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模型一致性~~研究~~

# A Survey on Shape Correspondence

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我们回顾了用于计算由三角形网格，轮廓或点集表示的几何形状之间的对应关系的方法。这项调查的部分动机是时空登记的最新发展，其中寻求非刚性和时变表面之间的对应关系，以及语义形状分析，这突出了最近将形状理解纳入分析管道的趋势。在形状之间建立有意义的对应关系通常是困难的，因为它通常需要在局部和全局层面理解形状的结构，并且有时也需要形状部分的功能。尽管其固有的复杂性，但形状对应是一个经常出现的问题，也是许多几何处理应用的重要组成部分。在本次调查中，我们讨论了对应问题的不同形式，并在问题定义的几个分类标准的帮助下，审查了主要的解决方法。主要的分类类别是根据输入和输出表示，目标函数和解决方案方法定义的。我们通过讨论开放性问题和未来前景来完成调查。

**Abstract**

*We review methods designed to compute correspondences between geometric shapes represented by triangle meshes, contours, or point sets. This survey is motivated in part by recent developments in space-time registration, where one seeks a correspondence between non-rigid and time-varying surfaces, and semantic shape analysis, which underlines a recent trend to incorporate shape understanding into the analysis pipeline. Establishing a meaningful correspondence between shapes is often difficult since it generally requires an understanding of the structure of the shapes at both the local and global levels, and sometimes the functionality of the shape parts as well. Despite its inherent complexity, shape correspondence is a recurrent problem and an essential component of numerous geometry processing applications. In this survey, we discuss the different forms of the correspondence problem and review the main solution methods, aided by several classification criteria arising from the problem definition. The main categories of classification are defined in terms of the input and output representation, objective function, and solution approach. We conclude the survey by discussing open problems and future perspectives.*

## 1. Introduction

在两个或更多个形状之间找到有意义的对应关系是基本的形状分析任务。 该问题通常可以表述为：给定输入形状S1; S2; :::; SN, 在它们的元素之间找到有意义的关系（或映射），例如，参见图。
在不同的情况下，问题也被称为注册，对齐或简单匹配。 形状对应是3D扫描对齐和时空重建等任务中的关键算法组件，也是各种应用中不可或缺的先决条件，包括属性转移，形状插值和统计建模。

Finding a meaningful correspondence between two or more shapes is a fundamental shape analysis task. The problem can be generally stated as: *given input shapes* $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N$, *find a meaningful relation (or mapping) between their elements*, e.g., see Figure 1. Under different contexts, the problem has also been referred to as registration, alignment, or simply matching. Shape correspondence is a key algorithmic component in tasks such as 3D scan alignment and space-time reconstruction, as well as an indispensable prerequisite in diverse applications including attribute transfer, shape interpolation and statistical modeling.

传统上在图像分析社区中研究了对应问题，但该调查针对的是计算机图形用户。 我们专注于对由三角形网格，轮廓或点集表示的几何形状进行操作的方法，而不是图像或体积。 我们特别关注3D形状 - 它们提供明确的几何信息，但通常缺少简单的参数化域。 另一个区别是图像分析通常受益于基于颜色和纹理的丰富的局部描述符，而为形状计算的描述符通常不那么独特。

The correspondence problem has been traditionally studied in the image analysis communities, but this survey is targeted towards the computer graphics audience. We focus on methods operating on geometric *shapes* represented by triangle meshes, contours, or point sets, as opposed to images or volumes. We pay special attention to 3D shapes — they provide explicit geometry information, but generally lack a simple parameterization domain. Another distinction is that image analysis often benefits from rich local descriptors based on color and texture, while the descriptors computed for shapes are generally not as distinctive.

Several specializations of the correspondence problem

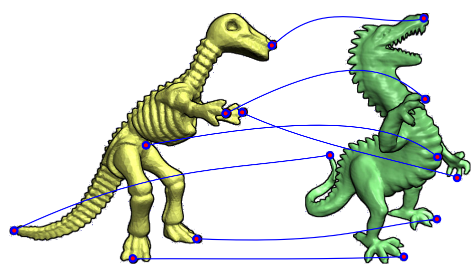

**Figure 1:** *A meaningful correspondence (blue lines) between a sparse set of feature points on two shapes. Note the large amount of geometric variations between the shapes which make the computation of such a correspondence difficult.*
图1：两个形状上稀疏的特征点集之间的有意义的对应关系（蓝线）。 注意形状之间的大量几何变化使得难以计算这种对应关系。

can be considered. One may ask whether it is meaningful to establish a correspondence in full, as in Figure 1, or only for part of the shapes, as in Figure 2. The additional need to find the common parts of the shapes not only increases the search space but also makes it difficult to define a proper objective function, making the partial correspondence problem harder [GSH*07,ZSCO*08]. One may also consider the density of the correspondence computed. Sparse correspondence only seeks to identify and match a small set of land-

可以考虑几种对应问题的专业化。 人们可能会问，完全建立一个对应关系是否有意义，如图1所示，或者只是部分形状，如图所示。另外需要找到形状的公共部分不仅会增加搜索空间，还会增加搜索空间。 难以定义适当的目标函数，使部分对应问题更难[GHS * 07, ZSCO * 08]。

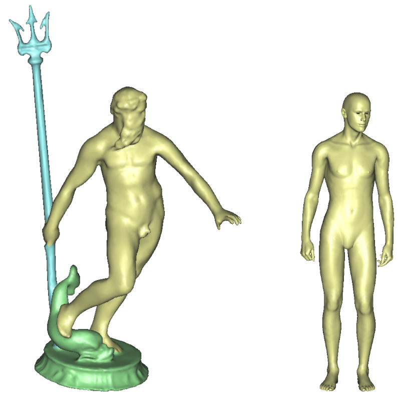

**Figure 2:** *An example of a partial correspondence problem: the goal here is to find a correspondence between Neptune's statue (left) and the human (right), by relating the parts in yellow and ignoring the extra parts shown in green and blue.*

图2：部分对应问题的一个例子：这里的目标是找到海王星雕像（左）和人（右）之间的对应关系，将黄色部分关联起来，忽略绿色和蓝色所示的额外部分。

mark points. However, it is as difficult as its dense counterpart, since the challenging aspect of the correspondence search remains essentially the same — in both cases it is necessary to consider the global structures of the shapes and possibly the semantics of their parts to obtain a meaningful solution. Often, a dense correspondence is derived from a sparse one via some form of interpolation [Ale02, KS04], although the computation of a dense correspondence can also present challenges in certain circumstances, e.g., in the case of partial matching or when there are topology differences between the shapes.

还可以考虑计算的对应关系的密度。 稀疏对应仅寻求识别和匹配一小组界标点。 然而，它与其密集对应物一样困难，因为对应搜索的挑战性方面基本上保持相同 - 在这两种情况下，有必要考虑形状的全局结构以及可能的部件的语义以获得有意义的解决方案。
通常，通过某种形式的插值[Ale02, KS04]从稀疏的对应性导出密集的对应关系，尽管密集对应的计算在某些情况下也可能存在挑战，例如，在部分匹配的情况下或存在拓扑时 形状之间的差异。

Defining what is a meaningful correspondence depends on the task at hand. The task can range from the simpler case of identifying portions of the shapes that are geometrically similar, to the more complex problem of relating elements that represent the same parts or serve the same function on the shapes. In the latter case, the matching parts may differ significantly in their geometry, structure within the context of the whole shape, or even topology. An example of such a semantic correspondence problem is shown in Figure 3. In general, computing such correspondences is difficult since it involves understanding the structure of the shapes at both the local and global levels, and possibly understanding the functionality of the shape parts.

定义什么是有意义的对应取决于手头的任务。该任务的范围可以从识别几何相似的形状的部分的更简单的情况到表示相同部分或在形状上起相同作用的相关元件的更复杂的问题。 在后一种情况下，匹配部分的几何形状，整个形状的上下文中的结构或甚至拓扑可能显着不同。图3中示出了这种语义对应问题的示例。通常，计算这种对应是困难的，因为它涉及理解局部和全局级别的形状的结构，并且可能理解形状部分的功能。

Classical applications of shape correspondence, such as 3D scan alignment or shape morphing, call for traditional approaches which include rigid alignment and feature matching. Rigid alignment is typically solved with methods based on sampling and verifying candidate transformations [IR96], or by applying the iterated closest point algorithm or one of its variations [RL01]. In the case of feature matching, shape descriptors are computed for representative points sampled from the shapes, and the correspondence is constructed by selecting the assignments that maximize the similarity be-

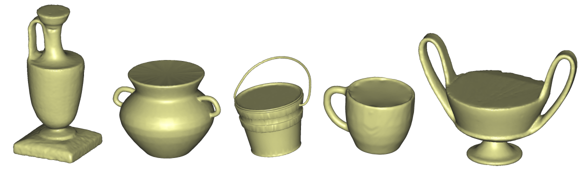

**Figure 3:** *An example of a collection of man-made shapes (liquid containers) for which computing a correspondence is a challenging problem. Note how the shapes can be constituted by different types and numbers of parts (e.g., one or two handles, neck, base), how the parts of a same type can vary in their geometry (e.g., long vs. short handles), and how they can connect to each other in different manners.*

图3：计算对应性是一个具有挑战性的问题的人造形状（液体容器）集合的示例。注意形状可以由不同类型和数量的部件（例如，一个或两个手柄，颈部，底部）构成，相同类型的部件如何在其几何形状上变化（例如，长柄与短柄），以及 他们如何以不同的方式相互联系。

tween the descriptors. Additional constraints can also be incorporated into this class of algorithms, such as the preservation of distances between points.

形状对应的经典应用，例如3D扫描对齐或形状变形，需要传统方法，包括刚性对齐和特征匹配。 刚性对齐通常通过基于采样和验证候选变换的方法[IR96]，或通过应用迭代的最近点算法或其变体之一[RL01]来解决。 在特征匹配的情况下，针对从形状采样的代表点计算形状描述符，并且通过选择使描述符之间的相似性最大化的分配来构造对应关系。 其他约束也可以包含在这类算法中，例如保持点之间的距离。

More recent works are attempting to deal with large variations in the shapes, to the point where the rigid alignment or feature matching criterion is no longer suitable. On one hand, emerging techniques for surface deformation have been successfully applied to non-rigid registration of surfaces [HAWG08, ZSCO*08]. On the other hand, progress in matching approximately isometric shapes has also been revealed in recent works [BBBK08, LF09]. Finally, there is an apparent recent trend to look beyond low-level geometric information and to incorporate high-level shape semantics into the shape analysis pipeline. Examples of such works include techniques for segmenting a mesh into parts [Sha08], finding analogies between these parts [SSSCO08], transferring information [SP04] or part styles [XLZ*10] from one shape to another, extracting the high-level structure of the shapes for manipulation or deformation [XWY*09, GSMCO09], and using prior knowledge to learn how to label a shape [KHS10] or establish a correspondence [vKTS*11]. Computing a correspondence between shapes is one of the key problems that can benefit from semantically-driven techniques, since the goal is to understand the structure of the shapes in order to find a meaningful correspondence between their parts.

最近的工作试图处理形状的大的变化，直到刚性对准或特征匹配标准不再适合的程度。 一方面，新兴的表面变形技术已成功应用于表面的非刚性配准[HAWG08, ZSCO 08]。另一方面，在最近的作品[BBBK08, LF09]中也揭示了匹配近似等距形状的进展。 最后，最近有一种趋势是超越低级几何信息，并将高级形状语义结合到形状分析管道中。 此类作品的示例包括将网格分割为零件的技术[Sha08]，在这些零件[SSSCO08]之间进行类比，将信息[SP04]或零件样式[XLZ 10]从一种形状转移到另一种形状，从而提取高级结构 操纵或变形的形状[XWY 09, GSMCO09]，并使用先前的知识来学习如何标记形状[KHS10]或建立对应关系[vKTS 11]。计算形状之间的对应关系是可以从语义驱动技术中受益的关键问题之一，因为目标是理解形状的结构以便在它们的部分之间找到有意义的对应关系。

The rest of the paper is organized as follows. In Section 2, we start by discussing the statement of the correspondence problem and its different specializations. Next, in Section 3, we give a more concrete overview of the problem in terms of some example applications. Our classification criteria are presented in Section 4, along with a detailed discussion of the literature. In Section 5, we review the main solution paradigms, while in Section 6, the validation of results is considered. Applications that make use of correspondence methods are summarized in Section 7. In Section 8, we review the previous surveys that covered the correspondence problem. Finally, in Section 9, we summarize our view on the state-of-the-art and elaborate on future perspectives.

本文的其余部分安排如下。在第2节中，我们首先讨论通信问题的陈述及其不同的专业。 接下来，在第3节中，我们根据一些示例应用程序更具体地概述了该问题。 我们的分类标准在第4节中介绍，并详细讨论了文献。 在第5节中，我们回顾了主要的解决方案范例，而在第6节中，考虑了结果的验证。 第7节总结了使用通信方法的应用程序。在第8节中，我们回顾了之前涉及通信问题的调查。 最后，在第9节中，我们总结了我们对最新技术的看法，并详细阐述了未来的观点。

## 2. Problem statement 问题陈述

A variety of problems can be classified under the scope of shape correspondence. As we will see in Section 3, significant differences exist between these problems and the computational paradigms used for their solution also differ. However, since all these different problems address the same fundamental task, we can view them all in a unified manner by considering the problem statement: given input shapes $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N$, establish a meaningful relation $\mathcal{R}$ between their elements. When two elements are related to each other (i.e., $(s,z) \in \mathcal{R}$, for elements $s \in \mathcal{S}_i$ and $z \in \mathcal{S}_j$, with $i \neq j$), we say that they are in correspondence or that they match to each other. The relation can be constrained in different manners, such as asking for a one-to-one, one-to-many, or many-to-many correspondence.

在形状对应的范围内可以将各种问题分类。 正如我们将在第3节中看到的，这些问题之间存在显着差异，并且用于其解决方案的计算范例也不同。 然而，由于所有这些不同的问题都解决了相同的基本任务，我们可以通过考虑问题陈述以统一的方式查看它们：给定输入形状S1；S2；....；SN，在它们的元素之间建立有意义的关系R.。 当两个元素彼此相关时（即，（s；z）E R，对于元素s E i和z E j，对于i！= j），我们说它们是对应的或者它们彼此匹配。 可以以不同的方式约束关系，例如要求一对一，一对多或多对多的对应。

The ultimate goal behind shape correspondence is to recognize the parts of the shapes and infer their purpose, so that the mapping established from one shape to the other is semantically meaningful. However, in several cases when geometry provides enough information about the shapes, it is sufficient to pose shape correspondence as the task of identifying homologous elements of two or more shapes, i.e., the elements that possess the same or similar structure in terms of their local appearance and context. The meaning of *elements* and *similar structure* depend on the problem at hand. However, a shape will be usually composed of multiple elements (primitives such as points, feature points, faces, skeletal features, or higher-level entities such as parts) and we seek to find a correspondence between these elements.

形状对应背后的最终目标是识别形状的各个部分并推断它们的目的，以便从一种形状到另一种形状建立的映射在语义上是有意义的。 然而，在几何提供关于形状的足够信息的几种情况下，将形状对应作为识别两种或更多种形状的同源元素的任务就足够了，即，就其局部而言具有相同或相似结构的元素。 外观和背景。 元素和类似结构的含义取决于手头的问题。 然而，形状通常由多个元素（诸如点，特征点，面，骨架特征或诸如部分的更高级实体之类的基元）组成，并且我们寻求找到这些元素之间的对应关系。

It is important to distinguish between the problems of shape correspondence and retrieval. Retrieval is the task where, given a query shape, the goal is to find shapes from a database that are similar to the query, tolerating a variety of transformations and possibly considering partial similarities. Therefore, the focus is only in quantifying the similarity between shapes. In correspondence, however, the goal is to find an explicit relation between the elements of two or more shapes. The evaluation of shape similarity is sometimes called *shape matching* in the literature (e.g., [VH01]), which conflicts with the nomenclature followed by other authors who refer to shape matching exclusively as computing a correspondence. Perhaps the reason for the interchangeable use of these terms in the literature is that retrieval and correspondence are closely related problems, since computing a correspondence and evaluating its quality is one possible manner of assessing shape similarity.

