Exercise 1: TDT4171 Artificial Intelligence Methods

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Problem 1

a)

Let the random variable S denote the number of siblings a child has. Then

$$P(S \leq 2) = 0.15 + 0.49 + 0.27 = \underline{0.91} \tag{1}$$

b)

Using the same notation, we get the probability

$$P(S > 2 | S \geq 1) = \frac{P(S > 2, S \geq 1)}{P(S \geq 1)} = \frac{P(S > 2)}{P(S \geq 1)} = \frac{0.09}{0.85} = \underline{0.106}. \tag{2}$$

c)

Let S_1, S_2 and S_3 denote each friend's number of siblings. We assume that these random variables are independent. We get:

$$P(S_1 + S_2 + S_3 = 3) = 0.49^3 + 0.15 \cdot 0.49 \cdot 0.27 \cdot 3! + 0.15^2 \cdot 0.06 \cdot 3 = \underline{0.241}. \tag{3}$$

The first term corresponds to everyone having 1 sibling, the second corresponds to the friends having 0, 1 and 2 siblings, and the last term corresponds to two of the friends having 0 siblings, while one has 3.

d)

Let E and J denote Emma and Jacob's number of siblings, respectively. We assume these random variables to be independent, and arrive at the following:

$$P(E = 0 | J + E = 3) = \frac{P(J = 3)}{P(J + E = 3)} = \frac{0.06}{0.15 \cdot 0.06 \cdot 2 + 0.49 \cdot 0.27 \cdot 2} = \underline{0.212} \tag{4}$$

Problem 2

a)

True. The CPT (Conditional Probability Table) of each node needs to store 2^k entries, where k is the number of parents. This results in $2 \cdot 1 + 4 \cdot 2 + 2 \cdot 4 = 18$ numbers. The reason why we don't need to store e.g. $P(A = true)$ and $P(A = false)$ is that we know these must sum to 1.

b)

True. A and G are marginally independent, as there is no way that information can "flow" between them given one or the other.

c)

True. H's Markov blanket is given, which implies that it is conditionally independent of all other nodes in the network, including E.

d)

False. We cannot conclude that this statement is true. For example, if the value of E is given, this could impact the distribution of G and in turn the distribution of H.

Problem 3

a)

$P(b) = P(b|\neg a)P(\neg a) + P(b|a)P(a) = \underline{0.44}$

b)

First calculate $P(\neg b) = P(\neg b|\neg a)P(\neg a) + P(\neg b|a)P(a) = 0.56$. Then we get

$$P(d) = P(d|\neg b)P(\neg b) + P(d|b)P(b) = \underline{0.712}. \tag{5}$$

c)

$$P(c|\neg d) = \frac{P(c, \neg d)}{P(\neg d)} = \frac{\sum_b P(c, \neg d, b)}{1 - P(d)} = \ldots = \underline{0.1778}. \tag{6}$$

See the appendix for details.

d)

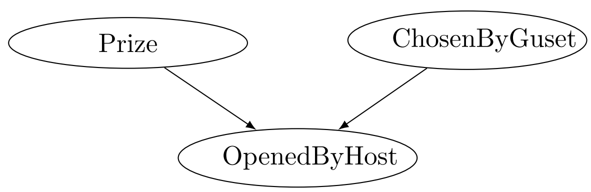
$$P(a|\neg c, d) = \frac{\sum_b P(a, \neg c, d, b)}{\sum_b P(\neg c, d, b)} = \ldots = \underline{0.7983}. \quad (7)$$

See the appendix for details.

Problem 4

Below is the Bayesian network with CPTs depicted.'P' is short for 'Prize', 'CBH' is short for 'ChosenByGuest' and 'OBH' is short for 'OpenedByHost'. Note: 0-indexing has been used, so the first door corresponds to 0, the second door corresponds to 2 and so on.

