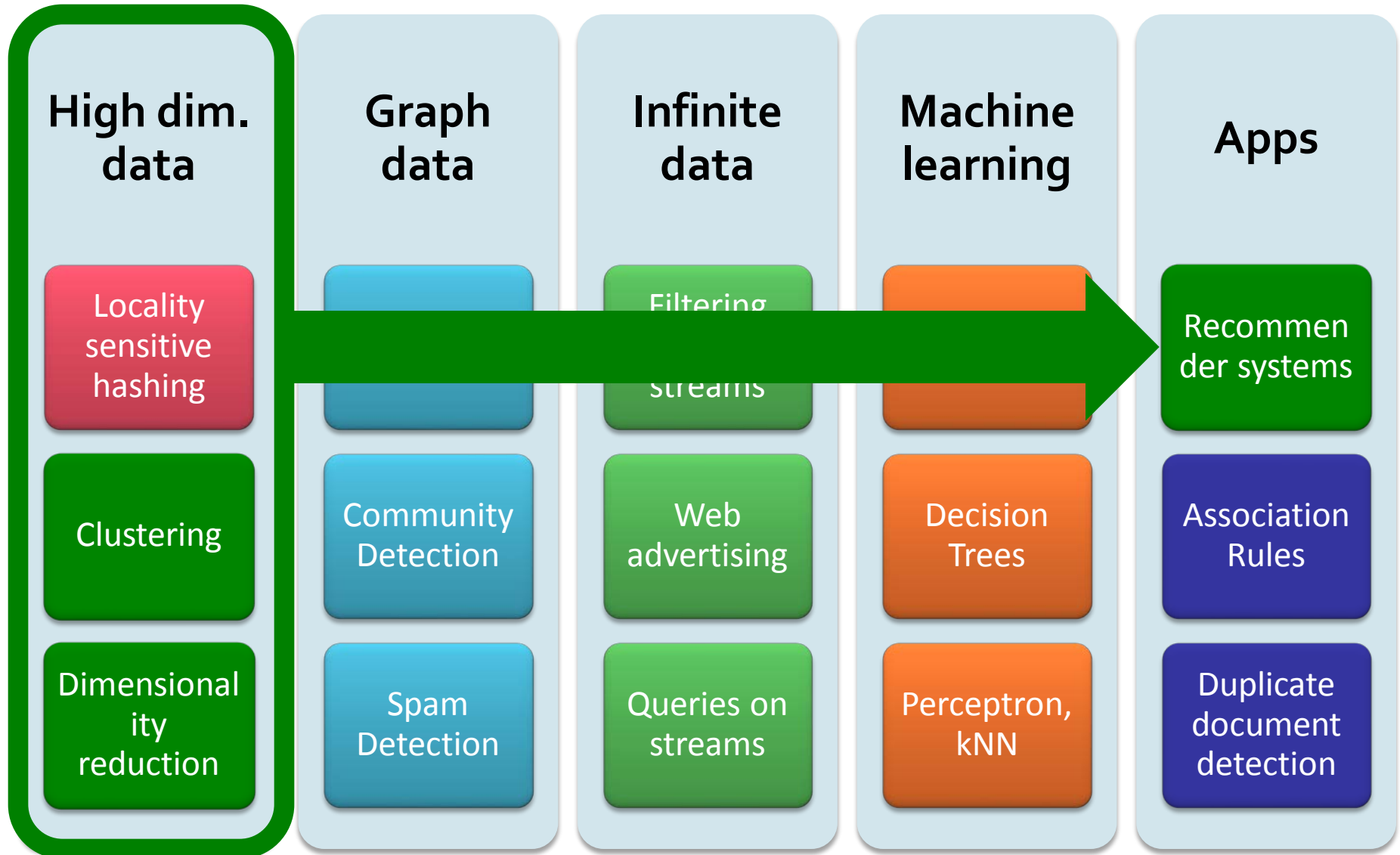


# Recommender Systems: Content-based Systems & Collaborative Filtering

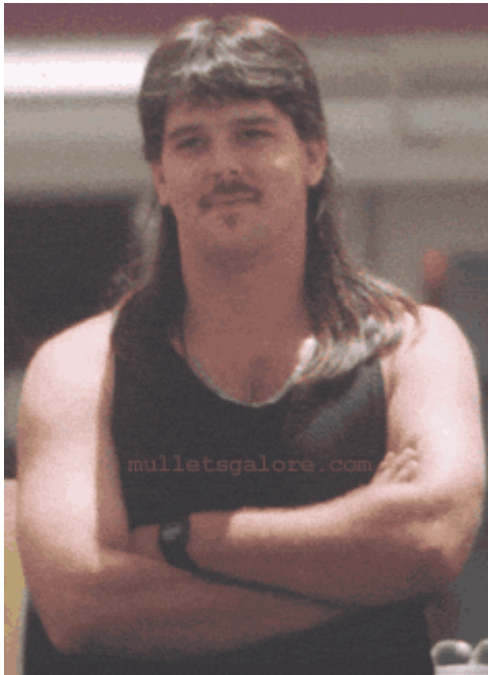
CS246: Mining Massive Datasets  
Jure Leskovec, Stanford University  
<http://cs246.stanford.edu>



