Homework 2

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Problem 1

1

.Derive the posterior distribution of precision matrix

Proof.

$$\begin{aligned}
P(\Lambda|X) &\propto P(X|\Lambda)P(\Lambda) && (1)\\
&\propto (\frac{|\Lambda|}{2\pi})^{n/2} exp(\sum_{i=1}^{n}(X_i-\mu)^T\Lambda(X_i-\mu))|\Lambda|^{\frac{n-p-1}{2}} exp(-trace(V^{-1}\Lambda)) && (2)\\
&\propto |\Lambda|^{\frac{2n-p-1}{2}} exp(\sum_{i=1}^{n}(X_i-\mu)^T\Lambda(X_i-\mu)-trace(V^{-1}\Lambda)) && (3)\\
&\propto |\Lambda|^{\frac{2n-p-1}{2}} exp(trace(\sum_{i=1}^{n}(X_i-\mu)(X_i-\mu)^T-V^{-1})\Lambda) && (4)
\end{aligned}$$

For the convenience to derive the MAP , we take log to $P(\Lambda|X)$ then calculate the MAP.

$$\begin{aligned}
\nabla lnP(\Lambda|X) &\propto \nabla\frac{2n-p-1}{2}ln|\Lambda| + trace((\sum_{i=1}^{n}(X_i-\mu)(X_i-\mu)^T-V^{-1})\Lambda) && (5)\\
&= \frac{2n-p-1}{2}\Lambda^{-1}+\sum_{i=1}^{n}(X_i-\mu)(X_i-\mu)^T-V^{-1} && (6)\\
&= 0 && (7)\\
& && (8)
\end{aligned}$$

Solve equation 8 ,then we can get MAP.

$$\Lambda_{MAP} = \frac{2}{2n-p-1}(V^{-1}-\sum_{i=1}^{n}(X_i-\mu)(X_i-\mu)^T)^{-1} \quad (9)$$

□

2

. For N = 10, 100 and 500 calculate Λ_{MAP}

$$\Lambda_{MAP10} = \begin{bmatrix} 0.045 & -0.0197 \\ -0.0197 & 0.0135 \end{bmatrix}$$

$$\Lambda_{MAP100} = \begin{bmatrix} 0.0006 & -0.00027 \\ -0.00027 & 0.0002 \end{bmatrix}$$

$$\Lambda_{MAP500} = \begin{bmatrix} 2.86*10^{-5} & -1.28*10^{-5} \\ -1.28*10^{-5} & 1.029*10^{-5} \end{bmatrix}$$

Problem 2

1

Compute the mean vector m_N and the covariance matrix S_N for the posterior distribution .

$$m_{N_{10}} = \begin{bmatrix} 0.65 \\ 6.56 \\ 4.47 \\ -5.19 \\ 0.30 \\ -2.70 \\ -12.93 \end{bmatrix} \quad S_{N_{10}} = \begin{bmatrix} 11.75 & -45.13 & 57.52 & -53.28 & 47.82 & -23.10 & 4.99 \\ -45.13 & 186.12 & -243.60 & 226.73 & -203.61 & 98.35 & -21.24 \\ 57.52 & -243.60 & 327.17 & -313.87 & 283.55 & -137.05 & 29.60 \\ -53.28 & 226.73 & -313.87 & 318.58 & -293.07 & 142.22 & -30.76 \\ 47.82 & -203.61 & 283.55 & -293.07 & 276.29 & -137.93 & 30.41 \\ -23.10 & 98.35 & -137.05 & 142.22 & -137.93 & 74.77 & -19.94 \\ 4.99 & -21.24 & 29.60 & -30.76 & 30.41 & -19.94 & 9.08 \end{bmatrix}$$

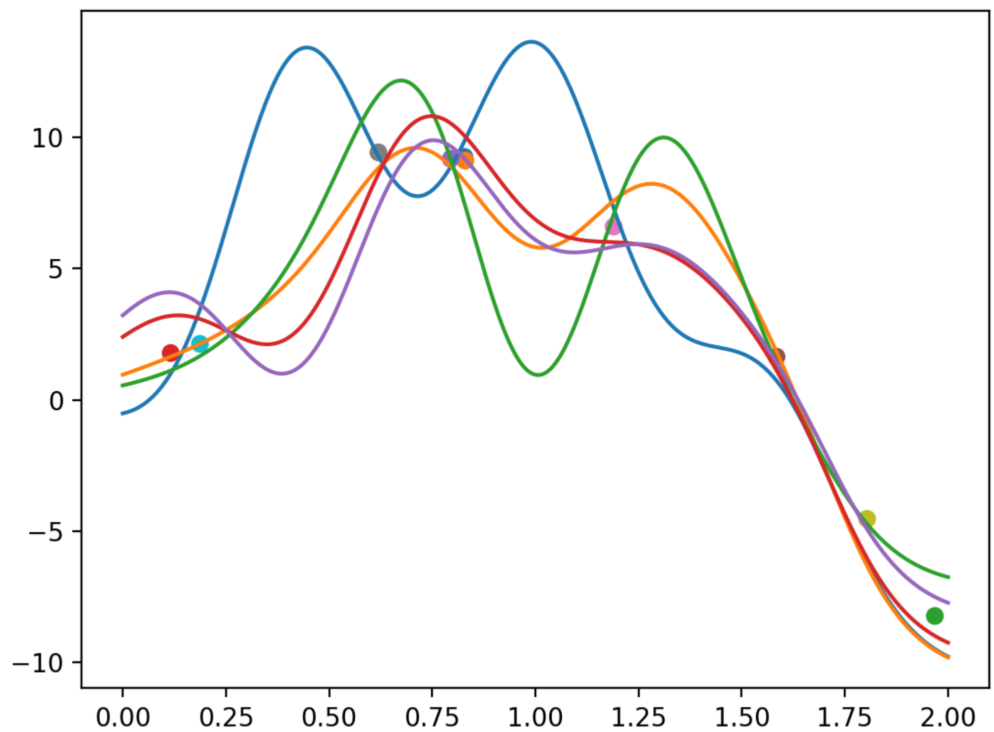
$$m_{N_{15}} = \begin{bmatrix} -1.30 \\ 14.94 \\ -7.79 \\ 8.32 \\ -12.03 \\ 2.84 \\ -13.79 \end{bmatrix} \quad S_{N_{15}} = \begin{bmatrix} 3.91 & -11.81 & 11.80 & -6.62 & 3.75 & -1.46 & 0.53 \\ -11.81 & 44.43 & -49.04 & 27.92 & -15.85 & 6.17 & -2.24 \\ 11.80 & -49.04 & 57.72 & -35.46 & 20.71 & -8.11 & 2.95 \\ -6.62 & 27.92 & -35.46 & 26.85 & -17.85 & 7.41 & -2.78 \\ 3.75 & -15.85 & 20.71 & -17.85 & 14.85 & -8.77 & 3.96 \\ -1.46 & 6.17 & -8.11 & 7.41 & -8.77 & 10.06 & -6.80 \\ 0.53 & -2.24 & 2.95 & -2.78 & 3.96 & -6.80 & 6.25 \end{bmatrix}$$

