Nonparametric Self-Exciting Point Processes: Benefits and Uses

James Molyneux

Assistant Professor

Department of Statistics

Slides: *https://github.com/jimmylovestea/wnar-2021-slides*

: jimmylovestea

: @jimmylovestea

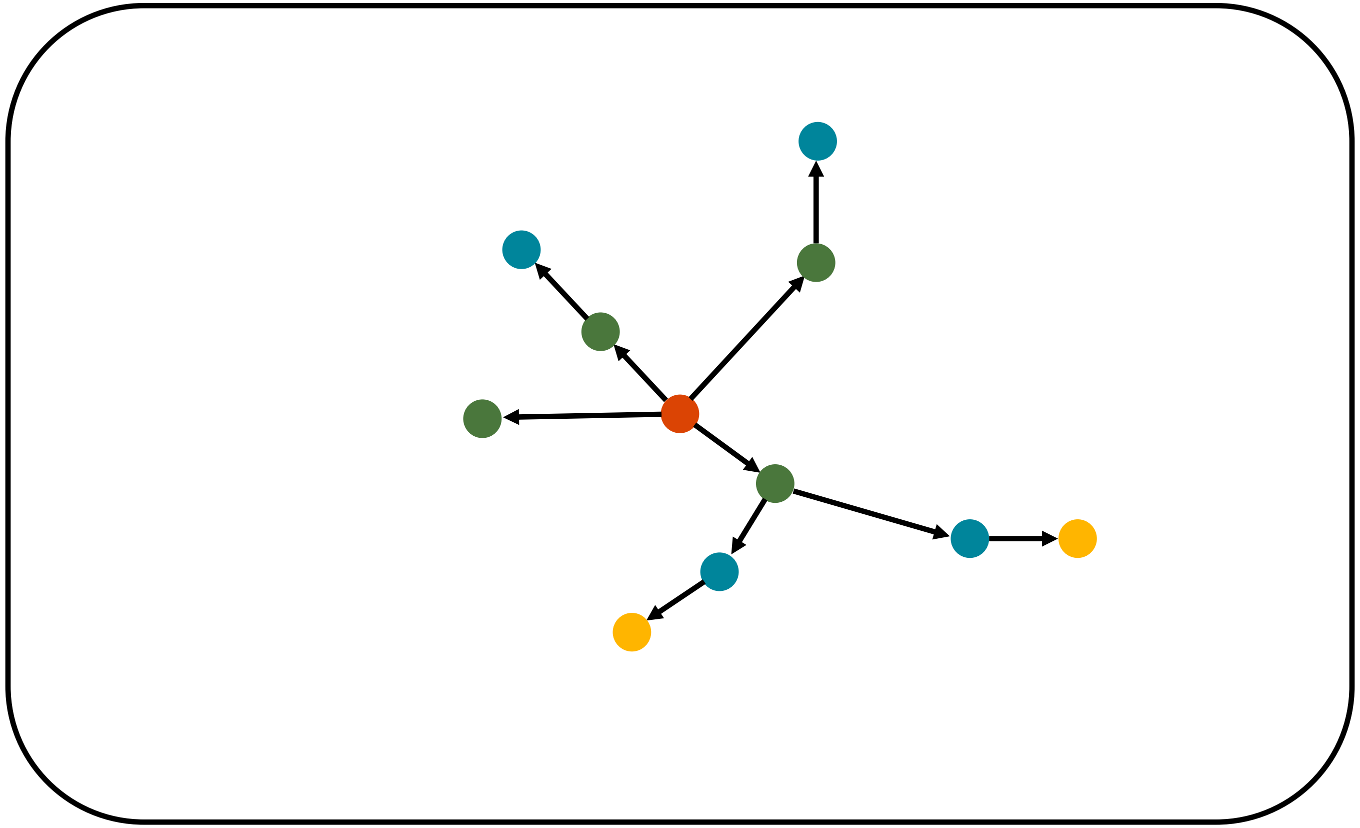
Oregon State University

Outline

- Self-exciting phenomenon
- Self-exciting point processes
- Recent nonparametric innovations
- Application: Hector Mine earthquake sequence
- Closing thoughts

