Compliant control of quadruped with VSA(Variable Stiffness Actuator)

Thesis Defense

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- Floating base dynamics of the robot
- Close loop kinematics chain between ground and trunk
- Highly non-linear dynamics of the robot
- Unknown contact force between the ground and robot feet

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})=\mathbf{T}\mathbf{u}+\mathbf{F}_C^T\boldsymbol{\lambda} \tag{1}$$

$$\mathbf{q}=\begin{bmatrix}\mathbf{q}_b\\ \mathbf{q}_j\end{bmatrix} \tag{2}$$

Full motion of the system:
6 unactuated+ 12 actuated = 18DOF

When the spring parameters are fixed the actuator is called a Serial-Elastic Actuator (SEA). In VSA, the string parameter is not fixed and is a controllable and desired term.

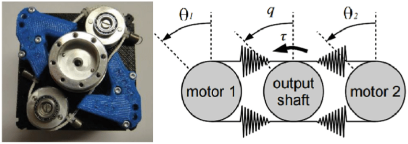


Figure 1: qbmove VSA motor

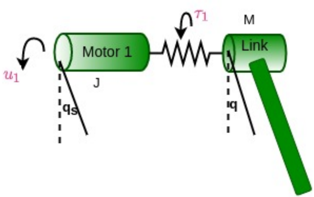


Figure 2: SEA

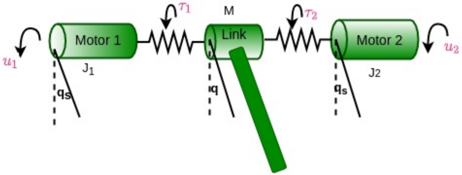


Figure 3: Antagonistic VSA

Constrained LQR

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{T}\mathbf{u} + {\mathbf{F_c}}^{\mathrm{T}}\boldsymbol{\lambda} \\ \mathbf{F_c}\dot{\mathbf{q}} = \mathbf{0} \\ \mathbf{F_c}\ddot{\mathbf{q}} + \dot{\mathbf{F_c}}\dot{\mathbf{q}} = \mathbf{0} \end{cases} \tag{3}$$

$$\mathbf{N} = \mathrm{null}\left(\begin{bmatrix} \mathbf{F}_c & \mathbf{0} \\ \dot{\mathbf{F}}_c & \mathbf{F}_c \end{bmatrix}\right) \tag{4}$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{c} \tag{5}$$

Project the linearized dynamics by projecting the linearized system into the nullspace of the jacobian constraint. We re-express the dynamics in first (3) in terms of $\mathbf{z}$ by multiplying it by $\mathbf{N}^\top$ on the left:

$$\mathbf{N}^\top\dot{\mathbf{x}} = \mathbf{N}^\top\mathbf{A}\mathbf{x} + \mathbf{N}^\top\mathbf{B}\mathbf{u} + \mathbf{N}^\top\mathbf{S}\lambda + \mathbf{N}^\top\mathbf{c} \tag{6}$$

$$\dot{\mathbf{z}} = \mathbf{A}_N\mathbf{z} + \mathbf{B}_N\mathbf{u} + \mathbf{c}_N, \quad \mathbf{u}(t) = -\mathbf{K}_N\mathbf{z} \tag{7}$$

where:

$$\mathbf{A}_N = \mathbf{N}^\top\mathbf{A}\mathbf{N} \quad \mathbf{B}_N = \mathbf{N}^\top\mathbf{B} \quad \mathbf{c}_N = \mathbf{N}^\top\mathbf{c} \quad K_N = \mathbf{N}^\top\mathbf{K_N}$$

$$\begin{cases} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})+\mathbf{K}(\mathbf{q}-\mathbf{q}_s)=\mathbf{0} \\ \mathbf{J}\ddot{\mathbf{q}}_s-\mathbf{K}(\mathbf{q}-\mathbf{q}_s)=\mathbf{u} \end{cases} \tag{8}$$