$$m_{N_{30}} = \begin{bmatrix} -1.91 \\ 17.13 \\ -9.56 \\ 8.02 \\ -10.11 \\ -1.71 \\ -9.89 \end{bmatrix} \quad S_{N_{30}} = \begin{bmatrix} 1.37 & -3.60 & 3.41 & -2.04 & 1.17 & -0.39 & 0.10 \\ -3.60 & 11.94 & -12.90 & 7.90 & -4.55 & 1.54 & -0.40 \\ 3.41 & -12.90 & 15.58 & -10.91 & 6.59 & -2.26 & 0.59 \\ -2.04 & 7.90 & -10.91 & 10.32 & -7.44 & 2.80 & -0.77 \\ 1.17 & -4.55 & 6.59 & -7.44 & 7.49 & -4.47 & 1.51 \\ -0.39 & 1.54 & -2.26 & 2.80 & -4.47 & 4.73 & -2.54 \\ 0.10 & -0.40 & 0.59 & -0.77 & 1.51 & -2.54 & 2.27 \end{bmatrix}$$

$$m_{N_{80}} = \begin{bmatrix} 0.26 \\ 9.39 \\ 0.08 \\ 0.42 \\ -4.64 \\ -4.21 \\ -9.33 \end{bmatrix} \quad S_{N_{80}} = \begin{bmatrix} 0.42 & -0.78 & 0.52 & -0.27 & 0.14 & -0.07 & 0.02 \\ -0.78 & 1.87 & -1.70 & 0.94 & -0.51 & 0.24 & -0.08 \\ 0.52 & -1.70 & 2.19 & -1.71 & 1.05 & -0.51 & 0.18 \\ -0.27 & 0.94 & -1.71 & 2.18 & -1.82 & 0.97 & -0.35 \\ 0.14 & -0.51 & 1.05 & -1.82 & 2.19 & -1.59 & 0.64 \\ -0.07 & 0.24 & -0.51 & 0.97 & -1.59 & 1.76 & -0.98 \\ 0.02 & -0.08 & 0.18 & -0.35 & 0.64 & -0.98 & 0.80 \end{bmatrix}$$

2

.Generate five curve samples from the parameter posterior distribution.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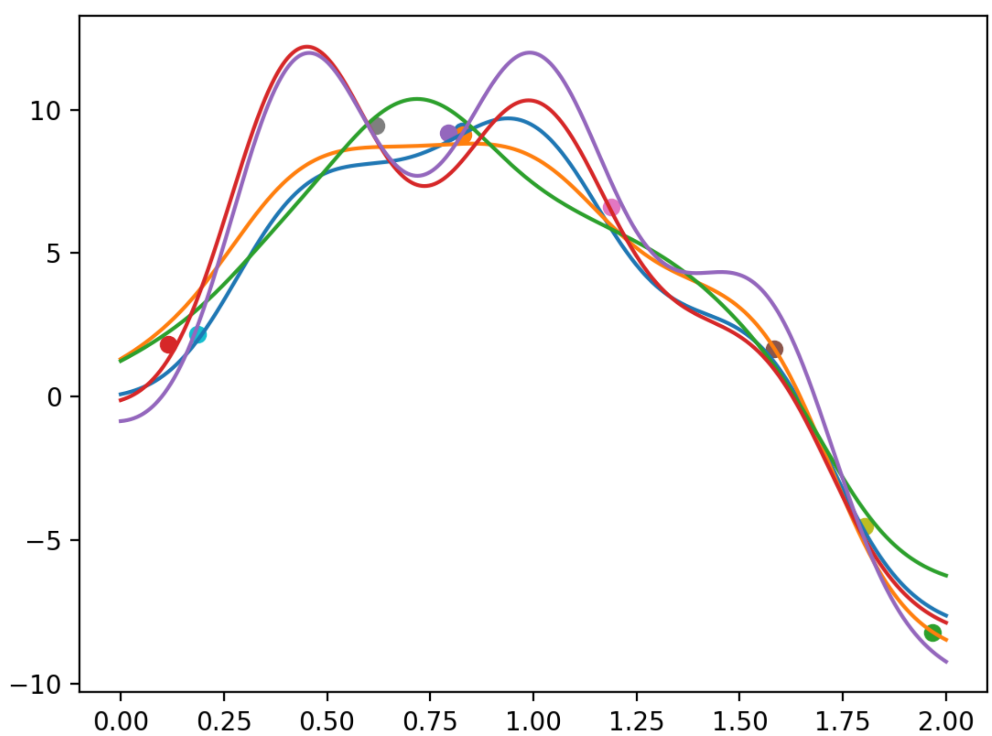
1.50
1.75
2.00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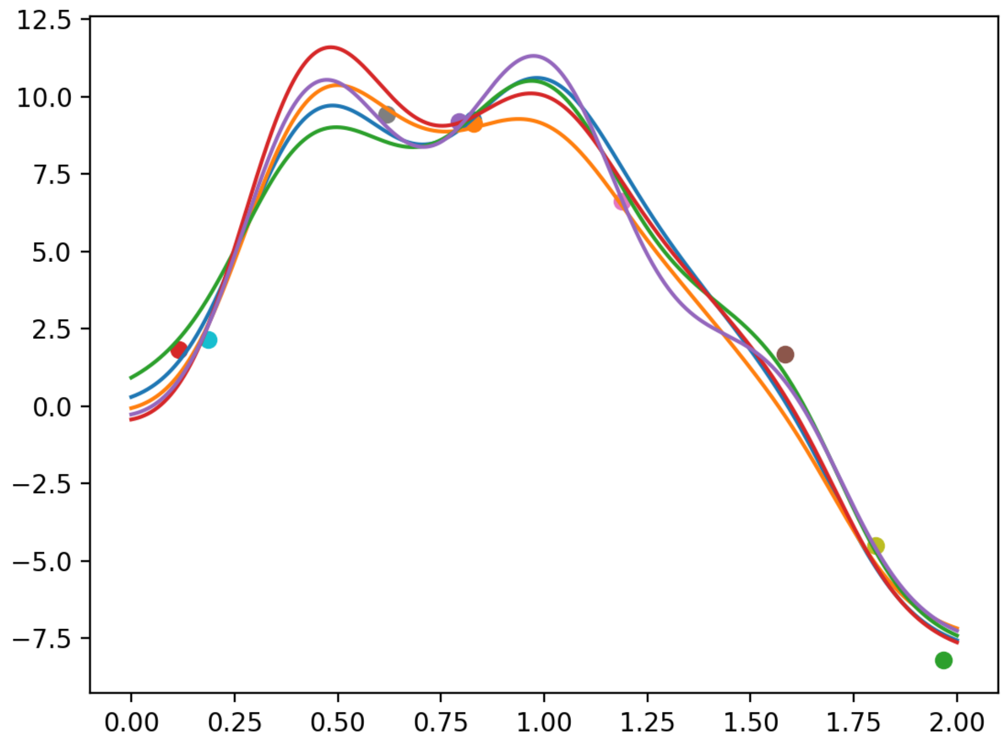
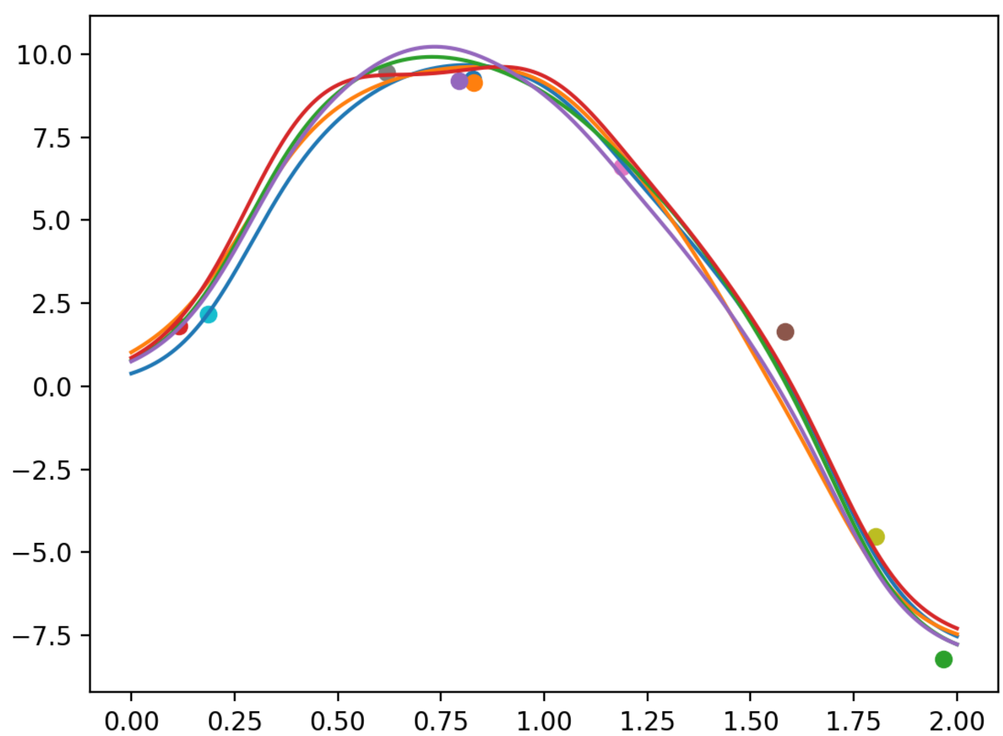
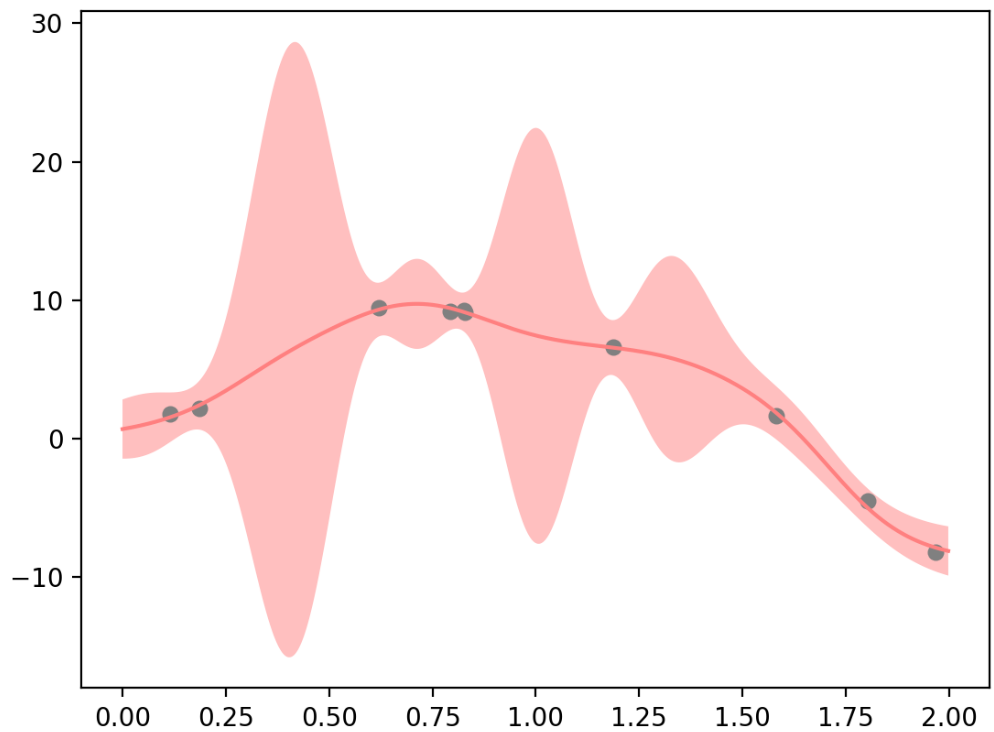




