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Structural losses, structural realism and the stability of Lie algebras

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ABSTRACT

One of the key assumptions associated with structural realism is the claim that successful scientific theories approximately preserve their structurally based content as they are progressively developed and that this content alone can explain their relevant predictions. The precise way in which these theories are preserved is not trivial but, according to this realist thesis, any kind of structural loss should not occur among theoretical transitions. Although group theory has been proven effective in accounting for preserved structures in the context of physics, structural realists are confronted with the fact that even group-theoretic structures are not immune to these structural discontinuities. Under such circumstances, my contribution consists in a two-fold task. Firstly, I will establish a general condition at the level of the group-theoretic structure to avoid the pessimistic induction argument by appealing to Lie algebra deformation and stability theory; and secondly, I will provide a case study associated with quantum-relativistic kinematics to demonstrate that this condition is actually satisfied. Specifically, through this case study I will support the claim that if the full Lie algebras of our current successful theories are stable, it is possible to disregard any kind of structural loss in the future and explain the relevant successful predictions in a way that we can support structural realism accordingly.

1. Introduction

Let us assume that in the history of physics there have been many successful physical theories that could not stand the test of time and are today considered false (i.e., with respect to currently known domains). It is a short inductive step from this assumption to the conclusion that in the future our current successful physical theories shall also, most likely, be found to be false (i.e., with respect to unknown domains). This is basically Larry Laudan's *pessimistic meta-induction argument* (PMI) against scientific realism in the context of physics (Laudan, 1981).

In the face of this (debatable) argument, the philosophical thesis known as *structural realism* (SR) strives to break the impasse associated with this pessimistic conclusion without the need to abandon scientific realism (e.g., different variants of SR include (Worrall, 1989; Ladyman et al., 2007; French, 2014)). According to structural realists (SRlists), the reason we regard our old physical theories as false is because in retrospect these theories posited objects and intrinsic properties whose ontological natures became incommensurable with those of the objects and intrinsic properties posited by current physics. Based on a realist commitment to at least the structural content of our best physical theories which governs the behaviour of such objects and intrinsic properties[1] (e.g., physical laws in the form of differential equations, physical symmetries in the form of abstract Lie groups and Lie-representations, etc.), SRlists believe they can avoid the force of the PMI argument by virtue of the putative fact that the structurally based ontology of these theories is approximately preserved across theory change as well as indispensably contributing to the explanation of their respective successful predictions. Thus, what the SRlist advises us to do against the PMI argument is to limit our commitments to at least the approximately preserved theoretic structures underlying our most successful physical theories.

Moreover, how much of a functional strategy SR has against the PMI argument depends upon the extent to which a theoretic structure is preserved. That SRlists are ontologically committed to a preserved structure (i.e., a structure that explains the relevant successful predictions obtained by different theories) does not mean that the complete set of structures underlying our best theories is immune to future change. As is the case for the ontology posited by any physical theory, SRlists are aware that the complete set of mathematical structures underlying successive theories inevitably changes. Nonetheless, contrary to the unexpected and unpredictable diachronic variations among the natures of posited objects and intrinsic properties, they argue that the set of

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[1] Different variants of SR differ by their ontological and (or only) epistemological commitments. Commitment with respect to the category of relations and structures suffices to block the PMI argument, regardless of the ontological status of the category of objects and intrinsic properties.

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structures as a whole progressively changes in a partial way. This means that only a shared core structure is fully preserved throughout the journey. Thus, although full preservation of the whole set of structures rarely occurs in actual practice, SRlists believe there is always partial preservation at the global structural level.[2]

Notwithstanding these expectations, SRlists still face serious concerns associated with the notion of partial preservation. If we pay special attention to the historical development of physics, and specifically, to actual scientific practices taking place at different times, we can identify certain patterns that lie behind the complex process of theory change, namely, *structural losses*. Although these patterns are unproblematic in the absence of any structurally based ontological commitment, they pose unfortunate consequences for SRlists. Following a structurally based Kuhnian analysis of the incommensurability question about the development of science (Bueno, 2008), argues that the presence of structural losses raises a serious objection to SR because the continuity of the theoretical structure that indispensably contributes to explaining the relevant predictions can be broken in the transition from one preceding theory to a more general successor theory.[3] According to Ótavio Bueno, SRlists do not have a problem with structural change as long as there is a shared core structure that is preserved and can explain the relevant predictions, but they do have a problem if the structural content of preceding theories, which was indispensable to the explanation of their respective successful predictions, is lost along the way. In the face of this objection, SRlists are compelled to identify within successful theories structural posits that do not experience such structural losses. Otherwise, the structure of the successor theory might not provide a complete explanation of the successful predictions obtained by its predecessor.[4]

Upon this issue, there are recently proposed formal strategies some of which rely on the explicit identification of a series of underlying or compatible structures which have been partially preserved throughout the development of our best physical theories (i.e., structures that either underlie the theories as they were originally formulated or are compatible with the theories as reformulated long after their creation). As is argued in (French, 2014, ch. 5), group theory is one of the most suitable mathematical languages in terms of which preserved structures can be identified. As an advocate of *group structural realism*, Steven French claims that while the ontology posited by our best theories shows discontinuities across theory change, group-theoretic structures underlying (or compatible with) these theories are likely to be partially immune to future changes, inferring that the category of group-theoretic relations is, in the ontological sense, more fundamental than the category of physical objects. Along these lines, some work has been done on trying to exemplify the alleged group-theoretic structures. For example, Brian Roberts put forward the claim that "The existing entities described by quantum theory are organized into a hierarchy, in which a particular symmetry group occupies the top, most fundamental position" (Roberts, 2011, p. 50). However, whereas Roberts characterises (and critically assesses) group structural realism in the domain of non-relativistic quantum mechanics, the work of (Auyang, 1995) can be seen as a legitimate attempt to reconcile SR with group theory in the framework of quantum field theory. As elucidated by her structuralist neo-Kantian account of this theory, neither the field-event structure nor the space-time structure is given ontological priority; conversely, "the event structure and the spatio-temporal structure of the objective world emerge together" as aspects of the worldly 'representation-transformation-invariant' structure (Auyang, 1995, p. 135). Furthermore, recent attempts have been made to bring group structural realism under broader scientific domains. As is claimed in my work (Manero, 2019), the effectiveness of group symmetries homeomorphic to *symplectic* structures stems from their ability to be preserved across different domains with no traces of structural losses. Unfortunately, as things currently stand, symplectic group symmetries are only effective for group-theoretic reformulations conducted within the scope of classical and quantum mechanics but do not entirely eliminate structural losses in transitions from classical to relativistic kinematics. At this level, some form of group contraction in the relativistic group structure operates to obtain the Galilean symplectic structure, preserving only a subgroup of it, namely, the *special orthogonal group* SO(3).[5]

Considering these important observations, the *main contribution of my paper* consists in fulfilling a two-fold task: firstly, to establish a general group-theoretic condition (i.e., a condition which can be applied to a general family of possible physical theories originally formulated or reformulated group-theoretically) that, should it be satisfied, allows us to block the PMI argument; and secondly, to demonstrate that this condition is satisfied by a concrete case study in current physics and thereby support SR. I seek to demonstrate that the *Lie algebra deformation and stability theory* (i.e., the inverse notion of group contractions in certain cases) provides such a mathematical condition. In the literature, this notion is associated with the stability properties of the underlying Lie algebraic structures of physical theories. The point is that if the *full Lie algebra* associated with a *current* successful theory is *stable* (i.e., if it is *semisimple*), regardless of whether it was developed from a preceding theory whose full Lie algebra was stable or unstable, then we can be sure that structural losses will not occur in the future among transitions taking place from this theory to a successor and that its relevant successful predictions will be exhaustively explained. As we shall see in detail, this condition is satisfied by a well-known case study in current physics, namely, *quantum-relativistic kinematics*.[6] From this two-fold analysis I will conclude that if the Lie-algebraic notion of stability is taken seriously both epistemically and ontologically, then we can solve the intriguing difficulties concerning structural losses in the context of SR. In order to arrive at this conclusion I shall rely on the following assumptions:

(1) The methodology to be followed shall be restricted to the branch of scientific realism called *selective realism* of the 'working posits' school.[7] With this restriction I do not claim that SRlists form a single homogeneous group nor that all SRlists are (and should be) selective realists of the working posit genre; my discussion shall be confined to the branch of SR that makes use of working posits for selection regardless of its merits and defects as a general philosophy of science.

(2) I shall confine the scope of my contribution to the problem of theory change as elucidated by the PMI argument. This means that my central aim is to block the PMI argument via selective SR without providing a full-fledged argument in favour of this philosophical thesis. Other relevant issues concerned with SR, such as the problem of structural underdetermination, representation, and metaphysical disputes on dependence and identity, shall lie beyond the scope of this paper.

[2] SRlists do not necessarily have to insist on the claim that a shared core structure is *always* preserved. However, as explained in Section 2, I think that selective SRlists of the 'working posits' school should assume this claim. I thank one of the reviewers for shedding light on this important point.

[3] NB: I shall rely on Ótavio Bueno's philosophical presupposition that structural losses are bad for SRlists without addressing the particular example suggested by him (i.e., von Neumann's rejection of Hilbert space).

[4] As we shall see in Section 2, this presupposes the endorsement of a particular selective approach to SR based on the explicit identification of preserved *working posits* (i.e., structural posits of successive theories indispensable to the explanation of their successful predictions). Although I acknowledge that the explicit identification of working posits is a problem for a selective approach to scientific realism —as discussed by (Vickers, 2017)—, I shall assume that we can identify these structural posits within a certain constrained domain. Granted this assumption, any discussion about the problematic identification of working posits shall lie beyond the scope of this paper.

[5] I shall explain the notion of group contraction in Section 4.

[6] I shall discuss what I mean by *full Lie algebra* in Section 6.

[7] I shall explain this notion in Section 2.

(3) I shall include in my analysis only current and future theories whose mathematical structures (do and will) take a relatively similar form based on a Lie algebraic formulation. More specifically, I shall establish the conditions under which stable Lie algebras may block the PMI argument for the restricted class of successful theories capable of being formulated in Lie algebraic terms, but I shall not address the question of whether it is reasonable to believe that these conditions (do and will) obtain through every possible theory change.
(4) I shall also include in my analysis only current and future theories whose mathematical structures (do and will) take a relatively similar form based on kinematical (as opposed to dynamical) structures, establishing with this emphasis a dynamic/kinematical distinction. As a result, our case study confines the total set of successful predictions obtained by quantum-relativistic mechanics to those derived from their kinematics.[8]

Considering these assumptions, the methodology of this paper shall be as follows. In Section 2, I explain why structural losses pose a problem for SRlists. In Section 3, I formally define the notions of *partial preservation* and structural losses in terms of the *partial structures* framework. In Section 4, I explain why group contractions involve structural losses via a group-theoretic analysis of the classical-relativistic transition. In Section 5, I argue that stable full Lie algebras can solve the problem of structural losses, and then I support this argument in the context of one case study in physics (i.e., quantum-relativistic kinematics). Finally, in Section 6, I represent quantum-relativistic kinematics in terms of the partial structures framework.