# High Dimensional Data



# Example: Recommender Systems



## ■ Customer X

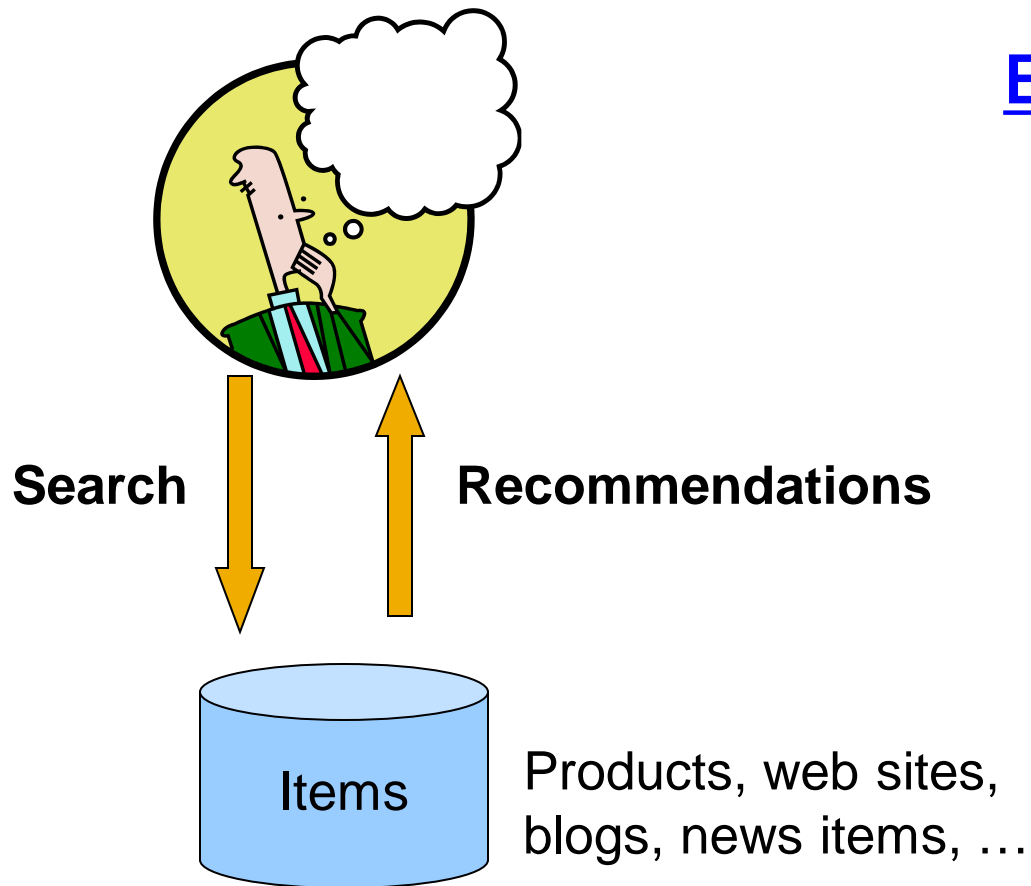
- Buys Metallica CD
- Buys Megadeth CD



## ■ Customer Y

- Does search on Metallica
- Recommender system suggests Megadeth from data collected about customer X

# Recommendations



## Examples:

amazon.com.



StumbleUpon



del.icio.us



movielens

helping you find the *right* movies

last.fm™  
the social music revolution

Google  
News

YouTube

XBOX  
LIVE

# From Scarcity to Abundance

- **Shelf space is a scarce commodity for traditional retailers**
  - Also: TV networks, movie theaters,...
- **Web enables near-zero-cost dissemination of information about products**
  - From scarcity to abundance
- **More choice necessitates better filters**
  - Recommendation engines
  - How **Into Thin Air** made **Touching the Void** a bestseller: <http://www.wired.com/wired/archive/12.10/tail.html>

# Sidenote: The Long Tail



Sources: Erik Brynjolfsson and Jeffrey Hu, MIT, and Michael Smith, Carnegie Mellon; Barnes & Noble; Netflix; RealNetworks  
Source: Chris Anderson (2004)

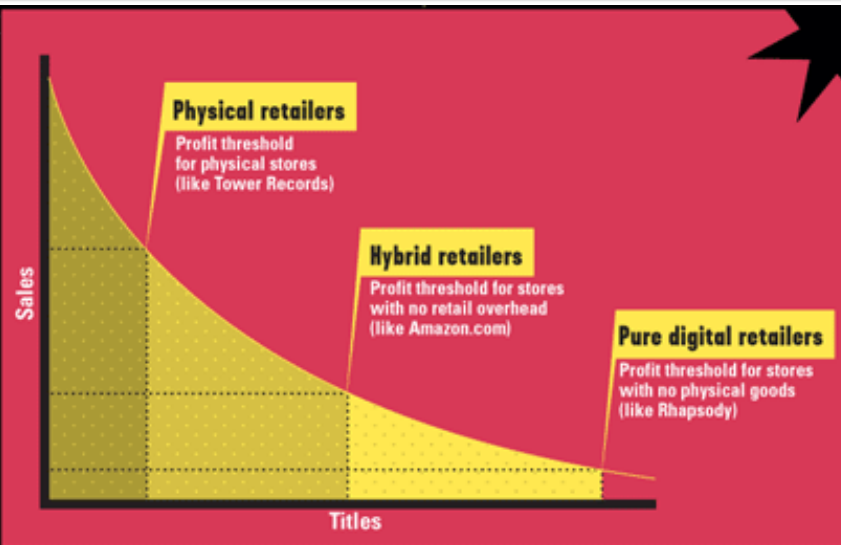


# Physical vs. Online

## THE BIT PLAYER ADVANTAGE

Beyond bricks and mortar there are two main retail models – one that gets halfway down the Long Tail and another that goes all the way. The first is the familiar hybrid model of Amazon and Netflix, companies that sell physical goods online. Digital catalogs allow them to offer unlimited selection along with search, reviews, and recommendations, while the cost savings of massive warehouses and no walk-in customers greatly expands the number of products they can sell profitably.