As the sample size increase , the sampling result of the posterior become more stable.

3

.Plot the predictive distribution of target value t and show the mean curve and the region of variance with one standard deviation on either side of the mean curve.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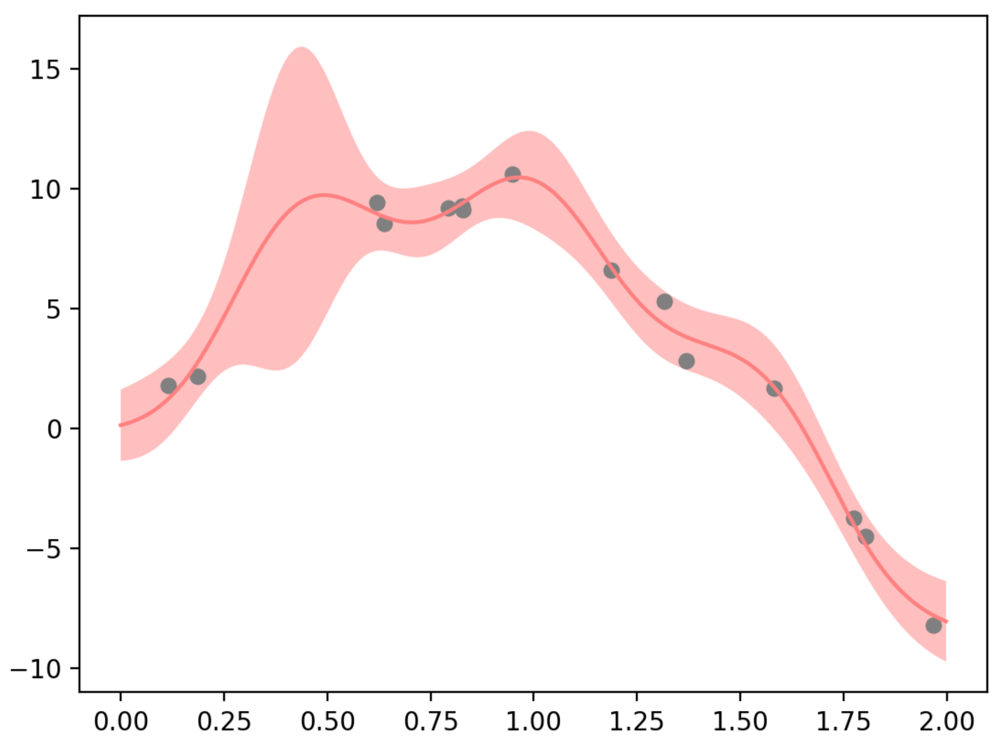
1.75
2.00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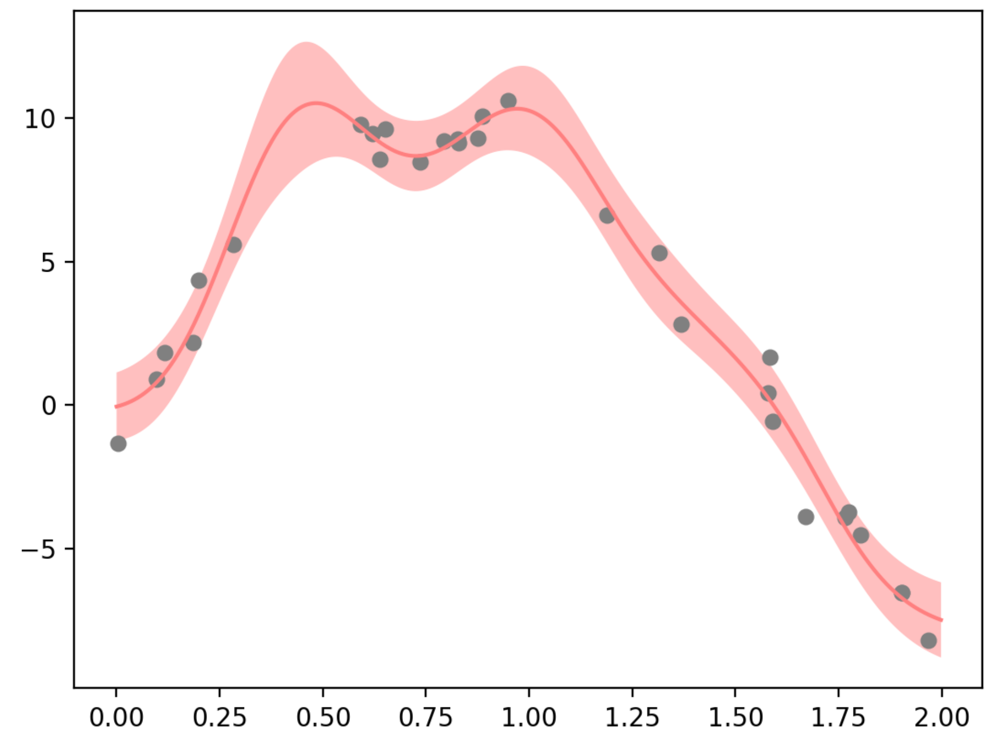
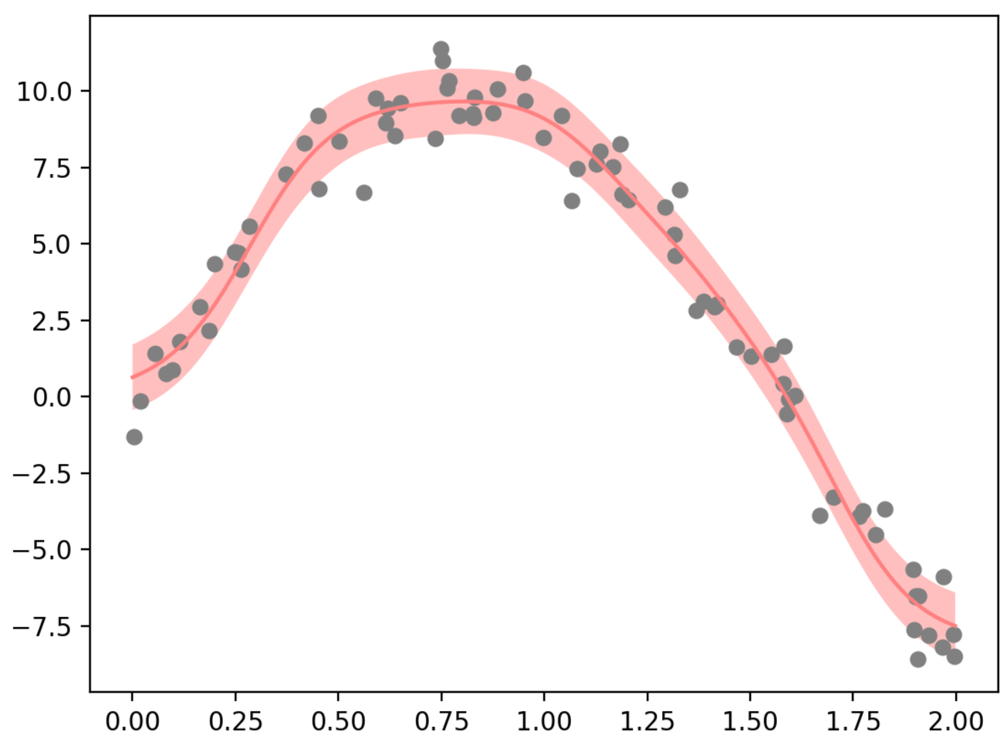
2.00





As the sample size increase , the prediction interval become smaller.

Problem 3

Implement the Newton-Raphson algorithm to construct a multiclass logistic regression model with the softmax transformation.

1

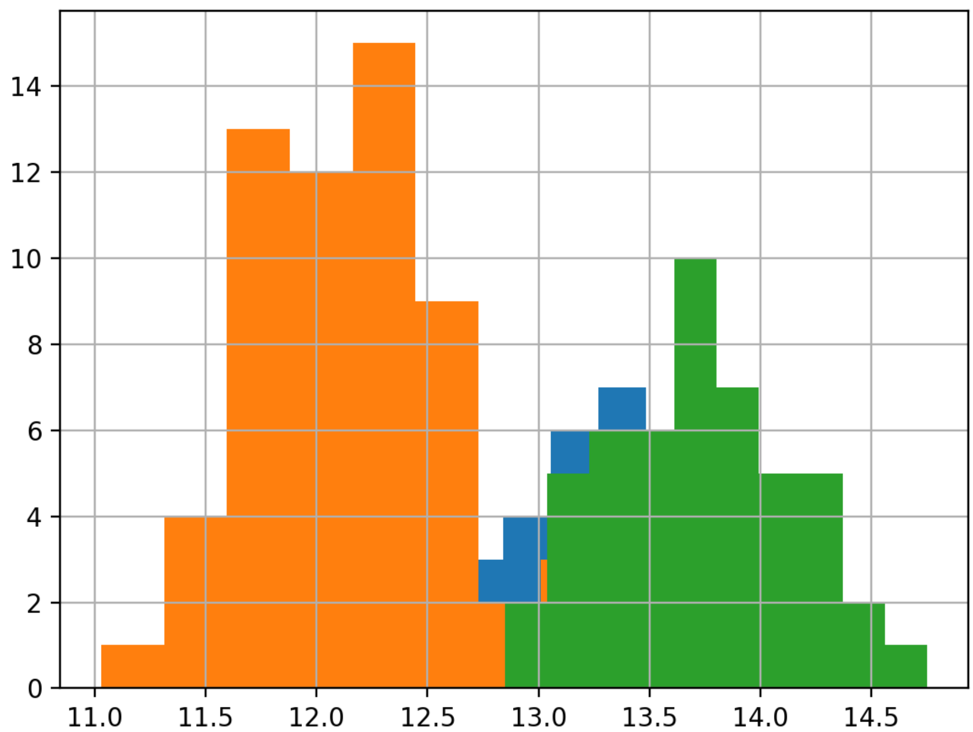
2

Show the classification result of test data.

