Below is the Mathematica code used for calculating values for the measurements PDF $f_Y(y)$, using said values to calculate approximate expectation values for bin counts $\{\nu_i\}$, using true PDF $f_X(x)$ to calculate exact expectation values for bin counts $\{\mu_j\}$, and exporting data to CSV files for use in `R` code.

Defined below are:

- The true PDF $f_X(x)$, **f[x]**.
- The Kernel $g(x, y)$, **g[x,y]**.
- The efficiency $\epsilon(x)$, **ϵ[x]**.

In[1]:=
```
f[x_] := 1/3 PDF[CauchyDistribution[12, 2], x] + 2/3 PDF[CauchyDistribution[19, 2], x];
g[x_, y_] := PDF[NormalDistribution[-2 (Log[Abs[x] + 1])^(1/3), 2 Exp[-Abs[x] / 30]], y - x];
ϵ[x_] := (1 - Exp[-Abs[x] / 80])^(1/4);
```

Performed below:

- The sequence of values for x and y from 0 to 30 with step sizes of 0.01 are generated for plotting, **xys**.
- The values of the true PDF $f_x(x)$ are calculated for plotting, **fxs**.
- The PDF $f_x(x)$ is integrated across bins of width Δx = 1 to produce its corresponding histogram, **histxs**.

In[43]:=
```
xys = Table[N[x / 100], {x, -600, 3600}];
fxs = N[f[xys]];
histxs = N[Table[∫_{i-1}^{i} f[x] ⅆx, {i, -5, 36}]];
```

Performed below:

- Point-by-point calculations of $\int_{-\infty}^{\infty} f_X(x)\,\epsilon(x)\,g(x, y)\,dx$ to get values of the PDF $f_Y(y)$ for each bin separately, **fysd[[i]]**.
- The mean for each bin is then found get the **histys**.
- The bins are combined into the values of $f_Y(y)$ to be plotted, **fys**.

The first item takes several minutes to perform.

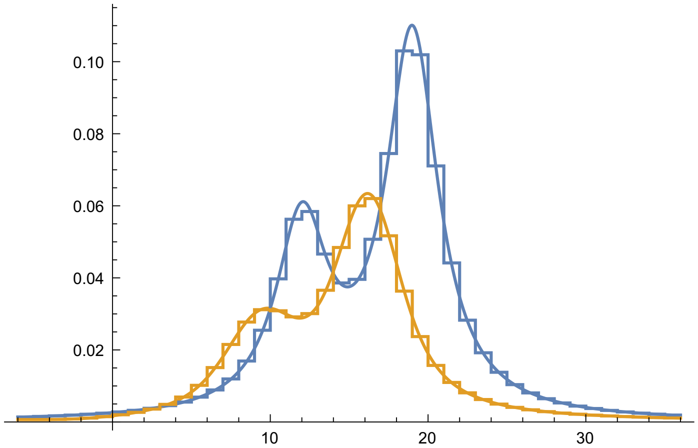
```
fysd = Table[Null, {x, -5, 36}];
For[i = 1, i < 43, i++,
  If[i - 6 == 24 || i - 6 == 26,
   fysd[[i]] =
    NIntegrate[ExpandAll[f[x] × ϵ[x] × g[x, N[Table[x / 100, {x, (i - 7) * 100, (i - 6) * 100}]]]],
     {x, -∞, ∞}, AccuracyGoal → 100, MaxRecursion → 800, MinRecursion → 600],
   fysd[[i]] = NIntegrate[ExpandAll[f[x] × ϵ[x] ×
       g[x, N[Table[x / 100, {x, (i - 7) * 100, (i - 6) * 100}]]]], {x, -∞, ∞}]]];
```

In[40]:=
```
histys = Table[Mean[fysd[[i]]], {i, 1, 42}];
seq = Table[x, {x, 2, 101}];
fys = Flatten[Join[{fysd[[1]]}, Table[fysd[[i]][[seq]], {i, 2, 42}]]];
```

Plotting **fxs**, **fys**, **histxs**, and **histys**.

In[46]:=
```
Show[ListLinePlot [{Transpose [{xys , fxs}], Transpose [{xys , fys}]}],
 ListStepPlot [
  {Transpose [{Table [x, {x, -6, 35}], histxs }], Transpose [{Table [x, {x, -6, 35}], histys }]}]]
```

Out[46]=



Generating column contents for the tibble `exp_hist` to be used for plotting **histxs** and **histxy** in `R`.

In[47]:=
```
hbinl  = Join[Table[x, {x, -6, 35}], Table[x, {x, -6, 35}]];
hbinh  = Join[Table[x, {x, -5, 36}], Table[x, {x, -5, 36}]];
hcountl  = Join[{0}, histxs [[Table [x, {x, 1, 41}]]],
    {0}, histys [[Table [x, {x, 1, 41}]]]];
hcount  = Join[histxs , histys ];
treat  = Join[Table["not  folded ", {x, 1, 42}], Table["folded ", {x, 1, 42}]];
```

Saving plotting data to their appropriate files.

In[54]:=
```
SetDirectory [NotebookDirectory  []];
Export["fyEstimate .csv",
  Transpose [{PrependTo [xys , Y], PrependTo [fys , Density ]}]];
Export["histExpected  .csv",
  Transpose [{PrependTo [hbinl , binLow], PrependTo [hbinh , binHigh ],
    PrependTo [hcountl , CountsL ], PrependTo [hcount , Counts],
    PrependTo [treat , Treatment ]}]];
```