Formally Verified C Code Generation from Hybrid Communicating Sequential Processes

***Abstract*—Hybrid Communicating Sequential Processes (HCSP) is a formal model for hybrid systems, including primitives for evolution along an ordinary differential equation (ODE), communication, and parallel composition. Code generation is needed to convert HCSP models into code that can be executed in practice, and the correctness of this conversion is essential to ensure that the generated code accurately reflects the formal model. In this paper, we propose a code generation algorithm from HCSP to C with POSIX library for concurrency. The main difficulties include how to bridge the gap between the synchronized communication model in HCSP and the use of mutexes for synchronization in C, and how to discretize evolution along ODEs and support interrupt of ODE evolution by communication. To prove the correctness of code generation, we define a formal semantics for POSIX C, and build transition system models for both HCSP and C programs. We then define an approximate bisimulation relation between traces of transition systems, and show that under certain robustness conditions for HCSP, the generated C program is approximately bisimilar to the original model. Finally, we evaluate the code generation algorithm on a detailed model for automatic cruise control, showing its utility on real-world examples.**

***Index Terms*—hybrid systems, multi-threaded C, approximate bisimulation, code generation**

I. Introduction

Cyber-Physical Systems (CPSs) can be complex, networked, systems of systems, and are often entrusted with safety-critical tasks. The efficient and verified development of safe and reliable CPSs is a priority mandated by many standards, yet a notoriously difficult and challenging field. To address design complexity under the necessity of verified CPS development, Model-Driven Development (MDD) has become a predominant approach in CPS development. MDD usually comprises different abstraction levels from top to bottom, e.g. graphical models, formal models, and code at the implementation level. There are two orthogonal principles followed by MDD:

- **Horizontal** Composition/Decomposition: $P \| Q$.
- **Vertical** Abstraction/Refinement: $P \sqsubseteq Q$.

The horizontal dimension requires a modelling mechanism that is compositional such that the complexity of modelling and verifying the system can be reduced by separately verifying its subcomponents. On the vertical dimension, specific modelling and analysis are performed at each abstraction level, and each level is refined by the more concrete level so that the behaviors are preserved. Therefore, if formal models are proved correct, the code generated from them is also guaranteed to be correct without further proof.

Especially, the MDD of hybrid systems that integrate traditional discrete models with dynamic models faces a central problem: how to transform an abstract hybrid control model to an algorithmic model at code level rigorously and automatically. The controller code determines how to sample data from the continuous plant and the entanglement between sampling data and computing control commands is intricate. An efficient approach is to discretize the continuous plant, and then, the discretized continuous plant together with the controller code constitute an embedded real-time implementation for the closed-loop hybrid system. How and according to which criterion to discretize the continuous behaviour and then generating correct C code, is addressed in this paper.

There are many industrial MDD tools targeting CPS design and development, such as Simulink/Stateflow [Mat13a], [Mat13b], SCADE [Dor08] and so on. Simulink/Stateflow and SCADE both support automatic code generation from control models, targeting at real-time applications. However, Simulink/Stateflow can only guarantee correctness of generated code by (incomplete) simulations. SCADE was founded on the synchronous dataflow language Lustre [HCRP91], and its formally verified compiler Vélus [BBD$^+$17] guarantees that the generated code faithfully implements the semantics of Lustre. However, it lacks support for continuous plants modelling. In the academic community, there are also a number of studies on formal modelling and verification of CPSs, for instance, hybrid and timed automata [Hen96], [FKL$^+$18], [WZA18], hybrid programs and dynamic differential logic [Pla18], Event-B and its hybridation [DASP21], and hybrid process algebra [CR05], [BM05]. Most of these approaches address verification only, while Event-B supports code generation with formal guarantee but only for discrete case.

Hybrid CSP (HCSP) [He94], [ZWZ17], a hybrid extension to the classic CSP (Communicating Sequential Processes) [Hoa78], is a compositional formalism in describing hybrid systems. It uses ordinary differential equations (ODEs) to model continuous evolutions and introduces interrupts to model interactions between continuous and discrete dynamics in a very flexible manner. The verification of HCSP is conducted by tools based on Hybrid Hoare Logic (HHL) [ZZW$^+$23], including verification within the interactive theorem prover Isabelle/HOL [ZZW$^+$13], [ZWZ17] as well as a more automatic tool HHLPy [SBZ23]. Graphical models for CPSs can be translated into HCSP [ZZW$^+$13], [XWZ$^+$22], and the verified HCSP can then be transformed to executable SystemC code [YJW$^+$20]. To guarantee the correctness of transformation from HCSP to SystemC, the notion of approximate bisimulation proposed by [GP07], [JDBP09] was used to measure the equivalence between hybrid and discrete systems, to allow a distance between the observations of two systems within a tolerable bound rather than exactly identical, which was proved to hold for HCSP and SystemC under some robustly-safety conditions in [YJW$^+$20].

This paper builds upon the work of Yan et al. in [YJW$^+$20]. It aims at generating code in C from HCSP models using the concurrency primitives of the POSIX pthreads library, with correctness guarantees. It adopts the notions of approximate bisimulation, robustly-safety, and some of the discretisation rules of HCSP proposed in [YJW$^+$20]. However, it is distinguished from previous work in the following aspects:

1) Although both works transform HCSP to its discretized version and then to code, in [YJW$^+$20], the discretization of the HCSP constructs related to communications, including inputs, outputs and continuous interrupt, is defined with the help of shared channel variables indicating their readiness to perform communication actions (see Table 2 and 3 of [YJW$^+$20]); and this work does not need to introduce extra shared variables at this step, thus keeps communications the same as original HCSP.
2) SystemC supports communication mechanisms similar to HCSP on its own right, so the communications in HCSP can be translated naturally to SystemC. Compared to this, communications are implemented in C using the POSIX pthreads library, in terms of mutexes and condition variables to achieve time and value synchronization. Hence, the method of translation to C is more involved. At semantic level, concurrency in HCSP follows a handshaking model while in C it follows an interleaving model controlled by mutexes and condition variables. Thus, the two concurrency mechanisms in HCSP and C are completely different, and how to prove the equivalence between them is one of the main challenges addressed in our work.

We prove the approximate bisimulation between an HCSP process and the generated C code in two steps: approximate bisimulation between the HCSP process and its discretised version, and bisimulation between the discrete HCSP process and the generated C code. Compared to the proofs of [YJW$^+$20], the first part proves the case for continuous interrupt based on its new discretization, and the second part is completely new: we introduce a new bisimulation relation, for which each transition step in HCSP may correspond to multiple atomic blocks of execution on the C side, but only one of them is considered as the essential step to perform the transition.

In summary, the main contributions of this paper comprise:

1) We present a formal semantics for a subset of C language with POSIX threads.
2) We implement the synchronized communications of HCSP with the use of mutexes in the pthreads library, based on which we realise the transformation from any HCSP process to C code.
3) Based on the notions of approximate bisimulation, we prove the correctness of the transformation from HCSP to C. The proof uses a new bisimulation relation for verifying equivalence between two concurrency mechanisms with synchronized and interleaving settings.
4) We apply our approach on a realistic Automatic Cruise Control System, including its HCSP model, the C code generated from the model, and the comparison with original HCSP and other C implementation by simulation.

After reviewing related works, the paper is organized as follows: Sect. II introduces some preliminary knowledge of this work. Sect. III introduces the syntax and semantics of C with POSIX threads. The translation from HCSP to C is specified in Sect. IV, and the correctness of the translation is justified in Sect. V. Sect. VI illustrates our approach by a realistically-scaled case study. Sect. VII concludes.