$$Y = \begin{bmatrix} 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 1.00 \\ 0.00 & 0.05 & 0.95 \\ 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 1.00 \\ 0.65 & 0.00 & 0.35 \\ 0.00 & 0.12 & 0.88 \\ 1.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 0.94 & 0.06 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 1.00 \\ 1.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 \end{bmatrix}$$

3

Plot the distribution (or histogram) of each variable in training data and map different colors to each class.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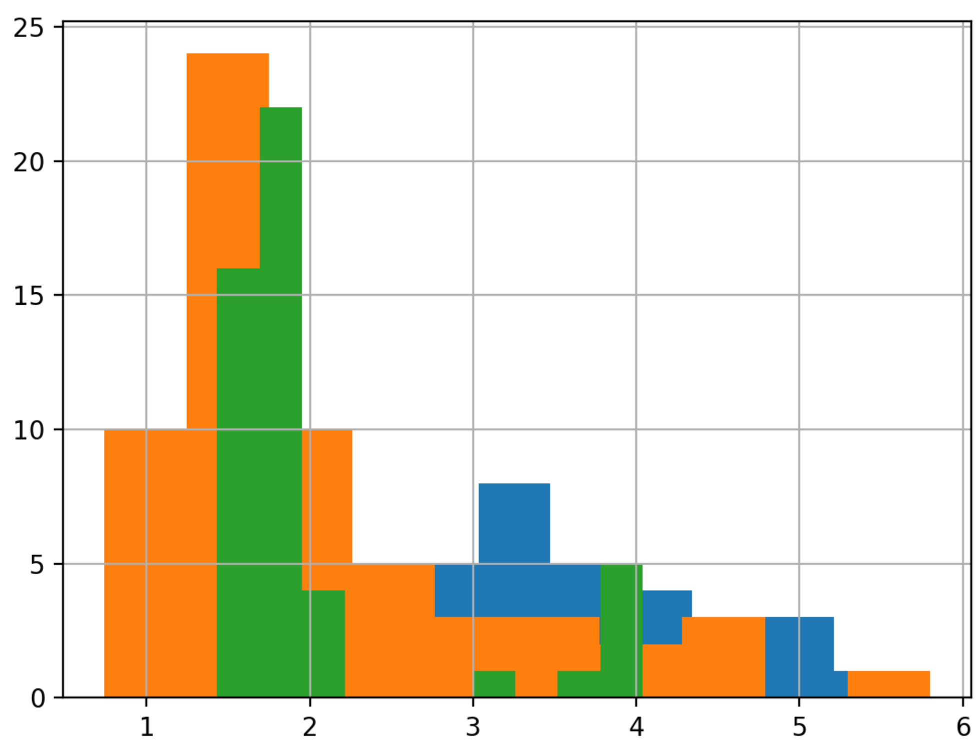
13.0
13.5
14.0
14.5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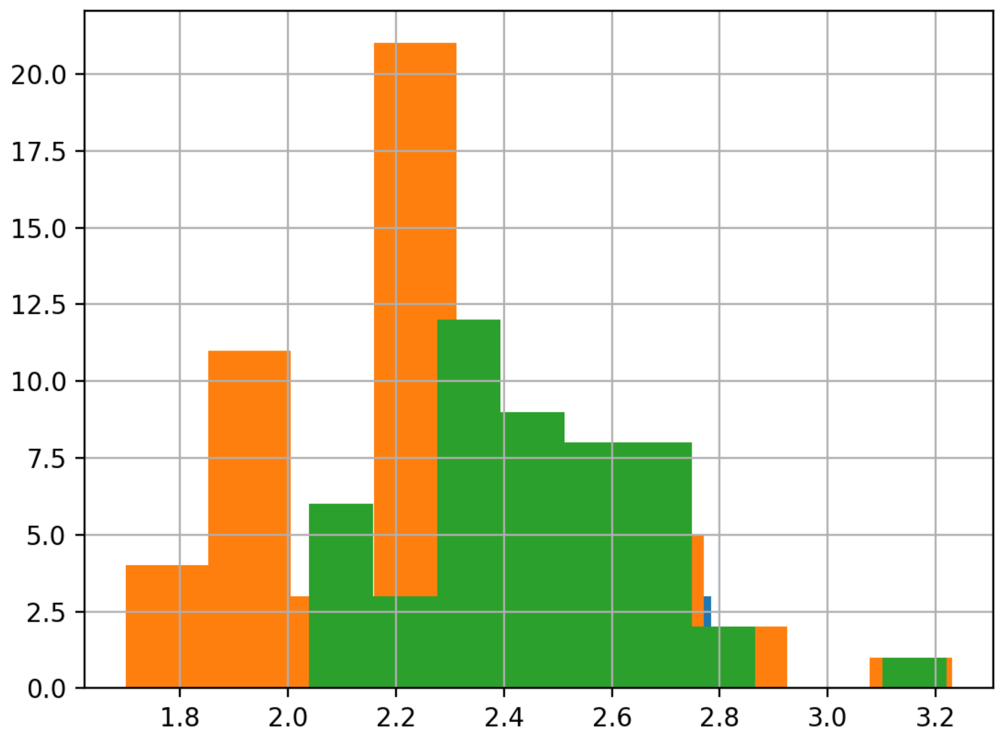
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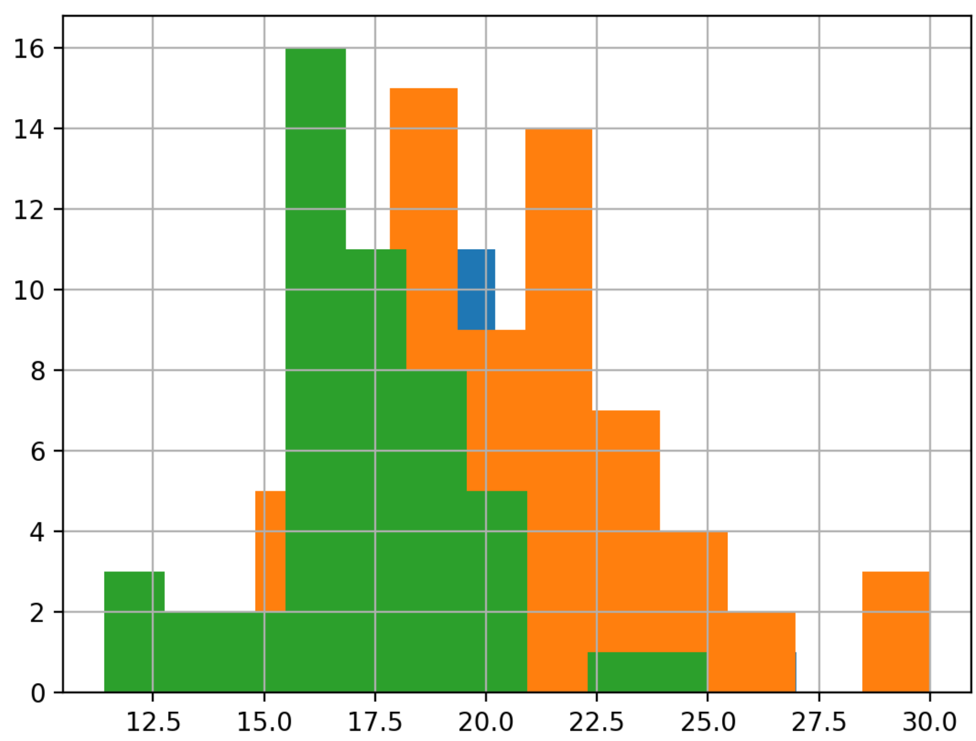
2.4
2.6
2.8
3.0
3.2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4
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0
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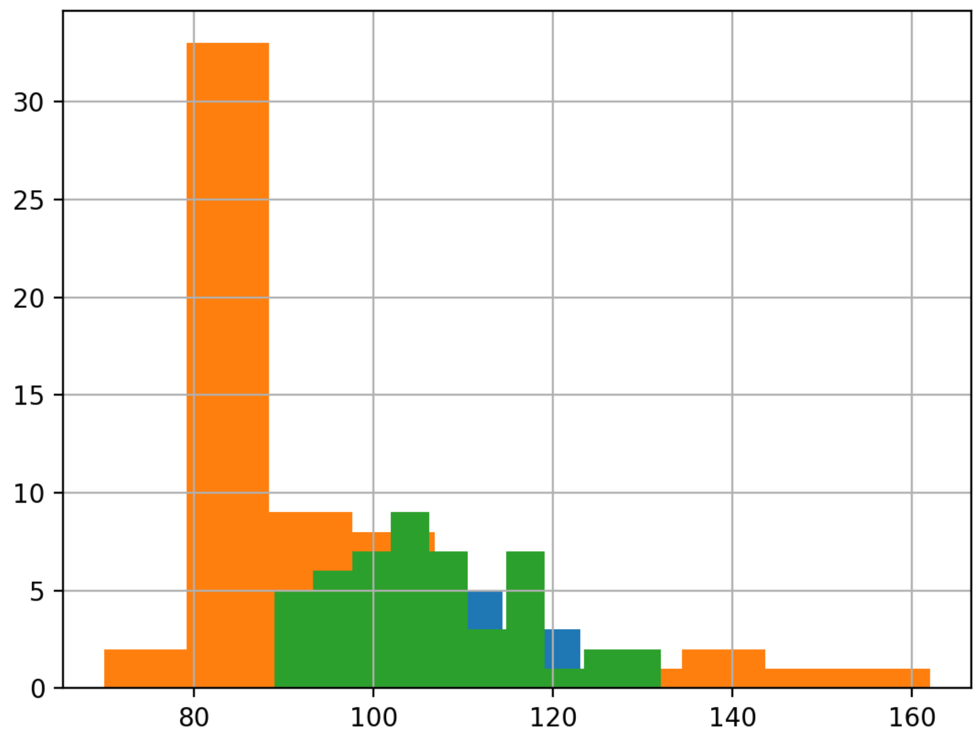
17.5
20.0
22.5
25.0
27.5
30.0