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{v} \tag{9}$$

$$\mathbf{x}=\begin{bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\end{bmatrix} \quad \mathbf{x}_s=\begin{bmatrix}\mathbf{q}_s\\ \dot{\mathbf{q}}_s\end{bmatrix}, \quad \mathbf{v}=\mathbf{K}(\mathbf{q}_s-\mathbf{q})$$

Let us define: $\mathbf{C}_1=\begin{bmatrix}\mathbf{I}_{n\times n} & \mathbf{0}_{n\times n}\end{bmatrix}$ and $\mathbf{C}_2=\begin{bmatrix}\mathbf{0}_{n\times n} & \mathbf{I}_{n\times n}\end{bmatrix}$

$$\mathbf{J}\ddot{\mathbf{q}}_s-\mathbf{K}(\mathbf{q}-\mathbf{q}_s)=\mathbf{u} \tag{10}$$

$$\ddot{\mathbf{q}}_s=-\mathbf{J}^{-1}\mathbf{K}(\mathbf{C}_1\mathbf{x}_s-\mathbf{C}_1\mathbf{x})+\mathbf{J}^{-1}\mathbf{u} \tag{11}$$

We take (9) and (11) and express it in form of a new linear state representation of:

$$\dot{\mathbf{z}} = \tilde{\mathbf{A}}\mathbf{z} + \tilde{\mathbf{B}}\mathbf{u} + \tilde{\mathbf{C}} \tag{12}$$

$$\dot{\mathbf{z}} = \begin{bmatrix} \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{K}(\mathbf{C}_1\mathbf{x}_s - \mathbf{C}_1\mathbf{x}) \\ [\mathbf{0} \quad \mathbf{I}]\mathbf{x}_s \\ -\mathbf{J}^{-1}\mathbf{K}(\mathbf{C}_1\mathbf{x}_s - \mathbf{C}_1\mathbf{x}) + \mathbf{J}^{-1}\mathbf{u} \end{bmatrix} \tag{13}$$

$$\dot{\mathbf{z}} \approx \underbrace{\begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{K}\mathbf{C_1} & \mathbf{B}\mathbf{K}\mathbf{C_1} \\ \mathbf{0} & \mathbf{C}_2 \\ \mathbf{J^{-1}}\mathbf{K}\mathbf{C_1} & -\mathbf{J^{-1}}\mathbf{K}\mathbf{C_1} \end{bmatrix}}_{\tilde{\mathbf{A}}} \mathbf{z} + \underbrace{\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{J^{-1}} \end{bmatrix}}_{\tilde{\mathbf{B}}} \mathbf{u} \tag{14}$$

where:

$$\mathbf{z} = \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_s \end{bmatrix} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \\ \mathbf{q}_s \\ \dot{\mathbf{q}}_s \end{bmatrix} \tag{15}$$

$$\begin{cases} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 \\ \mathbf{J}_1\ddot{\mathbf{q}}_{s1} + \boldsymbol{\tau}_1 = \mathbf{u}_1 \\ \mathbf{J}_2\ddot{\mathbf{q}}_{s2} + \boldsymbol{\tau}_2 = \mathbf{u}_2 \end{cases} \tag{16}$$

$$\boldsymbol{\tau}_i = \boldsymbol{\sigma}_i \sinh(\boldsymbol{a}_i(\mathbf{q}_{si} - \mathbf{q})) \quad , \quad i=1,2 \tag{17}$$

The stiffness of the VSA motor is given as:

$$\mathbf{K}_i = -\frac{\partial \boldsymbol{\tau}(\mathbf{q}_s - \mathbf{q}, \boldsymbol{\sigma})}{\partial \mathbf{q}} = \quad \boldsymbol{\sigma}_i \mathbf{a}_i \cosh(\boldsymbol{a}_i(\mathbf{q}_{si} - \mathbf{q})) \quad , \quad i=1,2 \tag{18}$$