Pushing this even further are pure digital services, such as iTunes, which offer the additional savings of delivering their digital goods online at virtually no marginal cost. Since an extra database entry and a few megabytes of storage on a server cost effectively nothing, these retailers have no economic reason not to carry *everything* available.



Read <http://www.wired.com/wired/archive/12.10/tail.html> to learn more!

# Types of Recommendations

- **Editorial and hand curated**
  - List of favorites
  - Lists of “essential” items
- **Simple aggregates**
  - Top 10, Most Popular, Recent Uploads
- **Tailored to individual users**
  - Amazon, Netflix, ...



# Formal Model

- $X$  = set of **Customers**
- $S$  = set of **Items**
- **Utility function**  $u: X \times S \rightarrow R$ 
  - $R$  = set of ratings
  - $R$  is a totally ordered set
  - e.g., **0-5** stars, real number in **[0,1]**

# Utility Matrix

	Avatar	LOTR	Matrix	Pirates
Alice	1		0.2	
Bob		0.5		0.3
Carol	0.2		1	
David				0.4

# Key Problems

- **(1) Gathering “known” ratings for matrix**
  - How to collect the data in the utility matrix
- **(2) Extrapolate unknown ratings from the known ones**
  - Mainly interested in high unknown ratings
    - We are not interested in knowing what you don't like but what you like
- **(3) Evaluating extrapolation methods**
  - How to measure success/performance of recommendation methods

# (1) Gathering Ratings

## ■ Explicit

- Ask people to rate items
- Doesn't work well in practice – people can't be bothered

## ■ Implicit

- Learn ratings from user actions
  - E.g., purchase implies high rating
- What about low ratings?

## (2) Extrapolating Utilities

- **Key problem:** Utility matrix  $U$  is **sparse**
  - Most people have not rated most items
  - **Cold start:**
    - New items have no ratings
    - New users have no history
- **Three approaches to recommender systems:**
  - 1) Content-based
  - 2) Collaborative
  - 3) Latent factor based

} **Today!**

# Content-based Recommender Systems



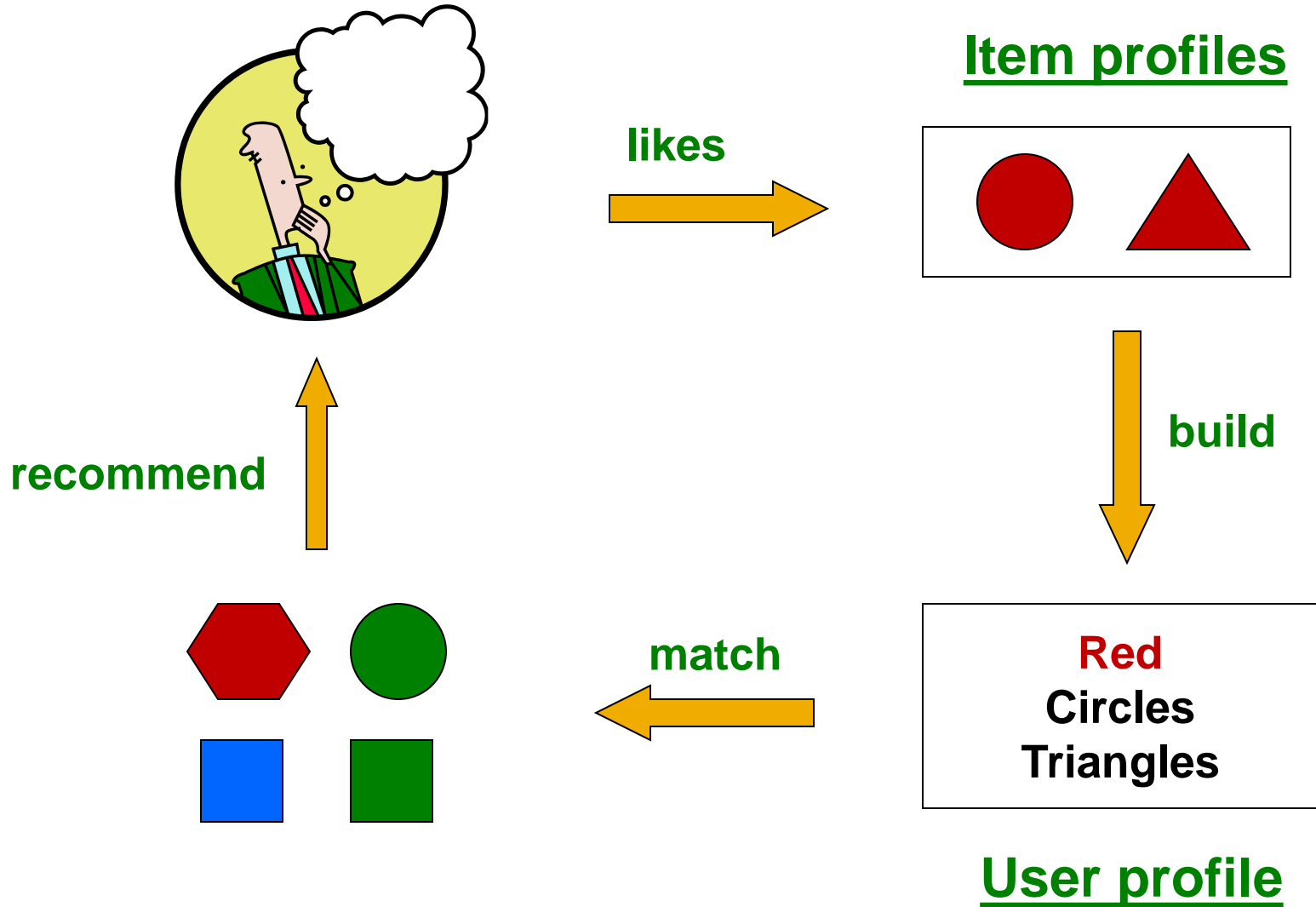
# Content-based Recommendations

- **Main idea:** Recommend items to customer  $x$  similar to previous items rated highly by  $x$

