TMA4300: Exercise 1

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Problem A

A.1

The exponential distribution has cumulative density function (CDF)

$$F(x) = 1 - e^{-\lambda x},$$

with rate parameter λ. By defining $u := F(x)$, we can express x as

$$x = -\frac{1}{\lambda}\ln(1-u) =: F^{-1}(u).$$

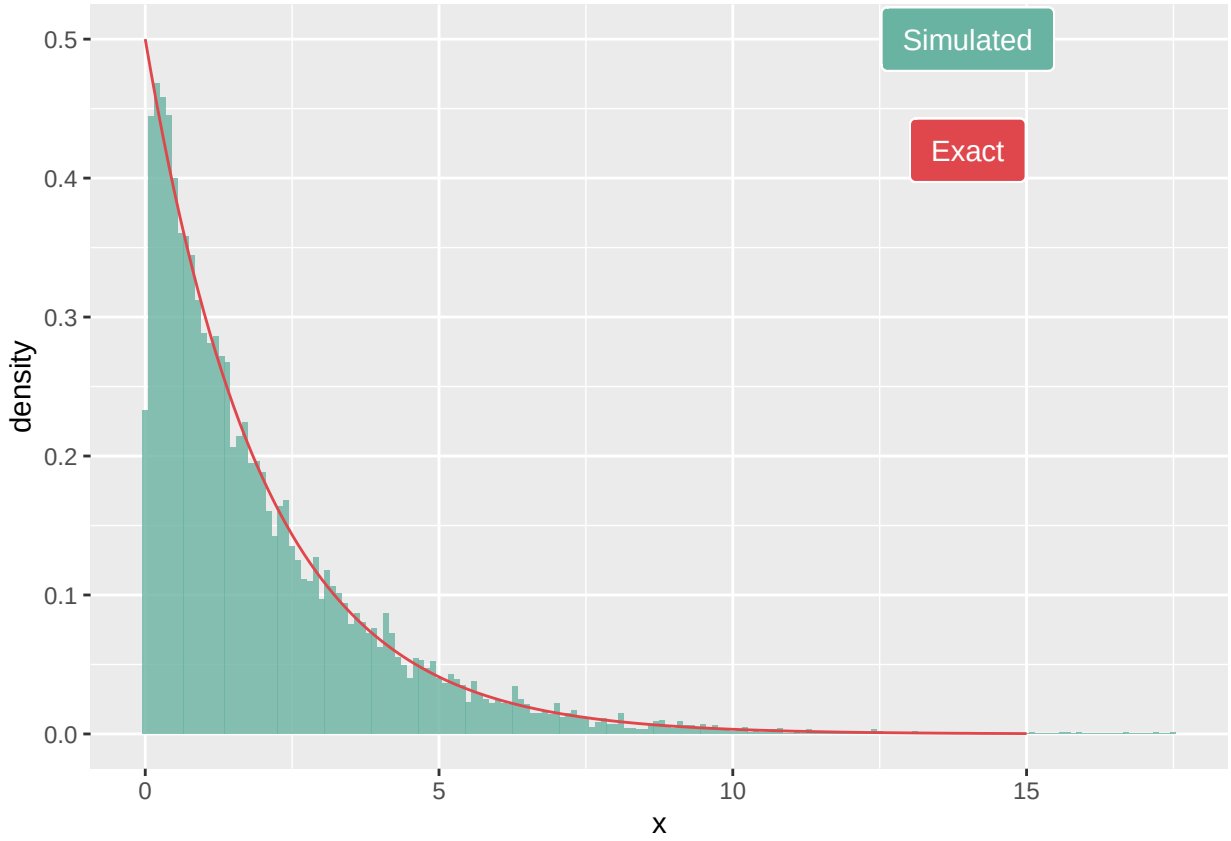
This means that we can use the *inversion method* to simulate from the exponential distribution. I.e., we let $U \sim \mathcal{U}_{[0,1]}$ and calculate $X = F^{-1}(U)$. Then, $X \sim \text{Exp}(\lambda)$. The function which simulates the exponential distribution is given below.

```
sim.exp <- function(rate, n){
  u <- runif(n,0,1)
  return(-1/rate * log(1 - u))
}
```

Next, we need to check if this gives reasonable results by comparing our simulated values to the theoretical knowledge.