$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}) + \mathbf{g}(\mathbf{z},\mathbf{u}) \tag{19}$$

$$\mathbf{f}(\mathbf{z}) = \begin{bmatrix} \mathbf{A}\mathbf{x} + \mathbf{B}(\boldsymbol{\sigma}_1 \sinh(\boldsymbol{a}_1(\mathbf{C}_1\mathbf{x}_{s1} - \mathbf{C}_1\mathbf{x}))) + \mathbf{B}(\boldsymbol{\sigma}_2 \sinh(\boldsymbol{a}_2(\mathbf{C}_1\mathbf{x}_{s2} - \mathbf{C}_1\mathbf{x}))) \\ \mathbf{C}_2\mathbf{x}_{s1} \\ -\mathbf{J}_1^{-1}(\boldsymbol{\sigma}_1 \sinh(\boldsymbol{a}_1(\mathbf{C}_1\mathbf{x}_{s1} - \mathbf{C}_1\mathbf{x}))) \\ \mathbf{C}_2\mathbf{x}_{s2} \\ -\mathbf{J}_2^{-1}(\boldsymbol{\sigma}_1 \sinh(\boldsymbol{a}_2(\mathbf{C}_1\mathbf{x}_{s2} - \mathbf{C}_1\mathbf{x}))) \end{bmatrix}$$

$$\mathbf{g}(\mathbf{z}) = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{J}_1^{-1}\mathbf{u}_1 \\ \mathbf{0} \\ \mathbf{J}_2^{-1}\mathbf{u}_2 \end{bmatrix} \tag{20}$$

$$\dot{\mathbf{z}} = \tilde{\mathbf{A}}\mathbf{z} + \tilde{\mathbf{B}}\mathbf{u} + \tilde{\mathbf{C}} \tag{21}$$

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{K}_1\mathbf{C}_1 - \mathbf{B}\mathbf{K}_2\mathbf{C}_1 & \mathbf{B}\mathbf{K}_1\mathbf{C}_1 & \mathbf{B}\mathbf{K}_2\mathbf{C}_1 \\ \mathbf{0} & \mathbf{C}_2 & \mathbf{0} \\ \mathbf{J}_1^{-1}\mathbf{K}_1\mathbf{C}_1 & -\mathbf{J}_1^{-1}\mathbf{K}_1\mathbf{C}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_2 \\ -\mathbf{J}_2^{-1}\mathbf{K}_2\mathbf{C}_1 & \mathbf{0} & -\mathbf{J}_2^{-1}\mathbf{K}_2\mathbf{C}_1 \end{bmatrix} \tag{22}$$

$$\tilde{\mathbf{B}} = \frac{\partial \mathbf{g}}{\partial \mathbf{u}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{J}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix} \tag{23}$$

The feedback component of $\mathbf{u}$ is given as:

$$\mathbf{u}_{fb} = -\mathbf{K}_s(\mathbf{z} - \mathbf{z}^{\mathbf{d}}) \tag{24}$$

$\mathbf{K}_s$ is obtained from solving algebraic ricatti equation The feed-forward force is given as:

$$\mathbf{u}_{ff} = \tilde{\mathbf{B}}^{+}(\dot{\mathbf{z}_d} - \tilde{\mathbf{A}}\mathbf{z}_d - \tilde{\mathbf{C}}) \tag{25}$$

$$\tilde{\mathbf{C}} = \dot{\mathbf{z}}_d - \tilde{\mathbf{A}}\mathbf{z}_d - \tilde{\mathbf{B}}\mathbf{u}_{ff} \tag{26}$$

where $\mathbf{z_d}$ is a vector the desired trajectories of joint states and elastic actuator states.
Thus, $\mathbf{u}$ in [14] is given as:

$$\mathbf{u} = \mathbf{u}_{fb} + \mathbf{u}_{ff} \tag{27}$$

Robot with constraint:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})=\mathbf{T}\mathbf{u}+\mathbf{F_c}^{\mathrm{T}}\boldsymbol{\lambda} \\ \mathbf{g}(\mathbf{q})=0 \end{cases} \tag{28}$$