区分形状对应和检索的问题很重要。 检索是一项任务，在给定查询形状的情况下，目标是从数据库中查找与查询类似的形状，容忍各种变换并可能考虑部分相似性。 因此，重点仅在于量化形状之间的相似性。 然而，相应地，目标是在两个或更多个形状的元素之间找到明确的关系。 形状相似性的评估有时在文献中称为形状匹配（例如，[VH01]），其与命名法冲突，其他作者仅将形状匹配称为计算对应关系。 也许在文献中可互换使用这些术语的原因是检索和对应是密切相关的问题，因为计算对应关系并评估其质量是评估形状相似性的一种可能方式。

The general correspondence problem stated above can be specialized in different ways, which we discuss as follows.

上述一般对应问题可以以不同方式专门化，我们将讨论如下。

**Full vs. partial correspondence:** following the problem definition, here we focus on the inclusion properties of the relation $\mathcal{R}$. The required correspondence can be full (defined for all the shape elements) or partial (defined for a subset of the elements). The motivation for the latter is that the shapes under consideration may be constituted by different parts (i.e., one shape can have missing or additional parts when compared to the others), and so it may not be meaningful to establish a correspondence between all their elements. These parts can differ by their geometry, scale, and connecting location on the object. An example is shown in Figure 2.

**完全与部分对应**：在问题定义之后，这里我们关注关系R的包含属性。所需的对应关系可以是完整的（为所有形状元素定义）或部分（为元素的子集定义）。 后者的动机是所考虑的形状可以由不同的部分构成（即，与其他形状相比，一个形状可以具有缺失或附加部分），因此在它们的所有元素之间建立对应关系可能没有意义。。 这些部件可能因其几何形状，比例和物体上的连接位置而不同。 一个例子如图2所示。

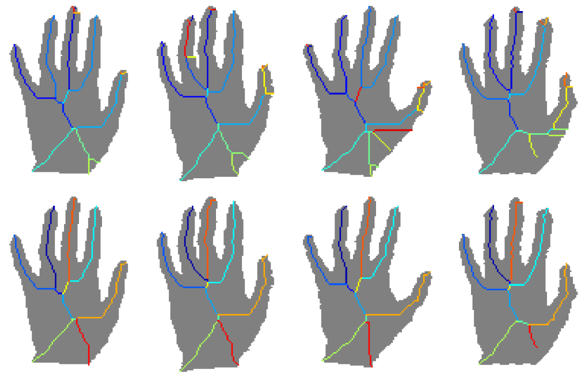

**Figure 4:** *Motivation for group correspondence: in this example, the skeletonization computed individually for each 2D shape (top row) is compared to the result of pruning these skeletons with the method in [WH09] (bottom row), which takes group information into account. It can be seen how using group information improves the coherence of the skeletons across shapes and also the quality of the branch correspondence (shown by coinciding colors).*

图4：组对应的动机：在这个例子中，将每个2D形状（顶行）单独计算的骨架化与使用[WH09]（底行）中的方法修剪这些骨架的结果进行比较，该方法将组信息输入 帐户。 可以看出使用组信息如何改善骨架在形状上的一致性以及分支对应的质量（由重合的颜色显示）。

Computing a partial correspondence is a more difficult problem than computing a full correspondence, due to the combinatorial explosion in the solution space. If we consider all one-to-one assignments between shapes with $n$ elements, the solution space is composed of $n!$ correspondences. If we add the possibility for partial matching, the solution space includes all the possible subsets of these $n!$. Moreover, searching for the right subset increases the complexity of the problem and also requires a careful definition of the optimality criterion.

由于解空间中的组合爆炸，计算部分对应是比计算完全对应更困难的问题。 如果我们考虑具有n个元素的形状之间的所有一对一分配，则解空间由n组成！对应。 如果我们添加部分匹配的可能性，则解空间包括这些n！的所有可能子集。 此外，搜索正确的子集会增加问题的复杂性，并且还需要仔细定义最优性标准。

**Dense vs. sparse correspondence:** another aspect to take into account is the *density* of the relation $\mathcal{R}$. A *dense correspondence* is defined for all the elements or primitives on the shape (e.g., mesh faces). A *sparse correspondence* is defined for a small number of pre-selected elements. The elements are usually a set of features. For example, to infer the semantics of human shapes, it is sufficient to match representative points located at the legs, arms, head and body of the shapes (a sparse correspondence). On the other hand, for applications such as morphing or attribute transfer between two shapes, a dense correspondence is required to guarantee global smoothness in the morphing or transfer result.

密集与稀疏对应：要考虑的另一个方面是关系R的密度。为形状上的所有元素或基元（例如，网格面）定义密集对应关系。 为少量预选元素定义稀疏对应关系。 元素通常是一组功能。 例如，为了推断人类形状的语义，匹配位于形状的腿，臂，头部和身体的代表点（稀疏对应）就足够了。 另一方面，对于诸如两个形状之间的变形或属性转移的应用，需要密集的对应以保证变形或转移结果中的全局平滑性。

Computing a sparse correspondence between representative points is a problem as hard as computing a dense one between two shapes, since both problems necessarily in-

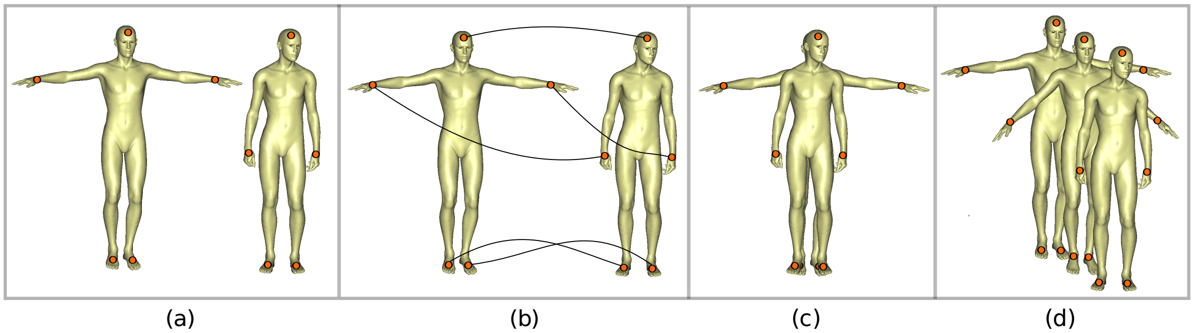


**Figure 5:** *Different manners of solving the correspondence problem for the input shapes shown in (a) and their feature points (indicated by the dots): (b) computing a correspondence without explicitly bringing the shapes into alignment, (c) computing a global rigid transformation to align the two shapes, and (d) computing local non-rigid transformations for the shape primitives to deform one shape into the other.* 图5：解决（a）中所示输入形状及其特征点（由点表示）的对应问题的不同方式：（b）计算对应而不明确地使形状对齐，（c）计算全局 刚性变换以对齐两个形状，以及（d）计算形状基元的局部非刚性变换以使一个形状变形为另一个形状。

计算代表点之间的稀疏对应是一个问题，就像在两个形状之间计算密集的问题一样困难，因为这两个问题必然涉及考虑形状的全局结构[ZSCO 08]。 计算密集对应关系的常用方法是使用交叉参数化或插值等技术扩展稀疏对应[Ale02]。

volve taking into consideration the global structure of the shapes [ZSCO*08]. A common approach to compute a dense correspondence is to extend a sparse one with techniques such as cross-parameterization or interpolation [Ale02].

组对应：形状对应的特殊形式同时涉及一组形状（即，在我们的问题定义中N> 2）。 尽管可以通过计算组中所有形状对之间的对应关系来直接解决该问题，但是考虑到所有形状一次可以在该过程中具有优势。 对于=示例，如图4所示，形状组可以提供信息以区分所有形状共有哪些结构或部分，并且应该在对应中考虑，哪些部分可以表征为噪声并且在计算中被忽略[WH09]。 另一个例子是同时注册一组形状，以便组信息可以作为解决方案搜索[GF09, XLZ 10]中的附加约束。此外，一组形状之间的稳健对应是任务的期望输入，例如建立一组形状的统计描述[DTT08]，尽管在这些方法中，对应和统计模型通常是联合计算而不是两个独立的程序。

**Group correspondence:** a specialized form of shape correspondence involves a group of shapes simultaneously (i.e., $N > 2$, in our problem definition). Although this problem can be straightforwardly solved by computing correspondences between all the pairs of shapes in the group, considering all the shapes at once can have advantages in the process. For example, as illustrated in Figure 4, the group of shapes can provide information to distinguish what structures or parts are common to all the shapes and should be considered in the correspondence, and which parts can be characterized as noise and ignored in the computation [WH09]. Another example is to register a group of shapes simultaneously, so that the group information can serve as an additional constraint in the solution search [GF09, XLZ*10]. Moreover, a robust correspondence between a group of shapes is the desired input for tasks such as building a statistical description of a group of shapes [DTT08], although in these methods the correspondence and statistical model are typically computed in conjunction and not as two independent procedures.

对应问题概述

## 3. Overview of correspondence problems

现在，我们通过更具体的方式讨论它们来概述通信问题，同时还描述一些示例应用程序。我们可以直接从元素的相似性获得对应关系，或者我们可以首先对齐形状，然后从对齐元素的接近度导出对应关系。 而且，我们也可以在两个程序之间进行迭代。 这些选项直接影响应选择哪种策略来查找对应关系。 值得注意的是，形状之间的对齐是计算的副产品，这对于底层应用是有用的并且有时是必要的。我们将首先考虑不使用对齐的情况，然后我们讨论关系R来源于以刚性或非刚性方式对齐形状的场景。 这些情况之间的区别如图5所示。我们最终确定了将时间维度添加到数据集的实例。

Now, we give an overview of correspondence problems by discussing them in a more concrete manner, while also describing some example applications. We can obtain a correspondence directly from the similarity of the elements, or we can first align the shapes and then derive a correspondence from the proximity of the aligned elements. Moreover, we can also iterate between the two procedures. These options directly affect which strategy should be selected to find the correspondence. It is worth noting that the alignment between the shapes is a side product of the computation which is useful and sometimes essential to the underlying application. We will start by considering the case in which no alignment is utilized, and then we discuss the scenarios where the relation $\mathcal{R}$ is derived from aligning the shapes in a rigid or a non-rigid manner. The distinction between these cases is illustrated in Figure 5. We finalize with the instance where time dimension is added to the datasets.

**Similarity-based correspondence:** one of the most fundamental ways of computing a correspondence is to estimate the similarity between pairs of shape elements or feature points collected from the shapes and derive a correspondence from those estimates, which is sometimes called the *feature matching* approach. The elements are commonly characterized by shape descriptors. A correspondence is then obtained by selecting assignments between pairs of elements while optimizing an objective function composed of two terms. The first term seeks to maximize the similarity between the descriptors of corresponding elements, while the second term seeks to minimize the distortion that would be introduced in the shapes if they were deformed to align their corresponding elements. However, the second term is estimated without explicitly aligning the shapes. Ideally, satisfying these objectives should translate into a solution that is geometrically or semantically meaningful. Such a solution is typically obtained with a standard optimization method (e.g., quadratic programming). Feature matching can be applied in any context where it is possible to compute a set of descriptors for the elements. Example applications include registration of 3D scans [CCFM08] and deforming surfaces [ASP*04], or skeleton matching [BMSF06].

**基于相似度的对应关系**：计算对应关系的最基本方法之一是估计从形状中收集的形状元素对或特征点之间的相似性，并从这些估计得出对应关系，这有时称为特征匹配方法。元素通常由形状描述符表征。然后通过选择元素对之间的分配同时优化由两个项组成的目标函数来获得对应关系。第一项旨在最大化相应元素的描述符之间的相似性，而第二项旨在最小化将在形状中引入的变形（如果它们变形以对齐其对应元素）。但是，在没有明确对齐形状的情况下估计第二项。理想情况下，满足这些目标应转化为几何或语义上有意义的解决方案。通常使用标准优化方法（例如，二次编程）获得这种解决方案。特征匹配可以应用于可以为元素计算一组描述符的任何上下文中。示例应用程序包括3D扫描[CCFM08]和变形曲面[ASP 04]的注册，或[BMSF06]的骨架匹配。

A correspondence problem that takes into account only the similarity of shape descriptors can be solved effectively by posing it as a linear assignment [PS82]. With the introduction of the distortion term, the problem becomes NP-hard. However, when the two shapes differ by a limited amount of deformation, good approximate solutions can still be obtained with heuristic algorithms, especially

只考虑形状描述符的相似性的对应问题可以通过将其作为线性赋值来有效地解决[PS82]。随着失真项的引入，问题变得非常难以解决。 然而，当两种形状因变形量有限而不同时，仍然可以通过启发式算法获得良好的近似解，特别是将问题放宽到连续域（例如，软化技术[GR95]或谱聚类[LH05]]）。 此外，特征匹配方法不仅限于其自己的域，可以与基于对齐的方法相结合，为这些方法[RL01]提供适当的初始化，或限制解空间的大小[GMGP05, AMCO08, CZ08, ACOT 10]，这也有助于解决这些问题。

with a relaxation of the problem to the continuous domain (e.g., the *softassign* technique [GR95] or spectral clustering [LH05]). Moreover, the feature matching approach is not restricted to its own domain and can be combined with alignment-based approaches to provide a proper initialization to these methods [RL01], or to restrict the size of the solution space [GMGP05, AMCO08, CZ08, ACOT*10], which also contributes to a practical solution of these problems.

**Rigid alignment:** under certain assumptions, it is possible to pose the correspondence problem as a search for a geometric transformation that aligns the shapes. One example application is the rigid alignment of geometry scans used for shape acquisition. The goal here is to capture a real-world static 3D shape and obtain its digital representation. However, it may not be possible to capture the entire object in a single scanning pass due to self-occlusions and physical constraints of the scanner, so it is often necessary to acquire multiple scans and optimally align them to reconstruct the full object [TL94, RL01, GMGP05, AMCO08]. The key characteristic of the rigid alignment problem is that the objects do not change from one scanning pass to another. Thus, it is assumed that each scan can be transformed with a *single rigid transformation* in order to align it perfectly with the other scans. Rigid transformations comprise mainly translations and rotations, and one of their important characteristics is that they reside in a low-dimensional space.

**刚性对齐**：在某些假设下，可以将对应问题构建为搜索对齐形状的几何变换。一个示例应用是用于形状获取的几何扫描的刚性对准。这里的目标是捕获真实世界的静态3D形状并获得其数字表示。但是，由于扫描仪的自遮挡和物理限制，可能无法在单次扫描过程中捕获整个对象，因此通常需要获取多次扫描并最佳地对齐它们以重建整个对象[TL94, RL01, GMGP05, AMCO08。刚性对齐问题的关键特征是对象不会从一个扫描传递到另一个扫描传递。因此，假设每次扫描可以用单个刚性变换进行变换，以使其与其他扫描完美对齐。刚性变换主要包括平移和旋转，它们的一个重要特征是它们位于低维空间中。

Scan alignment is just one example of many applications that rely on the assumption of rigidity in the datasets. For two shapes given as 3D point sets $\mathcal{S}$ and $\mathcal{Z}$, the problem of rigid alignment can be posed as: find the rigid transformation that, when applied to $\mathcal{S}$, maximizes the number of points in $\mathcal{S}$ that align to points in $\mathcal{Z}$. This goal is usually dependent on a threshold $\varepsilon$ that indicates when two points are close enough and can be considered as matching to each other [IR96]. Following this formulation, alignment with rigid or even affine transformations is a problem with a clear objective function, and efficient algorithms also exist to optimally solve such instances. However, as the complexity of these algorithms is still at least quadratic in the number of input elements, heuristics can be used to speed up the solution search, e.g., by exploring candidate assignments suggested by the feature matching approach, or randomizing steps of the algorithms [IR96]. Alternatively, a local-search algorithm such as the Iterated Closest Point (ICP) [RL01] can also be used.

扫描对齐只是许多应用程序的一个示例，这些应用程序依赖于数据集中的刚性假设。对于作为3D点集S和Z给出的两个形状，刚性对齐的问题可以提出为：找到刚性变换，当应用于S时，最大化S中与Z点中的点对齐的点数。该目标是通常依赖于阈值e，该阈值e指示两个点何时足够接近并且可以被认为彼此匹配[IR96]。在该公式之后，与刚性或甚至仿射变换的对齐是具有明确目标函数的问题，并且还存在有效算法以最佳地解决这种情况。然而，由于这些算法的复杂性仍然至少是输入元素数量的二次方，因此可以使用启发式来加速解决方案搜索，例如，通过探索由特征匹配方法建议的候选分配，或随机化算法的步骤[IR96]。或者，也可以使用诸如迭代最近点（ICP）[RL01]的局部搜索算法。

**Non-rigid alignment:** it might be necessary to lift the assumption that each scan can be perfectly aligned with a rigid transformation, e.g., when large amounts of noise are present in the scans. More significant examples of such datasets include the correspondence of articulated shapes [EK03, ASP*04, JZvK07, CZ08, HAWG08], where certain parts of the shapes can bend independently, the correspondence of anatomical shapes (e.g., organs) [AFP00], which can deform in an elastic manner and introduce stretching to localized portions of the shape, and finally the correspondence of shapes with different geometries but that represent a class of objects with parts that are semantically related [ACP03, ZSCO*08]. In the latter case, we can see the problem as that of establishing a correspondence between shapes that can differ in both local stretching and bending.