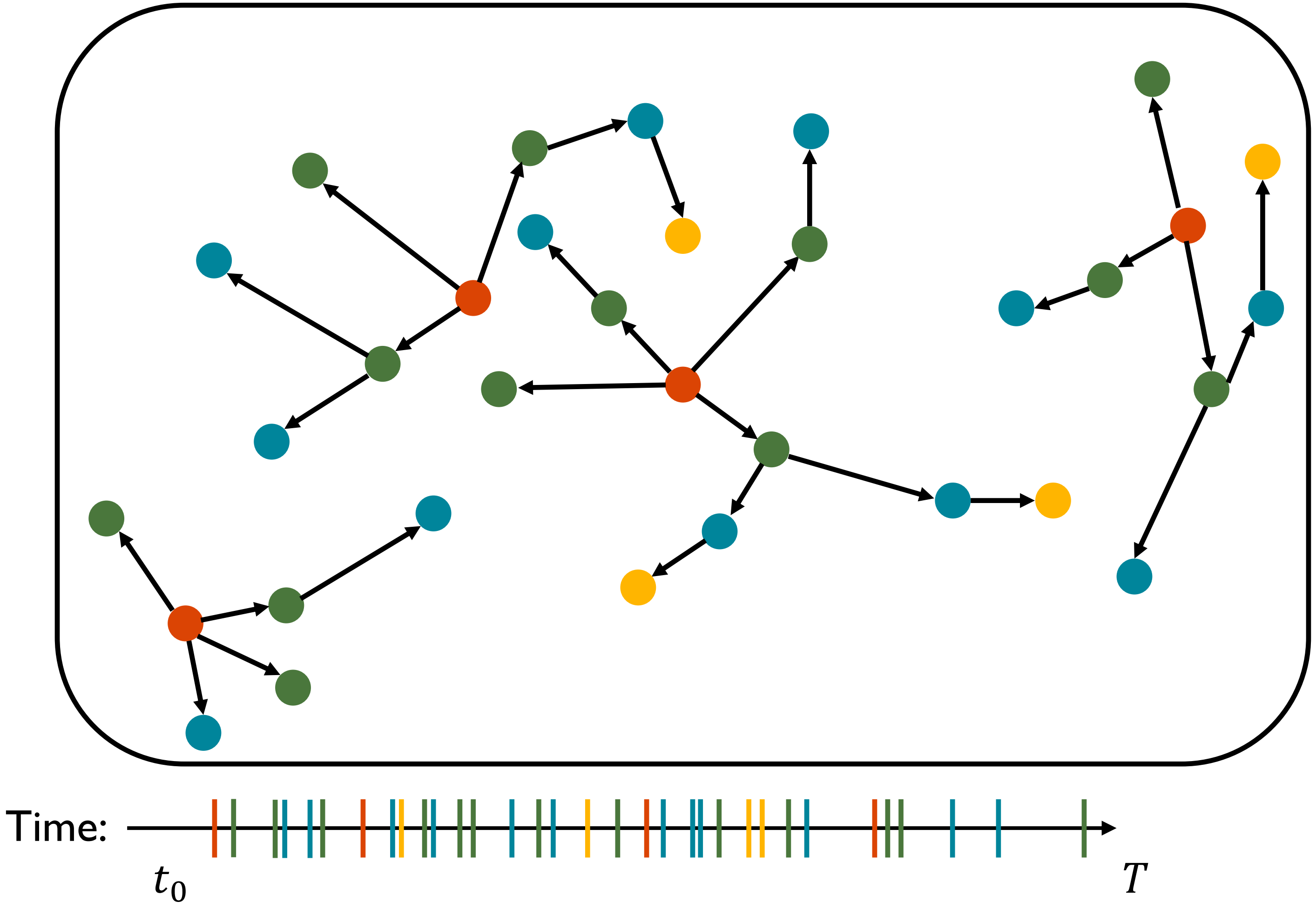
Self-exciting phenomenon

Self-exciting phenomenon

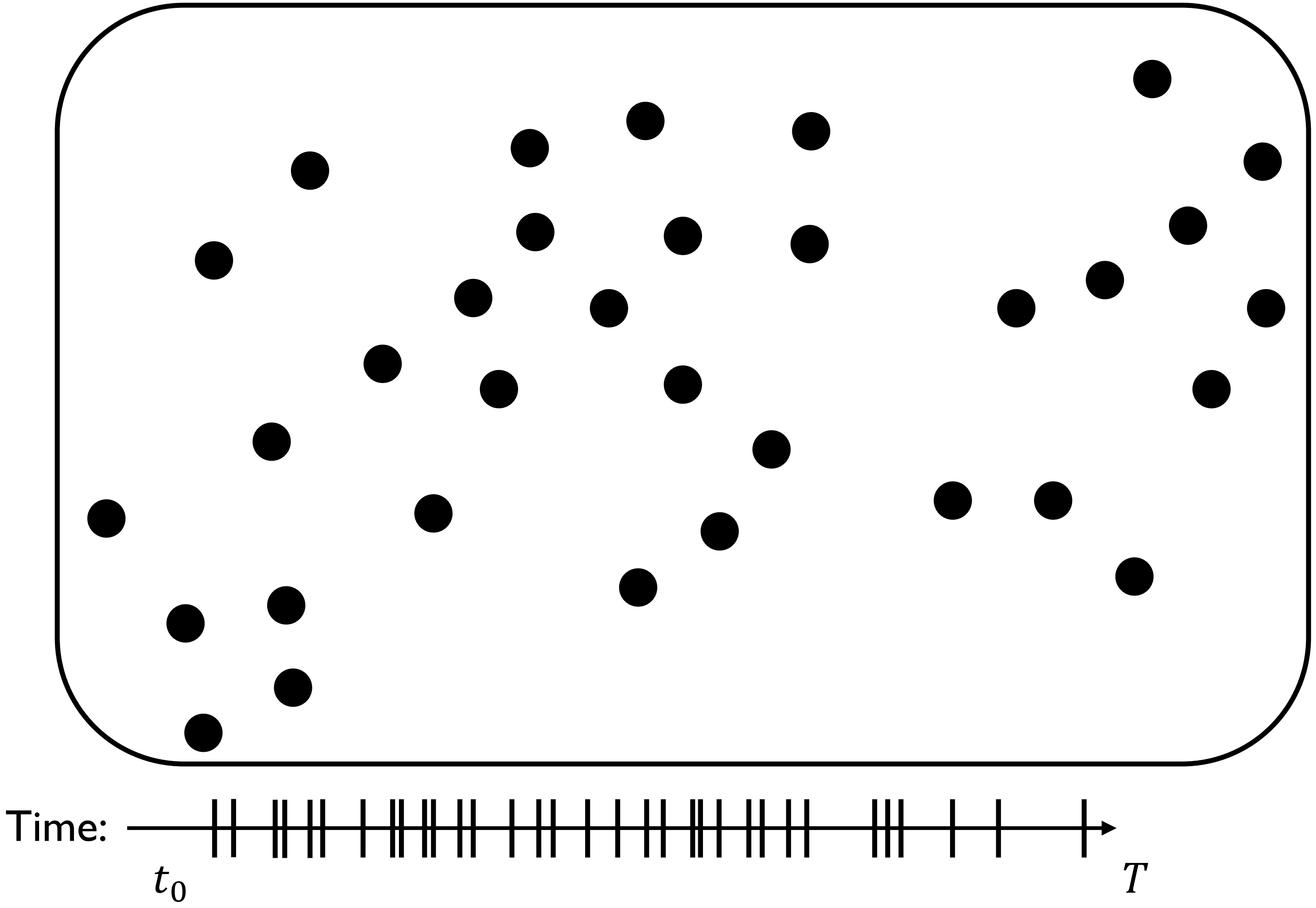


Time: $t_0 \longrightarrow t_1 \longrightarrow t_2 \longrightarrow t_3 \longrightarrow t_4 \longrightarrow \cdots \longrightarrow T$

Self-exciting phenomenon



Self-exciting phenomenon



Self-exciting phenomenon

- Invasive Species (Balderama, 2012)
  - Invasive species introduced by animals, floods, winds, people, etc.
  - Reproduce and spread
- Crime (Mohler, 2011)
  - Successful burglary occurs in a neighborhood
  - Successive burglaries attempted nearby
- Disease Epidemics (Park, 2020; Bertozzi, 2020)
  - A person is infected with a disease
  - The disease spreads from person to person
- Earthquakes (Ogata, 1999)
  - Mainshock earthquake occurs
  - Aftershocks occur afterwards