A. Related Work

Model-based automatic code generation has been extensively studied in both academic and industrial communities [AFH$^+$10], but code generation that supports hybrid systems and provides formal correctness guarantees of generated code at the same time is not well addressed. Some examples include the aforementioned Simulink [Mat13a], SCADE [Dor08] founded on synchronous Lustre, and the formal modelling languages [Hen96], [FKL$^+$18], [WZA18]. OSATE/AADL [OSA17] provides architecture modeling and analysis of real-time systems and furthermore supports the automated code generation from AADL models including runtime behavior and scheduling to C code. However, it validates the code generation by simulation, and moreover does not support continuous time modeling. The compiler Zélus [BP13] extends Lustre [HCRP91] with ODEs and implements code generation from the extended hybrid language, which has also been implemented in SCADE 6. It supports analysis of hybrid models by type systems and semantics, and handles the detection of zero-crossing events [BP13], [BBCP12]. But it does not explicitly support constructs related to communication and concurrency. VeriPhy [BTM$^+$18] automatically transforms verified formal models of CPSs modelled in differential dynamic logic (dL) [Pla18], [FMQ$^+$15] to controller implementations that preserve safety properties of original models rather than their semantics. Thus compared to our work, it does not consider the (approximate) equivalence between the source models with ODEs and the discrete implementation, and the zero-crossing problems caused by discretization.

II. Preliminaries

This section introduces the notion of transition systems, approximate bisimulation, discretization of ODEs, and HCSP.

A. Transition Systems

Definition 1 (Transition system). *A transition system is a tuple $T = \langle Q, L, \rightarrow, Q^0, Y, H\rangle$, where Q is a set of states, $L \subseteq \mathcal{ACT} \cup \{\tau\}$ is a set of labels, $\rightarrow \subseteq Q \times L \times Q$ is a set of transitions, $Q^0 \subseteq Q$ is a set of initial states, Y is a set of observations, and $H : Q \rightarrow Y$ is an observation function. $\mathcal{ACT}$ is a set of events and τ is an internal event ($\tau \notin \mathcal{ACT}$).*

Given a transition system, for any $a \in \mathcal{ACT}$, we define the τ-closed transition $q \overset{a}{\Rightarrow} q'$ to represent that q can reach q' via action a and a sequence of τ actions, i.e.$q(\xrightarrow{\tau})^* q_i \xrightarrow{a} q_{i+1}(\xrightarrow{\tau})^* q'$. We will define the semantics of HCSP and C using transition systems. In many cases we set $\mathcal{ACT} = \emptyset$ (such as when modelling the semantics for C), then we will define $\rightarrow \subseteq Q \times Q$ instead, and $\Rightarrow$ is equivalent to $\rightarrow^*$.

B. Approximate Bisimulation

The notion of approximate bisimulation was first proposed in [GP07] to measure the equivalence between hybrid systems, to allow a limited distance between the observations of two systems. Later in [JDBP09], it was extended to allow precision not only between the observations, but also between the synchronisation labels of two systems. In [YJW+20], the authors instantiate the precision parameters to be the time and value tolerances between two systems. Below we present the notion of approximate bisimulation in [YJW+20]. Let $TS_i = \langle Q_i, L_i, \rightarrow_i, Q_i^0, Y_i, H_i \rangle$, $(i = 1, 2)$ be two transition systems, h and ε the time and value precisions resp.

Definition 2 (Approximate Bisimulation). *$\mathcal{B}_{h,\varepsilon} \subseteq Q_1 \times Q_2$ is called a (h, ε)-approximate bisimulation relation between TS_1 and TS_2, if it is symmetric, and for all $(q_1, q_2) \in \mathcal{B}_{h,\varepsilon}$,*

- *The distance between the observations is within the given value precision, i.e. $|H_1(q_1), H_2(q_2)| \leq \varepsilon$, where $|H_1(q_1), H_2(q_2)|$ returns the maximum of Euclidean distances of observation variables in q_1 and q_2.*
- *$\forall q_1 \overset{l_1}{\rightarrow}_1 q_1'$, $\exists q_2 \overset{l_2}{\Rightarrow}_2 q_2'$ s.t. $(q_1', q_2') \in \mathcal{B}_{h,\varepsilon}$, and $|l_1, l_2| < h$, for $l_1 \in L_1, l_2 \in L_2$. Here $|l_1, l_2|$ is 0 if $l_1 = l_2$, $|l_1 - l_2|$ if $l_1, l_2 \in R$, and ∞ otherwise.*

The (h, ε)-approximate bisimulation requires the distance between the observations under the pair (q_1, q_2) must be within ε; furthermore, if one of them is able to reach a state via an event, the other one can also reach a state via an event such that the distance between the events is within h and the pair of resulting states also satisfy the approximate bisimulation.

Definition 3. *$\mathcal{TS}_1$ and $\mathcal{TS}_2$ are approximate bisimilar with respect to h and ε, denoted $\mathcal{TS}_1 \cong_{h,\varepsilon} \mathcal{TS}_2$, if there exists a bisimulation relation $\mathcal{B}_{h,\varepsilon}$ satisfying that for all initial configurations $q_1 \in {Q_1}^0$ there exists $q_2 \in {Q_2}^0$ such that $(q_1, q_2) \in \mathcal{B}_{h,\varepsilon}$ and vice versa.*

C. Discretization of ODE

Here we present the discretization of ODEs and its correctness, which have been studied in [YJW+20]. We briefly revisit the related results in [YJW+20] here.

We apply the 4-stage Runge-Kutta method to discretize the continuous dynamics, which is more effective with *global discretization error* $O(h^4)$. The ODE $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ on $[t_0, t_0+T_o]$ is discretized as

$$(\text{wait } h; \mathbf{x} := \mathbf{x} + h\mathbf{\Phi}(\mathbf{x}, h))^N; \text{wait } h'; \mathbf{x} := \mathbf{x} + h'\mathbf{\Phi}(\mathbf{x}, h')$$

where $N = \lfloor \frac{T_o}{h} \rfloor$, $h' = T_o - Nh$, and $\mathbf{\Phi}(\mathbf{x}, s) = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ with $k_1 = \mathbf{f}(\mathbf{x})$, $k_2 = \mathbf{f}(\mathbf{x} + \frac{1}{2}sk_1)$, $k_3 = \mathbf{f}(\mathbf{x} + \frac{1}{2}sk_2)$ and $k_4 = \mathbf{f}(\mathbf{x} + sk_3)$. With the initial state $\mathbf{x}_0$ at $h_0 = t_0$, the obtained sequence of approximate solutions $\{\mathbf{x}_i\}$ at time stamps $\{h_i\}$ is (below $1 \leq j \leq N$):

$$\begin{cases} \mathbf{x}_0, & h_0 = t_0, \\ \mathbf{x}_j = \mathbf{x}_{j-1} + h\mathbf{\Phi}(\mathbf{x}_{j-1}, h), & h_j = t_0 + j * h \\ \mathbf{x}_{N+1} = \mathbf{x}_N + h'\mathbf{\Phi}(\mathbf{x}_N, h'), & h_{N+1} = t_0 + T_o \end{cases}$$

Intuitively, T_o is divided into N intervals of length h and a possible residual interval of length h'. $\mathbf{\Phi}$ is computed based on the values of the vector field at the four points and used for approximating the value of $\mathbf{x}$. Below we present the global error of the discretization (see Theorem 7.2.2.3 in [SB13]).