## *Example:*

- **Movie recommendations**
  - Recommend movies with same actor(s), director, genre, ...
- **Websites, blogs, news**
  - Recommend other sites with “similar” content

# Plan of Action



# Item Profiles

- For each item, create an **item profile**
- **Profile is a set (vector) of features**
  - **Movies:** author, title, actor, director,...
  - **Text:** Set of “important” words in document
- **How to pick important features?**
  - Usual heuristic from text mining is **TF-IDF**  
(Term frequency \* Inverse Doc Frequency)
    - **Term ... Feature**
    - **Document ... Item**

# Sidenote: TF-IDF

$f_{ij}$  = frequency of term (feature)  $i$  in doc (item)  $j$

$$TF_{ij} = \frac{f_{ij}}{\max_k f_{kj}}$$

**Note:** we normalize TF to discount for “longer” documents

$n_i$  = number of docs that mention term  $i$

$N$  = total number of docs

$$IDF_i = \log \frac{N}{n_i}$$

**TF-IDF score:**  $w_{ij} = TF_{ij} \times IDF_i$

**Doc profile** = set of words with highest **TF-IDF** scores, together with their scores

# User Profiles and Prediction

- **User profile possibilities:**

- Weighted average of rated item profiles
- **Variation:** weight by difference from average rating for item
- ...

- **Prediction heuristic:**

- Given user profile  $\mathbf{x}$  and item profile  $\mathbf{i}$ , estimate

$$u(\mathbf{x}, \mathbf{i}) = \cos(\mathbf{x}, \mathbf{i}) = \frac{\mathbf{x} \cdot \mathbf{i}}{||\mathbf{x}|| \cdot ||\mathbf{i}||}$$

# Pros: Content-based Approach

- **+: No need for data on other users**
  - No cold-start or sparsity problems
- **+: Able to recommend to users with unique tastes**
- **+: Able to recommend new & unpopular items**
  - No first-rater problem
- **+: Able to provide explanations**
  - Can provide explanations of recommended items by listing content-features that caused an item to be recommended



# Cons: Content-based Approach

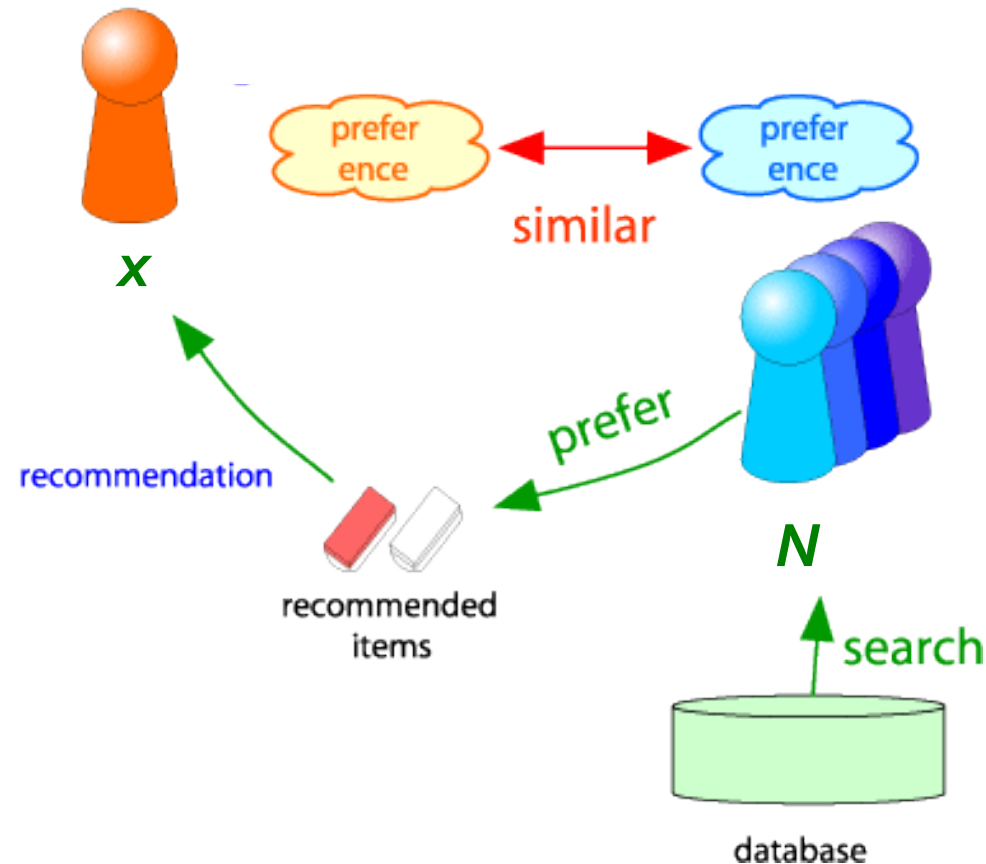
- —: Finding the appropriate features is hard
  - E.g., images, movies, music
- —: Recommendations for new users
  - How to build a user profile?
- —: Overspecialization
  - Never recommends items outside user's content profile
  - People might have multiple interests
  - Unable to exploit quality judgments of other users