2. Why are structural losses bad for structural realists?

Let us follow (Putnam, 1975, p. 75) and define scientific realism as a commitment, belief or attitude towards scientific theories that better explains their undeniable empirical success by virtue of the fact that these theories give us (or aim to give us) approximate knowledge of a mind-independent world. In order to support *explanationism realism* (as per the recent terminology used for the above definition), some contemporary philosophers, such as (Psillos, 1999), appeal to a 'selective' form of scientific realism, according to which only a subset of all theoretical terms associated with any given theory are successful referring terms. In other words, this group of philosophers (hereinafter *WP-selective realists*) provide an explanation for the success of science only in terms of a direct ontological commitment with respect to the *working posits* of the theory in question, namely, those theoretical referring terms that are primarily responsible for and indispensable to the explanation and prediction of the relevant phenomena. Moreover, assuming that any theoretical transition that has gained a relevant place in the history of science is, in an approximate way, a *progressive-cumulative development*, WP-selective realists seek to explain this progress (as a response to the PMI argument) in terms of preserved working posits.[9] More specifically, WP-selective realism blocks the PMI argument by stipulating that the working posits of successful theories are approximately preserved during theory change. In this way, an essential condition associated with this selective form of realism is that the associated, preserved working posits should be able to explain the alleged successes with respect to the restricted domain in which the theories are relevant. This condition suggests that there might be dispensable terms that are not preserved from one preceding theory to its successor, but there cannot be working posits that are lost among theoretical transitions.

Defined in this way, some SRlists would say that SR is a kind of WP-selective realism but it is one whose ontological commitments are not associated with the indispensable theoretical terms that putatively refer to unobservable objects or intrinsic properties, but to the structural content of the theories in question (French, 2014).[10] It mainly relies on the argument that standard scientific realism (i.e., 'object-oriented' scientific realism) is incapable of providing the required explanation of the relevant successful predictions across scientific change, and the only reasonable way to achieve this is through the ontological commitment with respect to the structurally based working posits of our best theories (or 'operative structures', as per the terminology used in (Votsis, 2010, p. 108)). For example, John Worrall's main point in (Worrall, 1989) is to associate these working posits with the mathematical equations of successive theories. As regards the explanatory and preservation capacities of structural working posits, WP-selective SR can be interpreted as the ontological, structural realisation of Heinz Post's heuristic *general correspondence principle*, according to which

> [...] any acceptable new theory L should account for the success of its predecessor S by 'degenerating' into that theory under those conditions under which S has been well confirmed by tests [...] We may divide a theory 'vertically' into well-confirmed working parts and others. Thus the phlogiston theory 'worked' in that it assigned consistent levels of phlogistication (explaining many features) to chemical substances related in more than one way by reactions. On the other hand it tried to establish a connection between colour and phlogistication, and this part of the theory was not successful even at the time (Post, 1971, p. 228).

As reinterpreted by Simon Saunders' structural notion of *heuristic plasticity*, this principle put forward the claim that "What is taken over from preceding theories is not only those laws and experimental facts which are well-confirmed, but also 'patterns' and 'internal connections', that in this way the successor theory accounts for whatever success its precursor enjoyed." (Saunders, 1993, p. 295).

Overall, the principal challenge for WP-selective SRlists is to justify the following *structural continuity argument* (appealing to a variation of the argument provided by (Votsis, 2010, p. 116)):

> If all and only the structurally based working posits underlying (or compatible with) successful scientific theories have been (and will be) approximately preserved through theory change, and if preservation is a reliable guide to (approximate) truth, then the approximate preservation of all and only the structurally based working posits

[8] One might argue that the third and fourth assumptions are statements of the denial of the PMI argument. If this were the case, one would be flatly assuming what needs to be shown. However, that the mathematical structures underlying (or compatible with) current and future successful physical theories do and will take a relatively similar form based on a Lie algebraic, kinematical formulation is not equivalent to the claim that current successful physical theories will be developed in a progressive and cumulative manner, that is, in a way that does not involve problematic structural losses. The reason is that it is logically possible to find a current successful physical theory whose Lie algebra is unstable, which is formulated in terms of a Lie-algebraic, kinematical structure but will not be developed in a progressive and cumulative manner and will involve structural losses. The point of recurring to the concrete case study of quantum relativistic kinematics whose Lie algebra is stable is to (partially) discard this logical possibility based on current physics. Thanks to one of the reviewers for pushing me to emphasize and clarify this point.

[9] As is well known, this assumption is precisely the target of the Kuhnian-Feyerabend incommensurability thesis. However, there is an implicit 'post-positivist' dogma (using Laudan's terminology) behind this assumption, namely, that "cumulative retention of confirmed explanatory successes is a precondition for judgments of progress" (Laudan, 1996, p. 23). Although this dogma is controversial and sufficiently significant for profound discussion, I will take it for granted in this paper.

[10] Philosophical issues concerned with the ontological commitment with respect to 'structural content', and the way physical structures are interpreted and related to mathematical structures, although significant in many respects, will be omitted in this contribution.

underlying (or compatible with) scientific theories through theory change is a reliable guide to their (approximate) truth.

In order to arrive at such a conclusion, WP-selective SRlists must, of course, confirm the respective two premises. Assuming that preservation is a reliable guide to (approximate) truth —an assumption associated with explanationism realism that we shall take for granted in this paper— WP-selective SRlists need to identify a core set of working posits that are approximately preserved but have not been lost among the theoretical transitions that have occurred throughout the history of science. It is from this structural continuity argument that we can see why structural losses are bad for WP-selective SRlists, for if they find that the structurally based working posits are lost in the transition, the structure underlying (or compatible) with the successor theories would be incapable of explaining the relevant domain entailed by the preceding theories. They are, in a sense, missing blocks in the chain of theoretical development.

It is important to note, however, that structural losses *per se* are not bad for WP-selective SRlists; structural losses pose a challenge to WP-selective SR if the structures involved in the theoretical transition are the working posits of the preceding theories. WP-selective SRlists are seemingly untroubled by the situation in which a theory experiences a considerable loss of structure during a theoretical transition, as long as the indispensable structural content that fully explains its respective predictions is fully preserved. In contrast, they are troubled by situations in which the preceding and successor theories share a common, preserved structure whilst the working posits of both theories are not preserved. In such a case, the successor theory can only explain a part of the successful predictions obtained by the preceding theory.

Two concluding observations are in order. Firstly, one might resist the claim that a shared core structure is *always* preserved and that we may countenance some occasional discontinuities, provided that this structural core is largely preserved. Although this claim might be resisted for certain more liberal forms of SR, WP-selective SRlists cannot accept *at any rate* that certain 'doomed' structural working posits are not preserved; otherwise, we could find an extremely successful theory (e.g., quantum electrodynamics) whose working posits shall not be preserved, something which would undermine the essential point of WP-selective SR in identifying and characterising (in a case-by-case basis) the preserved working posits of our most successful theories. To avoid this flaw, one would need to presuppose that the 'doomed' non-preserved working posits of certain theories are in some (non-ambiguous) sense 'less relevant' to the preserved working posits of the vast majority of theories as regards the explanation of the successful predictions. However, this seems in principle ill-suited as the essential role of working posits (as opposed to idle theoretical posits) is precisely to be relevant in this explanatory sense.

Secondly, behind the association between bad structural losses and working posits lies a well-known problem that predates Worrall. One might eventually realise that WP-selective SRlists are in some way again facing the Kuhn-Feyerabend incommensurability thesis, which is bound up with the claim that there are relevant transitions in the history of science that were not cumulative and that involved what is known as Kuhn's 'empirical losses' (i.e., the loss of empirical content through change of conceptual scheme). However, WP-selective SRlists do not interpret bad structural losses as Kuhn's losses[11] as bad structural losses are associated with 'explanatory losses' (i.e., losses that occur when a certain phenomenon is explained by an earlier theory but fail to be explained by its successor), an alternative notion better elaborated by (Laudan, 1996).[12] As implicitly suggested by (Bueno, 2008), this association leaves open the possibility of arguing, *contra* WP-selective SR, that bad structural losses are non-preserved working posits that involve theoretical transitions that are not in any reasonable way progressive, cumulative developments.

3. Defining and representing structural losses

Below, I formally define the notion of structural loss (together with the associated concepts employed herein) in terms of the partial structures framework.[13]

Mainly, we are interested in formally describing the continuity or *structural preservation* of mathematical structure across different successive theories or, in other words, to capture the net of 'horizontal' inter-theoretical relationships that hold between mathematical structures and the degree to which these relations take place. If we take a look at the history of physics, we can observe that in the transition from one preceding theory to its successor there are two possible kinds of mathematical change, one which implies a *full embedding*, and the other which implies a *partial embedding*. Full embeddings occur only when an expansive change in the preceding theory takes place at the level of its mathematical structure in which the structure underlying (or compatible with) the successor theory includes the structure underlying (or compatible with) its predecessor; partial embeddings happen precisely when structural losses appear on the scene, namely, when the structure underlying (or compatible with) the successor theory contains only a part but not the whole of its predecessor's structure. Both notions of theory change can be suitably represented in terms of the partial structures framework using different kinds of set-theoretical mappings, with *full homomorphisms* for full embeddings and *partial homomorphisms* for partial embeddings (Bueno, 2000). To appropriately disclose the notion of partiality inherent among these homomorphisms, let us make the notion of partial structure explicit. Following (da Costa & French, 2003), a partial structure S with full and partial homomorphisms is an ordered triple generally characterized (extensionally, rigorously) as:

$$S = \langle D, R_i, f_j \rangle_{i \in I, j \in J} \tag{1}$$

where D is a non-empty set that stands for the elements of the structure S, R is a non-empty set of n-place *partial relations* holding among the elements in D, and f are full and partial homomorphisms. In more detail, an n-place partial relation R over D is a triple (R_1, R_2, R_2), where R_1, R_2, and R_3 are mutually disjoint sets, such that R_1 is the set of n-tuples that (we know that) belong to R, R_2 is the set of n-tuples that (we know that) do not belong to R, and R_3 is the set of n-tuples for which it is not known whether or not they belong or not to R. Moreover, we say that a partial function f: $D \rightarrow D'$ is a partial homomorphism from S to S' if for every x and every y in D, $R_1xy \rightarrow R_1^{'}f(x)f(y)$ and $R_2xy \rightarrow R_2^{'}f(x)f(y)$. With respect to the notation, I is an index set that labels each of the n-place partial relations, and J is an index set corresponding to the labels of 'vertical' and 'horizontal' full and partial homomorphisms. Horizontal homomorphisms express inter-theoretical relations holding between different theories, whilst vertical ones express intra-theoretical relations holding between models of the same theory. In this contribution, I focus on the former. However, I shall include, for sake of clarity and precision, a brief discussion of the latter concerning structural underdetermination and empirical models.