Proposition 1 (Global Error). *Assume the ODE $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ satisfies the local Lipschitz condition, that is, for any compact set S of $\mathbb{R}^n$, $\|\mathbf{f}(\mathbf{y}_1) - \mathbf{f}(\mathbf{y}_2)\| \leq L\|\mathbf{y}_1 - \mathbf{y}_2\|$ for all $\mathbf{y}_1, \mathbf{y}_2 \in S$. Let $X(t, \widetilde{\mathbf{x}}_0)$ be the exact solution of the ODE with initial value $\widetilde{\mathbf{x}}_0$ on $[0, T_0]$. Suppose $\mathbf{x}_0 \in \mathbb{R}^n$ is a state with $\|\mathbf{x}_0 - \widetilde{\mathbf{x}}_0\| \leq \xi_1$. Then there exists a discretized step $h_e > 0$ s.t. for all $0 < h \leq h_e$ and all $i \leq \lceil \frac{T_o}{h} \rceil$, the global discretization error between $X(h_i, \widetilde{\mathbf{x}}_0)$ and $\mathbf{x}_i$ satisfies:*

$$\begin{aligned} &\|X(h_i, \widetilde{\mathbf{x}}_0) - \mathbf{x}_i\| \leq M(h), \ \textit{where} \\ &M(h) = \tfrac{e^{Lh'}-1}{L} C_2 (h')^4 + [1 + Lh' + \tfrac{(Lh')^2}{2} + \tfrac{(Lh')^3}{4} \\ &\qquad + \tfrac{(Lh')^4}{24}] M_N(h) \\ &M_N(h) = e^{NLh}\xi_1 + \tfrac{e^{NLh}-1}{L} C_1 h^4. \end{aligned}$$

Among them N, h', h_i and $\mathbf{x}_i$ are as defined previously, and C_1, C_2 are positive constants depending on the local discretization error of the 4-stage Runge-Kutta method. Here given a vector $\mathbf{x} \in \mathbb{R}^n$, $\|\mathbf{x}\|$ denotes the infinity norm of $\mathbf{x}$, i.e., $\|\mathbf{x}\| = \max\{|x_1|, |x_2|, ..., |x_n|\}$.

From the definition of $M(h)$, the global error is monotonically increasing with respect to step size h, the Lipschitz constant L, and the two local discretization error constants C_1 and C_2. In [YJW+20], the following theorem is proved for correctness of the discretization of an ODE.

Theorem 1 (Correctness of Discretization of ODEs). *Suppose L is the Lipschitz constant of $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ with initial condition $\mathbf{x}(t_0) = \widetilde{\mathbf{x}}_0$, and $\mathbf{x}_0$ satisfies $\|\mathbf{x}_0 - \widetilde{\mathbf{x}}_0\| \leq \xi_1$. For any $\xi > \xi_1 > 0$, there exists $h > 0$ s.t.*

$$\begin{aligned} &\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \ \mathbf{x}(t_0) = \widetilde{\mathbf{x}}_0. \qquad \textit{and} \\ &(\textit{wait } h; \mathbf{x} := \mathbf{x} + h\mathbf{\Phi}(\mathbf{x}, h))^N; \textit{wait } h'; \mathbf{x} := \mathbf{x} + h'\mathbf{\Phi}(\mathbf{x}, h') \end{aligned}$$

are (h, ξ)-approximately bisimilar on $[t_0, t_0 + T_o]$.

D. Source Language HCSP

The syntax for HCSP is given as follows.

$$\begin{array}{lll} p & ::= & \text{skip} \mid x := e \mid ch?x \mid ch!e \mid p_1; p_2 \mid B \rightarrow p \mid p_1 \sqcup p_2 \\ & & \mid p^* \mid \langle \dot{x} = e \& B \rangle \mid \langle \dot{x} = e \& B \rangle \trianglerighteq [\!]_{i \in I}(ch_i * \rightarrow p_i) \\ pc & ::= & p \mid pc_1 \|_{cs} pc_2 \end{array}$$

where e represents expressions, p a sequential HCSP process, and pc the parallel composition of processes. x denotes variables, B Boolean expressions, ch, ch_i channel names. The meanings of skip, assign, sequential, conditional, internal choice and repetition are as usual. We explain the intuitive meaning of the additional constructs as follows:

- The input $ch?x$ receives a value along the channel ch and assigns it to variable x; and the output $ch!e$ sends the value of e along ch. Each of them may block waiting for the corresponding dual party to be ready.
- The repetition p^* executes p for a nondeterministic finite number of times.
- $\langle \dot{x} = e \& B\rangle$ is the continuous evolution, which evolves continuously according to the ODE $\dot{\vec{x}} = \vec{e}$ as long as the *domain* B holds, and terminates when B becomes false.
- Interrupt $\langle \dot{x} = e \& B\rangle \trianglerighteq []_{i\in I}(ch_i* \to c_i)$ behaves like $\langle \dot{x} = e \& B\rangle$, except it is preempted as soon as one of the communication events ch_i* takes place, and then is followed by the corresponding c_i.
- $pc_1 \|_{cs} pc_2$ behaves as pc_1 and pc_2 run independently except that all communications along common channels cs are synchronized between pc_1 and pc_2. We assume that variables of pc_1 and pc_2 are disjoint, and no same channel direction (e.g. $ch!$) occurs in both pc_1 and pc_2.

Some other constructs can be defined as derived. For example, wait d is an abbreviation for $t := 0; \langle \dot{t} = 1 \& t < d\rangle$.

Fig. 1 presents part of the small-step semantics for HCSP. Two types of transitions are introduced: $(p, s) \xrightarrow{e} (p', s')$ defines that a sequential HCSP process p executes from initial state s in one step, produces event e and results in statement p' and state s'; and $(pc, s) \xrightarrow{e}_h (pc', s')$ defines one step execution of a parallel HCSP process. Here states $s, s' \in \mathit{Vars} \to \mathit{Values}$ assign values to variables of p. τ represents an internal discrete event. A *communication event* $\langle ch\triangleright, v\rangle$, where $\triangleright$ is one of ?, !, or nothing, indicating input, output, and synchronized input/output (IO) event, respectively, where v is the transferred value. A *wait event* $\langle d, \mathit{rdy}\rangle$, represents an evolution of time length $d > 0$ with a set of ready channels that are waiting for communication during this period. We denote the set of the above events by *HEvts*. A *ready set* is a set of channel directions, indicating that these channel directions are waiting for communication. Two ready sets rdy_1 and rdy_2 are *compatible*, denoted by $\mathrm{compat}(\mathit{rdy}_1, \mathit{rdy}_2)$, if there does not exist a channel ch such that $ch? \in \mathit{rdy}_1 \wedge ch! \in \mathit{rdy}_2$ or $ch! \in \mathit{rdy}_1 \wedge ch? \in \mathit{rdy}_2$.

For parallel composition, without loss of generality, suppose pc is a parallel composition of sequential processes $\{p_i\}_{i\in I}$, and s is a disjoint union of the states for all p_is, i.e. $s = \biguplus_{i\in I} s_i$. $pc[p'_i/p_i]$ returns a new process by substituting p'_i for p_i in pc, and $s[s'_i/s_i]$ the same. There are three cases: if one process among pc performs a τ step, then pc can perform the same τ step (G-Tau); if two processes synchronise over a same channel, then pc performs a communication immediately (G-comm); if all processes of pc can perform a wait duration d and their ready sets are mutually compatible, then pc performs a wait duration d, joining all ready sets together (G-delay).

