Below is the Mathematica code used for calculating values for the measurements PDF $f_Y(y)$, using said values to calculate approximate expectation values for bin counts $\{v_i\}$, using true PDF $f_X(x)$ to calculate exact expectation values for bin counts $\{\mu_j\}$, and exporting data to CSV files for use in **R** code.

Defined below are:

- The true PDF $f_X(x)$, **f[x]**.
- The Kernel $g(x, y)$, **g[x,y]**.
- The efficiency $\epsilon(x)$, **ϵ[x]**.

In[416]:=
```
alpha1 = 24; alpha2 = 42; beta = 0.4; p = 2 / 7;
f[x_] :=
  p PDF[GammaDistribution [alpha1 , beta], x] + (1 - p) PDF[GammaDistribution [alpha2 , beta], x];
g[x_ , y_] := PDF[NormalDistribution [-x^1/4, Log[(x + 10)/4]], y - x];
ϵ[x_] := 1 - Exp[- √x / 4];
```

Performed below:

- The sequence of values for x and y from 0 to 30 with step sizes of 0.01 are generated for plotting, **xys**.
- The values of the true PDF $f_x(x)$ are calculated for plotting, **fxs**.
- The PDF $f_x(x)$ is integrated across bins of width $\Delta x = 1$ to produce its corresponding histogram, **histxs**.

In[420]:=
```
xys = Table[N[x / 100], {x, 0, 3000}];
fxs = N[f[xys]];
histxs = N[Table[∫_(i-1)^i f[x] dx, {i, 1, 30}]];
```

Performed below:

- Point-by-point calculations of $\int_{-\infty}^{\infty} f_X(x)\,\epsilon(x)\,g(x, y)\,dx$ to get values of the PDF $f_Y(y)$ for each bin separately, **fysd[[i]]**.
- The mean for each bin is then found get the **histys**.
- The bins are combined into the values of $f_y(y)$ to be plotted, **fys**.

The first item takes several minutes to perform.

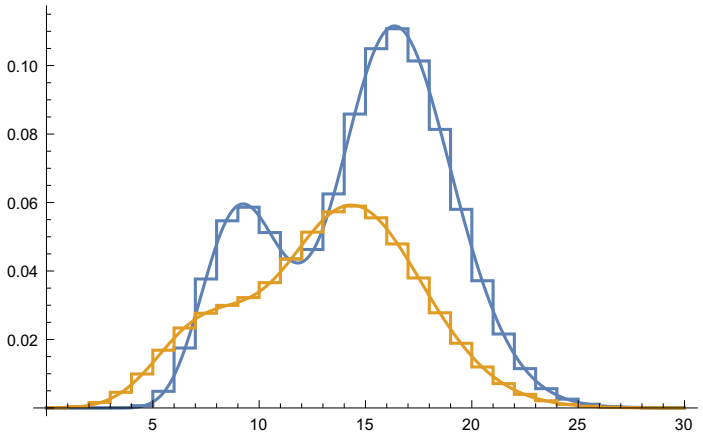
In[423]:=
```
fysd = Table[Null , {x, 1, 30}];
For[i = 1, i < 31, i++,
  fysd[[i]] =
   NIntegrate [ExpandAll [f[x] × ϵ[x] × g[x, N[Table[x / 100 , {x, (i - 1) * 100 , i * 100}]]]], {x, -∞, ∞}]];
```

In[425]:=
```
histys = Table[Mean[fysd[[i]]], {i, 1, 30}];
seq = Table[x, {x, 2, 101}];
fys = Flatten[Join[{fysd[[1]]}, Table[fysd[[i]][[seq]], {i, 2, 30}]]];
```

Plotting **fxs**, **fys**, **histxs**, and **histys**.

In[428]:=
```
Show[ListLinePlot [{Transpose [{xys , fxs}], Transpose [{xys , fys}]}],
 ListStepPlot [
  {Transpose [{Table[x, {x, 0, 29}], histxs }], Transpose [{Table[x, {x, 0, 29}], histys }]}]]
```

Out[428]=



Generating column contents for the tibble `exp_hist` to be used for plotting **histxs** and **histxy** in `R`.

In[429]:=
```
hbinl  = Join[Table[x, {x, 0, 29}], Table[x, {x, 0, 29}]];
hbinh  = Join[Table[x, {x, 1, 30}], Table[x, {x, 1, 30}]];
hcountl  = Join[{0}, histxs [[Table[x, {x, 1, 29}]]],
    {0}, histys [[Table[x, {x, 1, 29}]]]];
hcount  = Join[histxs , histys ];
treat  = Join[Table["not  folded ", {x, 1, 30}], Table["folded ", {x, 1, 30}]];
```

Saving plotting data to their appropriate files.

In[434]:=
```
SetDirectory [NotebookDirectory []];
Export["fyEstimate .csv",
  Transpose [{PrependTo [xys , Y], PrependTo [fys , Density ]}]];
Export["histExpected .csv",
  Transpose [{PrependTo [hbinl , binLow ], PrependTo [hbinh , binHigh ],
    PrependTo [hcountl , CountsL ], PrependTo [hcount , Counts ],
    PrependTo [treat , Treatment ]}]];
```