非刚性对齐：可能需要提升每个扫描可以与刚性变换完美对齐的假设，例如，当扫描中存在大量噪声时。 这些数据集的更重要的例子包括关节形状的对应[EK03，ASP04，JZvK07，CZ08，HAWG08]，其中形状的某些部分可以独立弯曲，解剖形状（例如，器官）的对应[AFP00]，可以弹性变形并将拉伸引入形状的局部部分，最后形状与不同几何形状的对应关系，但代表一类具有语义相关部分的物体[ACP03，ZSCO 08]。 在后一种情况下，我们可以将问题看作是在局部拉伸和弯曲两者中可以不同的形状之间建立对应关系的问题。

In this setting, it becomes necessary to add more freedom to how the shapes can be brought into correspondence. This can be achieved by generalizing two aspects of the problem. First, non-rigid (possibly non-linear) transformations can be taken into consideration, e.g, thin-plate splines [CR03]. Secondly, these transformations can be applied separately to local portions of the shape. For example, the transformation applied to a shape can be represented as a set of displacement vectors (one per shape vertex) [PMG*05]. Then, finding the best transformation amounts to computing the displacements that bring each vertex in correspondence with the target shape. The distinction from the rigid case is that the space of geometric transformations being considered is now inherently high-dimensional.

在这种情况下，有必要为形状如何对应增加更多的自由度。 这可以通过概括问题的两个方面来实现。 首先，可以考虑非刚性（可能是非线性）变换，例如薄板样条[CR03]。 其次，这些变换可以分别应用于形状的局部部分。 例如，应用于形状的变换可以表示为一组位移矢量（每个形状顶点一个）[PMG 05]。 然后，找到最佳变换相当于计算使每个顶点与目标形状相对应的位移。 与刚性情况的区别在于，所考虑的几何变换的空间现在具有固有的高维度。

Due to the high-dimensional nature of the solution space, these problems are typically solved with a form of local or approximate search, e.g., gradient descent [ACP03] or a combination of the non-linear transformations with the feature matching approach [CR03]. However, although heuristic solution methods are available, the quality of the results will typically depend on the complexity of the problem instance and the level of approximation introduced by the methods. As in the case of feature matching approaches, if there are more constraints available, they can be used to guide the algorithms more effectively to a correct solution.

由于解空间的高维性质，这些问题通常通过局部或近似搜索的形式来解决，例如，梯度下降[ACP03]或非线性变换与特征匹配方法的组合[CR03]。 然而，尽管可以使用启发式求解方法，但结果的质量通常取决于问题实例的复杂性和方法引入的近似水平。 与特征匹配方法的情况一样，如果有更多可用约束，则可以使用它们更有效地指导算法到正确的解决方案。

**Time-varying registration:** due to recent technological advances, an application that is attracting increasing attention is the reconstruction of 3D shapes acquired over time while moving and deforming. In this setting, a fixed number of scans is acquired per time step, and these scans have to be registered to allow the reconstruction of both the object and the motion sequence [MFO*07, WJH*07, SAL*08, PG08, dAST*08, LAGP09, GSdA*09, CZ09, TBW*09, ZST*10].

随时间变化的登记：由于最近的技术进步，吸引越来越多关注的应用是在移动和变形的同时重建随时间获得的3D形状。在此设置中，每个时间步骤获取固定数量的扫描，并且必须注册这些扫描以允许重建对象和运动序列[MFO 07，WJH 07，SAL 08，PG08，dAST 08，LAGP09，GSdA 09，CZ09，TBW 09，ZST 10]。

Although this may seem like another instance of the non-rigid alignment problem, there are certain particularities that make this problem unique. In the classic registration problem, it is assumed that all the scans can be registered to compose a single and coherent object. On the other hand, the time-varying setting introduces the additional difficulty that the shape might have deformed significantly from one frame to the other. Therefore, scans acquired later in time may only be registered to the earlier scans if the deformation is taken into account. Moreover, additional challenges are the large amount of missing data (due to occlusion) that can be present in each frame [PG08], and datasets that were captured over sparse time frames [CZ09, ZST*10]. However, the addition of temporal constraints can also help in reducing the size of the search space (e.g., kinematic constraints [MFO*07]).

虽然这看起来像是非刚性对齐问题的另一个例子，但是有一些特殊性使这个问题变得独特。 在经典注册问题中，假设可以注册所有扫描以组成单个且连贯的对象。 另一方面，时变设置引入了额外的困难，即形状可能从一帧到另一帧显着变形。 因此，如果考虑变形，则稍后获取的扫描可能仅被登记到较早的扫描。 此外，额外的挑战是每个帧[PG08]中可能存在的大量缺失数据（由于遮挡），以及在稀疏时间帧[CZ09，ZST 10]上捕获的数据集。 然而，时间约束的添加还可以帮助减小搜索空间的大小（例如，运动约束[MFO 07]）。

Having established the basic notions related to shape cor-

| | | | |
|---|---|---|---|
| *Input* | **Geometry representation** | | Points, feature points, surfaces, skeletons |
| | **Dimensionality of the data** | | 2D, 3D, 2D+time, 3D+time |
| *Output* | **Correspondence representation** | Correspondence + transformation | Translation, rigid, similarity, affine, nonlinear |
| | | Correspondence only | Bijective, injective, many-to-many, probabilistic, crisp |
| | **Full vs. partial** | | |
| | **Dense vs. sparse** | | |
| *Objective function* | **Similarity-based correspondence** | | Similarity only, similarity + distortion |
| | **Rigid alignment** | | Largest common pointset, geometric distance |
| | **Non-rigid alignment** | | Geometric distance + regularization |
| *Approach* | **Solution paradigm** | Transformation search | Alignment, pose clustering, non-rigid alignment |
| | | Correspondence search | Continuous optimization, combinatorial search |
| | | Hybrid search | ICP, prealignment + ICP, embedding + ICP |
| | **Fully-automatic vs. semi-automatic** | | |
| | **Global vs. local search** | | |
| | **Pairwise vs. groupwise** | | |

**Figure 6:** *Schematic view of the classification criteria followed in this work (a hierarchy read from left to right). The first entries are the components of the problem statement used to group the different criteria.*

本工作中遵循的分类标准的示意图（从左到右读取的层次结构）。 第一个条目是用于对不同条件进行分组的问题语句的组成部分。

在建立了与形状对应相关的基本概念后，我们接下来将问题分解为相关的组成部分，并更详细地分析每个问题。

respondence, we next divide the problem into its relevant components and analyze each of them in more detail.

对应方法的分类

## 4. Classification of correspondence methods

在本节中，我们将介绍几种对本报告中讨论的对应方法进行分类的方法。 这些分类的目的是让读者不仅可以通过算法方面来比较方法，还可以通过可以处理的问题的属性和方法的要求来比较这些方法。这些问题包括：方法可以处理哪些子问题（例如，部分对应，刚性配准等），可以处理哪些数据集，方法需要什么初始化等等。

In this section, we present several ways of classifying the correspondence methods discussed in this report. The purpose of these classifications is to allow readers to compare the methods not only by their algorithmic aspects, but also by the properties of the problems that can be handled and the requirements of the methods. These include questions such as: what subproblems the method can handle (e.g., partial correspondence, rigid registration, etc.), what datasets can be handled, what initialization the method requires, etc.

我们根据问题陈述对分类标准进行分组，我们在此再次回顾：给定输入形状S1；S2；...；SN，在它们的元素之间建立有意义的关系R．为了使此声明更具体，我们将其关键字与以下问题相关联。

We group the classification criteria based on the problem statement, which we review here once more: *given input shapes $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N$, establish a meaningful relation $\mathcal{R}$ between their elements*. To make this statement more concrete, we link its keywords to the following questions.

Keyword → Question

1. *shapes* → how are the input shapes represented?
2. *relation* → how is the output correspondence represented and what properties does it possess?

1.形状 - >输入形状如何表示？
2.关系 - >如何表示输出对应关系以及它具有哪些属性？

3. *meaningful* → which correspondence should be selected (which correspondence is closer to our objective)?
4. *establish* → what approach is used to compute the correspondence?

3.有意义的 - >应该选择哪种对应关系（哪种对应关系更接近我们的目标）？
4.建立 - >用什么方法计算对应关系？

Answering these questions leads to the different classification categories. We follow with a detailed discussion of these criteria, summarized in Figure 6.

回答这些问题会导致不同的分类类别。 我们将对这些标准进行详细讨论，如图6所示。

### 4.1. Input shapes

The geometry of the input datasets can be represented as point sets, oriented points (points with normals), surfaces, skeletons, images, volumes, or parts. Their dimensionality can be 2D, 3D, or they can be spatio-temporal: 2D + time or 3D + time. We focus on 2D and 3D shapes (point sets, surfaces, and skeletons). Surfaces can be represented implicitly (e.g., level sets) or explicitly (e.g., triangle meshes). One extra distinction in the case of triangle meshes is whether they need to represent well-defined manifolds or if they can be made up of triangle soups.

输入数据集的几何可以表示为点集，定向点（具有法线的点），曲面，骨架，图像，体积或零件。 它们的维度可以是2D，3D，也可以是时空的：2D +时间或3D +时间。 我们专注于2D和3D形状（点集，曲面和骨架）。 表面可以隐式表示（例如，水平集）或明确表示（例如，三角形网格）。在三角形网格的情况下，一个额外的区别是它们是否需要表示明确定义的流形或者它们是否可以由三角形汤组成。

Classic registration methods such as RANSAC [FB81], geometric hashing [WR97], pose clustering [Ols97], and alignment [HU90] typically work with point sets. Surfaces are the common representation for recent methods based

| Descriptor | Type of dataset |
|---|---|
| Shape context [BMP00, KPNK03] | Point sets |
| Spin images [JH99] | Oriented points |
| Multi-scale features [LG05] | Oriented points |
| Curvature maps [GGGZ05] | Surfaces |
| Integral invariants [GMGP05, MCH*06] | Surfaces |
| Spherical harmonics and wavelets [FS06, NSN*07, CHDD08, KH08] | Surfaces |
| Salient geometric features [GCO06] | Surfaces |
| Part-aware metric [LZSCO09] | Surfaces |
| Heat Kernel Signature [SOG09] | Surfaces |

**Table 1:** *A subset of the shape descriptors proposed in the literature that can be used for shape correspondence.*

表1：文献中提出的可用于形状对应的形状描述符的子集。

经典的配准方法，如RANSAC [FB81]，几何散列[WR97]，姿势聚类[Ols97]和对齐[HU90]通常适用于点集。 曲面是基于变形的最新方法的常见表示[HAWG08, ZSCO 08]，适用于等距曲面[BBBK08, LF09]或铰接形状[CZ08]的方法，以及基于模板匹配的图形应用[ACP03, PMG 05, LAGP09]。 时变表面是变形表面的运动重建工作的焦点[MFO 07, WJH 07, SAL 08, PG08, GSdA 09, LAGP09, ZST 10]。 Skeleton是形状表示的更通用的名称，例如中轴，Reeb图，曲线骨架或M-reps [SP08, CSM07, HSKK01, PCFea03, SSGD03, BMSF06, ACOT 10]。

on deformation [HAWG08, ZSCO*08], methods that work on isometric surfaces [BBBK08, LF09] or articulated shapes [CZ08], and graphics applications based on template matching [ACP03, PMG*05, LAGP09]. Time-varying surfaces are the focus of works on motion reconstruction of deforming surfaces [MFO*07, WJH*07, SAL*08, PG08, GSdA*09, LAGP09, ZST*10]. Skeleton is a more general name for shape representations such as the medial axis, Reeb graphs, curve skeletons, or M-reps [SP08, CSM07, HSKK01, PCFea03, SSGD03, BMSF06, ACOT*10].

These datasets can be obtained from a variety of sources, such as 3D scanners (based on LASER, structured light, physical contact, etc.) that provide point clouds, manual modeling via software which commonly results in triangle meshes, or images and volumes obtained with different imaging equipment, such as digital cameras, ultrasound, magnetic resonance imaging (MRI), and computed tomography (CT). Surfaces are typically extracted from images or volumes via image segmentation or isosurfacing. Skeletons are commonly obtained from surfaces by an extra processing step [CSM07], and they capture more structural information (such as shape parts represented as branches and their associated thicknesses).

这些数据集可以从各种来源获得，例如提供点云的3D扫描仪（基于激光，结构光，物理接触等），通过软件手动建模，通常产生三角形网格，或获得图像和体积使用不同的成像设备，如数码相机，超声波，磁共振成像（MRI）和计算机断层扫描（CT）。 通常通过图像分割或等面线从图像或体积中提取表面。 骷髅通常通过额外的处理步骤[CSM07]从表面获得，并且它们捕获更多结构信息（例如表示为分支的形状部分及其

*Shape descriptors:* instead of making use of the datasets in their original representation, an alternative characterization can be obtained by extracting representative points (features) from the shapes and computing descriptors for these points. The descriptors will typically be scalar values or vectors of scalars that capture some property of the shape around the neighborhood of the interest point. The shape descriptors are then used to indirectly establish the similarity between the datasets by assessing the similarity between the descriptors. Ideally, if two descriptors are similar, their corresponding points should also be similar. Alternatively, the descriptors can be used to guide the search for initial solutions, while the final verification of the correspondence quality is performed with the original dataset (e.g., [AMCO08]). Note that we can see the approach of comparing descriptors as a general correspondence framework, since the positions of the feature points themselves can be regarded as descriptors.

形状描述符：不是在原始表示中使用数据集，而是通过从这些点的形状和计算描述符中提取代表点（特征）来获得替代表征。描述符通常是标量值或标量的向量，其捕获兴趣点附近的形状的一些属性。然后，通过评估描述符之间的相似性，使用形状描述符间接地建立数据集之间的相似性。理想情况下，如果两个描述符相似，则它们的对应点也应相似。或者，描述符可用于指导搜索初始解决方案，而对应质量的最终验证是用原始数据集（例如，[AMCO08]）执行的。请注意，我们可以看到将描述符作为一般对应框架进行比较的方法，因为特征点本身的位置可以被视为描述符。

A variety of descriptors have been proposed in the literature and a discussion of their characteristics can be found in [VH01, BKS*05, TV08], while a comparison of descriptors for the image case is presented in [MS05]. In Table 1, we list a partial set of descriptors, which can be computed for each shape primitive and used in conjunction with correspondence algorithms for 2D surfaces or 3D point sets. Such a set of descriptors is the typical input to optimization- or search-based matching methods used in vision and graphics [MC03, ASP*04, BBM05, LH05, GMGP05, ZSCO*08]. A descriptor that has recently attracted a lot of interest is the Heat Kernel Signature [SOG09], related to the Laplace-Beltrami operator on surfaces and the spatial embeddings derived from it (see Section 5.4). This signature is preserved under isometries [OMMG10] and can be defined in a multi-scale manner to aid in the search of partial correspondences [OMMG10, DLL*10].

在文献中已经提出了各种描述符，并且可以在[VH01, BKS 05, TV08]中找到它们的特征的讨论，而在[MS05]中给出了图像情况的描述符的比较。 在表1中，我们列出了一组部分描述符，可以为每个形状基元计算这些描述符，并与2D表面或3D点集的对应算法结合使用。 这样的一组描述符是视觉和图形中使用的基于优化或搜索的匹配方法的典型输入[MC03, ASP04, BBM05, LH05, GMGP05, ZSCO 08]。最近引起很多兴趣的描述符是Heat Kernel Signature [SOG09]，它与表面上的Laplace-Beltrami算子和从中得到的空间嵌入有关（见5.4节）。 该签名保留在等距[OMMG10]下，并且可以以多尺度方式定义，以帮助搜索部分对应[OMMG10, DLL 10]。

Another interesting related problem is how to choose the set of descriptors that gives the best correspondence results, a problem known as *feature selection* in the machine learning literature [GE03, WH07, KHS10, vKTS*11].

另一个有趣的相关问题是如何选择能够提供最佳对应结果的描述符集，这在机器学习文献[GE03, WH07, KHS10, vKTS 11]中称为特征选择。

### 4.2. Output correspondence

输出通信

A correspondence between shapes can be represented in different manners and can possess different properties, which we discuss as follows.