Suppose that S is the model (i.e., partial structure) corresponding to the mathematical structure underlying (or compatible with) a preceding

[11] NB:on the assumption that theories are interpreted as a series of statements, the Kuhn-Feyerabend incommensurability thesis puts forward (in Laudan's terminology) a 'postpositivist' view against the invariance of 'meaning' among successive theories. However, while WP-selective SRlists interpret 'meaning' in the structural sense, Kuhn and Feyerabend do it in the 'object-oriented' sense.

[12] Of course, both notions are related by virtue of the well-documented historical fact that "the loss of one or more explanatory instances automatically carried with it the loss of empirical support" (Laudan, 1996, p. 121).

[13] See (da Costa & French, 2003) for a complete account of this framework.

theory and also suppose that there is another model S', which corresponds to the structure underlying (or compatible with) the successor theory. If we assume that in the transition from S to S' there are no structural losses, then there is a full homomorphism between the models. This means, formally speaking, that S is a substructure of S' in the sense that a subfamily of the known relations holding for S' turns out to be the set of all those known relations holding for S. However, this situation does not occur if there are structural losses, in which case there is a partial homomorphism between the models. This means that S is a partial substructure of S' in the sense that there are known relations that hold for S but do not hold for S'.

Two observations are in order. Firstly, note that the previous set-theoretic analysis presupposes that one already knows what the mathematical structure underlying (or compatible with) a given theory is. However, in the face of a structuralist realist commitment, such an analysis is not immune to problems of underdetermination of the relevant structures by the theory. As is well known, there could be two or more formulations consistent with a single theory which are mathematically inequivalent but empirically equivalent i.e., they imply the same observational sentences for all possible observations.[14] In order to break the underdetermination at the level of the mathematical structure, the most reasonable strategy consists in identifying some commonalities between the possible formulations of the theory in terms of a set of conditions for structural equivalence. In accordance with this strategy, let us introduce these conditions in terms of the partial structures framework. Suppose that S and S' are different models corresponding to two different mathematical structures underlying (or compatible with) the same theory. We say that two mathematical structures are equivalent iff there is a full homomorphism between S and S'. This means that there is an equivalence class of mathematical structures differing up to a structure-preserving mapping regardless of the diversity of models that lie at the interpretative (ontological) level.[15]

Secondly, as discussed by (Bueno et al., 2002), the structural and empirical equivalence between two or more formulations presuppose the notion of *empirical adequacy*, the primary role of which is to specify the manner in which the empirical facts are articulated throughout the entire theoretical edifice. In order to define this notion with precision and formality, we should proceed to extend its ‘narrow’ conception to an appropriate ‘broad’ view on the matter capable of being represented in terms of the partial structures framework. The essential idea of the latter view (as opposed to the semantic, ‘narrow’ interpretation of (van Fraassen, 1980, p. 64)) is to represent and characterise the complex nature of scientific practice (as regards its empirical significance) through a ‘vertical’ hierarchy of models located along both mathematical and empirical sides of the theoretical spectrum. At the ‘top’ of the hierarchy lie the abstract structures associated with the more general mathematical formulation, whereas at the ‘bottom’ lies the empirical dimension of the theory as expressed by empirical structures. More specifically, we say that a theory is empirically adequate iff there is a well-defined mapping between the underlying (or compatible) mathematical structure and the empirical results relevant to the theory, provided this mapping is represented in terms of two successive steps: firstly, via partial homomorphisms going from the models associated with the purely abstract mathematical structures to those associated with the empirical structures; and secondly, via partial homomorphisms going from the models associated with the empirical structures to those associated with the appearances i.e., experimental reports represented by data-structures, as proposed in (Bueno et al., 2002).[16]

4. Structural losses in the form of group contractions

Going back to the assumptions mentioned in the introduction, WP-selective SRlists might be inclined to say that these assumptions involve strong criteria about what the working posits are, and thus the putative structural losses are not actually losing any real working posits. However, I shall now provide historical evidence of the structural loss of the restricted class of working posits considered in this contribution (i.e., Lie algebraic structures with a dynamic-kinematic form). As we shall see, historical evidence demonstrates that structural losses of this special class of working posits are not fictional, uninteresting instantiations devoid of philosophical relevance but are well-documented processes in scientific practice which give rise to serious objections to WP-selective SR. I shall first introduce an atypical case study in which there is the full preservation of working posits. However, after considering more typical instances of theoretical transitions that involve a partial preservation of structure via infinite limits and approximations, I shall explicitly identify the group-theoretic form of certain structural losses that have been observed at the level of the kinematical structures involved in the classical-relativistic transition. Finally, I shall indicate the conditions under which kinematical structures can be interpreted as working posits, and based on this analysis, I shall conclude that the identified structural losses are working posits that are not fully preserved in the case considered.

One interesting case study which has widely been discussed in recent years is the theoretical transition between Fresnel's theory of light and Maxwell's wave electrodynamics. As is stressed by (Worrall, 1989; Redhead, 2001), this case study is a perfect example of a genuine transition that takes place at the level of the mathematical equations and that incorporates, *contra* Kuhn-Feyerabend's incommensurability thesis, a structural sense of progressive-cumulative development, in which a successor theory fully contains the preceding theory. Indeed, it is well-known that Maxwell's equations can be approximated to Fresnel's equations (or to the eikonal particle-like equation through the use of Hamilton-Jacobi theory) by virtue of the mathematical fact that there are terms in the former that become negligible when a certain scale parameter associated with the wavelength of electromagnetic plane waves continuously tend to some meaningful finite limit (i.e., short wavelengths). Since this approximation does not arise from a singular, infinite, discontinuous limit, it is a mathematical derivation allowed by the internal structure of Maxwell's equations (i.e., it is a model of Maxwell's equations). This means that there are no structural components in Fresnel's theory that get lost in the transition to Maxwell's theory.

However, even if the Fresnel-Maxwell transition can be considered one of the successful attempts to provide physics-based support for WP-selective SR (i.e., for structural progressive-cumulative development), in most cases such attempts only use particular case studies in a selective and convenient way without supplying an overall response against the PMI argument.[17] As stressed by (Votsis, 2010, p. 108), “the neat preservation of structure exhibited by the Fresnel-Maxwell case is atypical in the history of science (e.g., Redhead, 2001). More often a structure

[14] This syntactic notion of empirical equivalence is due to (Quine, 1975, p. 319).

[15] NB: structural equivalence is the formal, mathematical component of the more general notion of *theoretical equivalence*, which is amply discussed by Quine through the notion of ‘intertranslatability’ and by recent contributions via other additional or exclusive conditions. For example, (De Haro, 2021, p. 11) put forward the claim that two or more formulations are theoretically equivalent if “they are formally equivalent and, in addition, they have the same interpretations” i.e.,.the same ontology. Since my definition only considers the contribution of mathematical structures, I could be in agreement with De Haro's account if theoretical equivalence is distinguished from structural equivalence. The former (as opposed to the latter) asserts that equivalent formulations always share the same interpretation.

[16] This procedure shall be explicitly discuss in the context of group-theoretic structures in Section 5.

[17] Other selective examples, suggested by (Votsis, 2010, p. 108) and (Redhead, 2001), are Sadi Carnot's principle of maximum efficiency; the extension of electrostatics to electromagnetic phenomena; and the reduction of thermodynamics to statistical mechanics.

belonging to a superseded theory can be recovered only as a limiting case of a successor theory's structure." This important observation, already acknowledged by Worrall, invites us to differentiate between atypical cases of the full preservation of structure, such as the Fresnel-Maxwell transition, and more typical cases of transitions as infinite, discontinuous limits and approximations of certain physical properties or constants, where a partial preservation with structural losses occurs.[18] The important question here is whether these kinds of transitions involve working posits that are not preserved. Of course, Worrall's main point is to associate these structural approximations with actual cases of progressive-cumulative developments implicitly presupposing that working posits are preserved. However, as we shall now see, this is not actually the case in general.

As one reviews the history of physics in depth, one may come to realise that well-known theoretical transitions occurred which resulted in the acquisition of novel predictions via Post's correspondence principle without being an actual case of progressive-cumulative development. As pointed out by Feyerabend, the classical-relativistic transition is an instance of the incommensurability thesis based on the fact that classical mechanics addressed problems not solved by relativity theory (Feyerabend, 1962, pp. 80–81). Although his analysis was conducted at the level of its 'meaning' (i.e., object-oriented ontology) rather than at the structural level, we can analyse the structural patterns behind this transition and propose an analogous argument to Feyerabend's applied to the structural case. Let us elaborate this proposal by appealing to group structures.

Going back to the time Lie group theory was beginning to be applied to physics (and achieved important developments as a mathematical theory in turn), we should not ignore the fact that Eugene Wigner meticulously investigated the transition from classical to relativistic physics at the level of the group-theoretic kinematic structure. Together with his colleague Erdal Inönü, he developed a theory of *group contractions* to understand, in terms of the formal language of group theory, the transition that takes place between classical kinematics and special relativity kinematics (Inönü & Wigner, 1953). As written in the introduction to (Inönü & Wigner, 1953, p. 510): "Classical mechanics is a limiting case of relativistic mechanics. Hence the group of the former, the Galilei group, must be in some sense a limiting case of the relativistic mechanics' group." With this heuristic move in mind, Wigner and Inönü showed that the Galilean group *Gal* is the group contraction of the Poincaré group ISL(2,$\mathbb{C}$) with respect to the rotation group and time translations in the low velocity infinite limit $c \rightarrow \infty$. This means that the underlying group-theoretic structure of relativistic kinematics can be modified as a consequence of interpreting the velocity of light as a converging parameter within the associated Lie algebraic structure.