Self-exciting point processes

Self-exciting point processes

- *Conditional Intensity* : $\boxed{\lambda(x, y, t|\mathcal{H}_t)}$
  - Describes the infinitesimal rate of occurrences at time t and location (x, y)
- *Background rate* : $\boxed{\mu(x, y)}$
  - Function which describes the rate of *background* events at location (x, y)
- *Triggering function* : $\boxed{\nu(x - x_i, y - y_i, t - t_i)}$
  - Function which describes the amount of *excitation* from previous events

$$\boxed{\lambda(x, y, t|\mathcal{H}_t)} = \boxed{\mu(x, y)} + \sum_{i:t_i<t} \boxed{\nu(x - x_i, y - y_i, t - t_i)}$$

Self-exciting point processes

- In applications, we often let the triggering function be *seperable*
- The triggering function, $\nu(x - x_i, y - y_i, t - t_i)$, is a product of:
  -
  -
  -

$$\lambda(x, y, t|\mathcal{H}_t) = \mu(x, y) + \sum_{i:t_i<t} \nu(x - x_i, y - y_i, t - t_i)$$

Self-exciting point processes

- In applications, we often let the triggering function be *seperable*
- The triggering function, $\nu(x - x_i, y - y_i, t - t_i)$, is a product of:
  - Time : $\boxed{g(t - t_i)}$
  -
  -

$$\lambda(x, y, t|\mathcal{H}_t) = \mu(x, y) + \sum_{i:t_i<t} \boxed{g(t - t_i)}$$

Self-exciting point processes

- In applications, we often let the triggering function be *seperable*
- The triggering function, $\nu(x - x_i, y - y_i, t - t_i)$, is a product of:
  - Time : $g(t - t_i)$
  - Space : $\boxed{h(x - x_i, y - y_i)}$
  -

$$\lambda(x, y, t|\mathcal{H}_t) = \mu(x, y) + \sum_{i:t_i<t} g(t - t_i)\ \boxed{h(x - x_i, y - y_i)}$$

Self-exciting point processes

- In applications, we often let the triggering function be *seperable*
- The triggering function, $\nu(x - x_i, y - y_i, t - t_i)$, is a product of:
  - Time : $g(t - t_i)$
  - Space : $h(x - x_i, y - y_i)$
  - Marks : $\boxed{k(m_i)}$

$$\lambda(x, y, t|\mathcal{H}_t) = \mu(x, y) + \sum_{i:t_i<t} g(t - t_i)h(x - x_i, y - y_i)\boxed{k(m_i)}$$

Self-exciting point processes

- In applications, we often let the triggering function be *seperable*
- The triggering function, $\nu(x - x_i, y - y_i, t - t_i)$, is a product of:
  - Time : $g(t - t_i)$
  - Space : $h(x - x_i, y - y_i)$
  - Marks : $k(m_i)$
- Marks can be some additional information about an event
  - Earthquake magnitudes
  - Crime types
  - Plant size

$$\lambda(x, y, t | \mathcal{H}_t) = \mu(x, y) + \sum_{i:t_i<t} g(t - t_i) h(x - x_i, y - y_i) k(m_i)$$

Self-exciting point processes

- For earthquakes, the conditional intensity at time t and location (x, y) is:
  - $\mu(x, y) = \tilde{\mu}(x, y)$
  - $g(t - t_i) = (t - t_i + c)^{-p}$
  - $h(x - x_i, y - y_i) = \left((x - x_i)^2 + (y - y_i)^2 + d\right)^{-q}$
  - $k(m_i) = Ke^{\alpha(m_i - m_0)}$
- Estimate parameters $\{\tilde{\mu}, c, p, d, q, K, \alpha\}$ using maximum likelihood estimation

$$\lambda(x, y, t|\mathcal{H}_t) = \tilde{\mu}(x, y) + \sum_{i:t_i<t} (t - t_i + c)^{-p} \left((x - x_i)^2 + (y - y_i)^2 + d\right)^{-q} Ke^{\alpha(m_i - m_0)}$$