0
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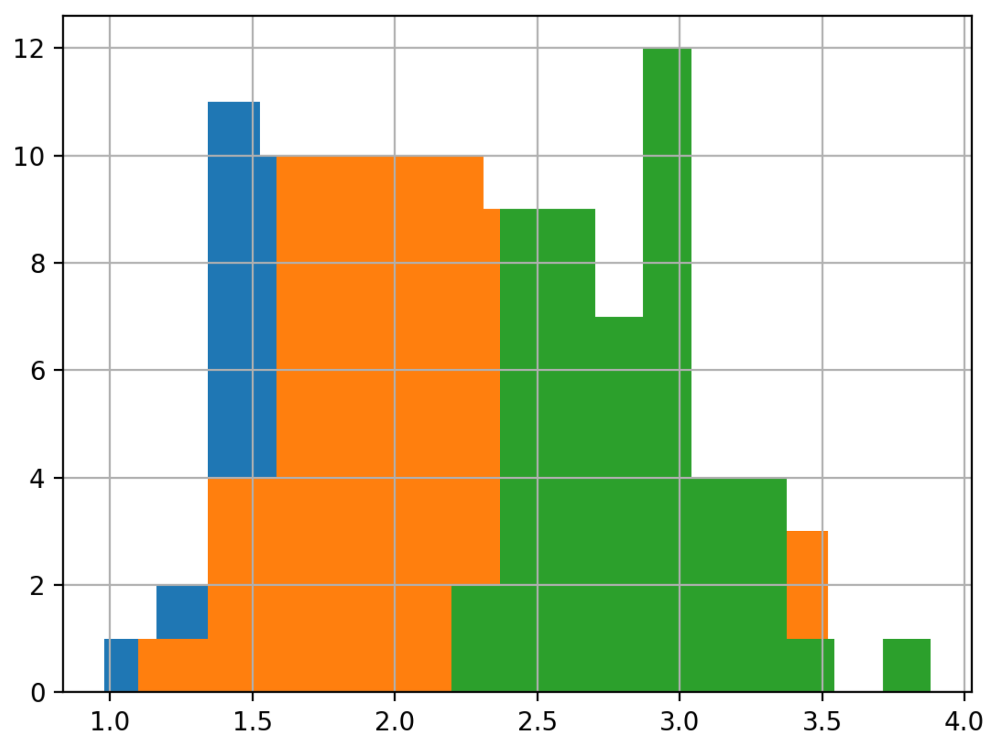
10
15
20
25
30
80
100
120
140
160