# Collaborative Filtering

Harnessing quality judgments of other users

# Collaborative Filtering

- Consider user  $x$
- Find set  $N$  of other users whose ratings are “**similar**” to  $x$ ’s ratings
- Estimate  $x$ ’s ratings based on ratings of users in  $N$



# Finding “Similar” Users

$$\begin{aligned} r_x &= [*, \_, \_, *, **] \\ r_y &= [*, \_, **, **, \_] \end{aligned}$$

- Let  $r_x$  be the vector of user  $x$ 's ratings

- Jaccard similarity measure**

- Problem:** Ignores the value of the rating

$r_x, r_y$  as sets:

$$r_x = \{1, 4, 5\}$$

$$r_y = \{1, 3, 4\}$$

- Cosine similarity measure**

- $\text{sim}(x, y) = \cos(r_x, r_y) = \frac{r_x \cdot r_y}{\|r_x\| \cdot \|r_y\|}$

$r_x, r_y$  as points:

$$r_x = \{1, 0, 0, 1, 3\}$$

$$r_y = \{1, 0, 2, 2, 0\}$$

- Problem:** Treats missing ratings as “negative”

- Pearson correlation coefficient**

- $S_{xy}$  = items rated by both users  $x$  and  $y$

$$\text{sim}(x, y) = \frac{\sum_{s \in S_{xy}} (r_{xs} - \bar{r}_x)(r_{ys} - \bar{r}_y)}{\sqrt{\sum_{s \in S_{xy}} (r_{xs} - \bar{r}_x)^2} \sqrt{\sum_{s \in S_{xy}} (r_{ys} - \bar{r}_y)^2}}$$

$\bar{r}_x, \bar{r}_y \dots$  avg.  
rating of  $x, y$

# Similarity Metric

$$\text{Cosine sim: } \text{sim}(x, y) = \frac{\sum_i r_{xi} \cdot r_{yi}}{\sqrt{\sum_i r_{xi}^2} \cdot \sqrt{\sum_i r_{yi}^2}}$$

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

- **Intuitively we want:**  $\text{sim}(A, B) > \text{sim}(A, C)$
- **Jaccard similarity:**  $1/5 < 2/4$
- **Cosine similarity:**  $0.386 > 0.322$ 
  - Considers missing ratings as “negative”

- **Solution: subtract the (row) mean**

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	2/3			5/3	-7/3		
B	1/3	1/3	-2/3				
C				-5/3	1/3	4/3	
D		0					0

**sim A,B vs. A,C:  
0.092 > -0.559**

Notice cosine sim. is correlation when data is centered at 0

# Rating Predictions

## From similarity metric to recommendations:

- Let  $\mathbf{r}_x$  be the vector of user  $x$ 's ratings
- Let  $N$  be the set of  $k$  users most similar to  $x$  who have rated item  $i$
- **Prediction for item  $s$  of user  $x$ :**
  - $r_{xi} = \frac{1}{k} \sum_{y \in N} r_{yi}$
  - $r_{xi} = \frac{\sum_{y \in N} s_{xy} \cdot r_{yi}}{\sum_{y \in N} s_{xy}}$
  - Other options?
- **Many other tricks possible...**

Shorthand:

$$s_{xy} = \text{sim}(x, y)$$



# Item-Item Collaborative Filtering

- So far: **User-user collaborative filtering**
- **Another view: Item-item**
  - For item  $i$ , find other similar items
  - Estimate rating for item  $i$  based on ratings for similar items
  - Can use same similarity metrics and prediction functions as in user-user model

$$r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

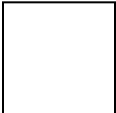
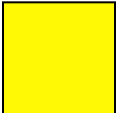
$s_{ij}$ ... similarity of items  $i$  and  $j$   
 $r_{xj}$ ... rating of user  $u$  on item  $j$   
 $N(i;x)$ ... set items rated by  $x$  similar to  $i$

# Item-Item CF ( $|N|=2$ )

users

movies

	1	2	3	4	5	6	7	8	9	10	11	12
1	1		3			5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

 - unknown rating       - rating between 1 to 5

# Item-Item CF ( $|N|=2$ )

		users											
		1	2	3	4	5	6	7	8	9	10	11	12
movies	1	1		3		?	5			5		4	
	2			5	4			4			2	1	3
	3	2	4		1	2		3		4	3	5	
	4		2	4		5			4			2	
	5			4	3	4	2					2	5
	6	1		3		3			2			4	



- estimate rating of movie 1 by user 5

# Item-Item CF ( $|N|=2$ )

		users												
		1	2	3	4	5	6	7	8	9	10	11	12	$\text{sim}(1,m)$
movies	1	1		3		?	5			5		4		1.00
	2			5	4			4			2	1	3	-0.18
	<u>3</u>	2	4		1	2		3		4	3	5		<u>0.41</u>
	4		2	4		5			4			2		-0.10
	5			4	3	4	2					2	5	-0.31
	<u>6</u>	1		3		3			2			4		<u>0.59</u>

## Neighbor selection:

Identify movies similar to  
movie 1, rated by user 5

Here we use Pearson correlation as similarity:

1) Subtract mean rating  $m_i$  from each movie  $i$

$$m_1 = (1+3+5+5+4)/5 = 3.6$$

row 1:  $[-2.6, 0, -0.6, 0, 0, 1.4, 0, 0, 1.4, 0, 0.4, 0]$

2) Compute cosine similarities between rows

# Item-Item CF ( $|N|=2$ )

		users												sim(1,m)
		1	2	3	4	5	6	7	8	9	10	11	12	
movies	1	1		3		?	5			5		4		1.00
	2			5	4			4			2	1	3	-0.18
	<u>3</u>	2	4		1	2		3		4	3	5		<u>0.41</u>
	4		2	4		5			4			2		-0.10
	5			4	3	4	2					2	5	-0.31
	<u>6</u>	1		3		3			2			4		<u>0.59</u>