形状之间的对应关系可以用不同的方式表示，并且可以具有不同的属性，我们将在下面讨论。

**Correspondence representation:** a correspondence can be represented as a transformation applied to the shapes (either a single transformation or multiple transformations) or simply as a relation between elements of the datasets, i.e., a set of pairwise assignments between vertices, parts, etc.

对应表示：对应关系可以表示为应用于形状的变换（单个变换或多个变换）或简单地表示为数据集的元素之间的关系，即顶点，部分等之间的成对分配。

*Correspondence + transformation:* when utilizing a single transformation or a set of them, one of the distinguishing factors is the type of transformation that is utilized. These transformations can be ordered by increasing number of degrees of freedom: translation, rigid transformation (translations and rotations), similarity transformation (includes isotropic scaling), affine transformation (adds shearing), and nonlinear deformation (includes nonlinear transformations).

对应+转换：当使用单个转换或它们的集合时，区别因素之一是所使用的转换类型。 这些变换可以通过增加自由度来排序：平移，刚性变换（平移和旋转），相似变换（包括各向同性缩放），仿射变换（增加剪切）和非线性变形（包括非线性变换）。

A rigid transformation preserves the pairwise distances between points and can be decomposed into translations, rotations and reflections. Therefore, it is the common choice when dealing with problems such as scan registration [RL01, GMGP05]. Similarity transformations incorporate the possibility of uniform scaling into the rigid transformations, which might be necessary in certain contexts such as matching patterns to limited portions of larger datasets (e.g., repeated pattern detection as in [PMW*08]). Moreover, affine transformations extend the previous set of transformations by also taking into consideration the possibility of shearing, which can be used at a global [IR96, AMCO08] or a

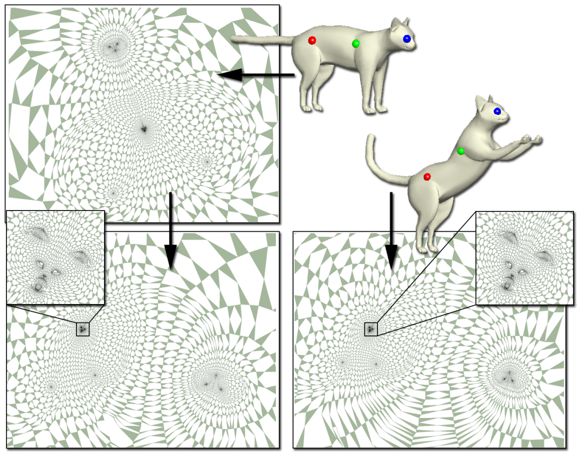

**Figure 7:** *Utilizing Möbius transformations for shape correspondence [LF09]: the meshes for a cat in two different poses are flattened onto the complex plane. Next, a Möbius transformation can be defined from three corresponding pairs of points (red, green, and blue). By applying the transformation to the flattening of the first pose (top-left), the result at the bottom-left is obtained, which looks similar to the flattening of the second pose (bottom-right).*

图7：利用Möbius变换进行形状对应[LF09]：将两个不同姿势的猫的网格平铺到复平面上。 接下来，可以从三对相应的点（红色，绿色和蓝色）定义莫比乌斯变换。 通过将变换应用于第一个姿势（左上角）的展平，获得左下角的结果，其看起来类似于第二个姿势的展平（右下

刚性变换保留了点之间的成对距离，并且可以分解为平移，旋转和反射。因此，在处理扫描注册[RL01，GMGP05]等问题时，这是常见的选择。相似性变换包括均匀缩放到刚性变换中的可能性，这在某些上下文中可能是必要的，例如将模式匹配到较大数据集的有限部分（例如，如[PMW 08]中的重复模式检测）。此外，仿射变换还通过考虑剪切的可能性来扩展先前的变换集，其可以在全局[IR96，AMC008]或本地[ACP03，SP04]级别使用。当匹配具有球体拓扑的非刚性表面时，要使用的自然变换是一组单调或莫比乌斯变换（图7），它们是共形（保留角度）并包含等距变换组（保留距离））[LF09]。最后，在非刚性对齐的更一般情况下，可能需要允许形状元素自由移动以与对应的数据集匹配，这可以被视为指定非线性可变形变换（例如，位移）到每个元素[PMG 05]。

local [ACP03, SP04] level. When matching non-rigid surfaces that possess the topology of a sphere, the natural transformation to use is the group of homographic or Möbius transformations (Figure 7), which are conformal (preserve angles) and contain the group of isometric transformations (which preserve distances) [LF09] . Finally, in more general cases of non-rigid alignment, it might be necessary to allow the shape elements to move freely in order to match with the corresponding dataset, which can be seen as assigning a nonlinear deformable transformation (e.g., a displacement) to each element [PMG*05].

我们还必须区分转换是应用于整体形状（全局转换），还是应用于局部方式，即形状的部分或元素。 全局案例是刚性对齐问题的核心概念，而本地案例通常用于非刚性性质的任务。

We must also distinguish whether the transformation is applied to the shape as a whole (a *global* transformation), or whether it is applied in a *local* manner, i.e., to parts or elements of the shape. The global case is a central concept in the problem of rigid alignment, while the local case is usual in tasks of a non-rigid nature.

仅对应：当仅使用对应关系时，关系R可以被限制为表示双射（一对一映射），注入（必须为作为参考的一个形状的每个元素定义关系， 但它可以是一对多映射），也可以允许完整关系（多对多）。 此外，我们可能只需要对一个元素子集进行一对一或一对多映射，例如在部分对应中，我们允许某些元素不属于任何赋值。

*Correspondence only:* when working only with a correspondence, the relation $\mathcal{R}$ can be limited to represent a bijection (one-to-one mapping), an injection (the relation has to be defined for every element of one of the shapes taken as reference, but it can be a one-to-many mapping), or a full relation can be allowed (many-to-many). Moreover, we might require a one-to-one or one-to-many mapping only for a subset of elements, such as in a partial correspondence where we allow some elements not to be part of any assignment.

Certain methods allow the user to select which type of mapping is desired, such as [LH05], where the final correspondence is obtained by filtering an initial result according to the mapping constraints. Other approaches assume that the focus of interest is only on a specific type of mapping and proceed to build their method over such an assumption (e.g. [MC03, BBM05], in which the correspondence problem is posed as an optimization where the mappings are constrained to be one-to-one).

某些方法允许用户选择期望哪种类型的映射，例如[LH05]，其中通过根据映射约束过滤初始结果来获得最终对应关系。 其他方法假设感兴趣的焦点仅在特定类型的映射上，并继续在这样的假设上构建它们的方法（例如[MC03，BBM05]，其中对应问题被作为优化，其中映射被约束为 是一对一的）。

A correspondence can also be characterized by whether a confidence value is assigned to the pairwise assignments. The assignments can be characterized as *crisp* (either an assignment is part of the correspondence or not) or *fuzzy* (they have a degree of confidence or probability attached to them). The first case can be seen as that where a binary confidence is associated to each assignment. The type of confidence measure that is available depends on the correspondence algorithm. For example, the method in [ZS08] formulates the correspondence problem in a probabilistic context and returns a probability associated to each assignment. The methods based on relaxation to the continuous domain also return a confidence weight attached to each assignment, e.g., the softassign technique [CR03, GRL*98] or spectral clustering [LH05]. On the other hand, the methods in [MC03, BBM05] return mainly binary outputs, since they formulate the problem in terms of integer optimization.

还可以通过是否将置信值分配给成对分配来表征对应关系。 分配可以被描述为清晰（分配是对应的一部分或不是）或模糊（它们具有一定程度的置信度或概率）。 第一种情况可以看作二元置信度与每个赋值相关联的情况。 可用的置信度测量类型取决于对应算法。 例如，[ZS08]中的方法在概率上下文中形成对应问题并返回与每个赋值相关联的概率。 基于对连续域的弛豫的方法也返回附加到每个分配的置信度权重，例如，softassign技术[CR03，GRL 98]或谱聚类[LH05]。 另一方面，[MC03，BBM05]中的方法主要返回二进制输出，因为它们根据整数优化来表示问题。

**Full vs. partial correspondence:** some methods are only suitable for contexts in which the full extent of the shapes is considered, while others are able to compute a partial correspondence. In general, if a method can find solutions to the partial case, it will also be applicable to the full correspondence case (while the reverse is not necessarily true). Since computing a partial correspondence is an important problem, we discuss such methods in more detail in Section 5.5.

完全对应部分对应：某些方法仅适用于考虑形状的完整范围的上下文，而其他方法则能够计算部分对应关系。 通常，如果方法可以找到部分情况的解，那么它也适用于完全对应的情况（反之则不一定正确）。 由于计算部分对应是一个重要问题，我们将在5.5节中更详细地讨论这些方法。

**Dense vs. sparse:** the advantage of defining the problem in terms of a sparse correspondence is that the complexity of the computation (in time and space) might be reduced by considering smaller sets of elements. Some techniques were designed by taking this view into consideration, such as the search-based methods described in [GMGP05, ZSCO*08]. Despite their associated exponential search space, these methods can be utilized in practice by considering a sparse set of feature points extracted from the shapes. Other methods do not make a strong distinction between these two cases and work interchangeably for computing both sparse and dense correspondences, since their complexity increases linearly with the number of elements in the shapes (e.g., some of the optimization methods discussed in Section 5). Finally, a collection of methods have been created to compute a dense correspondence from an initial sparse one, such as the cross-parameterization methods surveyed in [Ale02] or more general algorithms which take an initial set of markers as input for computing a non-rigid correspondence [ACP03, SP04, PMG*05, SSB05]. The definition of a distance met-

密集与稀疏：根据稀疏对应来定义问题的优点是可以通过考虑较小的元素集来减少计算的复杂性（在时间和空间上）。通过考虑这种观点来设计一些技术，例如[GMGP05，ZSCO 08]中描述的基于搜索的方法。尽管它们具有相关的指数搜索空间，但是通过考虑从形状中提取的稀疏特征点集，可以在实践中利用这些方法。其他方法在这两种情况之间没有很好的区别，并且可以互换地用于计算稀疏和密集对应，因为它们的复杂性随着形状中元素的数量线性增加（例如，第5节中讨论的一些优化方法）。最后，已经创建了一组方法来计算来自初始稀疏对应的密集对应关系，例如[Ale02]中调查的交叉参数化方法或更一般的算法，这些算法将一组初始标记作为输入来计算非刚性对应[ACP03，SP04，PMG 05，SSB05]。表面上距离度量的定义也可用于扩展稀疏对应，例如双调和距离[LRF10]。

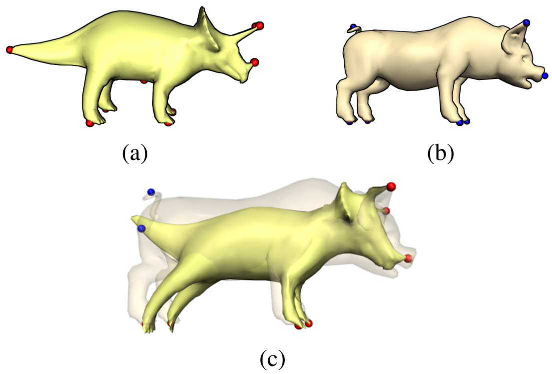


**Figure 8:** *Example of using a deformation to assess the quality of a correspondence (with the method described in [ZSCO*08]): the triceratops in (a) is deformed to match the pig in (b), according to a set of pairwise assignments between feature points (colored in red and blue). The result of the deformation is shown in (c).*

图8：使用变形评估对应质量的示例（使用[ZSCO 08]中描述的方法）：根据一组成对的方式，（a）中的三角龙变形以匹配（b）中的猪 特征点之间的分配（红色和蓝色）。变形的结果如（c）所示。

ric on the surfaces can also be used for extending a sparse correspondence, such as the biharmonic distance [LRF10].

### 4.3. Objective function 4.3目标函数

The objective function provides a measure of how good a given correspondence is, or how far it is from the desired solution. It is sometimes referred to as the *error measure*, the *cost function*, or the *energy*, in the case of methods that formulate the problem as the minimization of some energy function. Its formulation depends on the type of input dataset (points, surfaces, etc.), and also on the specific problems to be solved (partial correspondence, rigid alignment, etc.). Here, we will mainly look at the objective function from the perspective of the overview given in Section 3.

目标函数可以衡量给定对应关系的好坏程度，或者与所需解决方案的距离。 在将问题表述为一些能量函数的最小化的方法的情况下，有时将其称为误差测量，成本函数或能量。 它的公式取决于输入数据集的类型（点，表面等），还取决于要解决的具体问题（部分对应，刚性对齐等）。 在这里，我们将主要从第3节给出的概述的角度来看目标函数。

**Similarity-based correspondence:** when the goal is to find a correspondence between two datasets without first aligning the shapes, shape descriptors and intrinsic measures to quantify the quality of the correspondence must be used. Therefore, for two shapes $\mathcal{P}$ and $\mathcal{Q}$ and a correspondence relation $\mathcal{R}$, the objective takes the form

基于相似度的对应：当目标是在没有首先对齐形状的情况下找到两个数据集之间的对应关系时，必须使用形状描述符和量化对应质量的内在度量。 因此，对于两个形状P和Q以及对应关系R，目标采用该形式

$$\mathrm{Obj}(\mathcal{P},\mathcal{Q},\mathcal{R}) = \mathrm{Sim}(\mathcal{P},\mathcal{Q},\mathcal{R}) + \alpha\,\mathrm{Distor}(\mathcal{P},\mathcal{Q},\mathcal{R}), \quad (1)$$

with a similarity term that is linear on the number of feature points and a distortion term that is usually quadratic on the number of feature points, since it commonly involves comparing properties of pairs of points. The weight $\alpha$ controls the influence of each term in the objective function. Automatically setting $\alpha$ to a value that reflects the user's goal can also be a challenging problem [CMC*09].

具有与特征点数量成线性关系的相似项和通常在特征点数量上呈二次方的变形项，因为它通常涉及比较点对的属性。 权重a控制目标函数中每个项的影响。 自动将a设置为反映用户目标的值也可能是一个具有挑战性的问题[CMC 09]。

*Similarity term:* this term encodes the similarity of the shape descriptors of points in correspondence (Section 4.1). The descriptors can also include geometric attributes such as point normals or local frames, which can give an indication of whether the orientation of the points is coherent across the two matched point sets [ASP*04].

相似性术语：该术语编码对应点形状描述符的相似性（第4.1节）。 描述符还可以包括几何属性，例如点法线或局部帧，其可以指示点的方向是否在两个匹配的点集[ASP04]上是一致的。

*Distortion term:* the distortion term quantifies how much the shapes would be deformed if their corresponding elements were brought into alignment. A common candidate for a distortion measure is the disparity in the distances between pairs of matched points. The disparity is an approximate way of measuring the distortion introduced by the correspondence without having to first align the shapes. It can be expressed as

失真项：失真项量化了如果相应的元素对齐，形状将变形多少。 失真度量的共同候选者是匹配点对之间的距离的差异。视差是测量由对应引入的失真的近似方式，而不必首先对齐形状。 它可以表示为

$$\mathrm{Distor}(\mathcal{P},\mathcal{Q},\mathcal{R}) = \sum_{\substack{\{p_1,p_2\}\subset\mathcal{P}\\ \{q_1,q_2\}\subset\mathcal{Q}}} \mathrm{Dispar}(p_1,p_2,q_1,q_2), \quad (2)$$

where $(p_1,q_1)\in\mathcal{R}$ and $(p_2,q_2)\in\mathcal{R}$. The disparity term between two pairs $\{p_1,p_2\}$ and $\{q_1,q_2\}$ can be given by the difference in the distances between the pairs of points

其中（p1；q1）2 R和（p2；q2）2 R.两对fp1之间的差异项；p2g和fq1；q2g可以通过点对之间的距离的差异给出

$$\mathrm{Dispar}(p_1,p_2,q_1,q_2) = \left|\mathrm{dist}_{\mathcal{P}}(p_1,p_2) - \mathrm{dist}_{\mathcal{Q}}(q_1,q_2)\right|. \quad (3)$$

Any appropriate distance measure can be used. Examples include Euclidean distance [CZ08], or geodesic distance in the case of surfaces [ASP*04],

可以使用任何适当的距离测量。 例子包括欧几里德距离[CZ08]，或表面情况下的测地距离[ASP 04]，或者，可以使用交叉配置距离[SCF10]评估分配对之间的兼容性，它使用模糊测地线（表面的推广） 测地线）测量形状上点的结构排列的相似性。

Alternatively, the compatibility between pairs of assignments can be evaluated with the *intersection configuration distance* [SCF10], which utilizes *fuzzy geodesics* (a generalization of surface geodesics) to measure the similarity in the structural arrangement of points on the shapes.