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4
6
8
10
12
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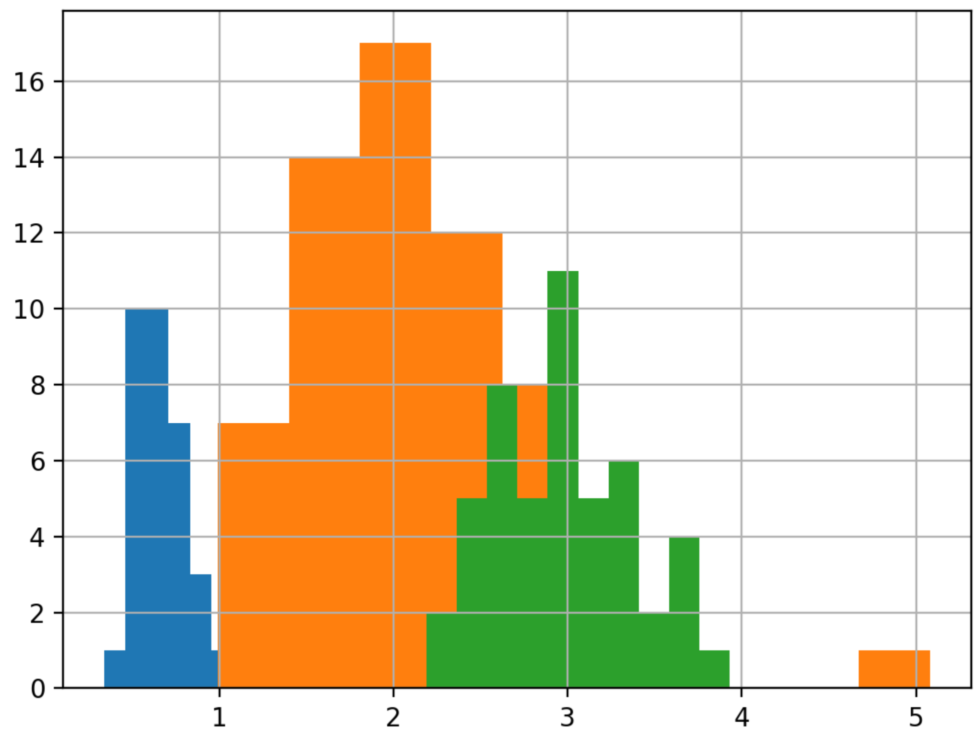
1.5
2.0
2.5
3.0
3.5
4.0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1
2
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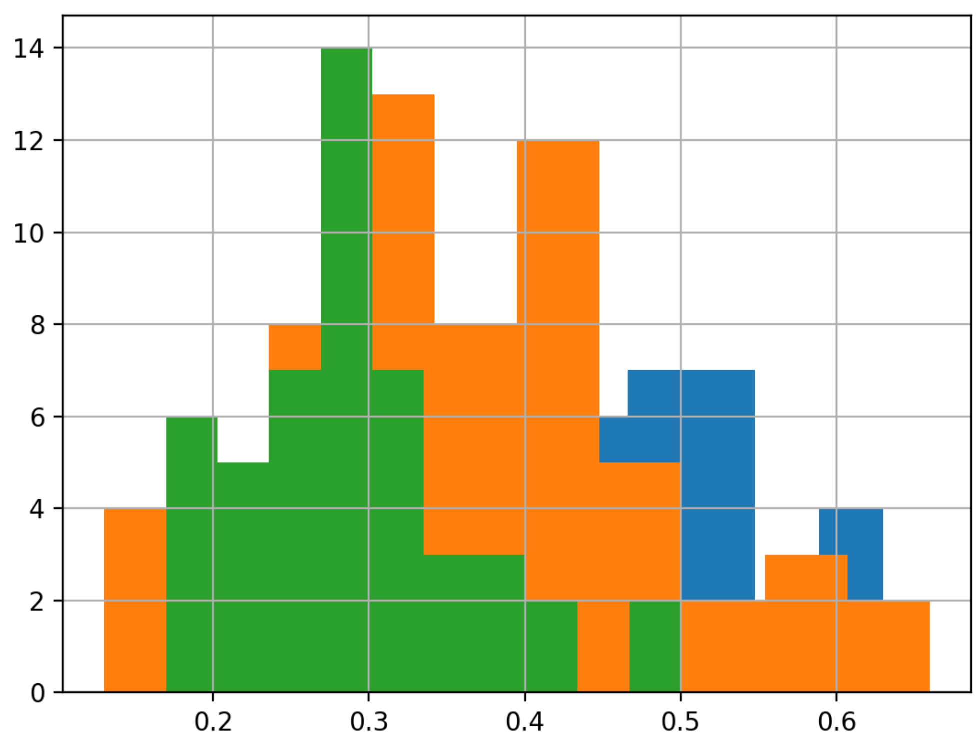
4
5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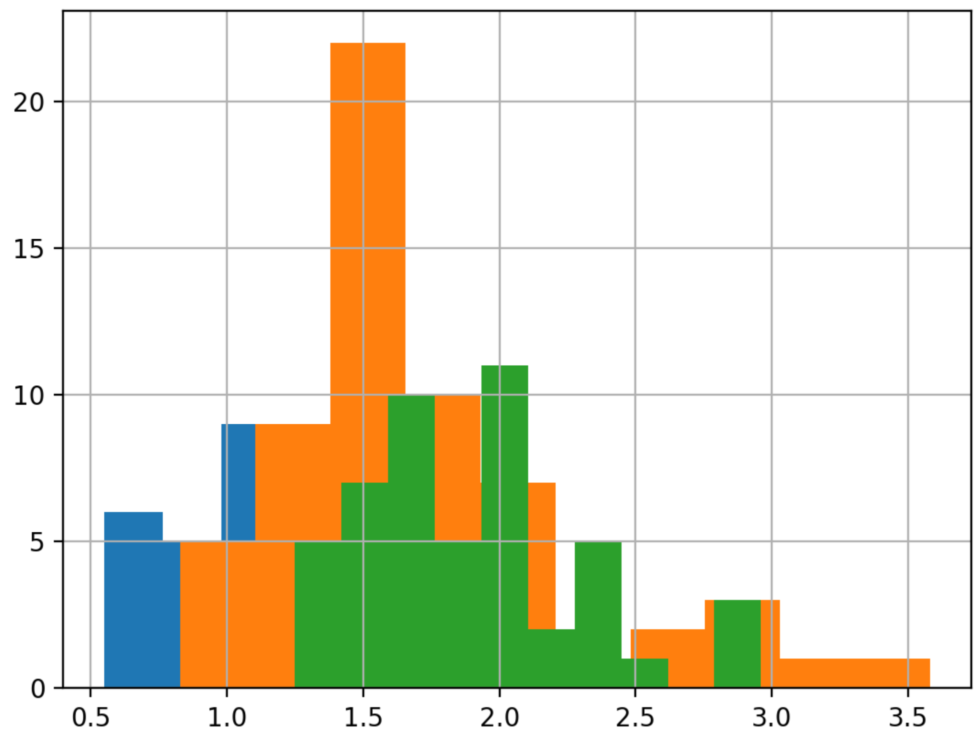