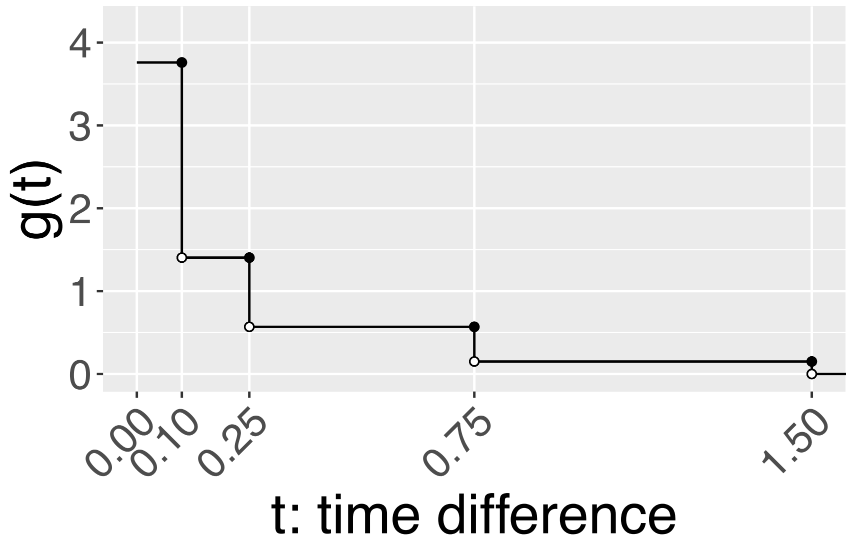
WHAT IF WE'RE UNSURE WE'VE SPECIFIED OUR MODEL CORRECTLY?

Self-exciting point processes

- Formulate the model nonparametrically (Marsan and Lengliné, 2008)
  - Introduce a latent variable, Z, which describes whether an event is a *background* event or a *triggered* event
    - Compute probability event i is a background event
    - Compute probability event i is triggered by previous event j
  - Use probabilities to update $\mu(x, y)$
  - Use probabilities to update *histogram estimators* of $g(t - t_i),\ h(x - x_i, y - y_i),\ k(m_i)$

Self-exciting point processes

- Histogram estimators:
  - Take domain of *pairwise* differences of points in time and space
  - Split the domains into disjoint sets of bins/intervals
  - Compute step functions (i.e. constant values) for each bin/interval based on the probabilities of points which fall into each bin/interval
  - Do something similar for the marks
- Example:



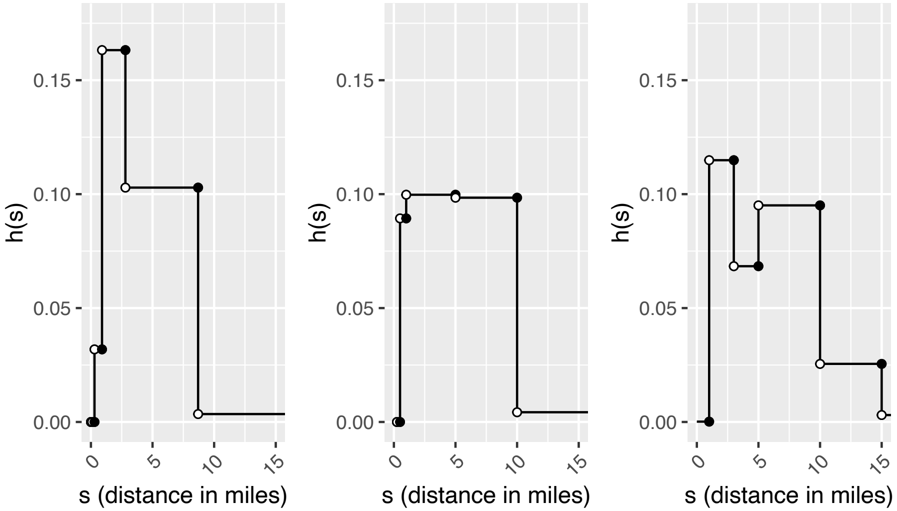
Self-exciting point processes

- With estimated histogram estimators and estimated background rate, $\mu(x, y)$
  - Update probability each event is a *background* event
  - Update probabilities each event was *triggered* by a previous event
- Iterate until convergence

Recent nonparametric innovations

Recent nonparametric innovations

- Peter Boyd, Ph.D. Candidate of Statistics, Oregon State University
- *Does how we decide to break up the domains of our histogram estimators matte*
  - Spoiler alert: Yes



- *Is there some way to resolve this issue which would also let us estimate the triggering functions in a less "clunky" manner?*
  - Spoiler alert: Also yes

