TMA4250 Spatial Statistics

Project 1 - Random Fields and Gaussian Random Fields

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Problem 1

a)

The positive semi-definite (PSD) property of the correlation function can be stated as follows. $\forall m \in \mathbb{Z}_+$, $\forall a_1, \ldots, a_m \in \mathbb{R}$ and $\forall \boldsymbol{s}_1, \ldots, \boldsymbol{s}_m \in \mathcal{D}$, we have

$$\sum_{i=1}^{m}\sum_{j=1}^{m} a_i a_j \rho(\boldsymbol{s}_i, \boldsymbol{s}_j) \geq 0.$$

To explain why this requirement is necessary, we observe that (in this case) $\rho(\boldsymbol{s}_i, \boldsymbol{s}_j) = \sigma^{-2} c(\boldsymbol{s}_i, \boldsymbol{s}_j)$, where c is the covariance function. Consequently,

$$\begin{aligned}\sum_{i=1}^{m}\sum_{j=1}^{m} a_i a_j \rho(\boldsymbol{s}_i, \boldsymbol{s}_j) &= \sigma^{-2} \sum_{i=1}^{m}\sum_{j=1}^{m} a_i a_j c(\boldsymbol{s}_i, \boldsymbol{s}_j) \\ &= \sigma^{-2} \mathrm{Var}\left[\sum_{i=1}^{m} a_i X(\boldsymbol{s_i})\right].\end{aligned}$$

Since the variance must be non-negative, it is clear that the PSD property above must be satisfied.

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