III. Target Language C: Syntax and Semantics

We consider a subset of concurrent C with POSIX threads as the target language of code generation. The abstract syntax is defined in Fig. 2. Here d denotes constants, x variables, **retv** and **ret** introduced for semantic use to record the return value of a function and check whether a return occurs so that the remaining code of the function will not execute, $a[k]$ array elements, **op** arithmetic or Boolean operators, B Boolean expressions, tid the ID of a thread, l mutex, cv condition variables, f, g function names, $\overline{T\ x}$ an abbreviation for a sequence of variable declarations with the form $T_1\ x_1; \cdots; T_n\ x_n$.

The remaining statements define the thread APIs for achieving concurrency, provided by the POSIX thread library of C:

- **create** $\mathit{tid}\ F\ \bar{e}$ spawns a new thread tid, and starts execution by invoking function F with arguments $\bar{e}$.
- **lock** l locks mutex l and gains exclusive access to the data protected by l.
- **unlock** l releases mutex l and in consequence another thread is allowed to acquire l and use the shared data.
- **cwait** $cv\ l$ blocks on condition variable cv and automatically releases mutex l. As soon as cv is signaled by another thread, it is unblocked on this signal and turns to re-acquire mutex l.
- **signal** cv signals the condition variable cv to the thread that is blocked on it and in consequence the thread is released to execute.
- **join** tid waits for the thread tid to terminate.

The last $x \leftarrow \mathbf{retv}.Es$ represents that x is set to be the return value **retv** in local states Es. Same as **retv** and **ret**, it is introduced for defining the small-step semantics of function calls with return values. A function declaration F includes a return type T_1, a function name f, and a body consisting of a sequence of local variable declarations and a command c. At the end, a C program P is composed of a sequence of global variable declarations decl, a sequence of function declarations $\overline{F}$, and the `main` function as the entry point of the program.

A. Small-step Semantics

1) Notations: The semantics of C statements is defined by two judgements. They are parameterized by a static environment Γ, a thread pool T, a global state G, and a local state set Es. Γ maps a function name to its declaration, T maps each active thread to its code, G maps global variables to their values, and Es maps each thread to its local state, which in turn maps local variables of the thread to values. Below list the judgements defining the one step execution of a thread and of a thread set consisting of multiple threads resp.:

- $\Gamma, \mathit{tid} \vdash (c, T, G, Es) \to_c (c', T', G', Es')$, stating that under static environment Γ and thread tid, statement c executes to c' in one step, changing thread pool T (due to thread creation), global state G, local states Es to T', G' and Es' resp.
- $\Gamma \vdash (T, G, Es) \to (T', G', Es')$, stating that under static environment Γ, thread pool T executes, leading to the continuation T', changing global state G and local states Es to G' and Es' resp.

2) Semantics: We present part of the small-step semantics of multi-threaded C in Fig. 3. Each statement is guarded by

$$\frac{}{(ch!e, s) \xrightarrow{\langle ch!, s(e)\rangle} (\text{skip}, s)} \quad \frac{}{(ch!e, s) \xrightarrow{\langle d, \{ch!\}\rangle} (ch!e, s)} \quad \frac{}{(ch!e, s) \xrightarrow{\langle \infty, \{ch!\}\rangle} (\text{skip}, s)}$$

$$\frac{\begin{array}{c}\vec{p} \text{ is a solution of the ODE } \dot{\vec{x}} = \vec{e} \\ \vec{p}(0) = s(\vec{x}) \quad \forall t \in [0, d).\, s[\vec{x} \mapsto \vec{p}(t)](B)\end{array}}{(\langle \dot{\vec{x}} = \vec{e} \& B\rangle, s) \xrightarrow{\langle d, \{\}\rangle} (\langle \dot{\vec{x}} = \vec{e} \& B\rangle, s[\vec{x} \mapsto \vec{p}(d)])} \quad \frac{\neg s(B)}{(\langle \dot{\vec{x}} = \vec{e} \& B\rangle, s) \xrightarrow{\tau} (\text{skip}, s)}$$

$$\frac{\vec{p} \text{ is a solution of the ODE } \dot{\vec{x}} = \vec{e} \quad \vec{p}(0) = s(\vec{x}) \quad \forall t \in [0, d).\, s[\vec{x} \mapsto \vec{p}(t)](B)}{(\langle \dot{x} = e \& B\rangle \trianglerighteq []_{i\in I}(ch_i* \to c_i), s) \xrightarrow{\langle d, rdy(\cup_{i\in I} ch_i *)\rangle} (\langle \dot{x} = e \& B\rangle \trianglerighteq []_{i\in I}(ch_i* \to c_i), s[\vec{x} \mapsto \vec{p}(d)]}$$

$$\frac{\neg s(B)}{(\langle \dot{x} = e \& B\rangle \trianglerighteq []_{i\in I}(ch_i* \to c_i), s) \xrightarrow{\tau} (\text{skip}, s)} \quad \frac{i \in I \quad ch_i* = ch!e}{(\langle \dot{x} = e \& B\rangle \trianglerighteq []_{i\in I}(ch_i* \to c_i), s) \xrightarrow{\langle ch!, s(e)\rangle} (Q_i, s)}$$

$$\frac{i \in I \quad ch_i* = ch?x}{(\langle \dot{x} = e \& B\rangle \trianglerighteq []_{i\in I}(ch_i* \to c_i), s) \xrightarrow{\langle ch?, v\rangle} (c_i, s[x \mapsto v])}$$

$$\frac{i \in I \quad (p_i, s_i) \xrightarrow{\tau} (p_i', s_i')}{(pc, s) \xrightarrow{\tau}_h (pc[p_i'/p_i], s[s_i'/s_i])} \text{G-Tau} \quad \frac{i, j \in I \quad (p_i, s_i) \xrightarrow{\langle ch!, v\rangle} (p_i', s_i') \quad (p_j, s_j) \xrightarrow{\langle ch?, v\rangle} (p_j', s_j')}{(pc, s) \xrightarrow{\langle ch, v\rangle}_h (pc[p_i'/p_i, p_j'/p_j], s[s_i'/s_i, s_j'/s_j])} \text{G-Comm}$$

$$\frac{\forall i \in I.(p_i, s_i) \xrightarrow{\langle d, rdy_i\rangle} (p_i', s_i') \quad \forall i, j \in I. i \neq j \Rightarrow \text{compat}(rdy_i, rdy_j)}{(\|_{i\in I} p_i, s) \xrightarrow{\langle d, \cup_{i\in I} rdy_i\rangle}_h (\|_{i\in I} p_i', \cup_{i\in I} s_i')} \text{G-delay}$$

Fig. 1. Part of small-step semantics of HCSP

Expressions	e	$::=$	$d \mid x \mid \textbf{retv} \mid \textbf{ret} \mid a[k] \mid e \textbf{ op } e \mid \ldots$
Statements	c	$::=$	$\text{skip} \mid x = e \mid x = f(\overline{e}) \mid g(\overline{e}) \mid c_1; c_2 \mid \textbf{if } B\ c_1 \textbf{ else } c_2$
			$\mid \textbf{while } B\ c \mid \textbf{for}(c_1, e, c_2)\ c_3 \mid \textbf{return } e \mid \textbf{create } tid\ F\ \overline{e}$
			$\mid \textbf{lock } l \mid \textbf{unlock } l \mid \textbf{cwait } cv\ l \mid \textbf{signal } cv \mid \textbf{join } tid \mid x \leftarrow \textbf{retv}.Es$
Variable Decls	$decl$	$:=$	$\overline{T\ x}$ Function Decls $F ::= T_1\ f(decl_1)\{decl_2; c\}$
Programs	P	$::=$	$decl; \overline{F}; \texttt{main}$

Fig. 2. Syntax of Subset of Multi-threaded C

ret $= 0$, to mean that no return occurs thus the continuation executes. For the contrary case when **ret** $= 1$, the continuation will be dropped, until the function call ends, indicated by rule (FunEnd), which transfers the return value back and resets **ret** to 0 again.