Recent nonparametric innovations

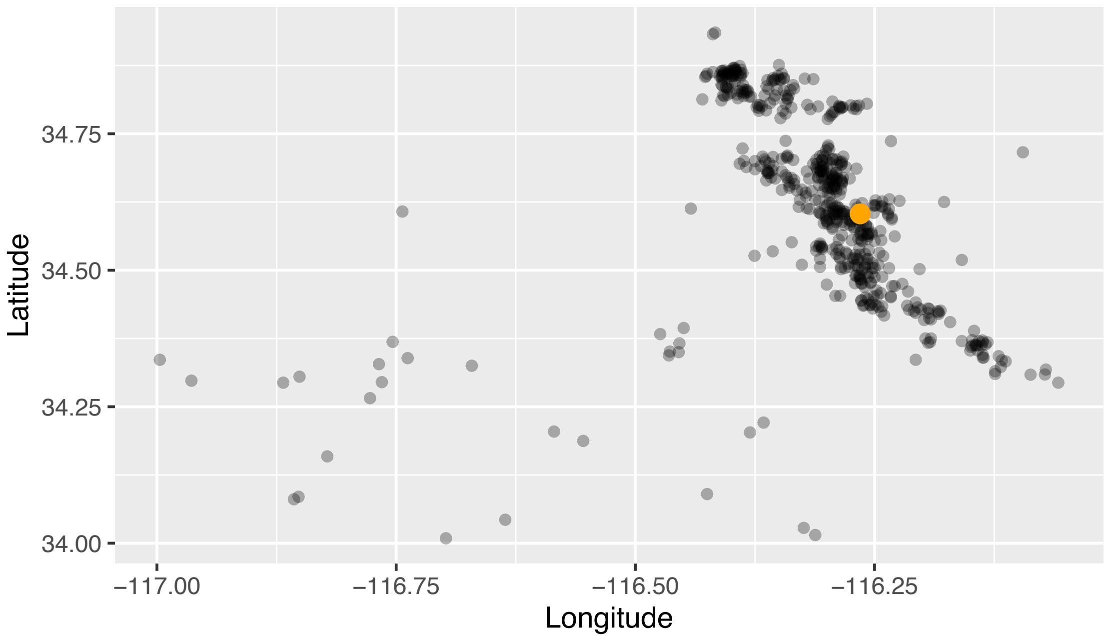
- Rather than specify a set of fixed bins, what if we instead:
  - Generate lots of random bins of varying size
  - Use these random bins to estimate the expected rates of triggering which occurs in each bin
  - Smooth these estimated expected rates to give us a sense of what happens, on average

- Benefits:
  - No longer need for users to specify, and then justify, bins themselves
  - Random bins of varying sizes give us a better *overall* sense of the variability of the triggering
  - Still flexible enough to focus on where we think more/less triggering occurs

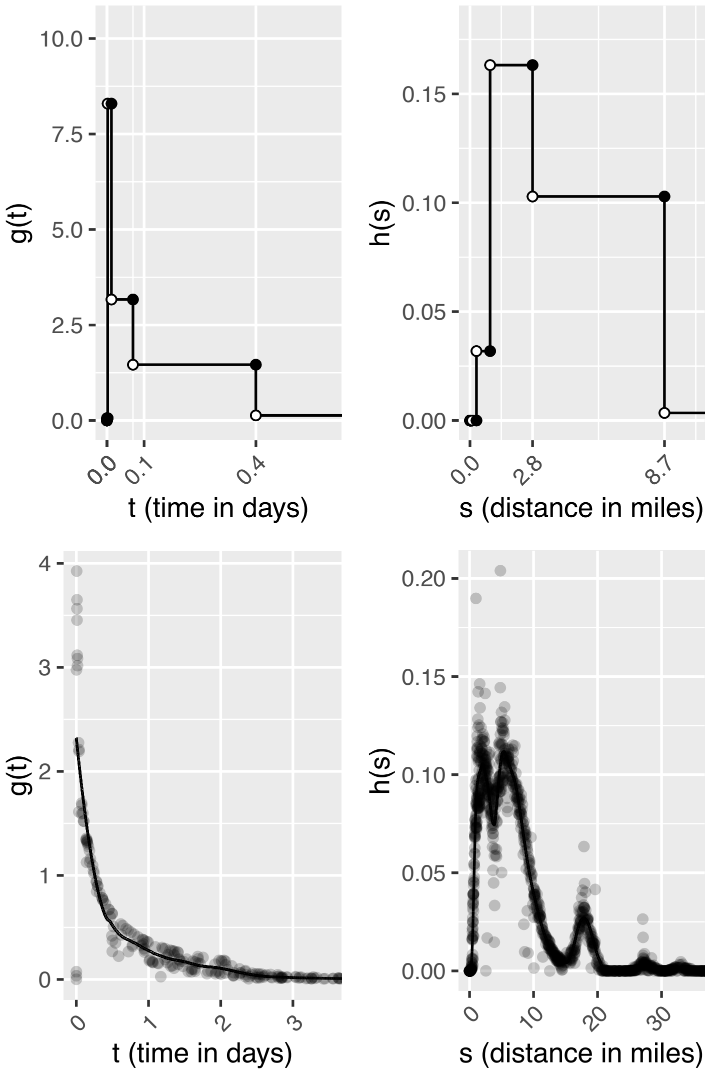
Application: Hector Mine earthquake sequence

