

# TMA4300: Exercise 1

Jim Totland, Martin Tufte

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## Problem A

### A.1

The exponential distribution has cumulative density function (CDF)

$$F(x) = 1 - e^{-\lambda x},$$

with rate parameter  $\lambda$ . By defining  $u := F(x)$ , we can express  $x$  as

$$x = -\frac{1}{\lambda} \ln(1 - u) =: F^{-1}(u).$$

This means that we can use the *inversion method* to simulate from the exponential distribution. I.e., we let  $U \sim \mathcal{U}_{[0,1]}$  and calculate  $X = F^{-1}(U)$ . Then,  $X \sim \text{Exp}(\lambda)$ . The function which simulates the exponential distribution is given below.

```
sim.exp <- function(rate, n){  
  u <- runif(n,0,1)  
  return(-1/rate * log(1 - u))  
}
```

Next, we need to check if this gives reasonable results by comparing our simulated values to the theoretical knowledge.

```
rate <- 0.5  
sim <- data.frame(x = sim.exp(rate, 10000))  
x = seq(from = 0, to = 10, by = 0.1)  
exact <- data.frame(x = x, y = rate*exp(-rate*x))  
  
ggplot(sim) + geom_histogram(aes(x = x, y = ..density.., alpha = 0.3), binwidth = 0.1) +  
  geom_line(data = exact, aes(x = x, y = y))
```