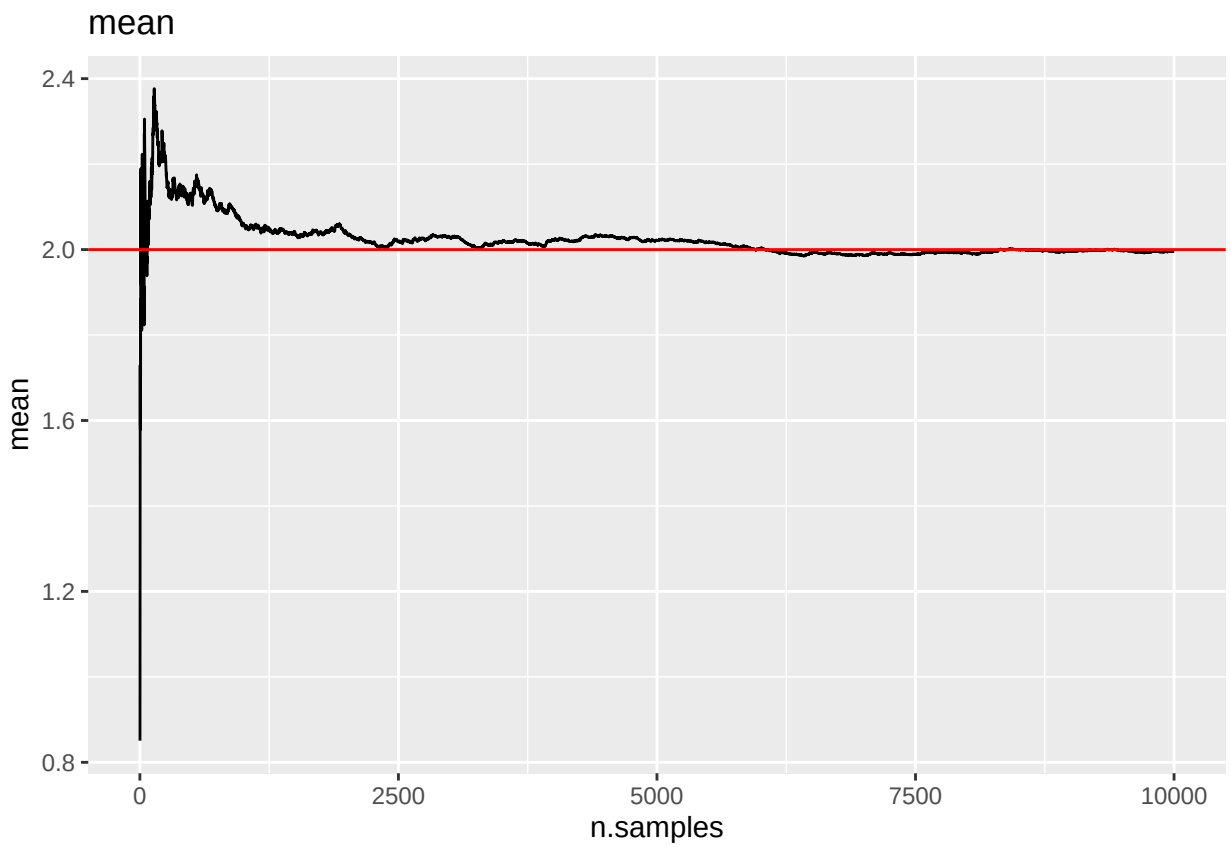
```
rate <- 0.5
n <- 10000
sim <- data.frame(x = sim.exp(rate, n))
x = seq(from = 0, to = 15, by = 0.1)
exact <- data.frame(x = x, y = rate*exp(-rate*x))

ggplot(sim) +
  geom_histogram(aes(x = x, y = ..density..),
                 alpha = 0.8, fill = "#69b3a2", binwidth = 0.1) +
  geom_line(data = exact , aes(x = x, y = y), color = "#e0474c")  +
  geom_label(
    label="Simulated",
    x=14,
    y=0.5,
    label.padding = unit(0.55, "lines"), # Rectangle size around label
```

```
    label.size = 0.35,
    color = "white",
    fill= "#69b3a2"
  ) +
  geom_label(
    label="Exact",
    x=14,
    y=0.42,
    label.padding = unit(0.55, "lines"), # Rectangle size around label
    label.size = 0.35,
    color = "white",
    fill = "#e0474c"
  )
```