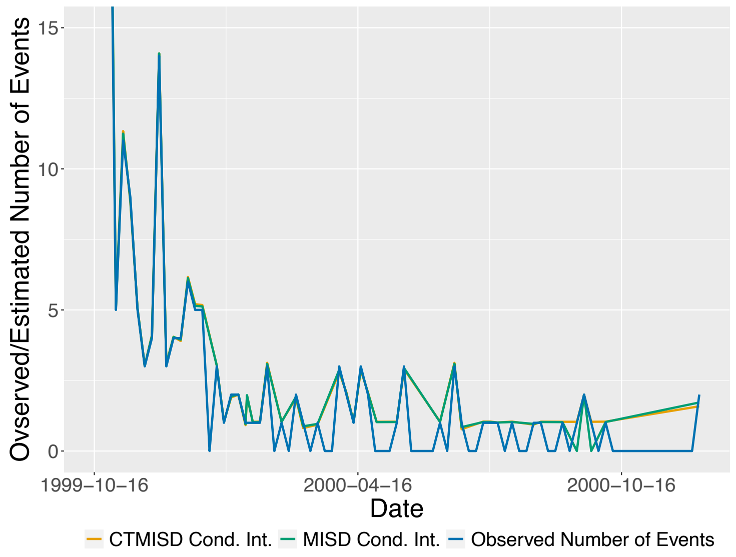
Application: Hector Mine earthquake sequence

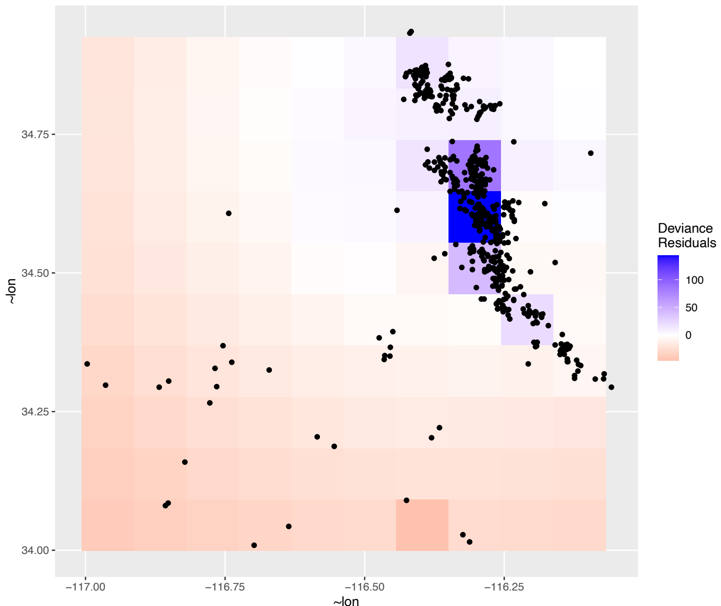
- Large, magnitude 7.1 earthquake which occurred on October 16, 1999
- ~50 miles southeast of Barstow, CA
- Felt throughout Southern California and Las Vegas
- A "fan favorite" for folks in statistical seismology



Application: Hector Mine earthquake sequence







Closing thoughts

Closing thoughts

- Self-exciting point processes are great models for data where points cluster in time or time and space.
- Nonparametric methods alleviate need to specify functional forms of temporal/spatial decay.
- Recent advances make nonparametric methods less "clunky" to implement/use.

- Check out Peter's Github for:
  - An R package which implements the histogram estimator version of the model.
  - Soon: An R package which implements the new method.

- Be on the lookout for our article about the new method

THANK YOU

: molyneuj@oregonstate.edu

: jimmylovestea

: jimmylovestea

Oregon State University