A 資工碩一 R10922132 吳峻銘
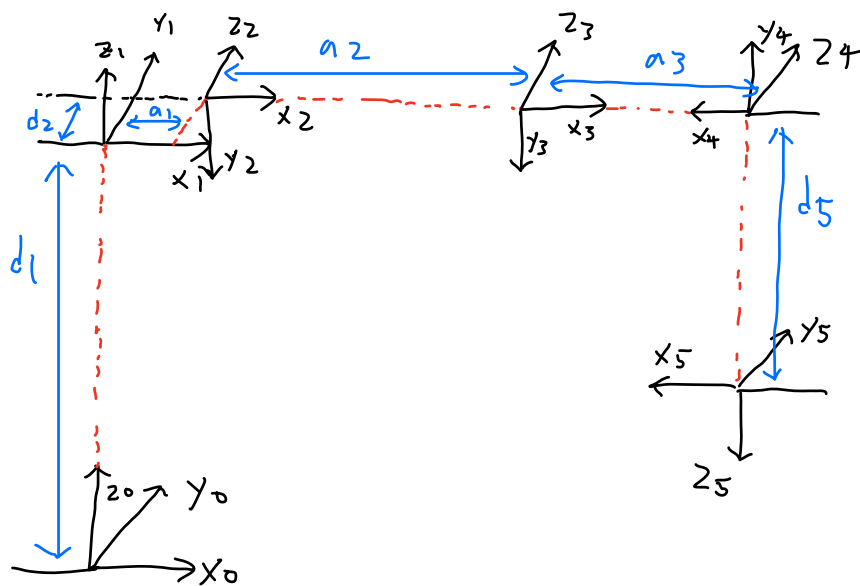
part A

(1)

(2)

joint	α_{i-1} (°)	a_{i-1} (mm)	d_i (mm)	θ_i
1	0	0	358.5	θ_1
2	−90	50	35.3	θ_2
3	0	300	0	θ_3
4	0	350	0	θ_4
5	90	0	251	θ_5

Part B

$$T_0^1 = \begin{bmatrix} C1 & -S1 & 0 & 0 \\ S1 & C1 & 0 & 0 \\ 0 & 0 & 1 & 358.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} C2 & -S2 & 0 & 50 \\ 0 & 0 & 1 & 35.3 \\ -S2 & -C2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} C3 & -S3 & 0 & 300 \\ S3 & C3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^4 = \begin{bmatrix} C4 & -S4 & 0 & 350 \\ S4 & C4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^5 = \begin{bmatrix} C5 & -S5 & 0 & 0 \\ 0 & 0 & -1 & -251 \\ S5 & C5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^5 = T_0^1 T_1^2 T_2^3 T_3^4 T_4^5$$

$$\begin{bmatrix} C1\,C_{234}\,C5 - S1S5 & -C1\,C_{234}S5 - S1C5 & C1S_{234} & C1(251S_{234} + 350C_{23} + 300C_2 + 50) \\ & & & -35.3\,S1 \\ S1\,C_{234}\,C5 + C1S5 & -S1\,C_{234}S5 + C1C5 & S1S_{234} & S1(251S_{234} + 350C_{23} + 300C_2 + 50) \\ & & & +35.3\,S1 \\ -S_{234}\,C5 & S_{234}\,S5 & C_{234} & 251C_{234} - 350S_{23} - 300S_2 + 358.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Part C

(1)

$$T_1^5 = [T_0^1]^{-1} T_0^5$$

$$= \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & -358.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\phi c\theta & c\phi s\theta s\psi - s\phi c\psi & c\phi s\theta c\psi + s\phi s\psi & x \\ s\phi c\theta & s\phi s\theta s\psi + c\phi c\psi & s\phi s\theta c\psi - c\phi s\psi & y \\ -s\theta & c\theta s\psi & c\theta c\psi & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1 - \phi)c\theta & \cos(\theta_1 - \phi)s\theta s\psi + \sin(\theta_1 - \phi)c\psi & \cos(\theta_1 - \phi)s\theta c\psi - s(\theta_1 - \phi)s\psi & c_1 x + s_1 y \\ -\sin(\theta_1 - \phi)c\theta & -\sin(\theta_1 - \phi)s\theta s\psi + \cos(\theta_1 - \phi)c\psi & -\sin(\theta_1 - \phi)s\theta c\psi - c(\theta_1 - \phi)s\psi & -s_1 x + c_1 y \\ -s\theta & c\theta s\psi & c\theta c\psi & z - 358.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$-x s_1 + y c_1 = 35.3 \quad \cdots \quad (1)$$

let $x = \rho \cdot c\alpha$ $\qquad y = \rho \cdot s\alpha$ $\qquad \rho = \sqrt{\rho x^2 + \rho y^2}$ $\qquad \alpha = \text{atan2}(y, x)$

