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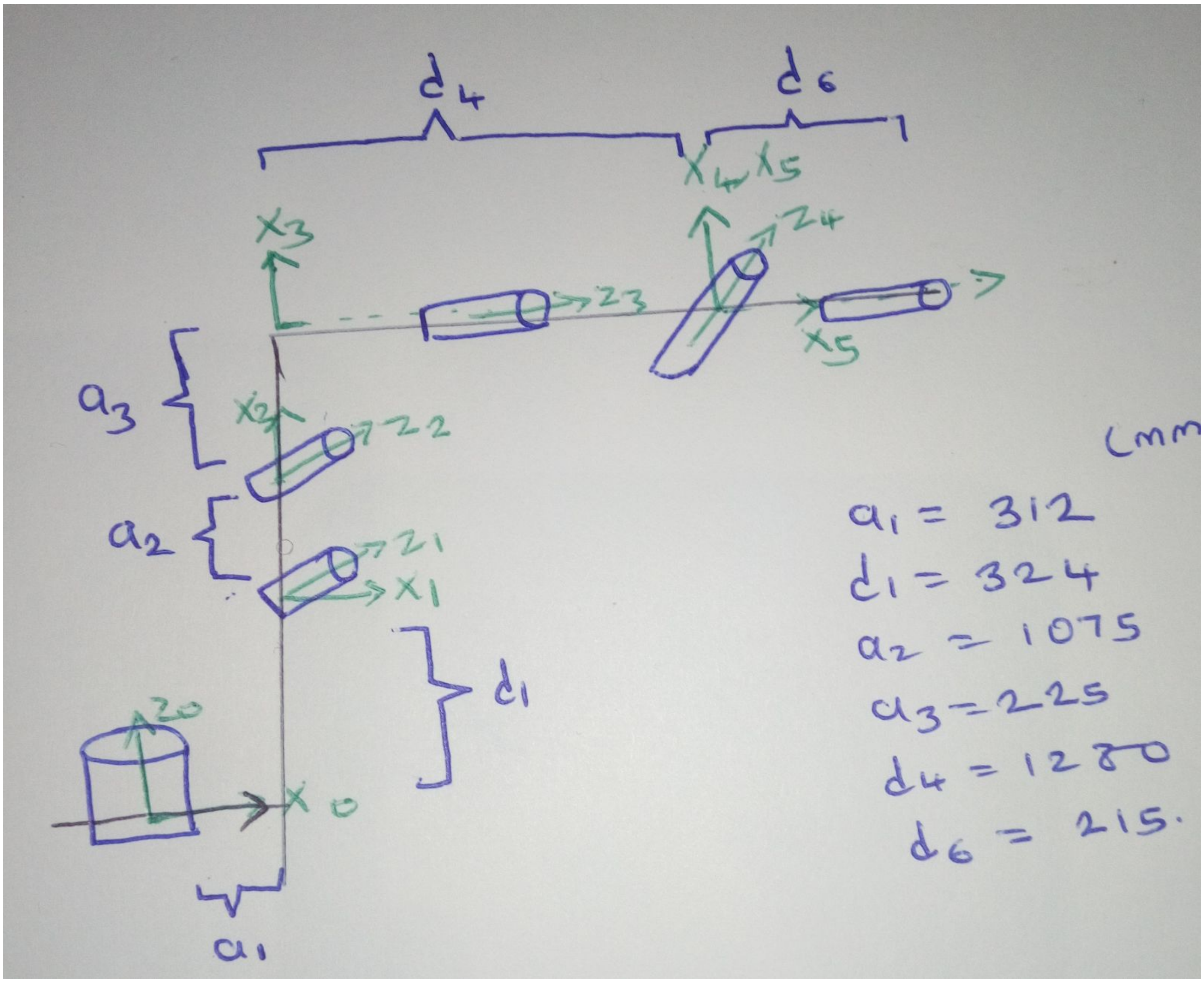
Report on Forward and Inverse Kinematics of the Fanuc R-2000iC/165F

Kinematic Analysis

Forward Kinematics

The joints coordinates and frames locations such that DH parameters were easily identified. Location of frame 3 was shifted to align with X axis of frame 2. Frame 4 and Frame 5 are also coinciding at frame 4 as shown in the diagram below.



