EE 517: Optimal Control **HW # 3** **Due: March 22, 2022**

Problem # 1:

Consider the following continuous LQR problem:

$$A=\begin{bmatrix}-1 & 2\\ 0 & 1\end{bmatrix}\quad B=\begin{bmatrix}-1\\ 1\end{bmatrix}\quad H=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}\quad Q=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\quad R=1\quad t_f=5\sec\quad x(0)=\begin{bmatrix}6\\ -8\end{bmatrix}$$

1. Write the HJB Equation for this problem. Assume that $J^*=\frac{1}{2}x^T(t)P(t)x(t)$, solve for P(t) and find $u^*(t)$, plot $u^*(t)$ and $x^*(t)$
2. Let $t_f\to\infty$. Find the steady state gain, Using this gain find and plot $u^*(t)$ and $x^*(t)$, compare with the results from above.
3. Repeat (2) with $Q^{new}=\begin{bmatrix}50 & 0\\ 0 & 0.01\end{bmatrix}$
4. Repeat (2) with $R^{new}=25$

You will need to discuss and compare the different cases, and draw conclusions. Code and figures alone are not enough.

Problem # 2:

Consider the following continuous time linear quadratic regulator problem:

$$\dot{x}(t)=1.5x(t)+0.5u(t);\quad x(0)=10$$

$$J=\frac{1}{2}x^2(t_f)+\frac{1}{2}\int\{x^2(\tau)+4u^2(\tau)\}d\tau;\quad t_f=10$$

1. Write the Riccati Equation
2. Solve it using the 2 different approaches discussed in class (numerical solution of the differential equation, and $P=EF^{-1}$)
3. Find and plot K(t)
4. Plot x(t) (closed loop)