Compute similarity weights:

$$s_{1,3}=0.41, s_{1,6}=0.59$$

# Item-Item CF ( $|N|=2$ )

		users											
		1	2	3	4	5	6	7	8	9	10	11	12
movies	1	1		3		2.6	5			5		4	
	2			5	4			4			2	1	3
	<u>3</u>	2	4		1	2		3		4	3	5	
	4		2	4		5			4			2	
	5			4	3	4	2					2	5
	<u>6</u>	1		3		3			2			4	

Predict by taking weighted average:

$$r_{1,5} = (0.41 \cdot 2 + 0.59 \cdot 3) / (0.41 + 0.59) = 2.6$$

$$r_{ix} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{jx}}{\sum s_{ij}}$$



# CF: Common Practice

Before:

$$r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

- Define **similarity**  $s_{ij}$  of items  $i$  and  $j$
- Select  $k$  nearest neighbors  $N(i; x)$ 
  - Items most similar to  $i$ , that were rated by  $x$
- Estimate rating  $r_{xi}$  as the weighted average:

$$r_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} s_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} s_{ij}}$$

baseline estimate for  $r_{xi}$

$$b_{xi} = \mu + b_x + b_i$$

- $\mu$  = overall mean movie rating
- $b_x$  = rating deviation of user  $x$   
= (avg. rating of user  $x$ ) -  $\mu$
- $b_i$  = rating deviation of movie  $i$

# Item-Item vs. User-User

	Avatar	LOTR	Matrix	Pirates
Alice	1		0.8	
Bob		0.5		0.3
Carol	0.9		1	0.8
David			1	0.4

- In practice, it has been observed that item-item often works better than user-user
- **Why?** Items are simpler, users have multiple tastes

# Pros/Cons of Collaborative Filtering

- **+ Works for any kind of item**
  - No feature selection needed
- **- Cold Start:**
  - Need enough users in the system to find a match
- **- Sparsity:**
  - The user/ratings matrix is sparse
  - Hard to find users that have rated the same items
- **- First rater:**
  - Cannot recommend an item that has not been previously rated
  - New items, Esoteric items
- **- Popularity bias:**
  - Cannot recommend items to someone with unique taste
  - Tends to recommend popular items

# Hybrid Methods

- **Implement two or more different recommenders and combine predictions**
  - Perhaps using a linear model
- **Add content-based methods to collaborative filtering**
  - Item profiles for new item problem
  - Demographics to deal with new user problem

# Remarks & Practical Tips

- Evaluation
- Error metrics
- Complexity / Speed

# Evaluation

Diagram illustrating a user-movie rating matrix. The vertical axis is labeled "users" and the horizontal axis is labeled "movies".

	1	3	4			
		3	5			5
			4	5		5
			3			
			3			
2				2		2
					5	
		2	1			1
		3			3	
1						

# Evaluation

Diagram illustrating a user-movie rating matrix for evaluation. The matrix is labeled "users" (vertical axis) and "movies" (horizontal axis).

The matrix is divided into two sections:

- Training Data (Yellow cells):** Contains known ratings for users 1, 2, and 3 across various movies.
- Test Data Set (Gray cells):** Contains unknown ratings (marked with "?") for users 2, 3, and 4 across various movies.

The matrix structure is as follows:

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6
User 1	1	3	4			
User 2		3	5			5
User 3			4	5		5
User 4			3			
User 5			3			
User 6	2			?		?
User 7					?	
User 8		2	1			?
User 9		3			?	
User 10	1					

# Evaluating Predictions

- **Compare predictions with known ratings**

- **Root-mean-square error (RMSE)**

- $\sqrt{\sum_{xi} (r_{xi} - r_{xi}^*)^2}$  where  $r_{xi}$  is predicted,  $r_{xi}^*$  is the true rating of  $x$  on  $i$

- **Precision at top 10:**

- % of those in top 10

- **Rank Correlation:**

- Spearman's *correlation* between system's and user's complete rankings

- **Another approach: 0/1 model**

- **Coverage:**

- Number of items/users for which system can make predictions

- **Precision:**

- Accuracy of predictions

- **Receiver operating characteristic (ROC)**

- Tradeoff curve between false positives and false negatives



# Problems with Error Measures

- **Narrow focus on accuracy sometimes misses the point**
  - Prediction Diversity
  - Prediction Context
  - Order of predictions
- **In practice, we care only to predict high ratings:**
  - RMSE might penalize a method that does well for high ratings and badly for others

# Collaborative Filtering: Complexity

- Expensive step is finding  $k$  most similar customers:  $O(|X|)$
- **Too expensive to do at runtime**
  - Could pre-compute
- Naïve pre-computation takes time  $O(k \cdot |X|)$ 
  - $X$  ... set of customers
- **We already know how to do this!**
  - Near-neighbor search in high dimensions (**LSH**)
  - Clustering
  - Dimensionality reduction

# Tip: Add Data

- **Leverage all the data**

- Don't try to reduce data size in an effort to make fancy algorithms work
- Simple methods on large data do best