DH Parameter Table

Link	**θ(i)**	**d(i) [mm]**	**a(i) [mm]**	**alpha(i) [mm]**
1	q1	0.324	0.312	$-pi/2$
2	q2	0	1.075	0
3	q3	0	0.225	$-pi/2$
4	q4	1.280	0	$pi/2$
5	q5	0	0	$-pi/2$
6	q6	0.215	0	0

Homogeneous Matrices

Homogeneous matrix transformation for each link were determined using the DH parameter

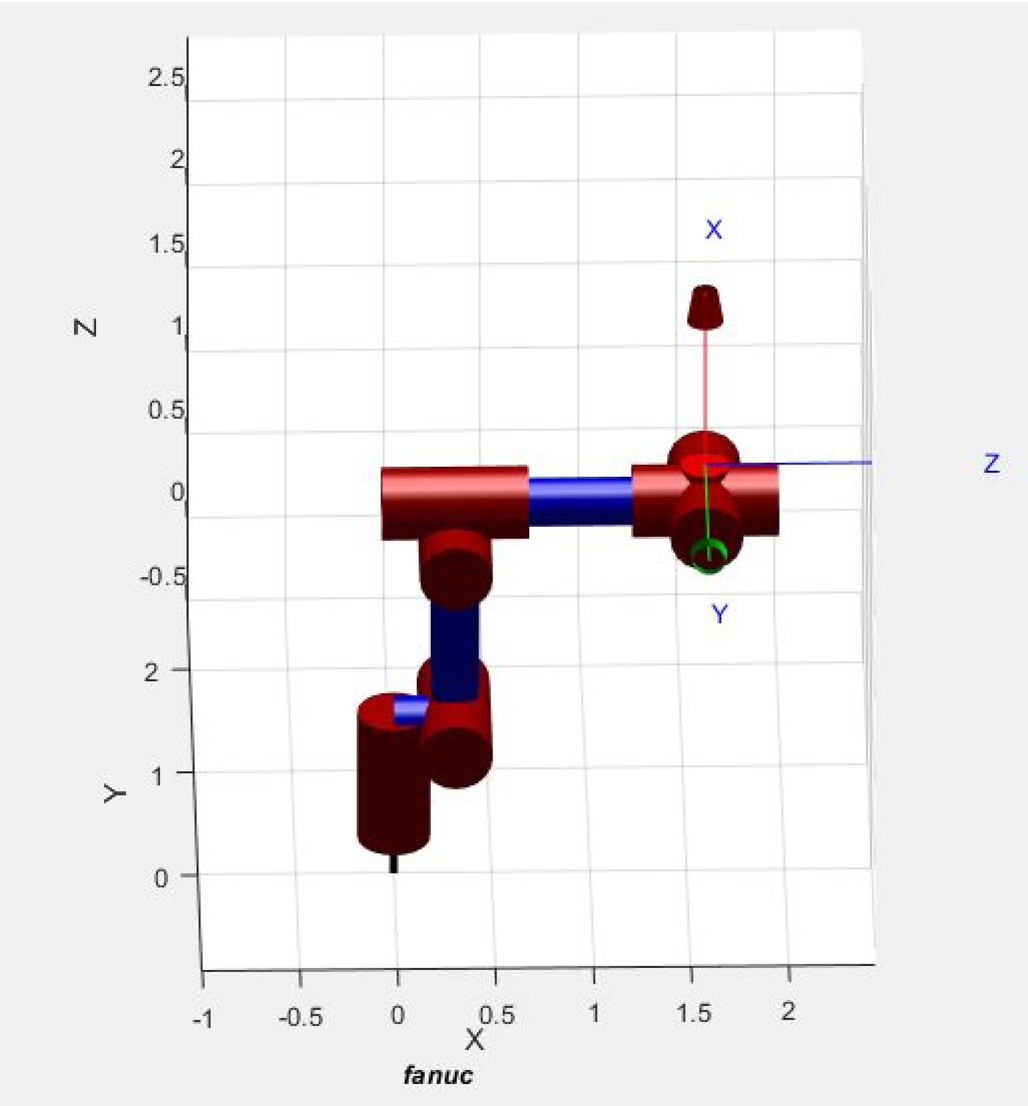
$$T = R_z(q)\, T_z(d)\, R_x(\propto)\, T_x(a)$$

Homogeneous matrix transformation was done for each link and the arm pose with respect to the base coordinate was obtained below in MATLAB.
This was implemented with *function DH* in DH.m file. This function was implemented in the *calcDH.m* file.

$$T_3^0 = \begin{bmatrix} cq_1q_2cq_1 & sq_1 & -sq_2q_3cq_1 & cq_1\left(a_1 + a_3.cq_2q_3 + a_2cq_2\right) \\ cq_1q_2sq_2 & -cq_1 & -sq_2q_3sq_1 & sq_1\left(a_1 + a3.cq_2q_3 + a_2cq_2\right) \\ -sq_2q_3 & 0 & -cq_{2q_3} & d1 - a_3sq_2q_3 - a_2sq_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} ----eqn(1)$$

In the same way, the forward kinematics of the end effector joint is obtained.