However, if we look closely at the notion of group contraction, we see that such an operation involves a loss of structure in the Lie algebra associated with the transition between different theories. Leaving out time translations (without loss of generality), the group contraction under consideration is a mapping between different elements of the ISL(2,$\mathbb{C}$)-Lie algebra with respect to a given parameter c (i.e., the velocity of light in this case), in such a way that if $c \rightarrow \infty$, the image of the corresponding Lie algebra of ISL(2,$\mathbb{C}$)\SO(3) yields the non-isomorphic *Gal*\SO(3)-Lie algebra. According to group theory, this mapping defines an injective immersion and group homomorphism but only with respect to a subgroup of *Gal*, namely SO(3). This is a subgroup of both *Gal* and ISL(2,$\mathbb{C}$), whose Lie algebras restricted to SO(3) are equivalent. In brief, we say that the group contraction of ISL(2,$\mathbb{C}$) with respect to SO(3) is *Gal*.[19] Physically speaking, this group contraction represents a process of 'Euclideanisation' of hyperbolic spacetime. This means that translations and boosts (as opposed to rotations) in Minkowski spacetime have a completely different physical interpretation to those defined in Galilean spacetime. Although classical and relativistic systems behave in the same way under rotations of the inertial frames (as these systems move isotropically), they behave differently under translations and boosts of inertial frames as revealed by the relativistic effects of length contraction and time dilation arising from the hyperbolic structure of Minkowski spacetime.

For the sake of clarity, let us now represent this group contraction in terms of the partial structures framework. Since a group contraction is not a group homomorphism, it can be represented as a partial homomorphism, which maps relations of a partial set-theoretic structure associated with the Lie group of a theory to those of another. Note that in this case it is evident that all the known relations holding (and not holding) for the partial structure of SO(3) ⊂ *Gal* (denoted by $S_{\mathrm{SO(3)}}$) are isomorphic to the corresponding relations holding (and not holding) for the partial structure of SO(3) ⊂ ISL(2,$\mathbb{C}$). However, the relations holding (and not holding) for the partial structure of the whole group *Gal* (denoted by S_{Gal}) are not isomorphic to those holding (and not holding) for the partial structure of ISL(2,$\mathbb{C}$) (denoted by $S_{\mathrm{ISL}(2,\mathbb{C})}$). Thus, at a certain pre-relativistic moment, it was known that some (physically-meaningful) relations did not belong to S_{Gal} as well as it not being known whether those relations belonged (or did not belong) to $S_{\mathrm{ISL}(2,\mathbb{C})}$, but when special relativity was put forward, it was known that they only belonged to $S_{\mathrm{ISL}(2,\mathbb{C})}$. These relations correspond to those holding for the partial structure of ISL(2,$\mathbb{C}$)\SO(3), which form part of the structural gain. Analogously, it was known that some (physically-meaningful) relations belonged to S_{Gal} as well as it not being known that such relations belonged (or did not belong) to $S_{\mathrm{ISL}(2,\mathbb{C})}$, but when special relativity was put forward, it was known that they did not belong to $S_{\mathrm{ISL}(2,\mathbb{C})}$. These relations correspond to those holding for the partial structure *Gal*\SO(3), which is the structural loss.

Overall, the previous analysis corroborates that certain structural losses have been observed at the level of the kinematical group-theoretic structures involved in the classical-relativistic transition. However, this analysis alone is not sufficient to warrant that the identified structural losses are working posits. Without establishing the conditions under which kinematical structures are regarded as working posits, one can always add to some successful theory a set of surplus kinematic structures that is dispensable to the explanation of the relevant phenomena. Under these circumstances, a two-fold task must be tackled: firstly, we must specify which elements of the kinematical structure indispensably contribute to explaining the relevant predictions; and secondly, we must show that these elements are empirically adequate. Hereinafter, the identified kinematical structure (considered to be an empirically adequate working posit) shall be named *full structure*.

With respect to the first task, I shall rely on a criterion of indispensability which prevents us from adding surplus kinematical structure but also ensures us that the relevant structure does not fail to explain the kinematical empirical predictions. In accordance with (Valentini, 1997), one must identify a minimum core set of kinematical structures (hereinafter *natural kinematics*) singled out by a given theory. This natural kinematics is associated with the underlying spacetime structure of the theory upon which the 'universal features of the dynamics' are based. One might eventually realise that the dynamics/kinematics distinction is problematic as one may in principle adopt any spacetime structure, provided one incorporates appropriate compensating dynamical factors. However, this problem can be solved by identifying and characterising the natural spacetime structure via the definition of a 'free' system according to the theory. In the case of both classical and special relativistic physics, for instance, a system is considered to be 'free' when external

[18] This distinction is drawn by (Post, 1971) in terms of consistent/inconsistent correspondence, and then further elaborated by (Redhead, 2001) as expanded structures or qualitatively new structures.

[19] Whereas SO(3) is the 'rotational' component (as it is Lie-represented in Galilean spacetime as rotations), the subgroups *Gal*\SO(3) and ISL(2,$\mathbb{C}$)\SO(3) are the 'translational' component of *Gal* and ISL(2,$\mathbb{C}$) (as they are Lie-represented in Galilean and Minkowski spacetime as space translations and velocity boosts).

interactions vanish. This means that the 'free' trajectories described by this system are completely independent of its state-independent parameters (e.g., mass, charge, etc.). It is therefore possible to identify and characterise the natural spacetime structure of classical and special relativistic physics by interpreting these 'free' trajectories as geodesics associated with appropriate affine structures, given by *Gal* and ISL(2,ℂ), respectively. Note that both Lie groups are full kinematic structures in the sense that there are not subgroups of *Gal* and ISL(2,ℂ) that are dispensable when neglecting any fundamental, external interaction in the classical and relativistic frameworks. Both the 'translational' and 'rotational' components of these groups do not form 'fictitious' kinematical structures which are not physically relevant; the isotropic, spacetime translation and boost symmetries are indispensable to the explanation of the phenomena of a 'free' body system.[20]

With respect to the second task, we should specify how natural kinematical structures of a theory are empirically adequate i.e., map onto all the kinematical empirical predictions of the theory. Let G be the symmetry group of the theory T.[21] As demonstrated by (Wigner, 1939), the mapping defined between G and the empirical results is provided by Lie-representations of the Lie algebra $\mathscr{G}$ associated with G (i.e., the tangent space of G at the identity) in the state space V of T.[22] $\mathscr{G}$ is said to be empirically relevant by virtue of being Lie-represented in V (i.e., what $\mathscr{G}$ 'looks like' when it is defined in V). The Lie-representation of $\mathscr{G}$ in V takes the form of infinitesimal transformations that represent the physical properties relevant to T. That is, physical quantities are associated with a class of Lie-representations defined in V. For example, in classical and quantum mechanics, the infinitesimal transformations induced by the generators of the associated Lie algebras are spelled out in terms of physical properties of the system (e.g., momentum, energy, etc.). As discussed in the last part of Section 3, we can represent the mapping defined between G and the empirical results in terms of two successive steps: firstly, via partial homomorphisms going from the models associated with the abstract Lie algebra $\mathscr{G}$ to the models associated with the induced transformations of $\mathscr{G}$ in V (some of which represent the physical quantities); and secondly, via partial homomorphisms going from the models associated with the empirical structures to those associated with the appearances. These Lie-representations are captured by partial homomorphisms because only a subfamily of the infinitesimal transformations induced by $\mathscr{G}$ is associated (under certain conditions set up by the theory) with the physical properties of the system and generates the empirical results.[23]

Going back to our main concern, what I would like to point out from this particular case study is that structural losses of working posits may arise from group contractions as limiting cases among theoretical transitions. However, the presence of group contractions and the mere characterisation of theoretical transitions as a theory approximating to another in the (infinite, discontinuous) limit involve theoretical developments that are neither progressive nor cumulative at the structural level. Thus, structural losses of this sort prevent us from embracing a WP-selective SRlist riposte to the PMI argument, at least in general.

5. Group-theoretic structures against the pessimistic argument

As mentioned in Section 2, WP-selective SRlists have no choice but to identify all and only the structurally based working posits underlying (or compatible with) our successful scientific theories that have been (and will be) approximately preserved through theory change. This identification is what lies behind the justification of the structural continuity argument. However, considering the previous case study, this identification cannot be conducted through a case-by-case, historic analysis in accordance with how preceding theories were formulated. One eventually realises that actual cases of working posit preservation are atypical if we consider only the structures in terms of which preceding theories were formulated. This critical observation seems to suggest that WP-selective SRlists are condemned by the history of science and that they only get support from the past by considering particular case studies —confined to restricted empirical domains—conveniently selected to fulfil their biased expectations.

Under these problematic circumstances, I shall suggest an alternative way to block the PMI argument (and support WP-selective SR) through a variation to the structural continuity argument under the WP-selective SR framework. In so doing, we should note that the PMI argument, as it has been formulated, is an inductive argument whose conclusions do not necessarily follow from its premises. Indeed, the fact that our preceding theories were successful but are regarded as false today does not necessarily imply that our current successful theories will subsequently be shown to be false. Although the premise of this inductive argument was assumed to be true, the inductive step can be blocked by providing against the conclusion a robust argument that can be supported by current physics. From the point of view of the WP-selective SRlist, this inductive step can be blocked by showing that the structurally based working posits underlying (or compatible with) our current successful theories, even if they followed from a transition where structural losses were involved, would be fully preserved and cumulatively expanded. Provided this demonstration is given, I do not see a problem with believing that the structural continuity argument cannot generally hold regarding the past but may indeed be sustained in the present. Let us shed light on this alternative strategy by appealing to recent attempts to do so in the broader philosophy of science literature.

The version of the PMI argument applicable to this context is roughly as follows: (i) if preceding theoretical transitions involved problematic structural losses, for all we know the transitions from current to future theories will involve such losses; (ii) we are justified in believing the reference of the structural working posits of current theories only if we are justified in believing that they will be preserved under future theoretical transitions; (iii) but preceding theoretical transitions have involved problematic structural losses; therefore, (iv) we are not justified in believing the reference of structural working posits of current theories. Under the framework of WP-selective SR, the standard objection against the conclusion (iv) assumes both (i) and (ii), but denies (iii) via the identification of preserved working posits that did not involve structural losses among past theoretical transitions and were indispensable to the explanation of the relevant predictions. Conversely, my alternative strategy is to block (iv) by denying (i) and endorsing both (ii) and (iii) based on the *privilege-for-current-theories strategy*.

As briefly sketched by (Psillos, 2018), this strategy (amply discussed in the philosophy of science literature) put forward the claim that in general physics has not been developed in a progressive and cumulative manner, whereas we have sufficient reasons to believe that the closer our successful physical theories are to the present, the less likely they are to be revised in the future; therefore, the more likely we are justified in believing the world conception of them. Note, however, that this strategy presupposes a sharp epistemic distinction between current theories and their predecessors, the former which enjoy of a privileged epistemic

[20] As (Valentini, 1997) interestingly argues, whereas the natural kinematics of Newtonian mechanics and general relativity is given by Galilean spacetime and curved spacetime, respectively, the natural kinematics of the pilot-wave approach to quantum mechanics is given by Aristotelian spacetime. The natural state of motion associated with this kinematical structure is undetected rest, as opposed to the inertial structure of the Galilean symmetry which is considered to be a fictitious symmetry.

