Things you will be able to do

1. Solve a linear system directly (when there are 3 or fewer unknown variables), and give an explicit parameterization of the set of solutions
2. Draw accurate diagrams of vectors, lines, and planes in $\mathbb{R}^2$ and $\mathbb{R}^3$
3. Convert a linear system to a matrix equation
4. Represent a linear system as an augmented matrix
5. Perform row operations (scaling, swapping, and in-place subtraction) on a matrix or augmented matrix
6. Perform the correct sequence of row operations on an augmented matrix leading to reduced row echelon form (Gaussian elimination)
7. Perform the correct sequence of row operations on a matrix leading to row echelon form (partial Gaussian elimination)
8. Determine an explicit parameterization of the set of solutions of a linear system using the reduced row echelon form of the augmented matrix
9. Write the solution of a linear system as a vector expression
10. Perform the correct sequence of row operations on the identity-augmented matrix leading to the matrix of the inverse transformation
11. Represent a change of basis as a matrix B (most often, from the standard basis to a given different basis)
12. Represent a vector v in $\mathbb{R}^n$ (given as a column vector with respect to the standard basis) as a column vector with respect to a given different basis (Bv)
13. Represent a vector v given as a column vector with respect to a given basis as a column vector with respect to another basis (Bv)
14. Represent a linear map or transformation as a matrix with respect to the standard basis (M)
15. Represent a linear map or transformation as a matrix with respect to a given basis (CMB^{-1} or BMB^{-1})
16. Represent a bilinear form $\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ as a matrix with respect to the standard basis (M)
17. Represent a bilinear form $\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ as a matrix with respect to a given basis ($B^T MB$)
18. Compute the value (output) of a linear map or transformation when applied to a given input vector
19. Compute the value of a bilinear form on a pair of input vectors
20. Compute the length of a vector in $\mathbb{R}^n$
21. Compute the length of the projection of a vector onto another vector
22. Write the matrix of a linear transformation $\mathbb{R}^n \to \mathbb{R}^n$ that orthogonally projects onto the line spanned by a given vector (i) in an easy basis, and (ii) in the standard basis
23. Write the matrix of a linear transformation that orthogonally projects onto a subspace given by an orthonormal spanning set (i) in an easy basis, and (ii) in the standard basis
24. Write the matrix of a linear transformation that orthogonally projects onto a subspace given by an arbitrary spanning set
25. Write the matrix of a linear transformation that rotates in a given 2-plane by a given angle (i) in an easy basis, and (ii) in the standard basis

26. Represent the composition (one after the other) of two linear maps or transformations as a matrix (matrix multiplication)
27. Compute the determinant of a 2 by 2 or 3 by 3 matrix
28. Instruct a computer to compute the determinant of an n by n matrix
29. (column space) Compute a basis for the image of a linear map $\mathbb{R}^n \to \mathbb{R}^m$ (the column space of the matrix; select the columns of the original matrix corresponding to pivots)
30. (null space) Compute a basis for the kernel of a linear map $\mathbb{R}^n \to \mathbb{R}^m$ (use the parameterization of the solution set of the homogeneous linear system $Ax = 0$, where A is the matrix of the map)
31. (row space) Compute a basis for the image of the dual map $\mathbb{R}^{m*} \to \mathbb{R}^{n*}$ (transpose then compute a basis for the column space)
32. ('left' null space) Compute a basis for the kernel of the dual map $\mathbb{R}^{m*} \to \mathbb{R}^{n*}$ (the kernel of the transpose matrix equation $A^T x = 0$)
33. Relate the dimensions of these 4 spaces and the rank of the original matrix:

$$\begin{aligned}
\text{rank} &= \dim(\text{col space}) = \dim(\text{row space}) \\
&\dim(\text{col space}) + \dim(\text{null space}) = n \\
&\dim(\text{row space}) + \dim(\text{left null space}) = m
\end{aligned}$$

34. Convert a list of vectors to an orthonormal basis for their span (Gram-Schmidt)
35. Decompose a full-rank matrix as QR, where the columns of Q are orthonormal and R is upper-triangular with positive entrires on the diagonal (remembering the moves of Gram-Schmidt)
36. Decompose a matrix as LU, where L is lower-triangular and U is upper-triangular, when such a decomposition exists (remembering the moves of Gaussian elimination)
37. Use an LU decomposition to solve a linear system $LUx = b$
38. Compute the least squares solution of an overdetermined linear system $Ax = b$ (the actual solution of $A^T Ax = A^T b$)
39. Perform symbolic matrix operations: product, inverse, transpose, determinant, trace, using linearity, basis invariance, associativity, homogeneity, etc.:

$$\begin{aligned}
&A + B = B + A \\
&(AB)C = A(BC) \\
&(AB)^{-1} = B^{-1}A^{-1} \\
&(AB)^T = B^T A^T \\
&(A^{-1})^T = (A^T)^{-1} \\
&\operatorname{tr}(A + B) = \operatorname{tr} A + \operatorname{tr} B \\
&\operatorname{tr}(BAB^{-1}) = \operatorname{tr} A \\
&\det(BAB^{-1}) = \det A \\
&\operatorname{tr}(\lambda A) = \lambda \operatorname{tr} A \\
&\det(\lambda A) = \lambda^{\text{size}\, A} \det A
\end{aligned}$$

40. Compute the characteristic polynomial of a linear transformation

41. Compute the eigenvalues of a linear transformation
42. Compute a basis of eigenvectors for the eigenspace of a given eigenvalue of a linear transformation
43. Find a basis with respect to which the matrix of a given linear transformation is diagonal, if one exists, and write the matrix equation relating the matrix with respect to the standard basis to the diagonal form