Rule (Create) spawns a new thread tid', recording the code of tid' in the thread pool to be $F(\overline{v})$, and meanwhile setting its initial local state as the one of parent thread tid. Rule (Lock) defines that if mutex l is available, then it can be obtained by thread tid, by mapping l to be the holding thread tid in the global state G. Rule (Unlock) defines that tid releases l. The semantics of **cwait** is specified by two rules: At first, tid locks l and cv is false, t releases l (WaitF); then as soon as cv is signaled thus becomes true, **cwait** stops waiting, and then it needs to acquire mutex l again and thus is equivalent to executing **lock**, and at the same time resetting cv to be false (WaitT). **signal** cv signals cv to some thread who is waiting on it (Signal). Rule (Join) defines that when thread tid' completes the execution of its code, **join** tid' terminates directly. Rule (Threads) defines the execution of multiple threads in T, which randomly selects an available thread tid to execute and updates the thread pool, the global and local states correspondingly.

IV. From HCSP to C

A. Auxiliary Variables and Functions

In order to transform HCSP to C, some auxiliary variables and functions are introduced in order to achieve synchronization between multiple threads.

1) Global and Local Clocks: In order to synchronize the executions of threads in parallel, we introduce a local clock for each thread i, denoted by `localTime[i]`, to record the local execution time of i. If a thread is waiting with a time limit (and possibly for some communications), its `localTime` is set to that time limit. If the thread is only waiting for communication, its `localTime` is set to infinity (with `DBL_MAX` used in practice). A global clock `currentTime` is used to record the global execution time, and to coordinate the execution of all threads. It equals the minimum of all local clocks, thus every time a local clock makes progress, `currentTime` will be updated and in consequence threads whose time limit has reached will be woken up and be notified to execute.

2) Thread States: There are six possible forms of thread states that indicate the execution status of a thread:

- `Stopped`, representing the thread has reached the end of execution;

$$\frac{Es(\mathbf{ret}) = 0 \quad [\![\overline{e}]\!]_{G,Es} = \overline{v}}{\Gamma, tid \vdash (\mathbf{create}\ tid'\ F\ \overline{e}, T, G, Es) \rightarrow_c (\epsilon, T[tid' \mapsto F(\overline{v})], G, Es[tid' \mapsto Es(tid)]}\ \text{Create}$$

$$\frac{Es(\mathbf{ret}) = 0 \quad G(l) = \bot}{\Gamma, tid \vdash (\mathbf{lock}\ l, T, G, Es) \rightarrow_c (\epsilon, T, G[l \mapsto tid], Es)}\ \text{Lock} \quad \frac{Es(\mathbf{ret}) = 0 \quad G(l) = tid}{\Gamma, tid \vdash (\mathbf{unlock}\ l, T, G, Es) \rightarrow_c (\epsilon, T, G[l \mapsto \bot], Es)}\ \text{Unlock}$$

$$\frac{Es(\mathbf{ret}) = 0 \quad G(cv) = 0 \quad G(l) = tid}{\Gamma, tid \vdash (\mathbf{cwait}\ cv\ l, T, G, Es) \rightarrow_c (\mathbf{cwait}\ cv\ l\ , T, G[l \mapsto \bot], Es)}\ \text{WaitF}$$

$$\frac{Es(\mathbf{ret}) = 0 \quad G(cv) = 1 \quad G(l) \neq tid}{\Gamma, tid \vdash (\mathbf{cwait}\ cv\ l\ , T, G, Es) \rightarrow_c (\mathbf{lock}\ l, T, G[cv \mapsto 0], Es)}\ \text{WaitT}$$

$$\frac{Es(\mathbf{ret}) = 0}{\Gamma, tid \vdash (\mathbf{signal}\ cv, T, G, Es) \rightarrow_c (\epsilon, T, G[cv \mapsto 1], Es)}\ \text{Signal} \quad \frac{Es(\mathbf{ret}) = 0 \quad T(tid') = \epsilon}{\Gamma, tid \vdash (\mathbf{join}\ tid', T, G, Es) \rightarrow_c (\epsilon, T, G, Es)}\ \text{Join}$$

$$\frac{tid \in dom(T) \quad T(tid) = c \quad Es(tid) = E \quad \Gamma, tid \vdash (c, T, G, Es) \rightarrow_c (c', T', G', Es')}{\Gamma \vdash (T, G, Es) \rightarrow_t (T'[tid \mapsto c'], G', Es')}\ \text{Threads}$$

$$\frac{Es(tid)(\mathbf{ret}) = 1}{\Gamma, tid \vdash (x \leftarrow \mathbf{retv}.Es', T, G, Es) \rightarrow_c (\epsilon, T, G, Es'[x \mapsto Es(\mathbf{retv}), \mathbf{ret} \mapsto 0])}\ \text{FunEnd}$$

Fig. 3. Part of small-step semantics of C

- `Waiting`, representing that the thread is waiting for the global clock with a time limit, specified by `localTime`. When the global clock reaches that time limit, the thread is released to run.
- `Available`, representing that the thread is waiting for a communication event. As soon as it receives the signal from another thread denoting that a compatible communication event is ready, it is woken up to run.
- `Waiting_Available`, representing that the thread is waiting both for the global clock to reach a time limit, and for a communication event. Either of them can release the thread to run.
- `Running`, representing that the thread is able to execute, not waiting for the global clock or communication events;
- A non-negative number i, representing that a communication is occurring on channel i. This is an intermediate state used to synchronize communication, meaning that the thread will carry out a communication on channel i, but has not finished doing so.

The thread states are shared by all threads and used for coordinating the execution of them. In our implementation, each thread is denoted by an ID among $\{0, 1, \cdots, N-1\}$, where N is the number of threads. An array `threadState`[N] is used to record the execution states of each thread.

3) Channels in C: In order to realize synchronized communication in C, we introduce global variables related to channels. In HCSP, each channel ch includes two ends: $ch?$ for input and $ch!$ for output, and they synchronize both on time and values transferred along ch. In C implementation, we introduce a structure `Channel` to define each single channel end, which is `(type, channelNo, pos)`, representing its type (0 for input and 1 for output), channel ID number (represented by natural numbers 0, 1, ...), and the pointer referring to the value it holds. Especially, each pair of input and output ends of a channel have the same channel ID. We also define three arrays for achieving synchronization between inputs and outputs: for each channel i, `channelInput[i]` and `channelOutput[i]` record the thread that is ready on corresponding input and output alone channel i, and thus available to participate in the communication. The default value is -1, denoting that no thread is available for the corresponding input or output; `channelContent` acts as a buffer to save the values transmitted along channels: the output ends write to it while the input ends read from it.

4) Locks and Condition Variables: A mutex `mutex` is introduced to protect the shared resources of threads. To access these resources, a thread must acquire `mutex` first. Moreover, to achieve synchronization between threads, an array `cond`[N] of condition variables are introduced, one for each thread.

