

TMA4250 Spatial Statistics

Project 1 - Random Fields and Gaussian Random Fields

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Problem 1

a)

The positive semi-definite (PSD) property of the correlation function can be stated as follows. $\forall m \in \mathbb{Z}_+$, $\forall a_1, \dots, a_m \in \mathbb{R}$ and $\forall \mathbf{s}_1, \dots, \mathbf{s}_m \in \mathcal{D}$, we have

$$\sum_{i=1}^m \sum_{j=1}^m a_i a_j \rho(\mathbf{s}_i, \mathbf{s}_j) \geq 0.$$

To explain why this requirement is necessary, we observe that (in this case) $\rho(\mathbf{s}_i, \mathbf{s}_j) = \sigma^{-2} c(\mathbf{s}_i, \mathbf{s}_j)$, where c is the covariance function. Consequently,

$$\begin{aligned} \sum_{i=1}^m \sum_{j=1}^m a_i a_j \rho(\mathbf{s}_i, \mathbf{s}_j) &= \sigma^{-2} \sum_{i=1}^m \sum_{j=1}^m a_i a_j c(\mathbf{s}_i, \mathbf{s}_j) \\ &= \sigma^{-2} \text{Var} \left[\sum_{i=1}^m a_i X(\mathbf{s}_i) \right]. \end{aligned}$$

Since the variance must be non-negative, it is clear that the PSD property above must be satisfied.

```
library(geoR)
```

```
## -----  
## Analysis of Geostatistical Data  
## For an Introduction to geoR go to http://www.leg.ufpr.br/geoR  
## geoR version 1.8-1 (built on 2020-02-08) is now loaded  
## -----
```

```
?cov.spatial
```