*Deformation:* a more elaborate form of quantifying distortion is to use a global deformation measure, such as described in [HAWG08, ZSCO*08]. Once the matching of feature points is known, one shape can be deformed into the other so that the matched points are aligned. An example of this procedure is shown in Figure 8. Notice that this is different from the non-rigid deformation case, since here a correspondence is already available and only an estimate of the distortion is needed, while on the former case the shapes are deformed to establish the actual correspondence. For this step, one of the recently proposed deformation methods can be utilized, as in the surveys [Sor06, SSP07]. Then, measuring how much the surfaces had to deform to align to each other (an intrinsic rigidity energy) gives an indication of the distortion introduced by the correspondence. One advantage of using a surface-based deformation energy is that it is able to differentiate between correspondences that switch symmetric parts of the shape (Figure 9), which usually pass undetected when only pairwise distances are utilized. Note that the deformation can also be performed at the part level and with the aid of binary relations between parts [XLZ*10].

变形：一种更精细的量化失真形式是使用全局变形测量，如[HAWG08，ZSCO 08]中所述。一旦知道了特征点的匹配，就可以将一个形状变形到另一个形状中，以使匹配的点对齐。图8显示了此过程的一个示例。请注意，这与非刚性变形情况不同，因为此处已经存在对应关系并且仅需要估计失真，而在前一种情况下，形状会变形建立实际的通信。对于该步骤，可以使用最近提出的变形方法之一，如在调查中[Sor06，SSP07]。然后，测量表面必须变形多少以彼此对齐（固有的刚性能量）给出由对应引入的失真的指示。使用基于表面的变形能量的一个优点是它能够区分切换形状的对称部分的对应关系（图9），当仅使用成对距离时，这通常不会被检测到。注意，变形也可以在零件级别并借助于零件之间的二元关系[XLZ 10]来执行。

**Rigid alignment:** for the problem of rigid alignment between two or more datasets (e.g., point sets), the objective is commonly defined in terms of the number of matching

刚性对齐：对于两个或多个数据集（例如，点集）之间的刚性对齐问题，目标通常根据匹配点的数量来定义，或者由量化数据集彼此对齐的度量的度量给出。

**Figure 9:** *When searching for a correspondence between the two dinosaur models shown on the left, a rigidity energy assigns a higher value to a correspondence that switches the limbs of the dinosaur (to the right), than to a correspondence that does not switch any symmetric parts of the shape [ZSCO*08].*

图9：当搜索左侧显示的两个恐龙模型之间的对应关系时，刚性能量为切换恐龙肢体（向右）的对应关系赋予更高的值，而不是对不切换任何恐龙的对应关系。 形状对称的部分[ZSCO 08]。

points, or given by a metric that quantifies how well the datasets align to each other.

最大公共点集（LCP）：这里的兴趣在于找到一个转换，它将最大数量的点带入对应[IR96, AMCO08]，给定一个阈值e，它指示两个点是否足够接近并且可以认为是与每个点匹配 其他。 因此，目标是最大化匹配点集的基数。 它可以表示为两个点集P和Q.

*Largest Common Pointset (LCP):* here the interest is in finding a transformation that brings the largest number of points into correspondence [IR96, AMCO08], given a threshold ε which indicates whether two points are close enough and can be considered as matching to each other. Therefore, the objective is to maximize the cardinality of the set of matched points. It can be expressed for two point sets $\mathcal{P}$ and $\mathcal{Q}$ as

$$LCP(\mathcal{P},\mathcal{Q}) = \sum_{p\in\mathcal{P}} \mathrm{Match}(p,\mathcal{Q}), \tag{4}$$

where

$$\mathrm{Match}(p,\mathcal{Q}) = \begin{cases} 1 & \text{if } \exists q \in \mathcal{Q}, \text{ s. t. } \|p-q\| < \varepsilon \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

for some distance measure $\|\ldots\|$.

几何距离：另一个常见的目标函数不依赖于参数e，而是最小化由点之间的平方距离之和给出的对齐误差。 也就是说，对于变换集中的每个点，找到参考集中的闭合点，并将这两个点之间的距离添加到误差测量中，表示为

*Geometric distance:* another common objective function does not rely on a parameter ε but minimizes the alignment error given by the sum of squared distances between points. That is, for each point in the transformed set, the closest point in the reference set is found and the distance between these two points is added to the error measure, expressed as

$$\mathrm{Dist}(\mathcal{P},\mathcal{Q}) = \sum_{p\in\mathcal{P}} \mathrm{Dist}(p,\mathcal{Q}), \tag{6}$$

where

$$\mathrm{Dist}(p,\mathcal{Q}) = \min_{q\in\mathcal{Q}} \|p-q\|. \tag{7}$$

是算法中使用的常用度量，例如迭代最近点（ICP）[RL01]。 也可以利用该方案的变体，例如，通过添加方向或表面信息[CM92]，其中当我们的数据集P和Q作为表面给出时，Dist（p; Q）被更精细的点对面测量替换。。

This is the common measure utilized in algorithms such as the Iterated Closest Point (ICP) [RL01]. Variants of this scheme can also be utilized, e.g., by adding orientation or surface information [CM92], where $\mathrm{Dist}(p,\mathcal{Q})$ is replaced by a more elaborate point-to-surface measure when our datasets $\mathcal{P}$ and $\mathcal{Q}$ are given as surfaces.

根据以上所述，LCP公式具有以下优点：部分匹配可以由目标函数直接处理，因为最大匹配点集将对应于两个点集之间的重叠区域。 平方距离之和必然会考虑目标中的所有点，除非用户向算法提供点集之间重叠量的估计或阈值以识别彼此相距太远的点[RL01]。

From the above, the LCP formulation has the advantage that partial matching can be directly handled by the objective function, since the largest set of matching points will correspond to the region of overlap between the two point sets. The sum of squared distances will necessarily consider all the points in the objective, unless the user provides the algorithm with an estimate of the amount of overlap between the point sets or a threshold to identify points that are too far away from each other [RL01].

非刚性对齐：在形状通过非刚性变形彼此对齐的情况下，目标必须包含术语以量化何时这种变换是有意义的。 也就是说，如果每个顶点可以根据其自身的变换或位移自由移动，则必须强制执行某种形式的全局一致性（正则化）。 这种正则化可以通过限制变换的自由度数或通过惩罚大的变形来获得。 例如，我们可以要求相邻顶点的变换相似（这提供了从一个顶点到另一个顶点的平移的平滑过渡）。 这种变换相似性可以以直接方式（例如，通过变换的矩阵表示之间的范数[ACP03, SP04]）或根据导数[PMG 05]来测量。

**Non-rigid alignment:** in the case that the shapes are aligned to each other by a non-rigid deformation, the objective will have to incorporate terms to quantify when such a transformation is meaningful. That is, if each vertex can move freely according to its own transformation or displacement, some form of global consistency (regularization) has to be enforced. Such a regularization can be obtained by limiting the number of degrees of freedom of the transformations or by penalizing large deformations. For example, we can demand the transformations of neighboring vertices to be similar (which provides a smooth transition of transformations from one vertex to the other). Such a transformation similarity can be measured in a direct manner (e.g., by the norm between the matrix representations of the transformations [ACP03, SP04]) or according to derivatives [PMG*05].

此外，与基于相似性的匹配的情况一样，非刚性情况下的误差测量还包括量化数据集的对齐程度。 这可以通过测量几何距离（类似于上面讨论的刚性情况）[SP04, PMG 05]或表面情况下的点到平面距离[ACP03]来给出。

Moreover, as in the case of similarity-based matching, the error measure in the non-rigid case also includes a quantification of how well the datasets are aligned. This can be given by a measure of geometric distance (similar to the rigid case discussed above) [SP04, PMG*05] or point to plane distance in the case of surfaces [ACP03].

### 4.4. Solution approach 4.4解决方案方法

可以使用各种技术来搜索最佳对应关系。 在解决方案范例方面，有一些方法可以搜索对齐形状的变换，只考虑元素之间的成对分配的方法，使用众所周知的优化或搜索技术找到解决方案，以及执行混合搜索的方法， 在对齐和对应计算之间交替。 我们将在第5节讨论这些解决方案策略。这里我们考虑基于解决方案方法的特定属性的其他分类标准。

A variety of techniques can be utilized to search for the best correspondence. In terms of the solution paradigm, there are methods that search for a transformation that aligns the shapes, methods that only consider the pairwise assignments between elements and find a solution using well-known optimization or search techniques, and methods that perform a hybrid search, alternating between alignment and correspondence computation. We discuss these solution strategies in

Section 5. Here we consider additional classification criteria based on particular properties of the solution approaches.

**Fully-automatic vs. semi-automatic:** semi-automatic methods require user input, such as a proper initialization or a set of corresponding landmarks between the shapes. Fully-automatic methods do not require any user input besides a few parameter values. Semi-automatic methods include the approaches for cross-parameterization [Ale02] and methods that take markers as input [ACP03, SP04, PMG*05, SSB05].

全自动与半自动：半自动方法需要用户输入，例如正确的初始化或形状之间的一组相应的界标。 除了一些参数值之外，全自动方法不需要任何用户输入。 半自动方法包括交叉参数化方法[Ale02]和将标记作为输入的方法[ACP03, SP04, PMG 05, SSB05]。

Although user input is required for the proper initialization of certain methods, this can also be seen as a necessary requirement when the semantics of the shapes cannot be easily inferred. Therefore, a track open for future research is that of methods based on a feedback loop, where the user gradually improves a correspondence based on his or her preferences. Ideally, such a method would minimize the amount of user interaction and provide hints on what information is missing to refine the correspondence.

尽管某些方法的正确初始化需要用户输入，但是当不能容易地推断出形状的语义时，这也可以被视为必要的要求。 因此，对于未来研究开放的轨道是基于反馈回路的方法，其中用户基于他或她的偏好逐渐改善对应关系。 理想情况下，这种方法可以最大限度地减少用户交互的数量，并提供缺少哪些信息以提炼信息的提示。

**Global vs. local search:** the distinction here is whether the method explores the whole solution space in search of a good solution (e.g., by performing an exhaustive search [GMGP05, ZSCO*08]) or whether the results of the method depend directly on its initialization. The initialization can be given by a user, as with semi-automatic methods, or by a fully-automatic method, whose solution will be used as the starting point for the local search of the algorithm.

全局与本地搜索：这里的区别在于该方法是否探索整个解空间以寻求一个好的解决方案（例如，通过执行穷举搜索[GMGP05, ZSCO 08]）或方法的结果是否直接依赖于它 初始化。 初始化可以由用户给出，如同半自动方法，或者通过全自动方法，其解决方案将用作本地搜索算法的起始点。

The most prominent example of the local search category is the ICP algorithm [RL01], which alternates the computation of correspondences between points (given by the closest points) with the computation of an aligning transformation. Since this iterative process follows a single path in the solution space, it can end up with a result that is a local minimum. Thus, the initial state clearly influences the final result of the algorithm, and therefore different forms of initialization have been proposed for this algorithm (which take the form of computing a prealignment between the shapes to be matched). These are discussed in detail in Section 5.

局部搜索类别的最突出的例子是ICP算法[RL01]，其交替计算点之间的对应关系（由最近点给出）与对齐变换的计算。 由于此迭代过程遵循解空间中的单个路径，因此最终结果可能是局部最小值。 因此，初始状态明显地影响算法的最终结果，因此已经为该算法提出了不同形式的初始化（其采用计算待匹配的形状之间的预对准的形式）。 这些将在第5节中详细讨论。

Another example of algorithms that perform local search are the methods for non-rigid alignment based on explicitly computing the transformation for each shape element. Since these transformations are computed with a method based on gradient descent or Newton's optimization, the initialization will also necessarily influence the final correspondence result [ACP03, SP04, PMG*05].

执行局部搜索的算法的另一示例是基于显式计算每个形状元素的变换的非刚性对齐的方法。 由于这些变换是使用基于梯度下降或牛顿优化的方法计算的，因此初始化也必然会影响最终的对应结果[ACP03, SP04, PMG 05]。

**Pairwise vs. groupwise:** methods for group correspondence appear predominantly in the computational anatomy community [HM09], where a coherent correspondence between a group of shapes is important for the accurate construction of a statistical model. A successful class of methods for this case is based on the minimum description length approach, where quality criteria of the statistical model are used to guide the computation of the group correspondence and simultaneous construction of the statistical model [DTT08].

成对与分组：组对应的方法主要出现在计算解剖学界[HM09]中，其中一组形状之间的连贯对应对于统计模型的精确构建是重要的。 针对这种情况的一类成功的方法是基于最小描述长度方法，其中统计模型的质量标准用于指导组对应的计算和统计模型的同时构建[DTT08]。

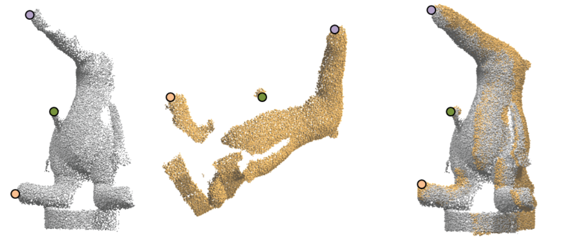

**Figure 10:** *Example of the alignment procedure in 3D for two partial scans of the Coati model (shown to the left): three points are sampled on each scan (the colored dots) and a rigid transformation is derived. The solution is found when the candidate transformation aligns the largest possible number of points (to the right) [AMCO08].*

图10：用于Coati模型的两次部分扫描的3D对准过程的示例（如左侧所示）：在每次扫描（彩色点）上采样三个点并且导出刚性变换。 当候选变换对齐最大可能点数（右侧）[AMCO08]时，找到解决方案。

Although the term *group correspondence* is not used in the field of time-varying reconstruction, a certain class of methods applied to this problem can also be seen as following this approach, since all scans are considered simultaneously in the registration. The difference to the case of anatomical shapes is that each scan can deform over time and there can be a significant amount of missing data between frames, while in the anatomy case the goal is typically a full correspondence between complete shapes, which are seen as variations from the same mean shape of an organ or bone. The time-varying reconstruction methods pose the problem as the reconstruction of a space-time surface [MFO*07, SWG08, SAL*08], or obtain a skeleton that is coherent for all the time frames [ZST*10]. The advantage of such formulations is that missing data can be filled in with data from frames that are further away in time.

尽管在时变重建领域中没有使用术语组对应，但是应用于该问题的某类方法也可以被视为遵循该方法，因为在登记中同时考虑所有扫描。 与解剖形状的情况的不同之处在于每次扫描都会随着时间的推移而变形，并且在帧之间可能存在大量的缺失数据，而在解剖学情况下，目标通常是完整形状之间的完全对应，这被视为变化。 从器官或骨骼的相同平均形状。 时变重建方法将问题定位为时空表面的重建[MFO 07, SWG08, SAL 08]，或者获得对于所有时间帧都是相干的骨架[ZST 10]。 这种配方的优点是可以用来自更远时间的帧的数据填充缺失的数据。

## 5. Representative methods

We use the classification based on how the correspondence is obtained as the starting point to cover the discussion of individual methods. Thus, the methods are primarily classified into those that search for an aligning transformation, those that search directly for a correspondence without performing alignment, and the ICP method, which works in a hybrid manner alternating between transformation search and correspondence search. Next, we discuss the use of embeddings, which can be applied for non-rigid alignment by combining them with methods for the rigid case, and then we conclude with a discussion on methods for computing partial correspondences.