- **Add more data**

- e.g., add IMDB data on genres

- **More data beats better algorithms**

<http://anand.typepad.com/datawocky/2008/03/more-data-usual.html>

# On Thursday: The Netflix prize and the Latent Factor Models

# On Thursday: The Netflix Prize

## ■ Training data

- 100 million ratings, 480,000 users, 17,770 movies
- 6 years of data: 2000-2005

## ■ Test data

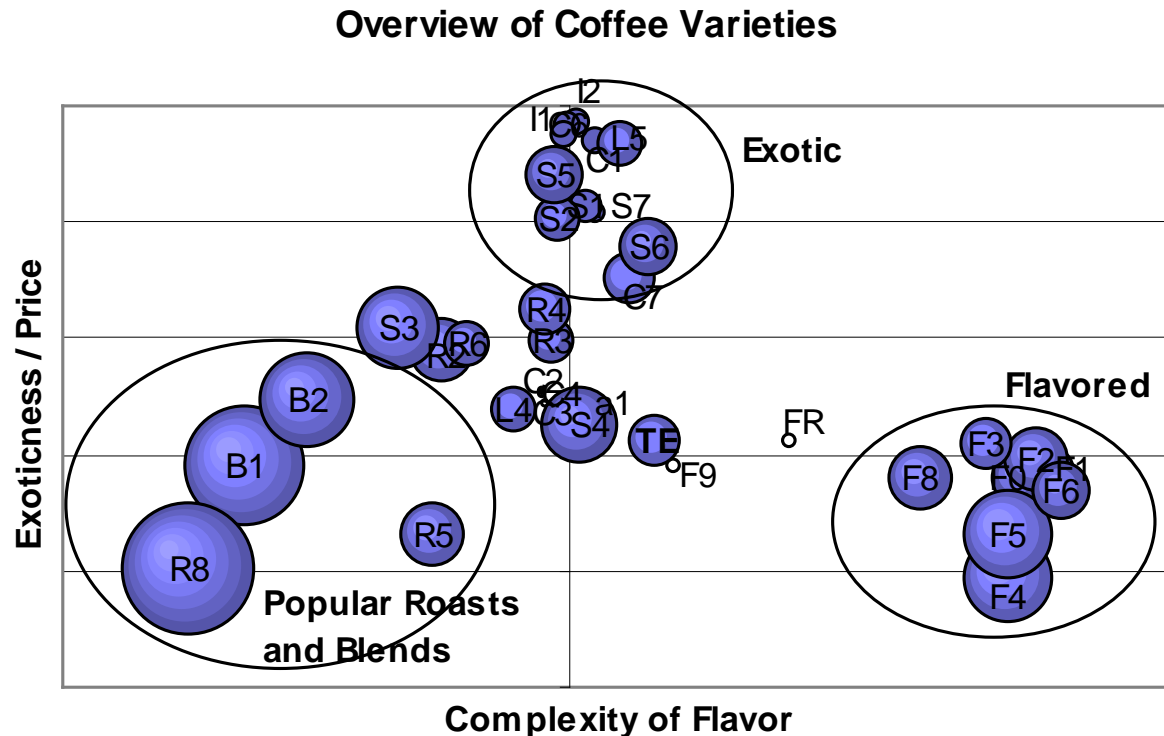
- Last few ratings of each user (2.8 million)
- Evaluation criterion: root mean squared error (RMSE)
- Netflix Cinematch RMSE: 0.9514

