MAT 200 project

Due dates:
Topic choice **email by Friday night June 6 12:00am**
Written submission **June 19**
Presentations **June 26 and July 1**

Choose one of the two options below.

I. Develop a formal mathematical theory of a non-mathematical subject, and prepare a 15-minute lecture explaining your theory. Decide what the 5-10 most logically elementary concepts are, and choose definitions and axioms for them within your framework. State the *most elementary results* about them, and give proofs directly from the definitions and axioms. You decide how (pictures, equations, projector, chalkboard, speaking). For at least the first 5 minutes, assume we don't know anything.
Also submit a written exposition with at least two sections: (1) the discussion or introduction, explaining the idea, and (2) precise articulation of the definitions, statement, and proof. (You may also wish to explicitly describe the successes and limitations of mathematical/analytical methods for understanding your chosen subject).
Discuss your topic with me early, so I can send you in the right direction. Here are some sample topics:

1. **Academic grades and ranking**. What is a grading system? What are the criteria for fair grading systems? Which systems satisfy them?

2. **Elections and voting**. What is a voting system? What are the criteria for fair voting systems? Which systems satisfy them?

3. **Queuing (waiting and task assignment)**. What is a queue? What are the criteria for effective task assignment systems? Which systems satisfy them?

4. **Permutations of finite configurations (e.g. card shuffling)**. What is shuffling? What are the criteria for a random or effective card shuffle? Which shuffling methods satisfy them?

5. **Information**. What is raw information? What is information processing? What does it mean for information to be gained or lost in a process? Which processes preserve or destroy information?

6. **Music**. What is a musical composition? What is the difference between interesting and boring music? Is there a classification of compositions? Does music contain or communicate information?

II. Choose a theorem from the list below and prepare a 15-minute lecture to explain the content (the statement) and the proof. Include: relevant definitions, axioms, results assumed, and the statement of related theorems. You decide how (pictures, equations, projector, chalkboard, speaking). For at least the first 5 minutes, assume we don't know anything.
Also submit a written exposition with at least two sections: (1) the discussion or introduction, explaining the idea, and (2) precise articulation of the definitions, statement, and proof.
Discuss your topic with me early, so I can send you in the right direction. Here are some sample topics:

1. **(Polynomial algebra)** The Newton identities: The elementary symmetric polynomials and the power sums are expressible in terms of each other.

2. **(Polynomial algebra)** Rational root theorem and the Gauss lemma.

3. **(Projective geometry)** Desargues' theorem, Pappus' hexagon theorem, or Pascal's theorem.

4. **(Projective geometry)** Spheres in visual perspective: When the image of a circle by a perspectivity is bounded, it is an ellipse.

5. **(Euclidean geometry)** The two-point equidistant cartographic projection: Prove that there is a projection of (some of) the points of the sphere onto a flat map such that the two distances to two chosen basepoints are all the same as on the sphere. Also give an interpretation in terms of practical navigation.

6. **(Euclidean geometry)** The classification of isometries of Euclidean 3-space: Every rigid motion is either a translation, rotation, reflection, glide reflection, glide rotation (screw), inversion, or a 'rotoinversion'.

7. **(Calculus)** Green's theorem. (Recommended: Learn many versions of this theorem.)

8. **(Complex analysis)** Marden's theorem, or Gauss-Lucas theorem: The complex roots of the derivative of a polynomial lie in the convex hull of the roots of the polynomial.

9. **(Combinatorics)** The generalized inclusion-exclusion principle.

10. **(Combinatorics)** Kastelyn's theorem: The number of perfect pairings subordinate to a graph is the Pfaffian of its oriented incidence matrix.

11. **(Topology)** There is a rotation of the 3-sphere that has no fixed points (Recommended: Learn many versions of the Hopf bundle construction)