$(1) \Rightarrow c_1 s\alpha - s_1 c\alpha = 35.3/\rho$ $\quad \sin(\alpha - \theta_1) = 35.3/\rho$ $\quad \cos(\alpha - \theta_1) = \pm\sqrt{1 - \left(\frac{35.3}{\rho}\right)^2}$

$$\theta_1 = \text{atan2}(y, x) - \text{atan2}\left(35.3, \pm\sqrt{x^2 + y^2 - 35.3^2}\right)$$

$$S5 = -\sin(\theta_1 - \phi) \, c\theta \quad \cdots (2)$$

$$L5 = -\sin(\theta_1 - \phi) \cdot S\theta \, S\psi + \cos(\theta_1 - \phi) C\psi \quad \cdots (3)$$

$(2), (3) \Rightarrow \theta_5 = a\tan2 \, (-\sin(\theta_1 - \phi) \, c\theta, \; -\sin(\theta_1 - \phi) \, S\theta \, S\psi + \cos(\theta_1 - \phi) \cdot C\psi)$

$$c_{234} \, L5 = \cos(\theta_1 - \phi) \, c\theta \quad \cdots (4)$$

$$-S_{234} \, L5 = -S_1 \quad \cdots \cdots (5)$$

$(4), (5) \Rightarrow \theta_2 + \theta_3 + \theta_4 = a\tan2 \, (S_1, \cos(\theta_1 - \phi) \cdot c\theta)$

$$350 \, c_{23} + 300 \, c_2 = L_1 X + S_1 Y - 50 - 251 \, S_{234} = A \quad \cdots (6)$$

$$350 \, c_{23} + 300 \, S_2 = 358.5 - Z + 251 \, c_{234} = B \quad \cdots (7)$$

$(6)^2 + (7)^2 \Rightarrow 350^2 + 306^2 + 2 \times 300 \times 350 \times c_3 = A^2 + B^2$

$\Rightarrow \theta_3 = a\tan2 \, (\pm \sqrt{1 - \left(\frac{A^2 + B^2 - 350^2 - 300^2}{210000}\right)^2}, \; \frac{A^2 + B^2 - 350^2 - 300^2}{210000})$

$$350 \, (c_2 c_3 - S_2 S_3) + 300 \, c_2 = A$$

$\Rightarrow c_2 (350 \, c_3 + 300) - S_2 (350 \, S_3) = A \quad \cdots (8)$

$$350 \, (S_2 c_3 + c_2 S_3) + 300 \, S_2 = B$$

$\Rightarrow S_2 (350 \, c_3 + 300) + c_2 (350 \, S_3) = B \quad \cdots (9)$

$(8) \cdot c_2 + (9) \cdot S_2 \Rightarrow 350 \, c_3 + 300 = A c_2 + B S_2$

$\Rightarrow \theta_2 = -a\tan2 \, (A, B) + a\tan2 \, (350 \, c_3 + 300, \; \pm \sqrt{A^2 + B^2 - 1350 (3 + 300)^2})$

$$\theta_4 = (\theta_2 + \theta_3 + \theta_4) - \theta_2 - \theta_3$$

(2)

(a) $(x, y, z, \phi, \theta, \psi) = (600, 100, 0, \frac{\pi}{4}, 0, \pi)$

$\theta_1 = 6.1354$

$\theta_2 = -20.8224$

$\theta_3 = 58.5446$

$\theta_4 = 142.2nnn$

$\theta_5 = 141.1354$

(b) $(x, y, z, \phi, \theta, \psi) = (600, 100, 100, \frac{\pi}{4}, 0, \pi)$

$\theta_1 = 6.1354$

$\theta_2 = -32.96118$

$\theta_3 = 62.1556$

$\theta_4 = 150.8055$

$\theta_5 = 141.1354$

(c) $(x, y, z, \phi, \theta, \psi) = (600, -100, 100, \frac{-\pi}{4}, 0, \pi)$
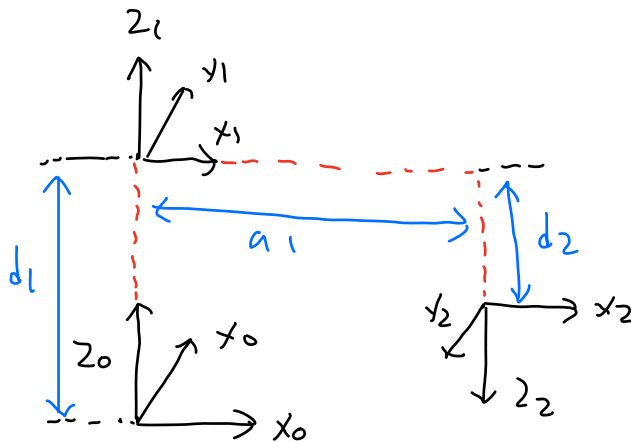
$\theta_1 = -12.n89$ $\theta_4 = 150.805$

$\theta_2 = -32.961$ $\theta_5 = -14n.n89$

$\theta_3 = 62.156$

part D

(1)

joint	$\alpha_{i-1}(^\circ)$	$a_{i-1}(mm)$	$d_i(mm)$	$\theta_i(^\circ)$
1	0	0	d_1	θ_1
2	π	a_1	d_2	0

(2)

θ_1 and d_2