B. Transformation of HCSP

1) Continuous evolution: For continuous evolution $\langle \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \& B \rangle$, we will first discretise it in HCSP and then transform the discretisation to C code. Before discretisation, the discretized time step h should be computed first. Given a HCSP process P, suppose the upper bound for the execution time of P is T_o, and the value precision is ε, then h is computed as follows:

- First, collect the set of ODEs of P, denoted by $\{ode_1(\overline{x_1}), \cdots, ode_k(\overline{x_k})\}$. Among them, for any $i \neq j$, $\overline{x_i}$ and $\overline{x_j}$ are continuous variables of the two ODEs resp. and they might have common variables;
- Next we compute the upper bound of the discretized step. On one hand, suppose the Lipschitz constants for the k ODEs exist and are $L^1, \cdots, L^k$, and the constants for the Runge-Kutta method are ${C_1}^1, \cdots, {C_1}^k, {C_2}^1, \cdots, {C_2}^k$, resp., as presented in Prop. 1. Then compute $M(h) \leq \frac{\varepsilon}{2}$ by assigning L, C_1, C_2 to be $\max\{L^i\}_{1 \leq i \leq k}$, $\max\{{C_1}^i\}_{1 \leq i \leq k}$,$\max\{{C_2}^i\}_{1 \leq i \leq k}$ resp., which finally obtains an upper bound for the discretized time step, denoted by h_1. We must have $h \leq h_1$.

- On the other hand, suppose for each ODE $ode_i(\overline{x_i})$ of the form $\langle \dot{\mathbf{x}}_i = \mathbf{f}_i(\mathbf{x}_i)\&B_i\rangle$, the derivative $\mathbf{f}_i(\mathbf{x}_i)$ is bounded over the interval $[0, T_o]$, satisfying $\|\mathbf{f}_i(\mathbf{x}_i)\| \leq U_i$. Then the distance between any two states at time a and b within one time step is bounded by $U_i \cdot \|a-b\|$. Let $U_i \cdot h < \frac{\varepsilon}{2}$, then $h < \frac{\varepsilon}{2U_i}$ for any i.
- At the end, let h be $\min(h_1, \min_{1\leq i\leq k}\{\frac{\varepsilon}{2U_i}\})$.

After h is calculated, for any continuous evolution $\langle \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})\&B\rangle$ occurring in p, it is transformed to the following discrete HCSP process according to Theorem 1:

$$\begin{aligned} &j := 1; (\neg(N(B,\varepsilon) \wedge N^n(B,\varepsilon)) \to j = 0;\\ &\qquad j = 1 \to \text{wait } h; \mathbf{x} := \mathbf{x} + h\mathbf{\Phi}(\mathbf{x}, h))^N; \qquad (1)\\ &N(B,\varepsilon) \wedge N^n(B,\varepsilon) \to \texttt{stop}; \end{aligned}$$

where $N = \lceil \frac{T-T_0}{h} \rceil$. Given a Boolean formula B, which can be considered as a set of states satisfying B, $N(B,\varepsilon)$ denotes the ε-neighbourhood of B, representing the set $\{a \mid \exists b. |a-b| < \varepsilon \wedge b \in B\}$ (obviously $B \subset N(B,\varepsilon)$); and $N^n(B,\varepsilon)$ is an abbreviation of $N(B[\mathbf{x} \mapsto \mathbf{x} + h\mathbf{\Phi}], \varepsilon)$, i.e. the ε-neighbourhood of B at next discretized time step. Instead of B, $N(B,\varepsilon) \wedge N^n(B,\varepsilon)$ is used to judge whether to continue evolving according to the ODE. With the help of robustly safe condition (to be explained in next section), the discretized implementation can always be guaranteed to escape within a tolerance. For the example shown in Fig. 5, the discretization escapes at next step $(n+1)h$ when the ODE violates B between nh and $(n+1)h$. With the C implementation of wait h by time delay, the C code is a direct translation of (1).

2) Continuous Interrupt: Continuous interruption $\langle \dot{x} = e\&B\rangle \trianglerighteq []_{i\in I}(ch_i* \to p_i)$ is first transformed to a sequential composition of discrete processes in HCSP, among which $n, N, \mathbf{\Phi}, \varepsilon$ are as defined before. For any HCSP process p, *HtoD*(p) transforms p to its discretised version.

$$\begin{aligned} &j_1 := 1; j_2 := -1;\\ &(\neg(N(B,\varepsilon) \wedge N^n(B,\varepsilon)) \to j_1 := 0;\\ &\ j_1 = 1 \wedge j_2 = -1 \to c := 0;\\ &\qquad \langle \dot{c} = 1\&c \leq h\rangle \trianglerighteq []_{i\in I}(ch_i* \to j_2 := i);\\ &\ j_1 = 1 \wedge j_2 = -1 \to \mathbf{x} := \mathbf{x} + h\mathbf{\Phi}(\mathbf{x}, h))^N;\\ &j_2 \geq 0 \to \mathbf{x} := \mathbf{x} + c\mathbf{\Phi}(\mathbf{x}, c); \textit{HtoD}(p_{j_2});\\ &j_1 = 1 \wedge j_2 = -1 \to \texttt{stop}; \end{aligned}$$

$c := 0; \langle \dot{c} = 1\&c \leq h\rangle$ is equivalent to wait h. We use it here in order to record the communication interrupt time by variable c, used to compute the state later (guarded by $j_2 \geq 0$). The C code corresponding to wait $h \trianglerighteq []_{i\in I}(ch_i* \to j_2 := i)$ is denoted by wait_comm$(h, \{ch_i*\}_{i\in I})$, to be explained next.

3) Wait communication: Fig. 4 presents the control flow for the C code corresponding to wait_comm$(h, \{ch_i*\}_{i\in I})$, after locking the mutex in the beginning: 1, Set `channelInput` and `channelOutput` for each communication to the current thread, indicating that $\{ch_i*\}_{i\in I}$ are all available, and meanwhile write values to the channel content buffer from outputs; 2, Look through the set of compatible channel ends to see if any is ready; 3, If some channel end is ready, set the state of the other thread to that channel (reserving communication on the other side); 4, Signal to the other thread and wait; 5, On returning from wait, reset thread state to be `RUNNING` and all channel sides corresponding to $\{ch_i*\}_{i\in I}$ to be unready; 6, In case a communication is performed, the function returns the channel index for later use; 7, If no channel is ready at the beginning, set target waiting time recorded by `localtime`; 8, Set thread state to be `AVAILABLE` and wait; 9, On returning from wait, check the thread state to determine which channel among $\{ch_i\}_i$ is matched to perform the communication; 10, Update local time to be the local time of the other thread, and reset all states as in step 5; 11, If no communication occurs during time h, return -1 and terminate. Input and output can be considered as special cases of wait_comm.

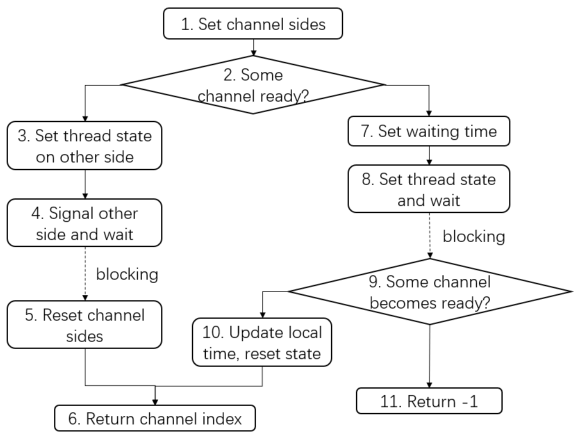

1. Set channel sides
2. Some channel ready?
3. Set thread state on other side
4. Signal other side and wait
blocking
5. Reset channel sides
6. Return channel index
7. Set waiting time
8. Set thread state and wait
blocking
9. Some channel becomes ready?
10. Update local time, reset state
11. Return -1

Fig. 4. The control flow of C code for wait_comm

Above we present the primitive functions related to communication and ODE used for transformation. By calling these primitives, each type of sequential compound processes of HCSP can be transformed inductively. At last, the top HCSP process, in the form of parallel composition, is transformed to a set of threads corresponding to all its sequential subprocesses.