Prize		
0	1	2
1/3	1/3	1/3



ChosenByGuest		
0	1	2
1/3	1/3	1/3

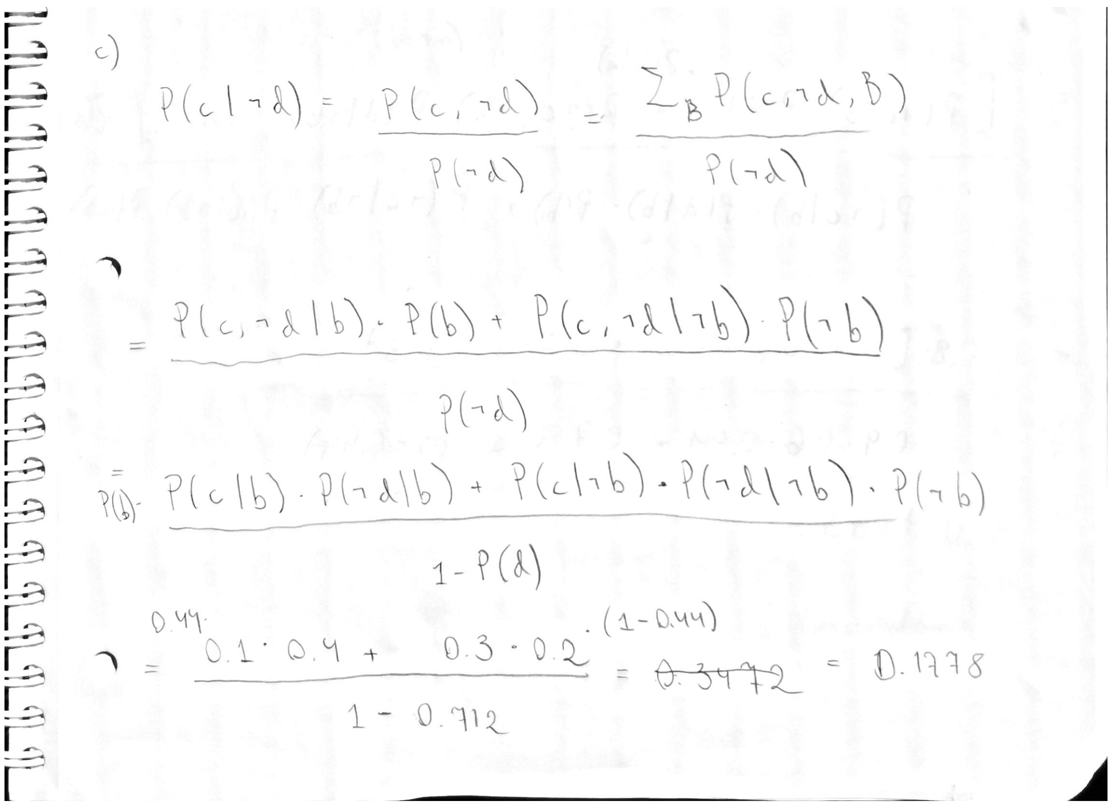
		OBH		
P	CBG	0	1	2
0	0	0	0.5	0.5
1	0	0	0	1
2	0	0	1	0
0	1	0	0	1
1	1	0.5	0	0.5
2	1	1	0	0
0	2	0	1	0
1	2	1	0	0
2	2	0.5	0.5	0

Using the above model, the distribution in question can be calculated as

$$\begin{aligned} P(Prize|ChosenByGuest = 0, OpenedByHost = 2) &= \langle \frac{1}{3}, \frac{2}{3}, 0 \rangle \\ &\approx \langle 0.333, 0.667, 0 \rangle \end{aligned}$$

My implementation yields the approximate result above, which clearly shows that it is advantageous to switch your choice.

Appendix



c)

$$P(c \mid \neg d) = \frac{P(c, \neg d)}{P(\neg d)} = \frac{\sum_B P(c, \neg d, B)}{P(\neg d)}$$

$$= \frac{P(c, \neg d \mid b) \cdot P(b) + P(c, \neg d \mid \neg b) \cdot P(\neg b)}{P(\neg d)}$$

$$= \frac{P(b) \cdot P(c \mid b) \cdot P(\neg d \mid b) + P(c \mid \neg b) \cdot P(\neg d \mid \neg b) \cdot P(\neg b)}{1 - P(d)}$$

$$= \frac{0.44 \cdot 0.1 \cdot 0.4 + 0.3 \cdot 0.2 \cdot (1 - 0.44)}{1 - 0.712} = \cancel{0.3472} = 0.1778$$

Figure 1: Detailed calculations for 3c.

d)

$$P(a \mid \neg c, d) = \frac{P(a, \neg c, d)}{P(\neg c, d)}$$

$$= \frac{\sum_B P(a, \neg c, d, B)}{P(\neg c, d)}$$

$$= \frac{P(\neg c, d \mid a, b) \cdot P(a, b) + P(\neg c, d \mid a, \neg b) \cdot P(a, \neg b)}{P(\neg c, d, b) + P(\neg c, d, \neg b)}$$

$$= \frac{\left[P(\neg c \mid b) \cdot P(d \mid b) \cdot P(b \mid a) + P(\neg c \mid \neg b) \cdot P(d \mid \neg b) \cdot P(\neg b \mid a) \right] \cdot P(a)}{P(\neg c \mid b) \cdot P(d \mid b) \cdot P(b) + P(\neg c \mid \neg b) \cdot P(d \mid \neg b) \cdot P(\neg b)}$$

$$= \frac{0.8 \cdot [0.9 \cdot 0.6 \cdot 0.5 + 0.7 \cdot 0.8 \cdot 0.5]}{0.9 \cdot 0.6 \cdot 0.44 + 0.7 \cdot 0.8 \cdot (1 - 0.44)}$$

$$= 0.7983.$$

Figure 2: Detailed calculations for 3d.