5.代表性方法
我们使用基于如何获得对应关系的分类作为覆盖单个方法的讨论的起点。 因此，这些方法主要分为搜索对齐变换的那些，直接搜索不进行对齐的对应的方法，以及在变换搜索和对应搜索之间交替工作的ICP方法。 接下来，我们讨论嵌入的使用，通过将它们与刚性情况的方法相结合，可以应用于非刚性对齐，然后我们讨论计算部分对应的方法。

It is worth noting that many of the methods discussed here are also applicable to images. The differences lie in the procedures that extract feature points, compute shape descriptors, and quantify the distortion introduced by a correspondence, which are dependent on the data representation. In the case of images, the problems of measuring distances and preserving the neighborhood structures of elements are sim-

值得注意的是，这里讨论的许多方法也适用于图像。 差异在于提取特征点，计算形状描述符和量化由对应引入的失真的过程，这些失真取决于数据表示。 在图像的情况下，通过由这些数据集强制执行的常规参数化来简化测量距离和保持元素的邻域结构的问题[TH08]。 此外，我们可能还必须考虑不同类型的变换（例如，将3D形状投影到2D平面上），以解决诸如匹配立体图像或从不同视点拍摄的图像的配准等问题[Bro92]。

| Approach | Method | 2D Time | 2D Space | 3D Time | 3D Space |
|---|---|---|---|---|---|
| Alignment | Naive algorithm | $O(m^3 n^2 \log n)$ | – | $O(m^4 n^3 \log n)$ | – |
| | Randomized | $O(mn^2 \log n)$ | – | $O(mn^3 \log n)$ | – |
| | Randomized verification | $O((n^2 r + lm) \log n) \approx O(n^2 \log n)$ | – | $\approx O(n^3 \log n)$ | – |
| | Sets of 4 coplanar points | $O(n^2 + k)$ | $O(n)$ | $O(n^2 + k)$ | $O(n)$ |
| Pose clustering | Naive algorithm | $O(m^2 n^2 + h)$ | $O(h)$ | $O(m^3 n^3 + h)$ | $O(h)$ |
| | Randomized | $O(mn^2 + h)$ | $O(h)$ | $O(mn^3 + h)$ | $O(h)$ |

| Approach | Method | Pre-processing | Space | Query |
|---|---|---|---|---|
| 2D Geometric hashing | Original algorithm | $O(m^3 \log m)$ | $O(m^3)$ | $O(n^3 \log n)$ |
| | Randomized | $O(r^3 \log r)$ | $O(r^3)$ | $O((n^3 + lm) \log n) \approx O(n^3 \log n)$ |

**Table 2:** *Complexity of rigid registration methods for two sets with m and n points. r is the size of a subset of points used for random verification. l is the number of times that such a subset of points match consistently and further verification is needed. k is the size of the output. h denotes the size of the accumulation table in pose clustering.*

表2：具有m和n点的两组的刚性配准方法的复杂性。 r是用于随机验证的点子集的大小。 l是这样的点子集始终匹配并且需要进一步验证的次数。 k是输出的大小。 h表示姿势聚类中的累积表的大小。

plified by the regular parameterization that is enforced by these datasets [TH08]. Furthermore, we might also have to consider different types of transformations (e.g., the projection of a 3D shape onto a 2D plane), for problems such as matching stereo images or registration of images taken from different viewpoints [Bro92].

### 5.1. Transformation and alignment search

5.1。 转换和对齐搜索
我们记得，该类中的方法首先搜索对齐形状的变换，然后从对齐元素的邻近度导出对应关系。

We recall that methods in this class first search for a transformation that aligns the shapes, and then derive the correspondence from the proximity of the aligned elements.

刚性对齐：刚性对齐的方法依赖于用于对齐的变换可以从一小组样本点导出的事实。 例如，如果考虑3D中两个点集之间的刚性变换，则其参数（由旋转矩阵和平移向量给出）可以从三个点的初始配置及其变换位置导出。 在对变换进行采样之后，算法可以验证对齐的质量或者对变换进行投票。

**Rigid alignment**: methods for rigid alignment rely on the fact that the transformations used for alignment can be derived from a small set of sample points. For example, if considering a rigid transformation between two point sets in 3D, its parameters (given by a rotation matrix and a translation vector) can be derived from an initial configuration of three points and their transformed positions. After sampling a transformation, the algorithm can either *verify* the quality of the alignment or *vote* on the transformation.

样本和验证：在刚性假设下，通过直接应用上述[HU90]的抽样思想给出了一种朴素对齐算法。 具体而言，算法从第一个形状中采样三个点，从第二个形状中采样三个点，导出刚性变换，并测试变换对齐两个形状的程度（例如，使用4.3节中讨论的目标之一）。 如图10所示。在测试两种形状中所有可能的点三元组后，返回最佳转换。 3D中的这种朴素算法具有0（m3 n3）的复杂度，用于对点的三元组进行采样，并且0（mlogn）用于验证，产生0（m4 n3 logn）的总复杂度，用于对齐两个大小为m和n的点集。。

*Sample and verify:* under assumptions of rigidity, a naive alignment algorithm is given by a direct application of the sampling idea above [HU90]. Specifically, the algorithm samples three points from the first shape and three points from the second shape, derives a rigid transformation, and tests how well the transformation aligns the two shapes (e.g., with one of the objectives discussed in Section 4.3). This is illustrated in Figure 10. After testing all possible triplets of points in the two shapes, the best transformation is returned. This naive algorithm in 3D has a complexity of $O(m^3 n^3)$ for sampling the triplets of points and $O(m \log n)$ for the verification, yielding a total complexity of $O(m^4 n^3 \log n)$ for aligning two point sets of size $m$ and $n$.

Clearly, such an algorithm is far from efficient. Thus, different modifications have been proposed to improve upon the idea. Approaches following the philosophy of the random sample consensus (RANSAC) method propose to randomize the different steps in the procedure described above [FB81]. Instead of sampling all possible triplets of points on one shape, only a constant-sized set of random samples has to be considered, reducing the complexity by a factor of $O(m^3)$. Furthermore, the verification step can also be randomized, reducing the complexity by another factor of $O(m)$ in the typical case [IR96].

显然，这样的算法远没有效率。 因此，已经提出了不同的修改来改进该想法。 遵循随机样本共识（RANSAC）方法的原则的方法建议使上述过程中的不同步骤随机化[FB81]。 不是在一个形状上对所有可能的三元组进行采样，而是必须仅考虑一组恒定大小的随机样本，从而将复杂度降低0（m3）。 此外，验证步骤也可以是随机的，在典型情况下将复杂度降低另一个因子0（m）[IR96]。

Other approaches propose to explore geometric invariances maintained by the transformations. One such case is the ratio of distances between three coplanar points, which is preserved by rigid and affine transformations. Thus, the problem of searching for triplets of points that provide the optimal transformation can be transposed to that of finding four sets of coplanar points that share the same ratios [Hut91]. By pre-processing these invariances and keeping them in appropriate data structures that allow for efficient retrieval, output-sensitive methods can be achieved [AMCO08], reducing the complexity of the alignment problem even further to $O(n^2 + k)$, where $k$ is the size of the reported output.

其他方法提出探索由变换维持的几何不变性。 一个这样的情况是三个共面点之间的距离比，其通过刚性和仿射变换来保持。 因此，搜索提供最佳变换的点的三元组的问题可以转换为找到共享相同比率的四组共面点的问题[Hut91]。 通过预处理这些不变性并将它们保存在允许有效检索的适当数据结构中，可以实现输出敏感方法[AMCO08]，将对齐问题的复杂性进一步降低到0（n2 + k），其中k是 报告输出的大小。

*Sample and vote:* instead of sampling a transformation and evaluating its quality, the verification step can be replaced by a voting procedure. For this purpose, pose clustering utilizes an accumulation table [Sto87, Ols97]. After enumerating two triplets of points and deriving a transformation, a vote indexed by the parameters of the transformation is stored in the table. At the end of this $O(m^3 n^3)$ process, the cells with most votes correspond to the best candidate transformations that align the point sets. Note that processing the accumulation table requires an extra $O(h)$ step dependent on the size $h$ of the table.

抽样和投票：验证步骤可以由投票程序取代，而不是对转换进行抽样并评估其质量。 为此，姿势聚类使用累积表[Sto87，Ols97]。 在枚举两个三元组点并导出变换之后，由变换的参数索引的投票存储在表中。 在该0（m3 n3）过程结束时，具有最多投票的单元格对应于对齐点集合的最佳候选变换。 请注意，处理累积表需要额外的0（h）步骤，具体取决于表的大小h。

Geometric hashing is a voting-based method that makes

几何散列是一种基于投票的方法，它利用预处理来加速对齐[WR97]。 其主要思想是在哈希表中存储一组参考点集的所有可能配置，以便当我们寻找与查询点集最佳匹配的参考点集时，可以有效地执行该搜索。 我们可以非正式地看一下这种方法，将朴素枚举的O（m3 n3）复杂度分解为O（m3 logm）预处理阶段（对参考集的所有可能配置进行采样并将它们存储在哈希表中）和 O（n3 logn）查询阶段（对查询集的所有可能配置进行采样并在哈希表中累积投票以允许检索最佳匹配参考集）。 查询阶段的速度增加是以使用更多内存为代价获得的。

use of pre-processing to speed up the alignment [WR97]. Its main idea is to store in a hash table all the possible configurations of a group of reference point sets, so that when we seek the reference point set that best matches to a query point set, this search can be performed efficiently. We can informally look at this method as breaking the $O(m^3 n^3)$ complexity of the naive enumeration into an $O(m^3 \log m)$ preprocessing phase (which samples all the possible configurations of a reference set and stores them in the hash table) and an $O(n^3 \log n)$ query phase (which samples all the possible configurations of the query set and accumulates votes in the hash table to allow the retrieval of the best matching reference set). The increase in speed in the query phase is gained at the cost of using more memory.

备注：正如我们从讨论中看到的，这些对齐技术都是密切相关的，它们的复杂性可以如表2所示进行比较。文献中也提出了几何散列和姿势聚类的随机化版本。 请注意，当目标是最小化两个点集之间的均方误差时，会出现刚性对齐的特殊情况。 在这种情况下，不需要搜索最佳对齐变换，因为可以通过协方差矩阵[Ume91]的奇异值分解直接获得变换。 另一个观察是，采样和验证变换的原理也适用于具有不同类型变换的其他上下文。 例如，在[LF09]中，Möbius变换用于建立近似等距的形状之间的对应关系。 这些变换也可以从每个形状采样的三元组中得出（如图7所示）。

*Remarks:* as we see from the discussion, these alignment techniques are all closely related and their complexity can be compared as in Table 2. Randomized versions of geometric hashing and pose clustering were also proposed in the literature. Note that a special case of rigid alignment arises when the goal is to minimize the mean squared error between two point sets. In this case, a search for the best aligning transformation is not needed, since the transformation can be obtained directly via the singular value decomposition of a covariance matrix [Ume91]. Another observation is that the principle of sampling and verifying transformations also applies to other contexts with different types of transformations. For example, in [LF09], Möbius transformations are used to establish a correspondence between shapes that are approximately isometric. These transformations can also be derived from triplets of points sampled from each shape (as illustrated in Figure 7).

分段刚性对齐：到目前为止讨论的方法利用一个全局变换来匹配一个形状与另一个形状。不同类的方法通过将变换应用于形状的局部部分来概括该想法。在[CZ08]中，这些变换以分段刚性的方式应用，以建立关节形状之间的对应关系。该问题被表述为用候选变换标记形状的顶点，这是通过将顶点的局部帧与相似的描述符对齐来估计的（图11）。由于现在顶点被限制为具有来自预定义集合的变换，因此与允许将任何变换分配给顶点的方法相比，解决方案搜索被大大简化。通过在标记优化中添加正则化项，可以保证将顶点分组为刚性分量。这种方法的另一种方法是明确地将形状拟合到关节骨骼的运动骨架上，这样骨架就可以用来跟踪形状的运动，并推断出哪些区域缺少数据[PG08, CZ09, GSdA 09 ]。

**Piecewise-rigid alignment:** the methods discussed so far utilize one global transformation to match one shape to another. A different class of methods generalizes this idea by applying transformations to local portions of the shapes. In [CZ08], these transformations are applied in a piece-wise rigid manner to establish a correspondence between articulated shapes. The problem is formulated as labeling the vertices of the shapes with candidate transformations, which were estimated by aligning the local frames of vertices with similar descriptors (Figure 11). Since now the vertices are restricted to possess a transformation from a pre-defined set, the solution search is greatly simplified, in contrast to the methods that allow any transformation to be assigned to a vertex. By also adding a regularization term to the labeling optimization, a grouping of the vertices into rigid components is guaranteed. An alternative to this approach is to explicitly fit the shapes to a kinematic skeleton of articulated bones, so that the skeleton can be used to track the movement of the shape and also infer in which regions there is missing data [PG08, CZ09, GSdA*09].

**Non-rigid alignment:** for the methods described in [ACP03, SP04, PMG*05], different transformations are assigned to each vertex on the shape. The problem is formulated as find-

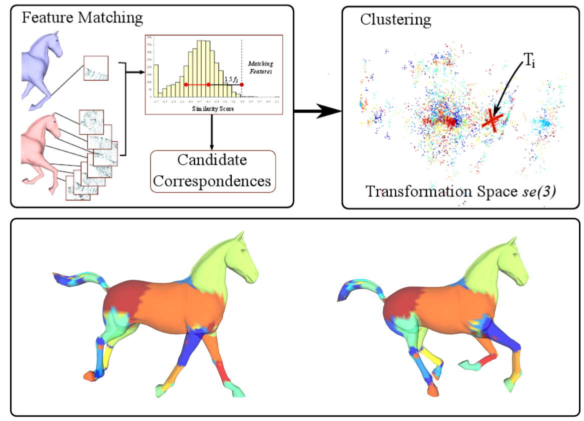


**Figure 11:** *Example of non-rigid alignment of articulated shapes: candidate correspondences are obtained by matching vertices with similar shape descriptors (top left). The candidates are used to derive transformations, which are then clustered (top right) and assigned to the shape vertices according to a labeling algorithm [CZ08]. The result of labeling two corresponding shapes is shown on the bottom.*

图11：关节形状的非刚性对齐示例：通过匹配具有相似形状描述符的顶点（左上）来获得候选对应关系。 候选者用于导出变换，然后将变换聚类（右上）并根据标记算法[CZ08]分配给形状顶点。 标记两个相应形状的结果显示在底部。

ing the best transformation that brings each vertex in a reference shape close to its counterpart in the target shape, and a regularization term is added to enforce the similarity of transformations across neighboring vertices. The difficulty in this setting is avoiding solutions that are local minima. This is achieved by initializing the methods with a set of corresponding marker points and solving the optimization in a multi-level fashion. The optimization can be posed as a nonlinear least squares problem and solved with a Newton-based method. This mainly involves the computation of derivatives of the high-dimensional problem and least-squares optimization at each step. Alternatively, the optimization can be performed by labeling a Markov random field [PH03].

非刚性对齐：对于[ACP03, SP04, PMG 05]中描述的方法，将不同的变换分配给形状上的每个顶点。 该问题被公式化为找到使参考形状中的每个顶点接近目标形状中的对应物的最佳变换，并且添加正则化项以强制跨越相邻顶点的变换的相似性。 这种设置的难点在于避免局部最小化的解决方案。 这是通过用一组相应的标记点初始化方法并以多级方式求解优化来实现的。 优化可以作为非线性最小二乘问题提出，并用基于牛顿的方法求解。 这主要涉及在每个步骤计算高维问题的导数和最小二乘优化。 或者，可以通过标记马尔可夫随机场[PH03]来执行优化。

Instead of computing local deformations or displacements for the vertices, in [SSB05], the displacements are implicitly obtained by learning a function that warps one shape into the other. The warp is obtained by solving a convex optimization problem similar to learning a support vector machine classifier (which includes a form of regularization in its definition). Thus, global minima are avoided.

在[SSB05]中，不是计算顶点的局部变形或位移，而是通过学习将一个形状扭曲成另一个形状的函数来隐式地获得位移。 通过求解类似于学习支持向量机分类器（其定义中包括正则化形式）的凸优化问题来获得扭曲。 因此，避免了全局最小值。

An extension of this class of methods which circumvents the need of marker points is proposed in [LSP08], where the alignment between two shapes is performed with two separated transformations: a global rigid transformation which roughly aligns the shapes, and per-vertex affine transformations that bring the non-rigid shapes into full alignment. A robust alignment can also be obtained by deforming one shape into the other in terms of a 3D optical flow [dATSS07] or a Laplacian deformation of the meshes [dAST*08].

在[LSP08]中提出了一种绕过标记点需求的这类方法的扩展，其中两个形状之间的对齐用两个分离的变换执行：全局刚性变换，大致对齐形状，以及每顶点仿射变换 使非刚性形状完全对齐。 通过在3D光流[dATSS07]或网格的拉普拉斯变形[dAST 08]方面将一个形状变形为另一个形状，也可以获得稳健的对准。

*Image registration:* deforming one shape into the other can also be achieved by adopting methods developed for image registration [MV98]. First, the shapes are transformed into 2D images (or 3D volumes) by mapping each feature point to its nearest pixel (or voxel). The value that is assigned to the pixel can be a vector of descriptors computed at the point [TH08], or the generated image can represent a level set function of the shape [HNM06]. Finally, the resulting images or volumes are registered by computing a global alignment followed by a non-rigid deformation. The advantage of such an approach is that the shapes are transformed into a parameterized representation where a variety of registration algorithms can be utilized [MV98]. However, creating a volume with enough resolution to capture all the details on the shapes can imply considerable memory consumption in 3D.