V. Correctness of the Transformation

In this section, we sketch the correctness proof of the transformation. We first introduce the notion of robustly safe HCSP processes (Sect. V-A). Then we state the theorem on approximate bisimulation between HCSP and C (Sect. V-B). The first part of the proof, that HCSP is approximately bisimilar to its discretization, is similar to [YJW+20] and omitted here. Finally, we sketch the proof of exact bisimulation between HCSP and the generated C code in Sect. V-C.

A. Robustly Safe HCSP Processes

Due to the discretization of the ODE, the behaviors of an HCSP process and its transformed C program are not exactly equivalent but allow a difference within given precisions. An HCSP process must be robust to tolerable errors. Especially, the discretization should preserve the control behavior in alternation like $B \to p$, and moreover, it should be able to detect the change of domain boundary in continuous evolution like $\langle \dot{x} = f(x)\&B\rangle$ and locate the changing point within a tolerance (which is the so-called zero-crossing detection and location in hybrid systems [ZYM08]). Hence, we introduce the notion of *robustly safe processes* with given precisions in Def. 4. Let ϕ denote a Boolean formula and ϵ a precision, define $N(\phi, -\epsilon)$ as the set $\{v \mid v \in \phi \wedge \forall u \in \neg\phi. |v - u| > \epsilon\}$, which is a subset of ϕ and furthermore the distance from all states in it to the boundary of ϕ are greater than ϵ.

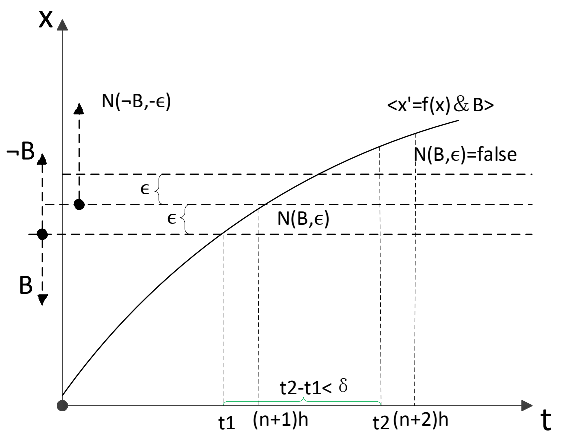


Fig. 5. The (δ, ϵ)-robustly safe continuous statement.

Definition 4 ((δ, ϵ)-robustly safe)**.** *An HCSP process P is (δ, ϵ)-robustly safe, for given time precision $\delta > 0$ and value precision $\epsilon > 0$, if the following two conditions hold:*

- *For every alternative process $B \to Q$ occurring in P with B depending on continuous variables of P, then the states v reached before the execution of $B \to Q$ satisfy $v \in N(B, -\epsilon)$ or $v \in N(\neg B, -\epsilon)$;*
- *For every continuous evolution $\langle \dot{\vec{x}} = \vec{e} \& B \rangle$ occurring in P, suppose its initial state is v_0, and B turns to false at state v and time t, then there exists $\widehat{t} \in (t, t + \delta)$ s.t. $U(v[\vec{x} \mapsto X(\widehat{t}, \widetilde{\vec{v}_0})], \epsilon) \subseteq N(\neg B, -\epsilon)$, where $X(t, \widetilde{\vec{v}_0})$ is the solution of $\dot{\vec{x}} = \vec{e}$ at t with initial value $\widetilde{\vec{v}_0}$.*

Def. 4 requires: (1) All reachable states before $B \to P$ are ϵ-far from the boundary of B, which is also the boundary of $\neg B$. (2) As shown by Fig. 5, when B turns false at time t_1, then before time t_2 (within δ tolerance), the ODE continues to go away from the boundary of B, at least 2ϵ further at t_2. According to our discretization (by checking $N(B, \epsilon) \wedge N^n(B, \epsilon)$), it is guaranteed to detect the change of B at next step of t_1, i.e. $(n+1)h$ in the example. Computing the robustly safe parameters (δ, ϵ) is very challenging and some methods for computing them are given in [YJW$^+$20].

B. Approximate Bisimulation between HCSP and C

We then state the main theorem for approximate bisimulation between HCSP and C. Next $\mathit{HtoD}_{h,\varepsilon}(P)$ and $\mathit{HtoC}_{h,\varepsilon}(P)$ denote the discretized version and the generated C code of HCSP process P with precisions h and ε resp.

Theorem 2. *Let P be a HCSP process and $T > 0$ is an upper bound of time. Suppose P is (δ, ϵ)-robustly safe, and for any ODE $\dot{x} = f(x)$ occurring in P, f is Lipschitz continuous. Then, for any precision $\varepsilon \in (0, \epsilon]$, there must exist $h > 0$ such that $P \cong_{h,\varepsilon} \mathit{HtoC}_{h,\varepsilon}(P)$ holds on $[0, T]$.*

We will prove Theorem 2 in two steps: $P \cong_{h,\varepsilon} \mathit{HtoD}_{h,\varepsilon}(P)$, and $\mathit{HtoD}_{h,\varepsilon}(P) \cong \mathit{HtoC}_{h,\varepsilon}(P)$. Due to page limit, we give a proof sketch for $\mathit{HtoD}_{h,\varepsilon}(P) \cong \mathit{HtoC}_{h,\varepsilon}(P)$ here, while the details for the whole proof can be found in the full version [**?**].

C. Proof of discrete bisimulation

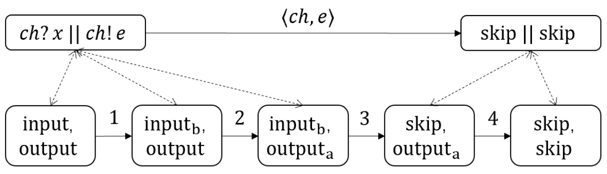
In this section, we sketch the proof of exact bisimulation between discretized HCSP and generated C code. The overall strategy is to define a bisimulation relation between the states of HCSP and concurrent C, and show that transitions on the HCSP side and C side correspond under this relation.

First, we note that there is only one mutex to protect all shared resources in the generated C code. The translation of wait_comm (and hence its special cases delay, input and output) all lock the mutex at the beginning and unlock it at the end. Hence, we can consider its execution to consist of a small number of atomic blocks. The execution of wait_comm may be blocked in the middle for two different reasons:

a) After setting the channel sides, at least one communication is ready to be performed. After nondeterministically choosing a communication to perform and reserve it by setting the thread state on the other side, it signals the other side and waits for it to respond. The thread returns from blocking when the other side responds with a signal.
b) After setting the channel sides, no communication is ready to be performed. Then it enters into waiting mode for both communications and a time limit. The thread returns from blocking when either the time limit is reached, or one of the matching communications is ready.

