MLF - HW2

By: B06902029 (Wu-Jun Pei)

1.

2.

We can know that $d_{VC} \geq 4 \Leftrightarrow$ there are some 4 inputs we can shatter.

Consider $X = \{(0,1),(1,0),(0,-1),(-1,0)\}$, four points on a unit circle.

- If none is `o`, we can draw a line somewhere else that contains none of these points.
- If there is only one `o`, we can find a thick line that is tangent to that point.
- If there are two `o`, no matter the two points are adjacent or not, we can simply line the two points that only them are contained in the thick line.
- If there are three `o`'s, we can draw on the three circles and leave the `x` alone.
- If there are four `o`'s, we can simply draw a very thick line that contains all the four points.

3.

Discuss with 蔡秉辰.

- First of all, we can rewrite α as a 4-base numbers. For example, $(13.875)_{10} = 3 \times 4^1 + 1 \times 4^0 + 3 \times 4^{-1} + 2 \times 4^{-2} = (31.32)_4$
- Secondly, if $x = 4^k$, then we can view αx as an operation that shift α leftward k units in 4-

base numbers. For example, if $\alpha = (31.32)_4, x = 4$, $\alpha x = (31.32)_4 \times 4 = (313.2)_4$.
- As for the $\mod 4$ part, we can view the operation as a mask that only sees the 4_0 part and the decimal part. For example, $(31.32)_4 \mod 4 = (1.32)_4$
- For simplicity, we define $\text{sign}(0) = -1$. And we can find that if the coefficient on 4^0 of αx is 0, then $h_\alpha(x)$ must be 1, if the coefficient on 4^0 of αx is 1, then $h_\alpha(x)$ must be -1.

For any finite n, there exist some n inputs that we can shatter.

Let $X = \{x_i = 4^i \text{ for } 1 \leq i \leq n\}$ and Y be any set composed by $\{-1, 1\}$. And we can yield an α with

$$\alpha = \sum_{i=1}^{n} 4^{-i} \times \begin{cases} 0, & \text{if } y_i = 1 \\ 1, & \text{if } y_i = -1 \end{cases}$$

Thus, we can shatter any finite n inputs, $d_{VC} = \infty$.

4.

Prove by contradiction.

Assume that $d_{VC}(H_1 \cap H_2) > d_{VC}(H_2)$. Let $n = d_{VC}(H_1 \cap H_2)$ and $m = d_{VC}(H_2)$, we have $n > m$.

By definition of VC-Dimension, we know that n inputs can be shattered by $H_1 \cap H_2$ while n inputs cannot be shattered by H_2. However, we have $H_1 \cap H_2 \subset H_2$, so n inputs must be shattered by H_2, contradicts to our assumption.

Thus, we have $d_{VC}(H_1 \cap H_2) \leq d_{VC}(H_2)$.

5.

We can observe that

$$\begin{aligned} m_{H_1 \cup H_2}(N) &= m_{H_1}(N) + m_{H_2}(N) - m_{H_1 \cap H_2}(N) \\ &= (N+1) + (N+1) - 2 \\ &= 2N \end{aligned}$$

since the intersection of $H_1 \cap H_2$ is all positive and all negative.

When $n = 2$, $m_{H_1 \cup H_2}(n) = 4 = 2^2$, we know that $H_1 \cup H_2$ can shatter 3 inputs.

When $n = 3$, $m_{H_1 \cup H_2}(n) = 6 \neq 8 = 2^3$, thus we know that $H_1 \cup H_2$ can not shatter 3 inputs.

Therefore, we have $d_{VC}(H_1 \cup H_2) = 2$.

6.

We can observe that

$$
\begin{aligned}
\mu &= \begin{cases} \frac{|\theta|}{2}, & \text{if } s = 1 \\ 1 - \frac{|\theta|}{2}, & \text{if } s = -1 \end{cases} \\
&= \frac{s+1}{2} \times (\frac{|\theta|}{2}) + \frac{1-s}{2} \times (1 - \frac{|\theta|}{2}) \\
&= \frac{s(|\theta| - 1) + 1}{2}
\end{aligned}
$$

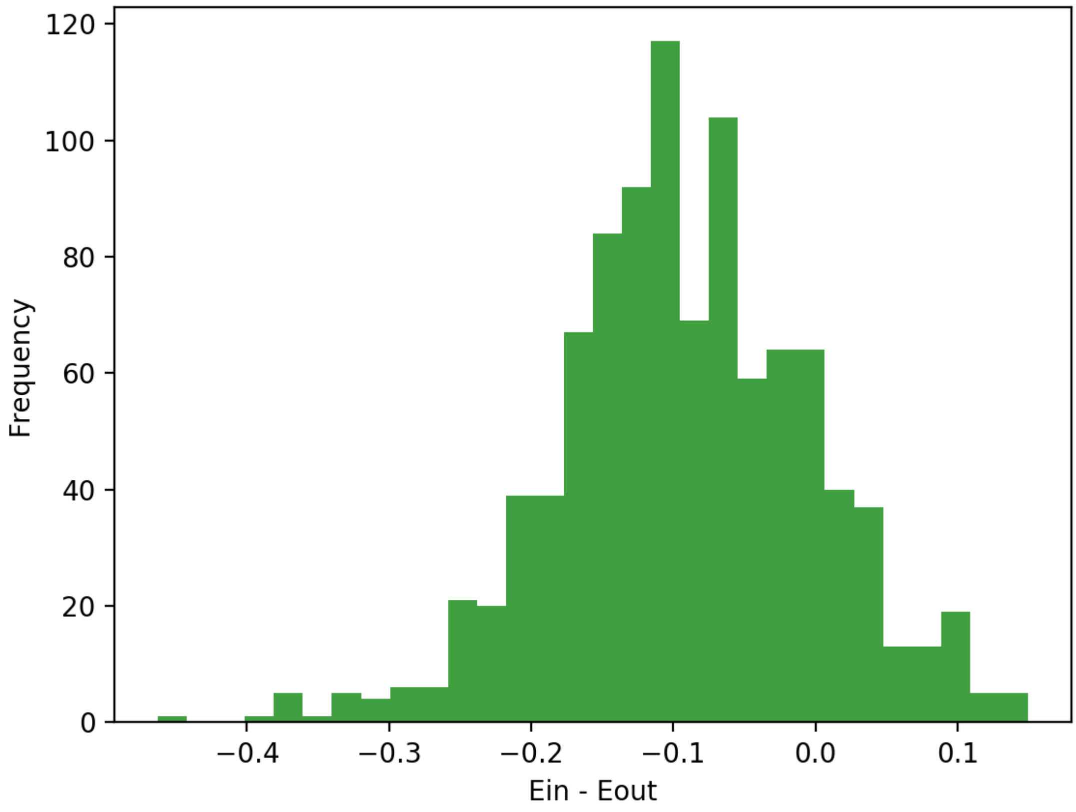
By *Problem 1 in Coursera*, we have

$$E_{out} = \mu\lambda + (1-\mu)(1-\lambda)$$

And $\lambda = 0.8$, we have

$$
\begin{aligned}
E_{out} &= 0.8\mu + 0.2(1-\mu) \\
&= 0.2 + 0.6\mu \\
&= 0.2 + 0.3 + 0.3s(|\theta| - 1) \\
&= 0.5 + 0.3s(|\theta| - 1)
\end{aligned}
$$

7.



The average $E_{in} - E_{out}$ falls around -0.95. And it looks like a normal distribution!