SEA dynamics:

$$\begin{cases} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})+\mathbf{K}(\mathbf{q}-\mathbf{q}_s)=\mathbf{0} \\ \mathbf{J}\ddot{\mathbf{q}}_s-\mathbf{K}(\mathbf{q}-\mathbf{q}_s)=\mathbf{u} \end{cases} \tag{29}$$

SEA dynamics with constraint:

$$\begin{cases} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})+\mathbf{K}(\mathbf{q}-\mathbf{q}_s)=\mathbf{F_c}^{\mathrm{T}}\boldsymbol{\lambda} \\ \mathbf{J}\ddot{\mathbf{q}}_s-\mathbf{K}(\mathbf{q}-\mathbf{q}_s)=\mathbf{u} \end{cases} \tag{30}$$

VSA dynamics:

$$\begin{cases} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})=\boldsymbol{\tau}_1+\boldsymbol{\tau}_2 \\ \mathbf{J}_1\ddot{\mathbf{q}}_{s1}+\boldsymbol{\tau}_1=\mathbf{u}_1 \\ \mathbf{J}_2\ddot{\mathbf{q}}_{s2}+\boldsymbol{\tau}_2=\mathbf{u}_2 \end{cases} \tag{31}$$

VSA dynamics with constraint:

$$\begin{cases} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})=\boldsymbol{\tau}_1+\boldsymbol{\tau}_2+\mathbf{F}_c^{\mathrm{T}}\boldsymbol{\lambda} \\ \mathbf{J}_1\ddot{\mathbf{q}}_{s1}+\boldsymbol{\tau}_1=\mathbf{u}_1 \\ \mathbf{J}_2\ddot{\mathbf{q}}_{s2}+\boldsymbol{\tau}_2=\mathbf{u}_2 \end{cases} \tag{32}$$

Linearized system:

$$\dot{\mathbf{z}}=\tilde{\mathbf{A}}\mathbf{z}+\tilde{\mathbf{B}}\mathbf{u}+\tilde{\mathbf{C}} \tag{33}$$

Linearized system with constraint:

$$\begin{cases} \dot{\mathbf{z}}=\tilde{\mathbf{A}}\mathbf{z}+\tilde{\mathbf{B}}\mathbf{u}+\tilde{\mathbf{C}}+\bar{\mathbf{F}}\lambda \\ \mathbf{G}\dot{\mathbf{z}}=0 \end{cases} , \quad \bar{\mathbf{F}}=\begin{bmatrix} \mathbf{0} \\ \mathbf{H}^{-\mathbf{1}}\mathbf{F}^T \end{bmatrix} \tag{34}$$

$$\mathbf{u}=-\mathbf{K}_s(\mathbf{z}-\mathbf{z}_d)+\underline{\tilde{\mathbf{B}}^+(\dot{\mathbf{z}}_d-\tilde{\mathbf{A}}\mathbf{z}_d-\tilde{\mathbf{C}})} \tag{35}$$

$$\tilde{\mathbf{C}}=\dot{\mathbf{z}}_d-\tilde{\mathbf{A}}\mathbf{z}_d-\tilde{\mathbf{B}}\mathbf{u}_{ff} \tag{36}$$

$$\begin{bmatrix} \dot{\mathbf{z}} \\ \lambda \end{bmatrix}=\begin{bmatrix} \mathbf{I} & -\bar{\mathbf{F}} \\ \mathbf{G} & \mathbf{0} \end{bmatrix}^+ \begin{bmatrix} \tilde{\mathbf{A}}\mathbf{z}+\tilde{\mathbf{B}}\mathbf{u}+\tilde{\mathbf{C}} \\ \mathbf{0} \end{bmatrix} \tag{37}$$