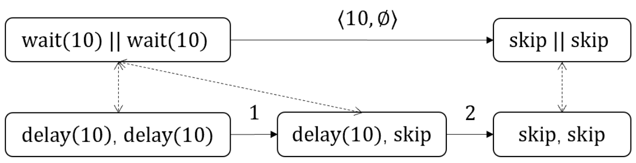
The translation of delay, input and output are simply special cases of the above. For delay it does not attempt to seek a matching communication. For input/output, the time limit for waiting is ∞. We use subscript a and b to denote the second atomic block of execution in the cases a) and b), respectively.

Hence, each execution of wait_comm consists of two atomic blocks. The execution of each matching communication then consist of four atomic blocks (two on the input side and two on the output side), one of which updates the value of the variable to be read on the input side. There are two cases, depending on whether the input side or the output side is ready first. The first case, when the input is ready first, is illustrated as follows.



The transition $ch.e$ on the HCSP side corresponds to four atomic steps on the C side. In step 1, the input side is ready and blocks on waiting for the output side. In step 2, the output side starts and detects the input is ready, copies the value to be sent into the channel, reserves the communication and signals the input side. In step 3, the input side copies the value in the channel into the variable to be read, cleans up after the communication, and finally signals back to the output side. In step 4, the output side cleans up the communication as well upon receiving the signal. In this sequence, the value to be read is updated on the third step. Hence, the proof strategy is as follows: the bisimulation relation is defined so that the HCSP program $ch?x \parallel ch!e$ is related to the first three states on the C side, while skip $\parallel$ skip is related to the final two states (as indicated by the dashed arrows).

The case when the output is ready first is mostly symmetric to the above. We next consider the case of bisimulation relation for delays. The transition $\langle d, rdy\rangle$ advances the clock on the HCSP side, and should correspond to updating the global variable `currentTime` on the C side. This may correspond to multiple atomic steps on the C side, as illustrated below.



In step 1, only the first thread is delayed by 10, incrementing `localTime` of that thread by 10 but leaving the global `currentTime` unchanged. In step 2, the second thread is delayed by 10, leading to the increment of `currentTime` as well. Hence, the bisimulation relation should be defined so that the HCSP program wait(10) $\parallel$ wait(10) is related to the first two states on the C side, while skip $\parallel$ skip is related to the final state.

In summary, the overall strategy of the bisimulation proof is as follows: (1) Bisimulation relation is defined between the states on the HCSP side and C side. There is a direct correspondence between processes in HCSP and threads in C. For each process/thread, if the HCSP program and C program that remains to be executed agree, then the other parts of the C state have their default values: thread state is `RUNNING` and `localTime` should agree with `currentTime`. There are several other cases where the program to be executed on the C side runs ahead or behind the HCSP program, which is indicated by different conditions on the other parts of the C state. (2) For each atomic block of execution in wait_comm, we prove that if the starting state satisfies the bisimulation relation, then the ending state satisfies it as well. There are several cases to consider, depending on whether the C program to be executed runs ahead, behind, or agrees with the HCSP program. We also make use of the fact that in certain states, some threads in C are blocked by the condition variable.

VI. Case Study

We adopt the realistically-scaled Automatic Cruise Control System (ACCS for short) from [XWZ+22] as the case study. The HCSP model of ACCS is moderately large and covers all the concerned features of HCSP and it is robustly safe (Sect. V-A) and hence can be discretized using our approach. In this section, we translate this HCSP model to the C code and compare its execution with the simulation of the HCSP model [XWZ+22] and the execution of the C code generated directly from the original AADL $\oplus$ S/S model [ZLW+19].

A. The Automatic Cruise Control System

The architecture of ACCS consists of three layers. The physical layer is the physical vehicle. The software level defines control of the system and it contains three processes for obstacle detection, velocity control, and panel control, and each process is composed of several threads. These processes interact with the environment (the physical layer) through devices. The platform layer consists of a bus and a processor. The connections between processes and devices could be bound to the bus and all the threads are bound to the processor, with HPF (High-Priority-First) scheduling policy.

B. Translation from HCSP to C

In work [ZLW+19], combined models of AADL $\oplus$ S/S are translated to C code directly. In this paper, we generate the C code of the case study from its HCSP model which has been verified in [XWZ+22]. Since the correctness of the translation is guaranteed (Sect. V), the generated code, especially the code for the controller (the velocity control mentioned in Sect. VI-A), satisfies the safety requirement and therefore is reliable. We use pthreads to implement the communication between processes and the correctness of the code generation has been proved in Sect. V. The generated C code is of 3500–4000 lines, longer than the C code (about 2500 lines) generated by [ZLW+19]. One major reason is that the generated C code in this paper is approximately bisimilar to the original AADL $\oplus$ S/S model of ACCS, which means the detailed behaviours of ACCS are reflected in the C code and vice versa, while it is not the case for [ZLW+19] where no such bisimulation can be guaranteed.

C. Comparison

We consider the following scenario. At the beginning, the driver pushes the `inc` button three times with time interval 0.5s in between to set a desired speed to 3m/s. After 30s, the driver pushes the `dec` button twice in 0.5s time intervals to decrease the desired speed. On the obstacle side, we assume that the obstacle appears at time 10s and position 35m, then moves ahead with velocity 2m/s, before finally moving away at time 20s and position 55m.

The top of Fig. 6 shows the execution results of the vehicle speed, where the black line denotes the desired velocity set by the driver, and the red, green, and blue lines denote the results of our work, the work of [ZLW+19], and the simulation of the HCSP model of ACCS [XWZ+22], respectively. We can see that the execution result (red line) of the generated C code is almost the same with the simulation (blue line) of its HCSP model. Specifically, the average relative error (ARE) between the time series of the velocity generated from the HCSP model and its C code is 0.138% with the variance 4.686×10^{-5}. Besides, we can also observe that there is negligible difference between the results of the C code generated by [ZLW+19] (green line) and the C code generated in this paper (red line): the ARE is 0.182% with the variance 4.232×10^{-3}. Readers can refer to [XWZ+22] for more details about this example.

It should be noted that although the generated C code in this paper is longer and somewhat less efficient than the C code generated directly from the original graphical model [ZLW+19], the former is more reliable because it is translated from the (formal) HCSP model, which can be verified by tools like HHLPy [SBZ23], and moreover, the correctness of the translation can be guaranteed (Sect. V).

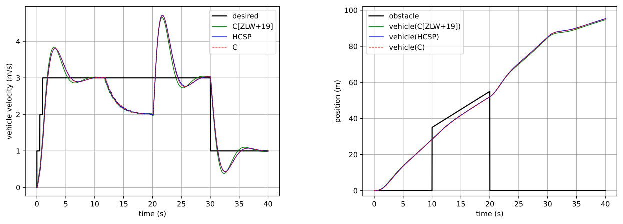


Fig. 6. Comparison of execution results

VII. Conclusions

This paper presents a formally verified C code generator from hybrid systems modelled using HCSP. In the transformation from HCSP to C, two main issues are addressed: the discretisation of continuous evolution and interrupts, and the realisation of synchronized communication of HCSP in concurrent C with the use of mutexes. For the first problem, we provide a method to compute a discretised time step such that under some robustness conditions, the continuous HCSP processes and the corresponding discrete ones are approximate bisimilar within the given precision. For the second problem, we propose a new notion of bisimulation relation: each transition step of HCSP corresponds to a sequence of atomic transitions of C, among which the substantial step to perform the equivalent transition in HCSP is determined. By transitivity of bisimulation relations, the correctness of the transformation from HCSP to C is guaranteed. Finally, we investigate a case study on Automatic Cruise Control System and its code generation using our approach. For future work, we consider to apply our approach to more practical case studies in industry.

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