

In[1161]:=
```
xys1 = xys; xys2 = xys;
GraphicsGrid[{{Show[ListLinePlot[{Transpose[{xys1, f1xs}], Transpose[{xys1, f1ys}]}],
    ListLinePlot[{Transpose[{xys2, f2xs}], Transpose[{xys2, f2ys}]}, PlotStyle → Dashed]],
   Show[ListStepPlot[{Transpose[{Table[x, {x, 0, 29}], hist1xs}],
      Transpose[{Table[x, {x, 0, 29}], hist1ys}]}],
    ListStepPlot[{Transpose[{Table[x, {x, 0, 29}], hist2xs}],
      Transpose[{Table[x, {x, 0, 29}], hist2ys}]}, PlotStyle → Dashed]]}}]
```

Out[1162]=



Saving $f_Y(y)$ estimates.

In[1163]:=
```
SetDirectory[NotebookDirectory[]];
Export["f1yEstimate.csv",
  Transpose[{PrependTo[xys1, Y], PrependTo[f1ys, Density]}]];
Export["f2yEstimate.csv",
  Transpose[{PrependTo[xys2, Y], PrependTo[f2ys, Density]}]];
```

Generating column contents for the tibble `exp_hist` to be used for plotting **histxs** and **histxy** in `R`.

In[1188]:=
```
lbin1 = Join[Table[x, {x, 0, 29}], Table[x, {x, 0, 29}]];
bin1 = Join[Table[x, {x, 1, 30}], Table[x, {x, 1, 30}]];
treat1 = Join[Table["Truth", {x, 1, 30}], Table["Measured", {x, 1, 30}]];
lcount1 = Join[{0}, hist1xs[[Table[x, {x, 1, 29}]]],
   {0}, hist1ys[[Table[x, {x, 1, 29}]]]];
count1 = Join[hist1xs, hist1ys];
lbin2 = Join[Table[x, {x, 0, 29}], Table[x, {x, 0, 29}]];
bin2 = Join[Table[x, {x, 1, 30}], Table[x, {x, 1, 30}]];
treat2 = Join[Table["Truth", {x, 1, 30}], Table["Measured", {x, 1, 30}]];
lcount2 = Join[{0}, hist2xs[[Table[x, {x, 1, 29}]]],
   {0}, hist2ys[[Table[x, {x, 1, 29}]]]];
count2 = Join[hist2xs, hist2ys];
```

Saving expected value histogram data.

In[1198]:=
```
Export["hist1Expected.csv",
  Transpose[{PrependTo[lbin1, LBin], PrependTo[bin1, Bin],
    PrependTo[lcount1, LCounts], PrependTo[count1, Counts],
    PrependTo[treat1, Treatment]}]];
Export["hist2Expected.csv",
  Transpose[{PrependTo[lbin2, LBin], PrependTo[bin2, Bin],
    PrependTo[lcount2, LCounts], PrependTo[count2, Counts],
    PrependTo[treat2, Treatment]}]];
```