## ■ Competition

- 2700+ teams
- \$1 million prize for 10% improvement on Cinematch

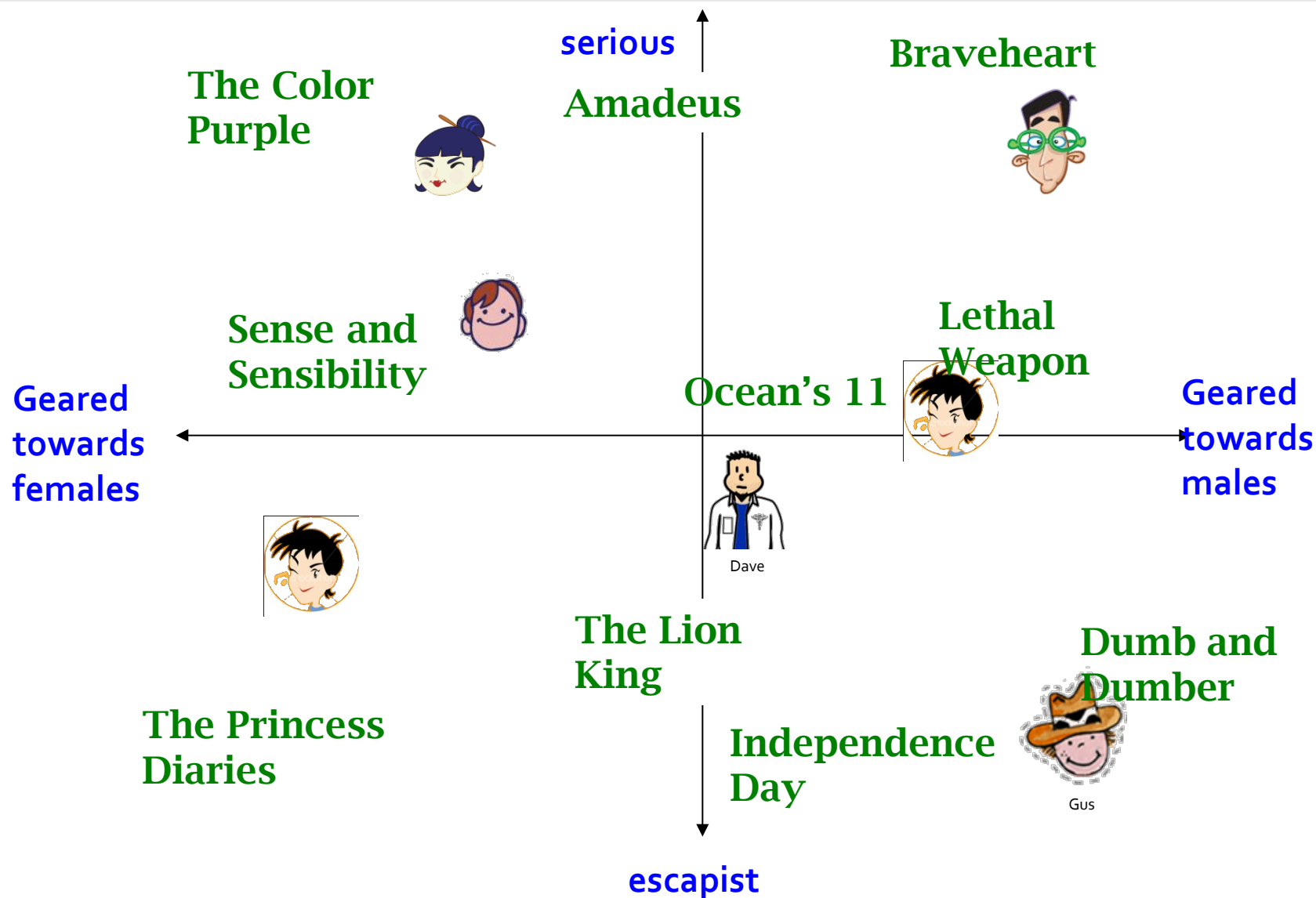
# On Thursday: Latent Factor Models

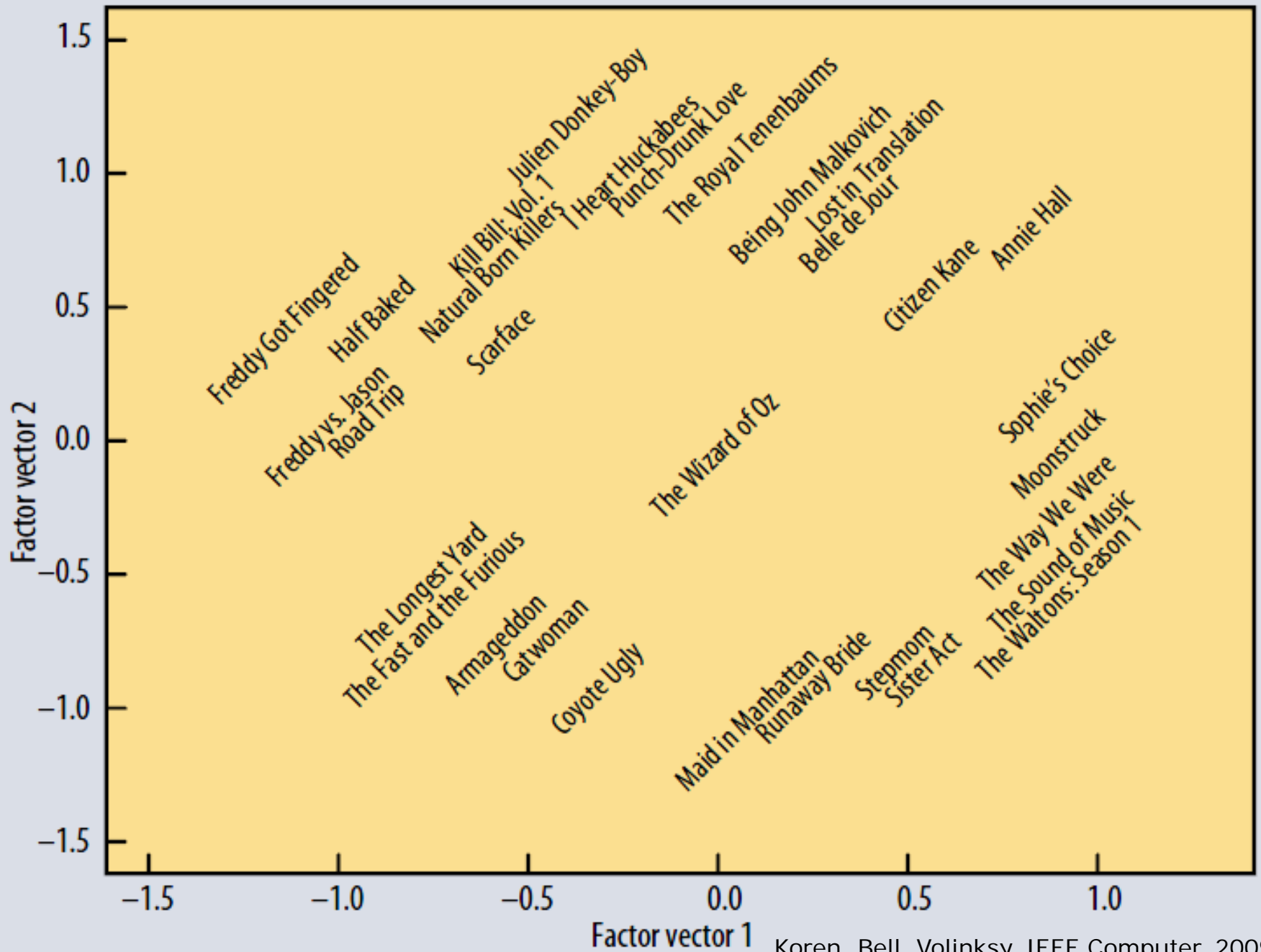
- **Next topic: Recommendations via Latent Factor models**



The bubbles above represent products sized by sales volume. Products close to each other are recommended to each other.

# Latent Factor Models (i.e., SVD++)





Koren, Bell, Volinsky, IEEE Computer, 2009