[21] A given Lie group G is a *symmetry group* of a theory T describing some physical system if the fundamental laws of T (i.e., partial differential equations) are invariant under the induced continuous transformations of G in the state space of T.

[22] The Lie-representation of a Lie group/Lie algebra G in a vector space V is a function that goes from G to the group of automorphisms of V. Those automorphisms are realized as a set of linear transformations defined in V.

[23] A more detailed analysis of the empirical relevance of Lie-representations from the perspective of the partial structures approach is given in my work (Manero, 2019).

status over and above the latter. Consequently, it seems that without providing a reasonable independent justification to draw such a distinction, we would be flatly assuming what needs to be shown. Under these circumstances, we have to justify the claim that a wide range of current physical theories (as opposed to preceding theories) will be developed in a progressive and cumulative manner, that is, in a way that does not involve problematic structural losses. In order to tackle this task from the perspective of WP-selective realism, let us consider the following observation:

The privilege-for-current-theories strategy has been generally interpreted as a different, independent reply to the PMI argument contrasted to the one endorsed by WP-selective realism. In view of the problem of structural losses addressed here, I shall take a different path based on Mario Alai's strategy, which put forward the claim that "The best strategy, instead, joins the selective idea there was both some truth and some falsity in discarded theories, like in current ones, with the moderate discontinuity idea that the truth rate in present best theories is much greater than in past ones." (Alai, 2017, p. 3267). Considering Alai's strategy, let us suggest the following alternative structural continuity argument:

> If all and only the structurally based working posits underlying (or compatible with) our *current* successful scientific theories are approximately preserved through theory change, and if preservation is a reliable guide to (approximate) truth, then the approximate preservation of all and only the structurally based working posits underlying (or compatible with) current scientific theories through theory change is a reliable guide to their (approximate) truth.

According to this 'balanced' strategy which conjoins WP-selective realism and the privilege distinction, the principal challenge is to *prospectively* identify all and only the structurally based working posits underlying (or compatible with) our current successful scientific theories that will be approximately preserved through theory change. Note, however, that in this alternative case we cannot identify such structures in hindsight or *retrospectively* because they are, in principle, working posits underlying (or compatible with) our current theories such as will be preserved in subsequent unknowable developments.[24] Under these circumstances, apart from the alternative structural continuity argument—an argument that does not tell us anything about the mathematical characteristics of the structures involved—we should provide a general recipe or condition that can inform us regarding the kind of mathematical features any given structure must possess in order to be regarded as a preserved working posit. Without providing this condition —a condition which serves not only to identify in advance which elements will be preserved in theory change and are indispensable to the explanation of the relevant predictions, but also to prospectively explain why preceding transitions involved structural losses—, we would not be able to justify the epistemic distinction drawn between current theories and their predecessors (as presupposed by my alternative strategy).

In view of the above observation, I intend to complete a two-fold task: firstly, I shall state a general condition P that may be applied to a broad family of possible structures $\mathbf{x}$ underlying (or compatible) with a theory or set of theories that, *if satisfied by current physics* (i.e., if $P(\mathbf{x}_0)$ is true, where $\mathbf{x}_0$ is a set of current theories), supports the alternative structural continuity argument (i.e., blocks the PMI argument and endorses WP-selective SR accordingly); and secondly, I shall provide a case study in current physics that satisfies this condition. Before making such a condition explicit, the previous discussion allows us to conclude that a conditional statement of the form

'If the condition $P(\mathbf{x})$ is true then $(W \wedge Q)(\mathbf{x})$ is true'

is true, where $\mathbf{x}$ ranges over a family of theoretic structures underlying (or compatible with) a certain set of successful physical theories, and $W \wedge Q$ is the conjunction of *two general statements* concerned with the ontological role played by $\mathbf{x}$. W states that $\mathbf{x}$ are working posits of these theories that can exhaustively explain their respective successful predictions; and Q states that $\mathbf{x}$ are approximately preserved and expanded in the course of a progressive-cumulative development without suffering structural losses. In terms of the partial structures framework, the true value of $(W \wedge Q)(\mathbf{x})$ may be expressed as follows: for any preceding and successor theory (whose models are S and S', respectively), there must be an underlying (or compatible) structurally based working posit $x_{G'}$ (whose model G' is a subset of S') that contains all known relations holding for a structurally based working posit x_G underlying (or compatible with) the preceding theory (whose model G is a subset of S), such that there is a full homomorphism between G and G'.

Before we launch into a search for such a general condition P, two observations are in order. Firstly, I am presupposing that both working posits x_G and $x_{G'}$ are full kinematical structures, that is, they are indispensable to the explanation of the relevant (kinematical) predictions and are empirically adequate with respect to the predicted (kinematical) phenomena (as specified in the last part of Section 4); and secondly, if P is satisfied only by a case study in physics, the risk remains of having a current theory with an underlying (compatible) structure which does not satisfy P. In fact, I accept that the alternative structural continuity argument provided here cannot be satisfied exhaustively, at least for now. As I shall show, my analysis will be carried out only in the particular context of the theoretical transition taking place from classical to quantum relativistic kinematics. However, since P is not associated with specific structures but with a general category of structures, P might be generally applied to any arbitrary structure involving theoretical transitions which could take place between more general theories that have been or will be developed. Although my aim is to state the precise condition P under which WP-selective SR might be supported, it is an interesting further step to investigate whether P can be satisfied in the case of massless particles or particles with spin, including dynamical symmetries in the picture—although it would even more interesting to investigate what happens when quantum field theory is considered and when gravitational effects are incorporated.

In order to establish P, I will proceed via two successive steps. Firstly, P given Q (which I shall denote as P_Q) will be addressed in subsections 5.1, 5.2, 5.3 and 5.4, and then P given $W \wedge Q$ (which I shall denote simply as P) will be addressed in subsections 5.5 and 5.6.

5.1. Antecedents of stability as preserved structure

There have been meaningful attempts to establish a certain stability condition P_Q that enables us to confirm the statement Q. One significant contribution in this respect is that proposed by (Redhead, 2001) according to which there are two exclusive kinds of structural transformations in scientific development: continuous and discontinuous. In his own words,

> Consider a one-parameter family of structures $\{S_p\}$ where the parameter p is a continuously variable real number. Let us suppose for values of p unequal to zero the structures S_p are all qualitatively the same, as p varies the structure changes, but in a continuous way. But suppose the change in structure suffers a discontinuity at the point $p = 0$, S_0 is qualitatively distinct from all the S_p with $p \neq 0$. We may say that the family of structures is stable for $p \neq 0$, but exhibits a singularity at $p = 0$ (Redhead, 2001, 86).

The essential idea behind Redhead's proposal is to conceive of actual cases of theoretical transitions (i.e., transitions where some novel structures and predictions are incorporated) as discontinuous transformations of unstable structures that give rise to non-isomorphic structures, while

[24] NB: it has been argued that the retrospective identification of the working posits of preceding theories seems to incur in fatal flaw as it begs the question by assuming from the start that current theories are true.

continuous transformations of stable structures give rise to isomorphic structures representing actual cases of structural preservations.[25] Thus, we can propose the following condition:

$P_{Q(Red)}$: **x** are *stable*, given that **x** ranges over a general family of possible structures underlying (or compatible with) a certain set of successful physical theories.

However, as pointed out by Redhead (and then analysed in detail by (Votsis, 2010)), whether or not a structural transformation is continuous is relative to the kind of stable successive structures with which one is dealing. Indeed, one should incorporate an identity criterion delimiting the equivalent class of essential features that constitute the successive structures differing under continuous transformations (what he calls the 'qualitative type of structure'). For example, if the essential features of a family of structures S_p are given by the equivalent class of qualities that are preserved under the group of homeomorphisms of the plane (e.g., that the shape of S_p completely encloses an area and that it has no endpoints), then circles and closed curves belong to S_p. However, if one includes the quality of 'having a centre' in the equivalent class then circles and closed curves become inequivalent structures. Considering this possibility (Votsis, 2010), proposes a distinction between *discontinuous* and *partially discontinuous* transformations in order to do justice to the varying degrees of discontinuity that is encountered in actual scientific practice. Whilst the first kind applies when none of the essential features of the successor structure are shared by its predecessor, the second kind applies only when some but not all of the essential features of the successor structure are shared by its predecessor. Thus, "a structure S′ and its predecessor structure S correspond if and only if with respect to a given parameter class there is a transformation from S′ to S that is either (a) continuous or (b) partially discontinuous" (Votsis, 2010, p. 110). Under this Redhead-Votsis scheme, one may interpret both the classical-relativistic and the classical-quantum transitions as prominent examples of partially discontinuous transformations. In the case of the former, the structures of both classical and relativistic kinematics do not share the relativity of simultaneity (given by the parameter $1/c \neq 0$) while they share the essential features that both absolute velocity and absolute spatial separation are forbidden. In the case of the latter, the structures of both classical and quantum dynamics do not share the non-commutativity of the structure (given by the parameter $\hbar \neq 0$) while they share certain other essential features, such as their symplectic structure.

Although the Redhead-Votsis scheme can be seen as a powerful, representational framework to elucidate the "[…] definite sense in which the new structures grow naturally, although discontinuously, out of the old structures" and that "[…] revolutions in physics, understood from the structural standpoint, can be understood progressively […]" (Redhead, 2001, p. 88), I do not see how it can give us the exact condition P_Q which we wish to obtain. We know that structural stability is a notion defined relative to a set of essential features that are constitutive of the structures underlying (or compatible with) successive theories. However, essential features are identified in this scheme by referring to the physical context and the relevant aspects involved in each case study (e.g., the relativity of simultaneity or the non-commutativity) without providing the exact condition P_Q that the corresponding mathematical structures satisfy in order to possess such essential physical features (e.g., the Lorentz invariance and the Moyal-Heisenberg invariance). In other words, the stability of a family of structures S_p relative to a set of essential physical features is defined in a merely vacuous, set-theoretical way, such as 'S_p are stable for $p \neq 0$ and unstable at $p = 0$', without supplying a definition of stability in terms of the mathematical properties of S_p that explain their essential physical features. Under these circumstances, my suggestion is to find an appropriate mathematical language underlying (or compatible with) any family of concrete structures S_p in terms of which the general condition P_Q can be established, i.e., a condition which enables us to explain the essential features relative to S_p in terms of the mathematical properties of a common language.