Cost on Experiments

$$\mathbf{x}_{cost} = max||\mathbf{x}(t_i) - \mathbf{x}^*(t_i)|| \tag{38}$$

$$\mathbf{u}_{cost} = max||\mathbf{u}_i - \mathbf{u}_i^*|| = max||\mathbf{K}(\mathbf{x}(t_i) - \mathbf{x}^*(t_i))|| \tag{39}$$

Result on Three-Link Manipulator

Table 1: x_{cost} on three link manipulator

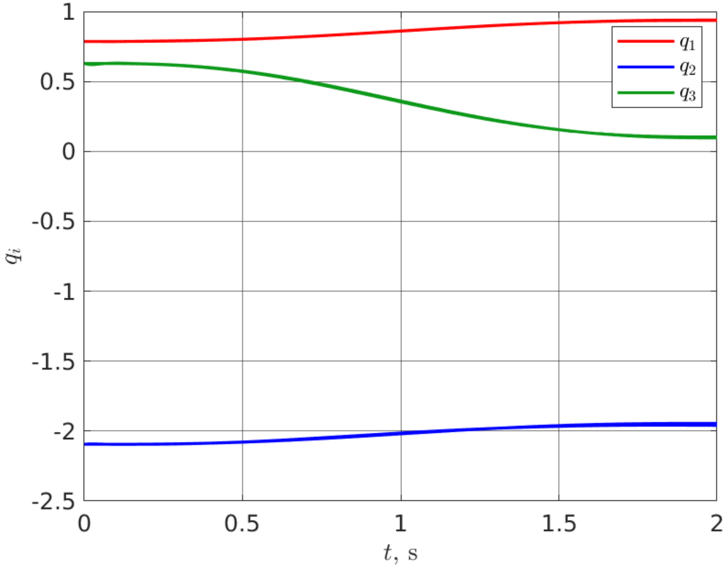
Sigma	CTC	CLQR
0.0001	3.2972	4.2956
0.001	8.2837	4.408
0.01	35.39	9.2947
0.1	263.87	57.833

Table 2: u_{cost} on three link manipulator

Sigma	CTC	CLQR
0.0001	30.696	4.2996
0.001	30.081	4.3216
0.01	25.339	4.9306
0.1	204.62	21.123

Note: result shows 100 experiments from different initial positions sampled from gaussian distribution

Result on Three-Link Manipulator



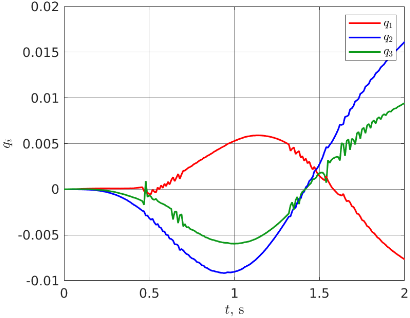


Figure 5: Position error on rigid joint

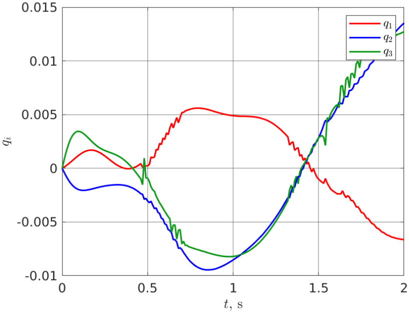
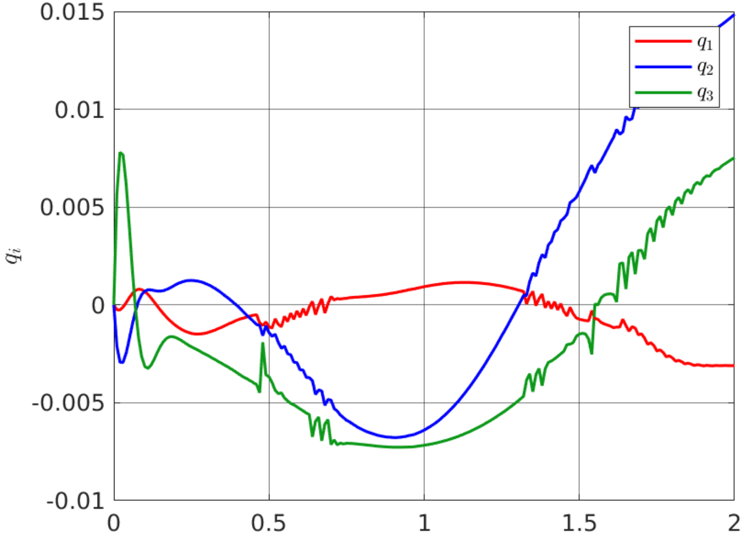