By using the values of the DH parameters in the datasheet and substituting the relevant values, And setting: **theta=[0 -pi/2 0 0 0 0]** the figure below was obtained. This also confirms the initial diagram shown in the datasheet.



The forward kinematics for the arm, wrist and end effector (T^0_3 , T^3_6, T^0_6) was equally obtained accordingly.
Note: theta corresponds to joint angles

Result of Forward Kinematics approach for different values of joint angles and the observed pose

	T^0_3 : arm	T^0_6 : end effector wrt to o
theta=[0 -pi/2 0 0 0 0]	$\begin{bmatrix} 0 & 0 & 1 & 0.312 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1.624 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 & 1.592 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1.839 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
theta=[0 -pi/2 pi/2 pi/3 0 pi/4]	$\begin{bmatrix} 0 & 0 & 1 & 0.5370 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 1.3990 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -0.2598 & -0.9659 & 0 & 0.4814 \\ -0.9658 & 0.2588 & 0 & -0.2077 \\ 0 & 0 & -1 & 0.1190 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
theta=[pi/8 -pi/6 pi/2 pi/3 0 pi/4]	$\begin{bmatrix} 04619 & 0.3827 & -0.8 & 1.2523 \\ 0.1913 & -0.9239 & -0.3314 & 0.5187 \\ -0.8660 & 0 & -0.5 & 0.666 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.2501 & -0.5452 & -0.8001 & 0.2819 \\ -0.9419 & 0.0543 & -0.3314 & -0.1080 \\ 0.2241 & 0.8365 & -0.5 & 0.0748 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Inverse Kinematics

To solve the Inverse kinematics problem, the robot was decoupled into an arm and wrist. Link1 -3 representing the arm and link 4-6 representing the wrist. This is in accordance to Piepers rule.

Thus, the following equation was obtained;

$$\mathbf{T}^0_6 = \mathbf{T}^0_3 \cdot \mathbf{T}^3_6 - - - - eqn(2)$$

Recall that the arm transformation matrix (T^0_3) was obtained in eqn(1) above;
Thus the joint1-3 angles corresponding to q1, q2 and q3 can be obtained.

$$T_3^0 = \begin{bmatrix} cq_1q_2cq_1 & sq_1 & -sq_2q_3cq_1 & cq_1\left(a_1 + a_2.cq_2q_3 + a_2cq_2\right) \\ cq_1q_2sq_2 & -cq_1 & -sq_2q_3sq_1 & sq_1\left(a_1 + a_2.cq_2q_3 + a_2cq_2\right) \\ -sq_2q_3 & 0 & -cq_{2q_3} & d1 - a_3sq_2q_3 - a_2sq_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} ----eqn(3)$$

Thus, we obtain

$$\boldsymbol{q1 = atan2(\mathbf{r12}, -\mathbf{r22})}$$

$$Px = cq_1\left(a_1 + a_2.cq_2q_3 + a_2cq_2\right)$$
$$Py = sq_1\left(a_1 + a_2.cq_2q_3 + a_2cq_2\right)$$
$$Pz = d1 - a_3sq_2q_3 - a_2sq$$

To obtain q2, observe that Pz, $\boldsymbol{sq2q3}$ *is unknown. But r31 gives sq2q3, this can then be substituted into Pz and sq2 make the subject of formula. From sq2, we obtain cq2 and obtain q2 as*

$q2 = a\tan2(s2, c2)$

i.e:

$c_{21} = +\sqrt{1 - s_2^{\,2}}$

$c_{22} = -c_{21}\sqrt{1 - s_2^{\,2}}$

Using the above in the matlab program, q2 is obtained

Meanwhile, q2q3 $\left(q_2 + q_3\right)$ *can be obtained and we get q3 directly from it simce q2 is already known.*

$$q23 = a\tan2\left(-r_{31}, -r_{33}\right)$$

$$\boldsymbol{q_{21} = atan2\left(\mathbf{s}_2,\ \mathbf{c}_{21}\right)}$$
$$\boldsymbol{q_{22} = atan2\left(\mathbf{s}_2,\ \mathbf{c}_{22}\right)}$$