图像配准：通过采用为图像配准开发的方法[MV98]，也可以实现将一种形状变形为另一种形状。 首先，通过将每个特征点映射到其最近的像素（或体素），将形状转换为2D图像（或3D体积）。 分配给像素的值可以是在点[TH08]处计算的描述符的矢量，或者生成的图像可以表示形状[HNM06]的水平集函数。最后，通过计算全局对齐，然后进行非刚性变形来记录所得到的图像或体积。 这种方法的优点在于，将形状转换为参数化表示，其中可以使用各种配准算法[MV98]。 但是，创建具有足够分辨率的卷以捕获形状上的所有细节可能意味着3D中的大量内存消耗。

### 5.2. Correspondence search

5.2。 通讯搜索

The characteristic of the methods discussed in this section is that they work primarily with the pairwise assignments between feature points, without searching for transformations that align the shapes. The correspondence problem is typically posed as optimizing an objective function of the form $\mathrm{Obj}(\mathcal{P},\mathcal{Q},\mathcal{R}) = \mathrm{Sim}(\mathcal{P},\mathcal{Q},\mathcal{R}) + \alpha \mathrm{Distor}(\mathcal{P},\mathcal{Q},\mathcal{R})$, as described in Section 4.3. The objective is based on the quality of pairwise assignments (a linear term) and the compatibility between pairs of such assignments (a quadratic term). The solution is found by using well-known discrete or continuous optimization methods. A special group of methods in discrete optimization utilize a tree-based search to explore the solution space.

本节讨论的方法的特点是它们主要用于特征点之间的成对分配，而不搜索对齐形状的变换。 对应问题通常被认为是优化形式Obj（P; Q; R）= Sim（P; Q; R）+ aDistor（P; Q; R）的目标函数，如4.3节所述。 目标是基于成对分配的质量（线性项）和这些分配对之间的兼容性（二次项）。 通过使用众所周知的离散或连续优化方法找到该解决方案。 离散优化中的一组特殊方法利用基于树的搜索来探索解空间。

**Optimization:** if the objective being optimized is only composed of a similarity term $\mathrm{Sim}(\mathcal{P},\mathcal{Q})$, then the formulation becomes a Linear Assignment Problem (LAP). This simplified objective can be solved by the simplex algorithm, since it is a special case of a linear program [PS82]. However, if the correspondence is constrained to a one-to-one mapping, the problem becomes that of finding an optimal matching in a weighted bipartite graph, which can be solved more efficiently by the Hungarian algorithm in $O(n^3)$ time, where $n$ is the number of feature points in each shape [PS82].

优化：如果被优化的目标仅由相似项Sim（P; Q）组成，则该公式变为线性指派问题（LAP）。 这个简化的目标可以通过单纯形算法求解，因为它是线性程序的一个特例[PS82]。 然而，如果对应关系被限制为一对一映射，则问题变为在加权二分图中找到最佳匹配的问题，这可以通过匈牙利算法在O（n3）时间内更有效地解决，其中n 是每个形状中的特征点数[PS82]。

On the other hand, if the objective comprises both the linear and quadratic terms, we arrive at a Quadratic Assignment Problem (QAP), which is known to be NP-hard [PRW94]. Several techniques have been proposed to compute approximate solutions to this problem. One group of methods poses the problem as an integer optimization, which is relaxed to the continuous setting and solved with a continuous optimization technique. Examples include the softassign technique [GR95] (which iteratively normalizes rows and columns of an affinity matrix), concave programming [MC03], approximations based on linear programming [BBM05], spectral clustering [LH05], or relaxation labeling [ZD06]. It can also be formulated in probabilistic terms and solved as a convex optimization problem [ZS08].

另一方面，如果目标包括线性项和二次项，我们得到二次分配问题（QAP），已知它是NP-hard [PRW94]。 已经提出了几种技术来计算该问题的近似解。 一组方法将问题作为整数优化提出，其被放宽到连续设置并且通过连续优化技术求解。 示例包括softassign技术[GR95]（迭代地标准化亲和度矩阵的行和列），凹面编程[MC03]，基于线性编程的近似[BBM05]，谱聚类[LH05]或松弛标记[ZD06]。 它也可以用概率术语表达，并作为凸优化问题解决[ZS08]。

Another group of methods solves the problem in the discrete setting without resorting to the continuous domain. One common solution approach in the discrete case is to solve the problem by computing an optimal labeling of a graph, e.g., the problem can be posed in terms of a Markov network where the set of labels corresponds to matching points on the target shape [ASP*04, ZST*10]. Other methods make use of metaheuristics for combinatorial optimization, such as ant colony optimization [vKHZW07]. One more option is to sample the space of correspondences in search of a solution, guided by geodesic distances and importance sampling [TBW*09].

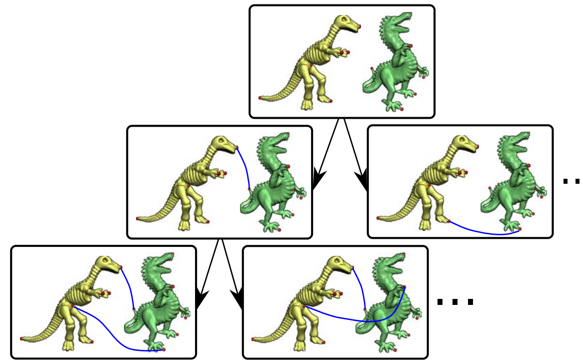

**Figure 12:** *Example of search-based correspondence for a set of feature points [ZSCO*08]: each node of the tree encodes a partial correspondence. All possible assignments are added when expanding a new level of the tree.*

图12：一组特征点的基于搜索的对应关系的示例[ZSCO 08]：树的每个节点编码部分对应关系。 扩展树的新级别时会添加所有可能的分配。

另一组方法在不依赖于连续域的情况下解决了离散设置中的问题。 离散情况下的一种常见解决方案是通过计算图的最佳标记来解决问题，例如，问题可以用马尔可夫网络提出，其中标记集对应于目标形状上的匹配点[ASP 04, ZST 10]。其他方法利用元启发式进行组合优化，例如蚁群优化[vKHZW07]。 另一个选择是在测地距离和重要性采样[TBW 09]的指导下，采集对应空间以寻找解决方案。

*2D contour correspondence:* a considerable part of the literature has also focused on the specific case of 2D contour correspondence, since these datasets can be easily extracted from 2D images. A collection of techniques were developed by taking into account the fact that the vertices on a contour can be linearly ordered. This observation is used in combination with optimization techniques such as dynamic programming [LWZ*04, SN06], which can also be used to solve the problem posed in terms of computing shortest paths [MCH*06], and graph cuts [STCB07]. The last two techniques are able to find an optimal correspondence for two contours.

二维轮廓对应：相当一部分文献也关注二维轮廓对应的具体情况，因为这些数据集可以很容易地从二维图像中提取出来。 通过考虑轮廓上的顶点可以线性排序的事实，开发了一组技术。 该观察与诸如动态编程[LWZ 04, SN06]的优化技术结合使用，其也可用于解决在计算最短路径[MCH 06]和图形切割[STCB07]方面提出的问题。 最后两种技术能够找到两个轮廓的最佳对应关系。

**Tree-based search:** one specific group of methods in discrete optimization finds a solution by making use of tree-based search techniques, such as branch-and-bound or priority search [GMGP05, FS06, ZSCO*08, ACOT*10, XLZ*10]. During the tree expansion, each node represents a partial solution. A full solution is found by following the path from the root of the tree to one of its leaves. Although the specific strategy in which the tree is expanded differs from method to method, these techniques usually involve three important steps: expanding a node that represents a new partial solution (branching), estimating how far the partial solution is from

基于树的搜索：离散优化中的一组特定方法通过利用基于树的搜索技术找到解决方案，例如分支定界或优先搜索[GMGP05, FS06, ZSCO 08, ACOT 10, XLZ 10]。 在树扩展期间，每个节点表示部分解决方案。 通过遵循从树的根到其中一个叶子的路径，找到完整的解决方案。 虽然扩展树的具体策略因方法不同而不同，但这些技术通常涉及三个重要步骤：扩展代表新的部分解（分支）的节点，估计部分解与最优解的距离（边界）），并消除不会导致最佳解决方案（修剪）的节点。

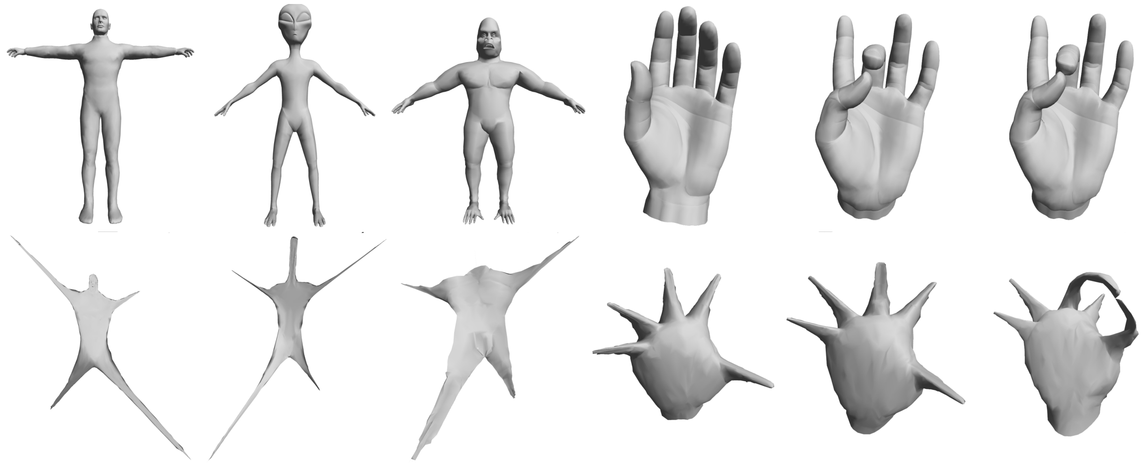

**Figure 13:** *Example of shape normalization: by applying Multi-Dimensional Scaling (MDS) to the meshes on the top row, the embeddings on the bottom row are obtained. Note how the pose of limbs and fingers is normalized, turning the non-rigid alignment problem into that of rigid alignment. The last column shows an example containing topological noise, where two fingers were connected after the reconstruction [GSCO07].*

图13：形状标准化的示例：通过将多维缩放（MDS）应用于顶行的网格，可以获得底行的嵌入。 注意肢体和手指的姿势是如何标准化的，将非刚性对齐问题转换为刚性对齐问题。 最后一列显示了一个包含拓扑噪声的示例，其中两个手指在重建后连接[GSCO07]。

the optimum solution (bounding), and eliminating nodes that will not lead to the optimum solution (pruning).

In the case of correspondence, solutions are mainly represented as collections of assignments between pairs of feature points, and the expansion step involves adding a new pairwise assignment to a given solution (Figure 12). Bounding and pruning can be performed by verifying the quality of the registration given by the current solution, either by aligning the shapes [GMGP05] or by deforming one shape into the other [ZSCO*08]. Other pruning methods include testing the compatibility between pairwise assignments, such as quantifying the distortion introduced in the Euclidean [GMGP05, FS06] or geodesic distances [ZSCO*08, ACOT*10] between pairs of points, or testing the agreement in the spatial configuration of the shapes [ACOT*10]. Naturally, the descriptors computed for the feature points are also considered in the bounding and pruning steps.

在对应的情况下，解决方案主要表示为特征点对之间的分配集合，并且扩展步骤涉及向给定解决方案添加新的成对分配（图12）。 可以通过验证当前解决方案给出的配准质量来执行边界和修剪，通过对齐形状[GMGP05]或通过将一个形状变形为另一个[ZSCO 08]。 其他修剪方法包括测试成对分配之间的兼容性，例如量化欧几里德[GMGP05, FS06]中引入的失真或者成对点之间的测地距离[ZSCO 08, ACOT 10]，或者测试空间配置中的协议。 形状[ACOT 10]。 当然，在边界和修剪步骤中也考虑为特征点计算的描述符。

When a hierarchical or multi-resolution structure is extracted from the shape representation, this information can also be considered in the solution search. Skeletons are commonly represented as trees or graphs for which a tree can be easily extracted. Therefore, in this context, it is common to resort to search-based algorithms that take this hierarchy into account [SSGD03]. Methods of a more greedy nature can also benefit from such hierarchical [BMSF06] or coarse-to-fine representations of the shapes [HSKK01].

当从形状表示中提取分层或多分辨率结构时，也可以在解决方案搜索中考虑该信息。 骷髅通常表示为可以容易地提取树的树或图。 因此，在这种情况下，通常采用基于搜索的算法来考虑这种层次结构[SSGD03]。 更贪婪的方法也可以受益于形状[HSKK01]的这种分层[BMSF06]或粗粒表示。

### 5.3. ICP and variants

This section discusses mainly the ICP method, which iteratively computes a correspondence by alternating between two steps. In the first step, the method searches for an alignment between the shapes. In the second step, a correspondence is derived from the alignment. Finally, this procedure is reiterated by using the correspondence to estimate a new aligning transformation. Thus, we call it a hybrid search method, since it searches for both alignment and correspondence solutions, which in turn affect each other. The different variants of the ICP algorithm are obtained when the two steps are solved in different manners (e.g., by changing the type of transformation or the way in which the correspondences are determined) [RL01].

5.3。 ICP和变体
本节主要讨论ICP方法，它通过两个步骤之间的交替迭代计算对应关系。 在第一步中，该方法搜索形状之间的对齐。 在第二步中，从对齐导出对应关系。 最后，通过使用对应关系来估计新的对齐变换来重复该过程。 因此，我们将其称为混合搜索方法，因为它搜索对齐和对应解决方案，这反过来又相互影响。 当以不同方式解决这两个步骤时（例如，通过改变变换的类型或确定对应关系的方式），获得ICP算法的不同变体[RL01]。

**Rigid alignment:** in the "classic" variant of the ICP algorithm for rigid alignment, given two point sets $\mathcal{P}$ and $\mathcal{Q}$, an assignment is established between every point $p \in \mathcal{P}$ and its closest point in $\mathcal{Q}$, according to a given distance metric. Next, from all the pairwise assignments that were defined in the previous step, the best rigid transformation that aligns the two point sets is estimated by solving a linear system and the point sets are then realigned according to this transformation. Finally, the two-step procedure is repeated until there is no significant change in the alignment.

刚性对齐：在用于刚性对齐的ICP算法的“经典”变体中，给定两个点集P和Q，根据给定的距离度量，在每个点p 2 P与其在Q中的最近点之间建立分配。 接下来，从上一步中定义的所有成对分配中，通过求解线性系统来估计对齐两个点集的最佳刚性变换，然后根据该变换重新对齐点集。 最后，重复两步程序，直到对齐没有显着变化。

The initial positions of the point sets tremendously influence the final result of the ICP algorithm, since the first correspondence is derived from this initial configuration. Thus, a crucial step in ICP-based methods is to perform a prealignment of the shapes so that the algorithm does not get trapped in local minima. Different forms of prealignment have been proposed in the literature to address this issue. The classic solutions are to rely on a set of matching feature points, an initial set of markers given by a user, or to automatically prealign the shapes with Principal Component Analysis (PCA) [RL01]. Recently, prealignment based on the reflec-

点集的初始位置极大地影响ICP算法的最终结果，因为第一对应关系源自该初始配置。 因此，基于ICP的方法中的关键步骤是执行形状的预对准，使得算法不会陷入局部最小值。 为了解决这个问题，文献中已经提出了不同形式的预对准。 经典解决方案依赖于一组匹配的特征点，用户给出的初始标记集，或者使用主成分分析（PCA）[RL01]自动预先对齐形状。 最近，基于形状的反射对称轴的预对准已被建议作为另一种有效的解决方案[PSG 06]。

tional symmetry axes of the shapes has been suggested as another effective solution [PSG*06].