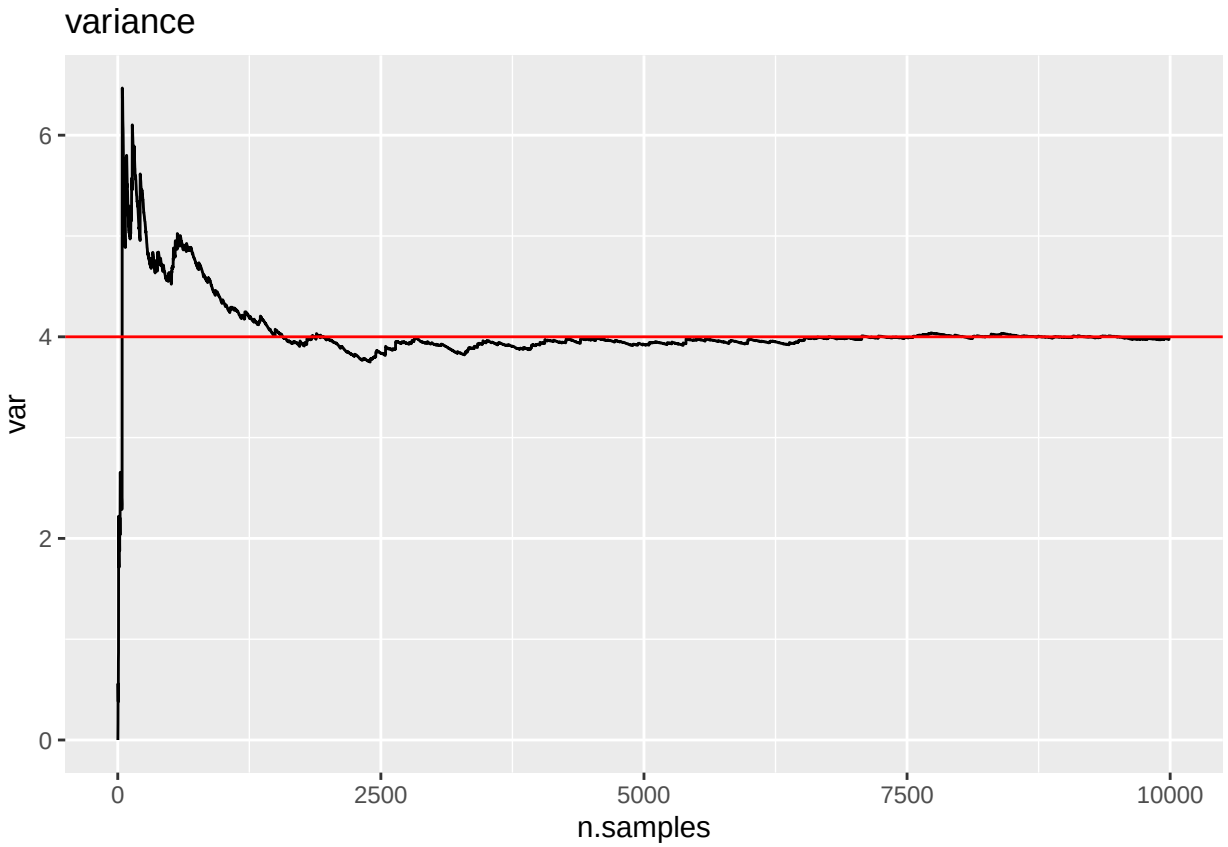


We also compare the estimated mean and variance to first and second central moments of the exponential distribution

```
# mean
data.frame(n.samples = 1:n,
           mean = cumsum(sim$x)/(1:n)) %>%
  ggplot() + geom_line(aes(n.samples, mean)) +
  geom_hline(yintercept = 1/rate, color = "red") +
  ggtitle("mean")
```




```
  # variance
data.frame(n.samples = 1:n,
           mean = cumsum(sim$x)/(1:n),
           mean2 = cumsum(sim$x^2)/(1:n)) %>%
  mutate(var = mean2-mean^2) %>%
  ggplot() + geom_line(aes(n.samples, var)) +
  geom_hline(yintercept = 1/rate^2, color = "red") +
    ggtitle("variance")
```


variance
6
4
2
0
var
0
2500
5000
7500
10000
n.samples