$q3 = q23 - q2$

But we have two solutions for q2,

Thus :

$q_{31} = q_{23} - q_{21}$

$q_{32} = q_{23} - q_{22}$

Alternatively, q2 and q3 could be obtained as shown below. (The result here is only an analytical solution and was not implemented in matlab)

$$Px = cq_1\left(a_1 + a_3.cq_2q_3 + a_2cq_2\right)$$
$$Py = sq_1\left(a_1 + a3.cq_2q_3 + a_2cq_2\right)$$
$$Pz = d1 - a_3sq_2q_3 - a_2sq_2$$
$$\frac{Px}{cq_1} - a_1 = cq_1\left(a_3.cq_2q_3 + a_2cq_2\right)$$
$$\frac{Py}{sq_1} - a_1 = sq_1\left(a_3.cq_2q_3 + a_2cq_2\right)$$
$$Pz - d1 = -a_3sq_2q_3 - a_2sq_2$$

Combining the above equations, it can be obtained:

$$\left(\frac{Px}{cq_1} - a_1\right)^2 + \left(\frac{Py}{sq_1} - a_1\right)^2 + (Pz - d1)^2 = a_2^2 + a_3^2 + 2a_2a_3c_3$$

$$c_3 = \frac{\left(\frac{Px}{cq_1} - a_1\right)^2 + \left(\frac{Py}{sq_1} - a_1\right)^2 + (Pz - d1)^2 - a_2^2 - a_3^2}{2a_2a_3} \in (-1, 1)$$

$$\pm s_3 = \pm\sqrt{1 - c_3^2}$$
$$q_{3+} = a\tan2\left(+s_3,\ c_3\right)$$
$$q_{3-} = a\tan2\left(-s_3,\ c_3\right)$$

Result of Inverse Kinematics to obtain wrist angles

This is tested in matlab by using the result of a known pose through forward kinematics and then compare the results of q1, q2 and q3 to the input of the forward kinematics. The same angles were obtained, with the difference that the inverse kinematics approach does not always give unique solution.

Once q1 q2 and q3 are obtained, q4 q5 and q6 can then be obtained.

$$\mathbf{T}_6^0 = \mathbf{T}_3^0 \cdot \mathbf{T}_6^3$$

From the above, $T_6^{\ 3}$ *is obtained and the rotational part is extracted to have:*

$$R_6^3 = R_3^{0-1} \cdot R_6^0 = R_3^{0T} \cdot R_6^0$$

Meanwhile the wrist rotation corresponds to euler angle formation. Thus if the transformation matrix of T^3_6 Is obtained, using euler angle approach, q4, q5 and q6 could be obtained.

Recall , euler angle approach:

$$R_{q4q5q5} = R_z R_y R_z$$

$$= \begin{bmatrix} cq_4cq_5cq_6 - sq_5sq_6 & -cq_4cq_5sq_6 - sq_4sq_6 & cq_4sq_5 \\ sq_4cq_5cq_6 + cq_5sq_6 & -sq_4cq_5sq_6 + cq_4cq_6 & sq_4sq_5 \\ -sq_5cq_6 & sq_5q6 & cq_5 \end{bmatrix}$$

$$q_5 = a\tan2\left(r_{13},\ r_{23} \right)$$

$$or$$

$$q_5 = a\tan2\left(r_{33},\ -\sqrt{1 - {r_{33}}^2} \right)$$

$$q_4 = a\tan2\left(r_{13},\ r_{23} \right)$$

$$q_6 = a\tan2\left(-r_{31},\ r_{32} \right)$$

If sin(q5) = 0; q5 = 0
Then :
q4+q6 = atan2(r11, r21)= atan2(r11, -r21)
When this occurs, there are infinite solutions for q5 and q6 in the inverse kinematic problem.

The solution for q4 , q5 and q6 was not implemented in matlab.
The analytical solution is provided in the report.

Noteby that calcDH.m would be ran in matlab and DH.m would only be placed in the same folder as calcDH.m. FanucAnalytical gives the result without substituting actual values of joint variables and constants in terms of q1 q2 q3 q4 q5 q6 and other dh parameters.