As we shall now see, the group-theoretic notion of stability gives us the required language. Thus, the language of group theory enables us to confirm the statement Q not in a vacuous way, as in the Redhead-Votsis scheme, but relativized to a given group-theoretic structure responsible for the essential physical features associated with any concrete S_p.

5.2. Lie algebra deformation and stability theory

In order to establish a certain condition P_Q that enables us to confirm the statement Q, I shall appeal to Lie algebra deformation and stability theory. I will follow José Figueroa-O'Farrill's heuristic guide in associating this theory with the inverse notion of group contraction (i.e., the operation of deforming a given Lie algebra to reach its associated contracted, non-isomorphic Lie algebra), for "The theory of Lie algebra deformations provides us with a systematic procedure which is an inverse to the more common Lie algebra contractions" (Figueroa-O'Farrill, 1989, p. 2735). Although the semantic connotation of an algebraic deformation as 'the inverse of group contraction' is not generally accurate, it is mathematically appropriate in specific cases. As we shall see in the next subsections, this heuristic guide will serve to identify the source of structural losses among a broad family of possible theoretical transitions, and on the basis of this identification we will be able to reconstruct such transitions as do not involve structural losses but which have the property of being stable transitions (i.e., those whose preceding Lie algebras are stable). Following the work of (Nijenhuis & Richardson, 1966; Mendes, 1994, 2016), I shall first introduce a brief outline of the primary foundations of Lie algebra deformation and stability theory.

It is a mathematical fact that Lie algebras can be deformed. Indeed, they can be deformed in a similar way to vectors or operators, denoted by ξ, which are perturbed by a linear transformation of the form $\xi' = \xi + \mathbb{I}\varepsilon$ which is infinitely close to being trivial. However, the manner and the extent to which they are deformed can either lead to isomorphic Lie algebras or open the way to deriving completely different algebraic structures. Considering both cases, one can draw a line between two equivalent classes of Lie algebras depending on their behaviour with respect to these deformations: one class is that in which all algebras are isomorphic (up to deformations); in the other, they are non-isomorphic. To know where to draw this line in the space of Lie algebras, the notion of stability is introduced:

Definition 5.1 A Lie algebra $\mathscr{G}_P$ is called stable if, under any *infinitesimal deformation*, it necessarily leads to an isomorphic Lie algebra. Otherwise, it is unstable.

Instead of inquiring into the details of these infinitesimal deformations, I shall state an important theorem that determines not only the specific topological and geometrical conditions Lie algebras have to satisfy in order to be stable, but also the way in which infinitesimal deformations lead to isomorphic Lie algebras.[26]

Theorem 5.1 *Semisimple* Lie algebras are stable (although not the converse).[27]

This theorem shows that the topological property of being semisimple is intrinsically connected to the stability of the Lie algebra in question. Thus, we can evaluate each of our theories against their group-theoretic property of semisimplicity.

[25] Robert Batterman draws a similar distinction between reduction (where the limit is regular) and inter-theoretic relations (where it is singular) (Batterman, 2002).

[26] The proof of this theorem can be seen in (Nijenhuis & Richardson, 1966).

[27] Semisimplicity is a topological property of a general topological object. A semisimple Lie algebra is one that can be decomposed into a sum of simple Lie algebras, while simple Lie algebras are those that do not contain non-trivial proper subalgebras.

5.3. Theoretical preservations and theoretical transitions

Once the above formal framework has been established, let us differentiate the deformation of the Lie algebra from the notion of group contraction. As noted above, algebraic deformations are not, strictly speaking, the inverse of group contractions. There are also Lie algebras that can be deformed to yield isomorphic algebras. Although group contractions are inverse deformations between non-isomorphic algebras, deformations that are between isomorphic algebras do not count as group contractions. In this sense, the theory of deformations is broader in scope.

Considering this distinction, we can apply the theory of Lie algebra deformations to our problem of theoretical transitions and suggest that actual cases of theoretical transitions, where some novel structures and predictions are incorporated, can be represented, at the group-theoretic level, as deformations of Lie algebras that give rise to non-isomorphic Lie algebras. In the same way, we can suggest that any deformation of a Lie algebra that gives rise to an isomorphic Lie algebra represents, in group-theoretic terms, an actual case of structural preservation. Yet we can make a further, important point. As discussed above, the distinction between deformations giving rise to non-isomorphic and isomorphic Lie algebras depends on whether the Lie algebra in question is unstable or stable. If the algebra is unstable, then its deformation yields a non-isomorphic algebra, while if it is stable, its deformation yields an isomorphic algebra. This fact implies that we can characterize theoretical transitions that involve novel structures and predictions as deformations of unstable Lie algebras to reach stable or unstable ones, while the preservation of structure can be characterized in terms of deformations of stable Lie algebras to reach isomorphic stable ones. To put it schematically, we have two different kinds of deformations:

(1) *Theoretical transition*: Lie-algebraic deformations from unstable Lie algebras to non-isomorphic stable (or unstable) ones.
(2) *Theoretical preservation*: Lie-algebraic deformations from stable Lie algebras to isomorphic stable ones.

Thus, it seems that this powerful technique can give us a precise and formal account of the way theoretical transitions arise and the way theoretical structures are preserved. To illustrate this with concrete examples in physics, I will draw upon the particular case study suggested by Inönü and Wigner. Let us begin by illustrating how theoretical transitions arise in this particular case study, and then proceed to address theoretical preservation.[28]

The Lie algebra of *Gal*, denoted by $\mathscr{G}_{Gal}$, is the set of generators of rotations, boosts and spacetime translations. Considering only the generators of rotations J_a and boosts K_b, a = 1,2,3, there is a way to find all possible deformations of the Lie algebra associated with this restricted group (that we will denote as $\mathscr{G}^{'}_{Gal}$). It can be demonstrated that $\mathscr{G}^{'}_{Gal}$ is not semisimple, and therefore, it is unstable. It is however possible to stabilize this Lie algebra. The Lie product corresponding to $\mathscr{G}^{'}_{Gal}$ can be deformed in a special way such that we end up with a different but stable Lie algebra. It can be shown that if $\mathscr{G}^{'}_{Gal}$ is deformed through the finite one-parameter *t*, then the following algebra is obtained

$$[J_a, J_b] = i\varepsilon^c_{ab}J_c \quad ; \quad [J_a, K_b] = i\varepsilon^c_{ab}K_c \quad ; \quad [K_a, K_b] = it\varepsilon^c_{ab}J_c \tag{2}$$

which is the Lorentz algebra if $t = -1/c^2$. In a nutshell, by deforming the Lie algebra of the homogenous part of classical kinematics we obtain the Lie algebra of the homogenous part of relativistic kinematics. The latter algebra is semisimple, and thus stable. This means that if we deform it, we end up with an isomorphic Lie algebra, contrary to the case of $\mathscr{G}^{'}_{Gal}$.

Note that this result only holds for the homogenous restricted algebra $\mathscr{G}^{'}_{Gal}$ without spacetime translations. The deformation of the whole inhomogeneous algebra $\mathscr{G}_{Gal}$ (i.e., an unstable Lie-algebra) giving rise to the algebra of the whole inhomogeneous Poincaré group ISL(2,ℂ) is not a stabilization process. The Lie algebra of ISL(2,ℂ), denoted by $\mathscr{G}_{\mathrm{ISL}(2,\mathbb{C})}$, is unstable. Thus, the interesting point to make from this example is that a stabilization process only arises from deforming a subalgebra of $\mathscr{G}_{Gal}$ giving rise to a subalgebra of $\mathscr{G}_{\mathrm{ISL}(2,\mathbb{C})}$, that is, the Lorentz algebra. This means that only the Lorentz subalgebra (as opposed to the full algebra $\mathscr{G}_{\mathrm{ISL}(2,\mathbb{C})}$) will subsequently be preserved because any deformation of it yields an isomorphic algebra.

Let us now illustrate how theoretical preservation arises in this case study. As emphasized above, the unique substructure that is fully preserved from classical to relativistic kinematics is the special orthogonal group SO(3). Since this group is the 'rotational' component of the wider Galilean structure, this means that any transformation defined by the rest of the elements of the Galilean algebra turns out to be isotropic. During the transition, the other substructures are either lost, such as the inhomogeneous component of the Galilean group *Gal* (i.e., space-time translations and Galilean boosts), or they are incorporated, such as the inhomogeneous component of the Poincaré group ISL(2,ℂ) (i.e., space-time translations and Lorentzian boosts). Surprisingly, this isotropic preservation can be explained on the grounds that the unique substructure that is strictly shared by classical and relativistic kinematics is SO(3), which has a stable Lie algebra, while the substructure that forms the inhomogeneous component of *Gal* has an unstable one. Indeed, the Hamiltonian algebra of classical mechanics associated with the inhomogeneous component of the standard Galilean group (the *Poisson bracket* formalism) is not semisimple and thus is unstable.

Consequently, the stability of the Lie algebra associated with a group-theoretic structure is directly related to its full preservation when transitions from one theory to another take place, whilst instabilities in full algebraic structures involve structural losses.

5.4. Stable and unstable theoretical transitions

Considering the above case study, we can make sense of a general distinction of theoretical transitions (which are, by definition, between non-isomorphic algebras) that involve some kind of deformation in the associated Lie algebraic structure: they are either from unstable to stable algebras, as is the case for the homogenous subalgebras given above, or from unstable to unstable algebras, as is the case for the whole inhomogeneous algebras.

However, these theoretical transitions are not the only transitions in town. There are transitions of another kind that are not, strictly speaking, algebraic deformations, but that involve non-isomorphic algebras that take place from stable Lie algebras to stable (or unstable) ones. So, we can make a further and broader distinction encompassing different possible theoretical transitions between non-isomorphic Lie algebras:

(1.1) *Stable transitions*: Theoretical transitions from stable Lie algebras to stable (or unstable) ones.
(1.2) *Unstable transitions*: Theoretical transitions that involve a deformation from unstable Lie algebras to stable (or unstable) ones.

Although, according to this distinction, the examples given above fall under the category of unstable transitions, no details of stable transitions have been given. As we shall see, these are precisely the kinds of transitions that, at the group-theoretic level, do not involve structural losses. Thus, by establishing a condition of the form

$P_{Q(Lie)}$: **x** are *stable*, given that **x** ranges over a general family of possible *Lie algebras* underlying (or compatible with) a certain set of successful physical theories.