Figure 6: Position error due to SEA

Position tracking



Result on Three-Link Manipulator

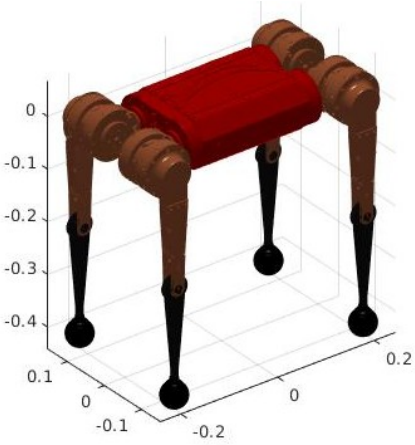


Figure 8: Initial Pose

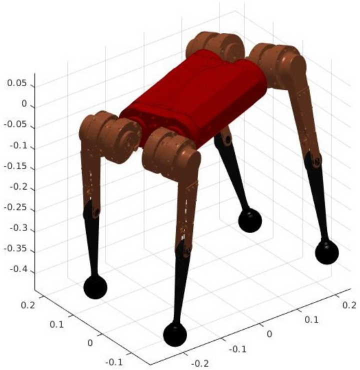


Figure 9: Final Pose

Table 3: x_{cost} on cheetah quadruped

Sigma	CTC	CLQR
0.0001	0.11991	0.14218
0.001	0.12275	0.14683
0.01	0.21808	0.18284
0.1	2.2156	2.6047

Table 4: u_{cost} on cheetah quadruped

Sigma	CTC	CLQR
0.0001	52.974	16.013
0.001	53.144	15.985
0.01	91.30	16.013
0.1	291.75	20.001

Note: result shows 100 experiments from different initial positions sampled from gaussian distribution

Result on actuator stiffness

Comparison of stiffness trajectory based on initial actuator stiffness

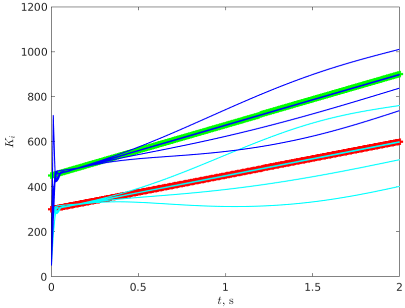


Figure 10: Stiffness trajectory-exp1

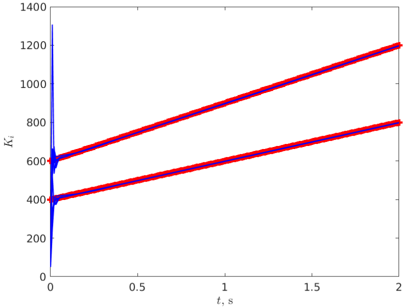


Figure 11: Stiffness trajectory-exp2

Conclusion

- Enforce contact consistency between feet and the ground
- Re-forumulate linearized system in SEA and VSA in terms of the normal robot dynamics form linear form.
- Apply VSA for quadruped motion control.
- Use SEA and VSA to stabilize robot motion control with a linear controller

Thank you