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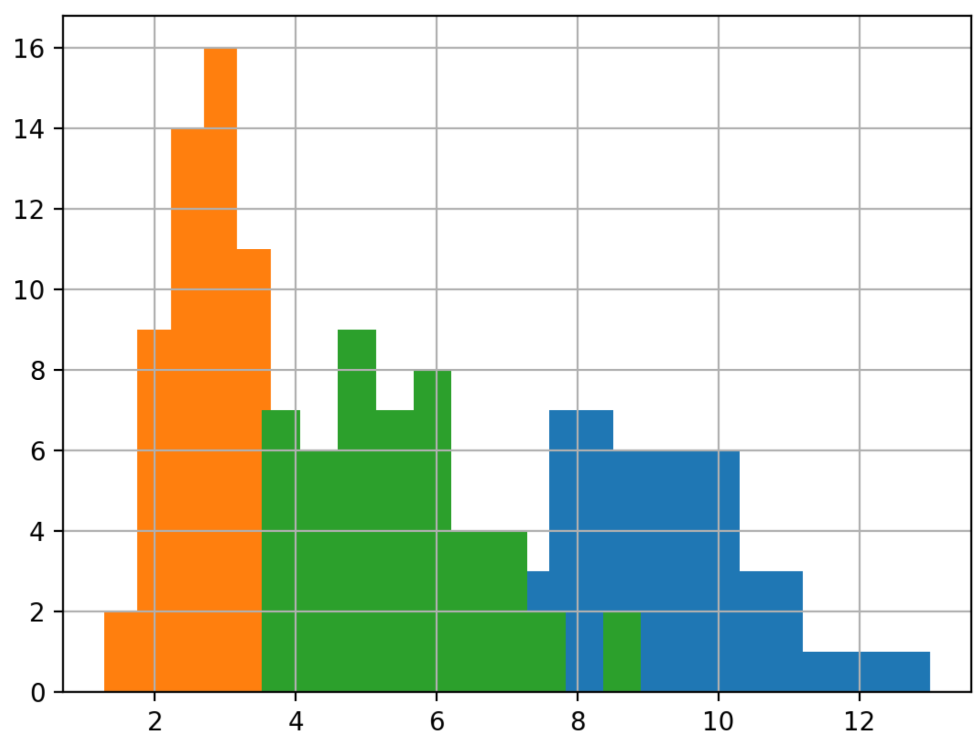
1.5
2.0
2.5
3.0
3.5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10
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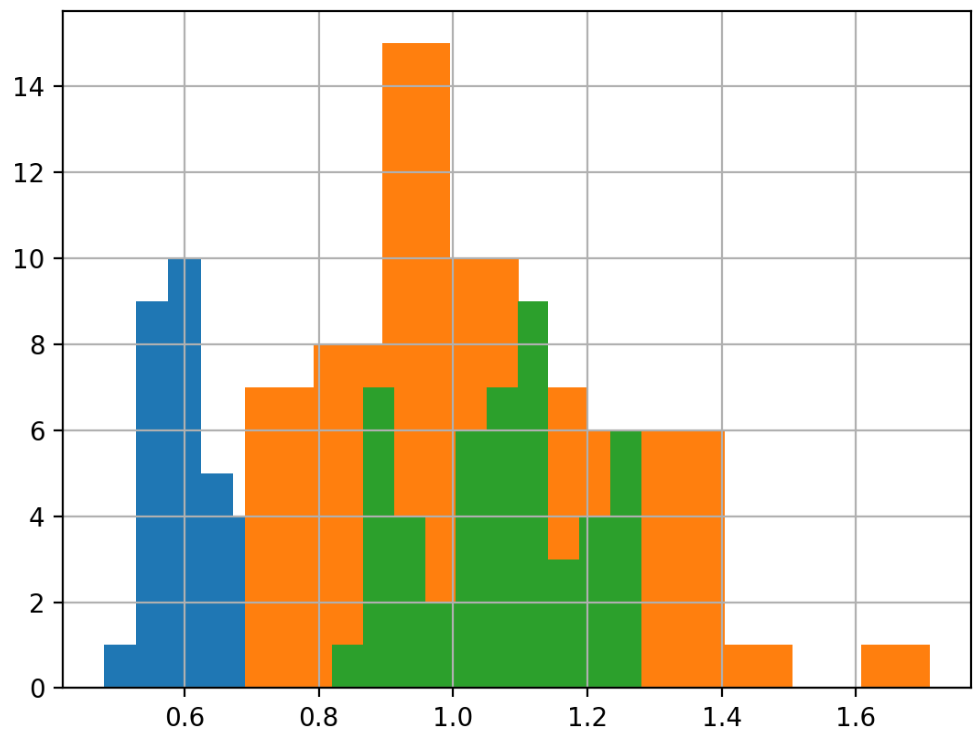
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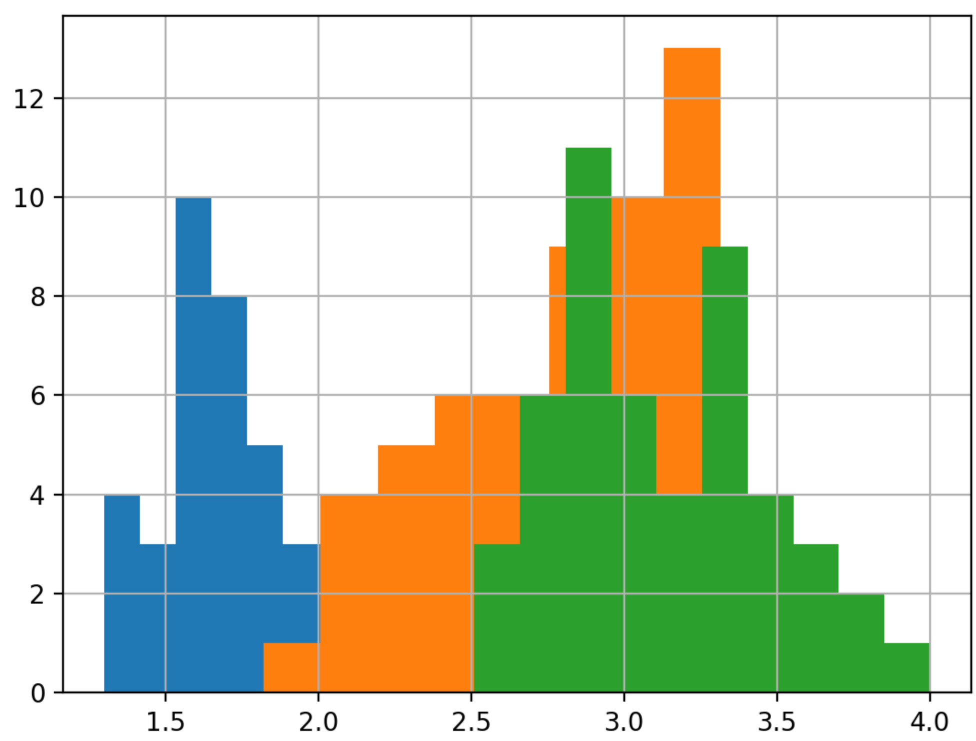
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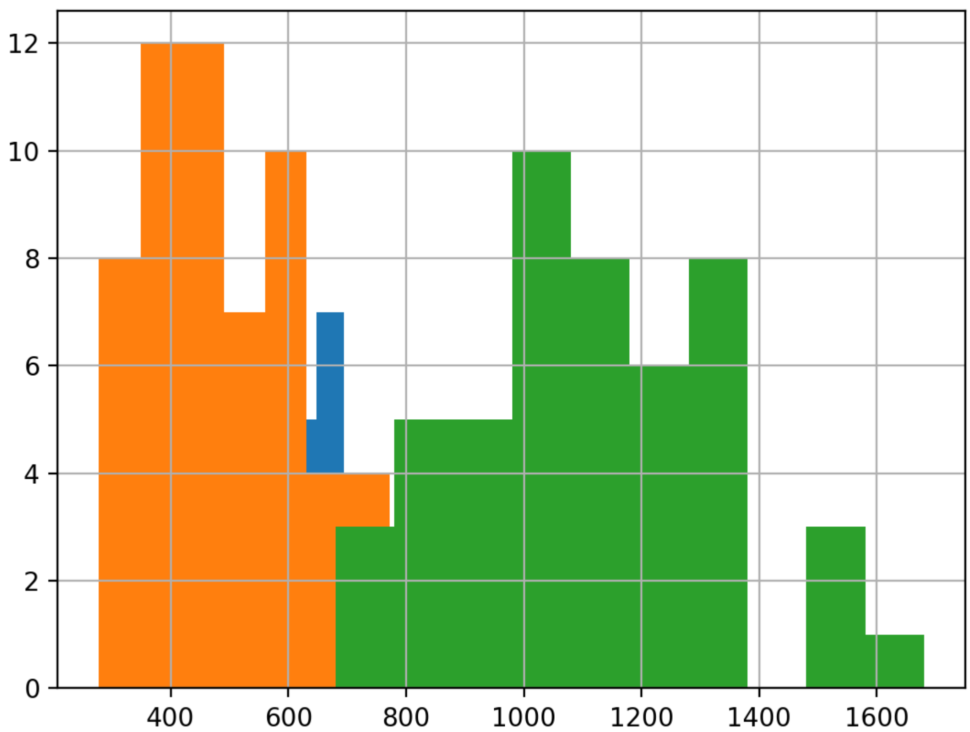
4.0



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