[28] A more detailed characterization of both kinds of deformation will be elaborated in terms of the partial structures approach in Section 5.

we might make Q true. To see how these new kinds of transitions arise, let us consider again the case study of classical and relativistic kinematics. However, this analysis will reveal that not all structures that involve a stable transition can provide an answer to the PMI argument. This is because, as discussed in Section 2, condition P should be such that when it is true it makes not only Q true but also W (i.e., the statement concerned with working posits). Stable transitions are neither sufficient (nor necessary) conditions for supporting WP-selective SR. On the basis of a general condition P that not only implies Q but also implies W, I will conclude with a better strategy that provides a full answer to our concerns.

Consider the transitions that occur from classical kinematics, restricted to the algebra of SO(3), denoted by $\mathscr{G}_{\mathrm{SO}(3)}$, to relativistic kinematics, either restricted to the homogenous Lorentz algebra $\mathscr{G}_{Lorentz}$ or fully captured in terms of the inhomogeneous Poincaré algebra $\mathscr{G}_{\mathrm{ISL}(2,\mathbb{C})}$. Since $\mathscr{G}_{\mathrm{SO}(3)}$ differs from $\mathscr{G}_{\mathrm{ISL}(2,\mathbb{C})}$ and $\mathscr{G}_{Lorentz}$ by virtue of the fact that these algebras introduce relativistic boosts (in addition to spacetime translations in the case of the Poincaré algebra), the preceding and the succeeding algebras are non-isomorphic. Moreover, $\mathscr{G}_{\mathrm{SO}(3)}$ and $\mathscr{G}_{Lorentz}$ are stable while $\mathscr{G}_{\mathrm{ISL}(2,\mathbb{C})}$ is unstable. This means that there are two kinds of stable transitions with no structural losses: one which takes place from a stable algebra to a non-isomorphic stable algebra (from $\mathscr{G}_{\mathrm{SO}(3)}$ to $\mathscr{G}_{Lorentz}$), and the other which takes place from a stable to an unstable algebra (from $\mathscr{G}_{\mathrm{SO}(3)}$ to $\mathscr{G}_{\mathrm{ISL}(2,\mathbb{C})}$). What we have in both cases is a cumulative expansion of the stable algebraic structure of SO(3) that is preserved and fully contained in the successor non-isomorphic Lorentz or Poincaré algebra. In particular, these kinds of stable transitions not only take place between the whole non-isomorphic algebras but also between stable isomorphic subalgebras as preserved structures.

However, although it is true that $\mathscr{G}_{\mathrm{SO}(3)}$ is preserved, and that the transitions in which it is involved are stable, stable transitions of this sort cannot provide a complete answer to the objection posed by the PMI. This preserved algebraic structure alone cannot explain all the phenomena successfully predicted by classical kinematics and even those predicted by the restricted homogeneous Lorentz algebra. It only defines the 'rotational' component of the structure, and thus incorporates the isotropy associated with the rest of the elements of the algebra. As such, a correct response in favour of WP-selective SR would have to identify the preserved working posits of the *full* Galilean Lie algebra $\mathscr{G}_{Gal}$ that not only disregard structural losses, but also lead to group-theoretic structures playing the role of the working posits of relativistic kinematics. In fact, the Lie algebra preserved under this transition should, by itself, explain all phenomena that fall under the successful predictions of classical kinematics, and when it is expanded it should incorporate the complete successful predictions of relativistic kinematics. Nevertheless, this condition does not apply in the case considered. One might reply that neither $\mathscr{G}_{\mathrm{SO}(3)}$ nor $\mathscr{G}'_{Gal}$ and $\mathscr{G}_{Gal}$ incorporate the complete successful predictions of classical mechanics, in the same way that $\mathscr{G}_{\mathrm{ISL}(2,\mathbb{C})}$ or $\mathscr{G}_{Lorentz}$ do not incorporate the complete predictions of special relativity. However, we should note that this particular case study limits the empirical predictions of these theories to those derived from their kinematical structure. In this way, by all of the successful predictions of classical and relativistic kinematics I do not mean all of the predictions of the full theories but only those restricted to their space-time symmetries.

5.5. Stable Lie algebras against the induction argument

Under these circumstances, let us recall that $P_{Q(Lie)}$ is a condition that has to be satisfied to attain Q. However, in order to satisfy $(W \wedge Q)$, let us consider stable transitions through the following group-theoretic condition[29]

P: $\mathbf{x_0}$ are *stable*, given that $\mathbf{x_0}$ ranges over a general family of possible *full Lie algebras* underlying (or compatible with) a certain set of *current* successful physical theories.

If P is true, regardless of whether a current theory has been developed from a preceding theory whose full Lie algebra was stable or unstable, then we can be sure that any transition from this theory to a successor will be stable, and will preserve, at the physical level, the working posits of all of the successful predictions of the theory, and will not involve, at the group-theoretic level, structural losses. In other words, the condition of the form 'if group-theoretical condition $P(\mathbf{x_0})$ is true then $(W \wedge Q)(\mathbf{x_0})$ is true' is true. In the same way, one would expect the full algebra of the successor theory after the transition to be stable and to exhaustively explain its empirical predictions because only in this way is it capable of being cumulatively developed and expanded through subsequent transitions without suffering structural losses.

Once condition P has been established, we shall see that it is satisfied by a well-known case study in contemporary physics, namely, quantum-relativistic kinematics.

5.6. Case study: The classical-quantum-relativistic transition

Considering that the transition from $\mathscr{G}_{Gal}$ to $\mathscr{G}_{\mathrm{ISL}(2,\mathbb{C})}$ is unstable, and thus involves structural losses, our task is to search for a stable full Lie algebra succeeding the full algebra $\mathscr{G}_{Gal}$, and to characterize the transition between classical and relativistic kinematics as the preservation and expansion of $\mathscr{G}_{Gal}$ to yield this non-isomorphic stable full Lie algebra. Originally introduced by (Mendes, 1994) but developed by (Chryssomalakos & Okon, 2004), we shall see that such an algebraic structure exists, and that the key assumption that may seem to complete our task is the introduction of the group-theoretic structure underlying quantum kinematics. In other words, the only way to preserve and expand the full algebra of the full Galilean group *Gal* to yield a stable full relativistic algebra, and thereby explain all relativistic predictions and prevent structural losses (at subsequent transitions), is by incorporating quantum kinematics.

Let us embark upon the search for a stable full Lie algebra encompassing relativistic and quantum effects that embraces their kinematical properties (Chryssomalakos & Okon, 2004). Start this analysis with the 'Poincaré plus positions' Lie algebra $\mathscr{G}_{CR}$ associated with classical relativistic kinematics for a massive particle:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(g_{\mu\sigma}J_{\nu\rho} + g_{\nu\rho}J_{\mu\sigma} - g_{\mu\rho}J_{\nu\sigma} - g_{\nu\sigma}J_{\mu\rho}) \tag{3}$$

$$[J_{\rho\sigma}, P_\mu] = i(g_{\mu\sigma}P_\rho - g_{\mu\rho}P_\sigma) \tag{4}$$

$$[J_{\rho\sigma}, Z_\mu] = i(g_{\mu\sigma}Z_\rho - g_{\mu\rho}Z_\sigma) \tag{5}$$

where J is the Lorentz algebra generators, P is the momentum generator, and Z is what is called the 'moment generator' $Z_{mu} = X_{mu}M$, with X and M, the position and central generators, respectively.[30] This Lie algebra, not being semisimple, is unstable. Actually, it can be proven that there are five non-trivial generators associated with this algebra, each one representing the direction of a possible infinitesimal deformation. After some calculations, they conclude that the only way $\mathscr{G}$ can be stabilized is by deforming $\mathscr{G}_{CR}$ along the Heisenberg generator (i.e., introducing the modified Heisenberg commutator):

$$[P_\mu, Z_\nu] = i\hbar g_{\mu\nu}M. \tag{6}$$

[29] Recall that the notion of *full Lie algebras* is explained in the last part of Section 4.

[30] Vanishing commutators are omitted, the metric is $g = \mathrm{diag}(1, -1, -1, -1)$ and the speed of light c is conventionally taken equal to 1. Also, M is introduced to obey the condition of linearity in the generators, whose physical interpretation is of a mass-generator representing a phase-time shift between different observers.

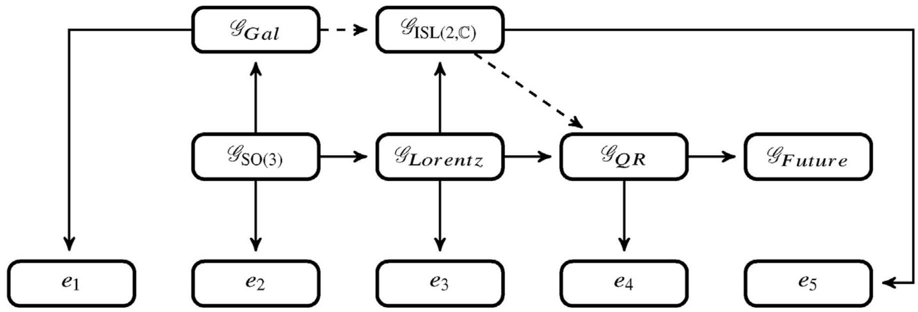

Fig. 1. *The whole Lie algebraic structure*. The hierarchy of the Lie algebraic structures underlying (or compatible with) different theories.

to yield the 'Poincaré plus Heisenberg' Lie algebra $\mathscr{G}_{QR}$, which is stable. However, a meticulous analysis of the non-uniqueness of stable deformations of this algebra and the primitiveness of the momentum operator Z shows that $\mathscr{G}_{QR}$ must also satisfy:

$$[Z_\mu, M] = -i\hbar P_\mu \quad ; \quad [Z_\mu, Z_\nu] = i\hbar^2 J_{\mu\nu}. \tag{7}$$

The proof of this can be found in (Chryssomalakos & Okon, 2004) but regardless of the details, $\mathscr{G}_{QR}$ can be considered the actual Lie algebra of quantum-relativistic kinematics for a massive and spinless particle, where the Z-Z non-commutativity is a purely quantum phenomenon and $i\hbar^2 J_{\mu\nu}$ is (a multiple of) the covariant form of the orbital angular momentum generator. Physically speaking, the deformation of the 'position' algebra to yield the Moyal-Heisenberg Lie algebra amounts to the incorporation of the quantum uncertainty relations between position and momentum into the 'translational' component of $\mathscr{G}_{QR}$. As is well known, these relations constrain the measured value of the position and momentum of a quantum-relativistic system and are responsible for the purely quantum behavior observed in moving bodies.

This case study demonstrates the existence of a stable Lie algebra $\mathscr{G}_{QR}$, a point which confirms two statements needed to address the objection posed by the PMI argument. Firstly, concerning statement Q, $\mathscr{G}_{QR}$ is a stable Lie algebra and, as such, any deformation of it does not lead to a non-isomorphic algebra, meaning that it will be preserved when novel and future predictions are incorporated. Secondly, concerning statement W, $\mathscr{G}_{QR}$ represents the physical structure embracing the quantum and relativistic domains in such a way that it can be considered the working posit not only of both theories but also of the effective, restricted domain of classical kinematics. It is fair to acknowledge that this algebraic structure ultimately underlies the complete relativistic and quantum effects, with the exception of massless and spin particles (whose generalization is still under investigation), but we can still restrict our analysis to this (still) very broad domain, and in this way appreciate that there are no more phenomena not successfully explained by this theory.

6. Concluding remarks: the overall picture

Considering the above analysis, we can characterize the unstable transition between the complete domain of classical kinematics and that of quantum-relativistic kinematics as the inverse process of a group contraction going from the group associated with an unstable full Lie algebra to that of a stable full Lie algebra. However, we can also characterize the stable transition that takes place between the stable subalgebras of these full structures that does not involve any kind of structural loss. Specifically, we can characterize with precision the transition occurring between the restricted domain of the 'rotational' component of classical mechanics (whose algebra is $\mathscr{G}_{\mathrm{SO}(3)}$), the restricted domain of the 'rotational' and 'boost' components of relativistic kinematics (the algebra of the Lorenz group $\mathscr{G}_{Lorentz}$), and the full domain of quantum-relativistic kinematics (the Poincaré plus Heisenberg Lie algebra $\mathscr{G}_{QR}$) as a hypothetical case of theoretical development that does not involve any kind of structural loss. This characterization is important because it can provide a clear and precise idea of how stable and unstable transitions are involved in the case study considered. Let us now see how the overall picture looks in terms of the partial structures framework.

From the previous section, we can conclude that something like Fig. 1 (below) happens in the transition between classical and quantum-relativistic kinematics.where each box is a set-theoretical model (i.e., a partial structure) representing the Lie algebraic structure of the corresponding physical theory.[31] The dashed lines are partial homomorphisms holding between models representing partial embeddings, while the bold lines are full homomorphisms holding between models representing full embeddings. Likewise, the models e_i (where i has values 1, 2, 3, 4 and 5) represent empirical data that correspond to the corresponding theory's successful predictions.

As we can see, the transition that starts from $\mathscr{G}_{Gal}$, goes through $\mathscr{G}_{\mathrm{ISL}(2,\mathbb{C})}$, and ends at $\mathscr{G}_{QR}$ is a successive partial embedding between algebraic structures. This means that there are structural elements that are lost along the way. So, let e_1 be the complete set of empirical data predicted by classical kinematics. These data were entailed by partial relations that physicists of the time knew they were holding for $\mathscr{G}_{Gal}$. However, after new empirical evidence was acquired in the full framework of relativistic kinematics e_5, some relations that they knew were holding for $\mathscr{G}_{Gal}$ turned out to be relations not holding for $\mathscr{G}_{\mathrm{ISL}(2,\mathbb{C})}$. These relations, expressed by $\mathscr{G}_{Gal}\backslash\mathrm{SO}(3)$, form structural losses. In the same way, we can see that after new empirical evidence was acquired in the framework of quantum-relativistic kinematics e_4, some relations that physicists knew that held for $\mathscr{G}_{\mathrm{ISL}(2,\mathbb{C})}$ turned out to be relations which do not hold for $\mathscr{G}_{QR}$. These relations are expressed by $\mathscr{G}_{\mathrm{ISL}(2,\mathbb{C})}\backslash\mathscr{G}_{Lorentz}$, which are structural losses too. Instead, the transition that starts from $\mathscr{G}_{\mathrm{SO}(3)}$, goes through $\mathscr{G}_{Lorentz}$, and ends at $\mathscr{G}_{QR}$ is a successive full embedding between algebraic structures. This means that, contrary to the last transition, there are no structural losses. So, if we confine the set of empirical data to what is predicted by the isotropic classical symmetry e_2, we can claim that physicists of the time knew that some relations held (and did not hold) for $\mathscr{G}_{Gal}$, but when new empirical evidence was acquired in the framework of relativistic kinematics, confined to rotations and boosts e_3, and in that of quantum relativistic kinematics e_4, all relations that they knew held for $\mathscr{G}_{\mathrm{SO}(3)}$ and $\mathscr{G}_{Lorentz}$ turned out to be relations which held for $\mathscr{G}_{Lorentz}$ and hold for $\mathscr{G}_{QR}$, respectively. These relations were preserved and additional relations pertaining to relativistic and quantum effects were incorporated among the structures. In particular, in the transition from classical to Lorentzian relativistic kinematics, Lorentzian boosts $\mathscr{G}_{Lorentz}\backslash\mathrm{SO}(3)$, were incorporated while in the transition from Lorentzian relativistic kinematics to quantum-relativistic kinematics, it was the non-commutative momentum and position translations (in the form of the Heisenberg algebra) $\mathscr{G}_{QR}\backslash\mathscr{G}_{Lorentz}$. Finally, let us note that the subsequent hypothetical transition from $\mathscr{G}_{QR}$

[31] I will assume here that the notation for Lie algebras is the same notation as for its associated partial structures.

to $\mathscr{G}_{Future}$ is a full embedding represented by a full homomorphism, meaning that all relations that physicists currently know to hold for $\mathscr{G}_{QR}$ will turn out to be relations which shall hold for the algebra of a future theory $\mathscr{G}_{Future}$.

Following the discussion addressed in Section 3, we can conclude that there will be a kinematical structurally based working posit of a future theory $\mathscr{G}_{Future}$ that will contain all known relations holding for the structurally based working posit $\mathscr{G}_{QR}$, such that there will be a full homomorphism between $\mathscr{G}_{QR}$ and $\mathscr{G}_{Future}$. It is the stable nature of this hypothetical transition that prevents the PMI argument being a means to support WP-selective SR.

Author declaration statement

Studies in History and Philosophy of Science

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Availability of data and material

Not applicable.

Code availability

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Declaration of competing interest

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References

Alai, M. (2017). Resisting the historical objections to realism: Is Doppelt's a viable solution? *Synthese, 194*, 3267–3290.

Auyang, S. (1995). *How is quantum field theory possible?* Oxford: Oxford University Press.

Batterman, R. (2002). *The devil in the details: Asymptotic reasoning in explanation, reduction, and emergence*. New York: Oxford University Press.

Bueno, O., French, S., & Ladyman, J. (2002). On representing the relationship between the mathematical and the empirical. *Philosophy of Science, 69*(3), 452–473.

Bueno, O. (2000). Empiricism, scientific change and mathematical change. *Studies in History and Philosophy of Science Part A, 31*(2), 269–296.

Bueno, O. (2008). Structural realism, scientific change, and partial structures. *Studia Logica, 89*(2), 213–235.

Chryssomalakos, C., & Okon, E. (2004). Generalized quantum relativistic kinematics: A stability point of view. *International Journal of Modern Physics D, 13*(10), 2003–2034.

da Costa, N. C. A., & French, S. (2003). *Science and partial truth: A unitary approach to models and scientific reasoning*. Oxford: Oxford University Press.

De Haro, S. (2020). *The empirical under-determination argument against scientific realism for dual theories*. Erkenntnis. https://doi.org/10.1007/s10670-020-00342-0

Feyerabend, P. (1962). *Explanation, reduction, and empiricism* (Vol. 3, pp. 30–97). University of Minnesota Press.

Figueroa-O'Farrill, J. (1989). Deformations of the Galilean algebra. *Journal of Mathematical Physics, 30*(12), 2735–2739.

French, S. (2014). *The structure of the world: Metaphysics and representation*. Oxford: Clarendon Press.

Inönü, E., & Wigner, E. (1953). On the contraction of groups and their representations. *Proceedings of Natural Academy of Science, 39*(6), 510–524.

Ladyman, J., Ross, D., & Spurret, D. (2007). *Every thing must go: Metaphysics naturalized*. Oxford: Oxford University Press.

Laudan, L. (1981). A confutation of convergent realism. *Philosophy of Science, 48*(1), 19–49.

Laudan, L. (1996). *Beyond positivism and relativism: Theory, method, and evidence*. Michigan: Avalon Publishing.

Manero, J. (2019). Imprints of the underlying structure of physical theories. *Studies in the History and Philosophy of Modern Physics, 68*, 71–89.

Mendes, V. (1994). Deformations, stable theories and fundamental constants. *Journal of Physics, A27*, 8091–8104.

Mendes, V. (2016). The stability of physical theories principle. In I. Licata (Ed.), *Beyond peaceful coexistence* (pp. 153–200). London: Imperial College Press.

Nijenhuis, A., & Richardson, R. (1966). Cohomology and deformations in graded Lie algebras. *Bulletin of the American Mathematical Society, 72*, 1–29.

Post, H. (1971). Correspondence, invariance and heuristics. *Studies in History and Philosophy of Science, 2*, 213–255.

Psillos, S. (1999). *Scientific realism: How science tracks truth*. London: Routledge.

Psillos, S. (2018). Realism and theory change in science. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy (Summer 2018 Edition)*. URL https://plato.stanford.edu/archives/sum2018/entries/realism-theory-change/.

Putnam, H. (1975). *Mathematics, matter and method*. Cambridge: Cambridge University Press.

Quine, W. (1975). On empirically equivalent systems of the world. *Erkenntnis, 9*(3), 313–328.

Redhead, M. (2001). The intelligibility of the universe. *Royal Institute of Philosophy Supplement, 48*, 73–90.

Roberts, B. (2011). Group structural realism. *The British Journal for the Philosophy of Science, 62*(1), 47–69.

Saunders, S. (1993). To what physics corresponds. In S. French, & H. Kaminga (Eds.), *Correspondence, invariance, and heuristics: Essays in honour of Heinz Post* (pp. 295–325). Dordrecht: Kluwer.

Valentini, A. (1997). On Galilean and Lorentz Invariance in Pilot-Wave dynamics. *Physics Letters A, 228*, 215–222.

van Fraassen, B. C. (1980). *The scientific image*. Oxford: Clarendon Press.

Vickers, P. (2017). Understanding the selective realist defence against the PMI. *Synthese, 194*(9), 3221–3232.

Votsis, I. (2010). Structural realism: Continuity and its limits. In A. Bokulich, & P. Bokulich (Eds.), *Scientific structuralism* (pp. 105–117). Dordrecht: Springer.

Wigner, E. (1939). On unitary representations of the inhomogeneous Lorentz group. *Annals of Mathematics, 40*, 149–204.

Worrall, J. (1989). Structural realism: The best of both worlds? *Dialectica, 43*(1), 99–124.