非刚性对齐：ICP方法也可以通过修改其某些组件用于非刚性对齐。 一组方法计算加权对应，其中每个分配具有相关的置信度值。 这些值对于强大的离群值检测至关重要。 在将形状与加权的ICP严格对齐之后，通过计算基于薄板样条的翘曲函数[CR03, BR07]，形状彼此非刚性地变形。 在[HAWG08]中，原始ICP算法的刚性变换被基于刚体分量的变形代替，以便解决近似等距形状的配准问题。 此外，铰接形状的对齐也可以通过首先将形状嵌入到将其标准化以进行弯曲的空间中来获得（第5.4节），然后在该空间中刚性地对齐形状。

**Non-rigid alignment:** the ICP method can also be used for non-rigid alignment by modifying some of its components. One set of methods computes a weighted correspondence where each assignment has an associated confidence value. These values are crucial for robust outlier detection. After rigidly aligning the shapes with the weighted ICP, the shapes are non-rigidly deformed into each other by computing a warp function based on thin-plate splines [CR03, BR07]. In [HAWG08], the rigid transformation of the original ICP algorithm is substituted by a deformation based on rigid-body components, in order to address the registration of approximately isometric shapes. Moreover, the alignment of articulated shapes can also be obtained by first embedding the shapes in a space that normalizes them for bending (Section 5.4), and then rigidly aligning the shapes in this space.

最近，ICP的变体也已用于时变表面重建的背景下，以对准相邻时间帧的几何形状[WJH 07, PG08, WAO 09, LAGP09]。 如果每单位时间获得足够数量的扫描，则可以假设在形状的空间配置中仅发生小的变化（即，可以一致地跟踪刚体分量），并且因此每个的初始对准 框架在注册时不是一个强大的问题。

Recently, variants of ICP have also been utilized in the context of time-varying surface reconstruction to align the geometry of adjacent time frames [WJH*07, PG08, WAO*09, LAGP09]. If a sufficient number of scans is acquired per unit of time, it can be assumed that only small changes take place in the spatial configuration of the shapes (i.e., rigid-body components can be consistently tracked), and so the initial alignment of each frame is not a strong issue in the registration.

### 5.4. Use of embeddings

5.4。使用嵌入
形状的非刚性对齐，尤其是铰接形状的对齐，也可以通过首先将形状嵌入到刚性部件的构造被标准化的空间中，然后将该问题简单地处理为刚性对准的情况来实现。嵌入空间（如图13所示）。然后可以通过目前讨论的任何方法获得刚性对准。创建这种嵌入的关键是获得对弯曲不变的形状的内在表示（例如，通过收集表面点之间的测地距离），然后利用该表示将形状嵌入新的环境空间中，以便在这个新空间中，形状的内在几何被转化为它的外在几何。该嵌入可以通过诸如多维缩放（MDS）[EK03, BBK06, BBBK08]或谱变换[JZvK07, MHK08, SY10]的技术获得。 [ZvKD10]给出了对不同形式嵌入的全面介绍。嵌入用于规范化的形状也可以用于其他类型的数据集（图14）。

The non-rigid alignment of shapes, especially that of articulated shapes, can also be accomplished by first embedding the shapes in a space where the configuration of the rigid parts is normalized, and then treating the problem simply as a case of rigid alignment in this embedding space (shown in Figure 13). The rigid alignment can then be obtained by any of the methods discussed so far. The key to create such an embedding is to obtain an intrinsic representation of the shape which is invariant to bending (e.g., by collecting geodesic distances between surface points) and then utilize this representation to embed the shape in a new ambient space, so that the intrinsic geometry of the shape is translated into its extrinsic geometry in this new space. This embedding can be obtained with techniques such as Multi-Dimensional Scaling (MDS) [EK03, BBK06, BBBK08] or the spectral transform [JZvK07, MHK*08, SY10]. A comprehensive coverage of different forms of embeddings is given in [ZvKD10]. Embedding the shapes for normalization can also be used for other types of datasets (Figure 14).

另一组方法还利用形状的光谱特性进行对应，尽管这些方法用于计算姿势不变点签名，而不是将形状明确地嵌入姿势标准化空间中。 示例包括基于Laplace-Beltrami算子[Rus07]的全局点签名，使用热核进行形状匹配[SOG09, OMMG10, DLL 10]，以及用于扩展稀疏对应的双调和距离的定义[LRF10]。

Another collection of methods also makes use of spectral properties of shapes for correspondence, although these are utilized to compute pose-invariant point signatures and not to explicitly embed the shapes in a pose-normalizing space. Examples include the global point signatures based on the Laplace-Beltrami operator [Rus07], the use of the heat kernel for shape matching [SOG09, OMMG10, DLL*10], and

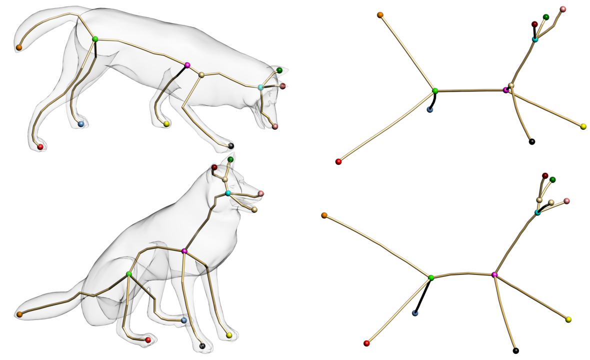

**Figure 14:** *The skeletons of the dog in two different poses (to the left) are normalized by applying MDS (to the right). The embeddings are used to measure the spatial consistency of a correspondence, which acts as a pruning test in the search-based method of [ACOT*10].*

图14：通过应用MDS（向右）对两个不同姿势（左侧）的狗的骨骼进行归一化。 嵌入用于测量对应的空间一致性，其在[ACOT 10]的基于搜索的方法中用作修剪测试。

the definition of the biharmonic distance for extending a sparse correspondence [LRF10].

### 5.5. Partial correspondence

5.5。 部分通信
由于计算部分对应关系是形状对应的一个重要特征，因此我们在本节中讨论为此任务提出的一些策略。 该问题被定义为找到可以计算有意义的对应关系的形状元素的子集，而不是找到可以包括在两个形状中不存在的附加部分或特征的完整对应。 该任务可以看作由两个子问题组成：搜索一致匹配的k个特征点的最佳子集，并找到这些k个元素之间的对应关系。

Since computing a partial correspondence is an important specialization of shape correspondence, we discuss in this section a few strategies proposed for this task. The problem is defined as finding a subset of shape elements for which a meaningful correspondence can be computed, as opposed to finding a full correspondence that could include additional parts or features which do not exist in both shapes. This task can be seen as composed of two subproblems: searching for an optimal subset of $k$ feature points that match consistently, and finding the correspondence between these $k$ elements.

确定子集的一种方法是检查目标函数以寻找对准误差的急剧增加，当将异常点添加到匹配点集合[GMGP05, ZSCO 08]时出现。 或者，可以向优化提供对异常值特征的数量的估计，这限制了在计算的对应关系[MC03, BBM05]中出现的点的数量。 这样的估计可以从数据本身得出[OEK08]。

One approach to determine the subset is to examine the objective function in search of sharp increases in the alignment error, which appear when an outlier point is added to the set of matched points [GMGP05, ZSCO*08]. Alternatively, an estimate on the number of outlier features can be provided to the optimization, which limits the number of points that appear in the computed correspondence [MC03, BBM05]. Such an estimate can be derived from the data itself [OEK08].

确定特征子集的另一种策略是依靠投票[LF09, ACOT 10]。 这里，计算一系列候选对应关系，并对构成每个候选者的成对分配进行投票。 在此过程结束时，确定的任务将成为投票数最高的任务，而与没有任何有意义匹配的点相关的任务具有可忽略的投票数量。 此过程充当群组强化，我们只对分配进行投票，如果它可以是一致通信的一部分。

Another strategy to determine the subset of features is to rely on voting [LF09, ACOT*10]. Here, a series of candidate correspondences is computed and votes are cast on the pairwise assignments that constitute each candidate. At the end of this process, the assignments that are certain emerge as the ones with the highest number of votes, while the assignments relating points that do not have any meaningful matching possess a negligible quantity of votes. This procedure acts as a group reinforcement, where we only vote on an assignment if it can be part of a consistent correspondence.

It is also natural to look at the partial correspondence problem as that of matching two graphs. The feature points on a shape or skeleton can be seen as the nodes in a graph, where every pair of nodes is connected with an edge whose weight is proportional to some geometric quantity (e.g., distance between the nodes). Then, partial matching becomes the problem of subgraph isomorphism, whose decision variant is known to be NP-complete. Since different heuristics have been proposed to address this problem, it is also possible to use these methods for shape correspondence. For example, the notion of matching two graphs by finding a set of operations that transform one graph into the other (e.g., by merging nodes [NB07]) is used to derive heuristic algorithms for skeleton matching in 2D [SKK04] and 3D [BMSF06]. Then, the subset of matching features is given by the nodes that are not removed during the editing of the graph.

将部分对应问题看作匹配两个图的问题也是很自然的。 形状或骨架上的特征点可以被视为图中的节点，其中每对节点与边缘连接，边缘的权重与某个几何量（例如，节点之间的距离）成比例。 然后，部分匹配成为子图同构的问题，其决策变量已知为NP完全。 由于已经提出了不同的启发法来解决该问题，因此也可以将这些方法用于形状对应。 例如，通过查找将一个图形转换为另一个图形的一组操作（例如，通过合并节点[NB07]）来匹配两个图形的概念用于导出用于2D [SKK04]和3D [BMSF06]中的骨架匹配的启发式算法。]。 然后，匹配特征的子集由在编辑图形期间未被移除的节点给出。

## 6. Validation of correspondence methods

6.验证通信方法

Validation is an important and necessary aspect of the correspondence problem, since we need to be able to effectively compare results from different methods. The most common form of validation is visual inspection of results. Displaying a morph between shapes is a similar way of assessing the visual quality of correspondences, where we expect to see a smooth transition from one shape to the other for a good correspondence [KS04, ZSCO*08]. Such procedures allow for a qualitative comparison of the results. However, since this form of evaluation can be subjective, more objective or quantitative procedures are also sought.

验证是对应问题的一个重要且必要的方面，因为我们需要能够有效地比较不同方法的结果。 最常见的验证形式是对结果进行目视检查。 在形状之间显示变形是评估对应的视觉质量的类似方式，我们希望看到从一种形状到另一种形状的平滑过渡以实现良好的对应[KS04, ZSCO 08]。 这些程序允许对结果进行定性比较。 然而，由于这种评估形式可能是主观的，因此也寻求更客观或定量的程序。

One possibility for more objective comparisons is to use the output of an objective function (Section 4.3) to derive a similarity measure, so that the correspondence method can be indirectly evaluated in terms of retrieval [FS06, JZ07]. This assumes that the accuracy of the retrieval results will be proportional to the accuracy of the computed correspondences. Additionally, in the computational anatomy community, it is common to assess the computed correspondences in terms of the quality of the statistical shape models that they generate, which can be evaluated with measures such as generality, specificity, or compactness [DTC*02, SRN*03, KE06]. As in the case of retrieval, this assumes that more accurate correspondences will lead to better models.

更客观比较的一种可能性是使用目标函数的输出（第4.3节）来导出相似性度量，以便可以根据检索[FS06, JZ07]间接评估对应方法。 这假设检索结果的准确性将与计算的对应关系的准确性成比例。 此外，在计算解剖学界，通常根据它们生成的统计形状模型的质量来评估计算的对应关系，这可以通过诸如一般性，特异性或紧凑性的度量来评估[DTC 02, SRN 03 , KE06]。 与检索的情况一样，这假设更准确的对应将导致更好的模型。

However, to allow for a direct comparison of results, the most reasonable procedure is to utilize a set of shapes already in correspondence, so that the computed correspondences can be compared against the ground-truth. This can be achieved with a discrete measure such as the Hamming loss (counting the number of points incorrectly matched [CMC*09]) or more continuous measures such as the endpoint error (where, for each point, we compute the distance from its matching point to its known ground-truth). We can add up the endpoint errors [KE06] or compute statistics on them [CMC*09].

然而，为了允许直接比较结果，最合理的程序是利用已经对应的一组形状，以便可以将计算的对应关系与地面实况进行比较。 这可以通过离散测量来实现，例如汉明损失（计算错误匹配的点数[CMC 09]）或更多连续测量，例如端点误差（对于每个点，我们计算距其匹配点的距离） 它的已知基础事实）。 我们可以添加端点错误[KE06]或计算它们的统计数据[CMC 09]。

These considerations lead us to the question of a benchmark for shape correspondence. A number of datasets developed for shape retrieval and analysis have been utilized for evaluating results. Shapes from the well-known *Princeton Shape Benchmark* [SMKF04] and *McGill 3D Shape Benchmark* [SZM*08] have been widely used for the comparison of methods that work on articulated shapes. Moreover, the datasets related to the various tracks of the *Shape Retrieval Contest* (SHREC) have also been considered. These datasets are aimed at various problems such as partial shape retrieval and retrieval of specific models, e.g., CAD models and human faces. Recently, datasets involving non-rigid deformations have been made available, such as the *SCAPE dataset* [ASK*05], the *Non-rigid World Dataset* [BBK06], and the *TOSCA* dataset [BBK08].

这些考虑因素使我们想到了形状对应的基准问题。 已经开发了许多用于形状检索和分析的数据集来评估结果。 来自着名的普林斯顿形状基准[SMKF04]和麦吉尔3D形状基准[SZM 08]的形状已被广泛用于比较用于关节形状的方法。 此外，还考虑了与形状检索竞赛（SHREC）的各种轨道相关的数据集。 这些数据集针对各种问题，例如部分形状检索和特定模型的检索，例如CAD模型和人脸。 最近，已经提供了涉及非刚性变形的数据集，例如SCAPE数据集[ASK 05]，非刚性世界数据集[BBK06]和TOSCA数据集[BBK08]。

From the above list, the TOSCA dataset can be characterized as a true benchmark for correspondence, since the models have a compatible triangulation (i.e., the ordering of vertices corresponds across all the deformations of a same model), and thus the correct correspondences can be obtained. This dataset has been further extended into a correspondence benchmark for the 2010 Shape Retrieval Contest (SHREC'10) [SHR10]. It includes 138 meshes divided into 3 classes (human, dog, and horse), modified by different types of deformations (e.g., global and local scaling) or topological changes (e.g., introduction of holes). The creation of a dataset with ground-truth correspondences has also been considered for anatomical shapes, where a reduced set of shapes in correspondence was enlarged by applying random (yet realistic) deformations on the original shapes [HJT08].

从上面的列表中，TOSCA数据集可以被表征为真实的对应基准，因为模型具有兼容的三角测量（即，顶点的排序对应于同一模型的所有变形），因此正确的对应关系可以是获得。 该数据集已进一步扩展为2010形状检索竞赛（SHREC'10）[SHR10]的对应基准。 它包括138个网格，分为3类（人类，狗和马），通过不同类型的变形（例如，全局和局部缩放）或拓扑变化（例如，引入孔）进行修改。 具有地面实况对应关系的数据集的创建也被考虑用于解剖学形状，其中通过在原始形状上应用随机（但真实的）变形来扩大对应的减少的形状组[HJT08]。

However, there is still the absence of a benchmark directed at more *general* correspondence problems (e.g., including man-made shapes that possess significant geometric and topological differences, where semantic information or functionality play a more essential role in linking the parts of different shapes). Such a dataset would have to include a collection of challenging correspondence cases and also a list of prominent features or parts on the shapes with their ground-truth correspondence across the dataset.

然而，仍然缺乏针对更一般的对应问题的基准（例如，包括具有显着几何和拓扑差异的人造形状，其中语义信息或功能在链接不同形状的部分中起更重要的作用）。 这样的数据集必须包括一系列具有挑战性的通信案例，还包括形状上的突出特征或部分列表，以及它们在整个数据集中的地面实况对应关系。

## 7. Applications of correspondence

7.通信的应用

In this section, we discuss what we view as the most important applications that make use of correspondence methods. Examples of these applications are illustrated in Figure 15.

在本节中，我们将讨论我们认为使用对应方法的最重要的应用程序。 这些应用的例子如图15所示。

**Shape registration:** given a number of scans in arbitrary initial positions, the goal of registration is to match regions that correspond across the scans, so that the scans can be aligned and the target object can be fully reconstructed. It can be assumed either that the shapes do not change during the scanning (rigid registration) [RL01, AMCO08] or that they are free to deform during the acquisition (non-rigid registration) [ASP*04, JZvK07, BR07, CZ08, HAWG08].

形状配准：在任意初始位置进行多次扫描时，配准的目标是匹配扫描对应的区域，以便可以对齐扫描并完全重建目标对象。 可以假设在扫描（刚性配准）[RL01, AMCO08]期间形状不会改变，或者在采集过程中它们可以自由变形（非刚性配准）[ASP 04, JZvK07, BR07, CZ08, HAWG08]。

**Time-varying surface reconstruction:** the goal in this task

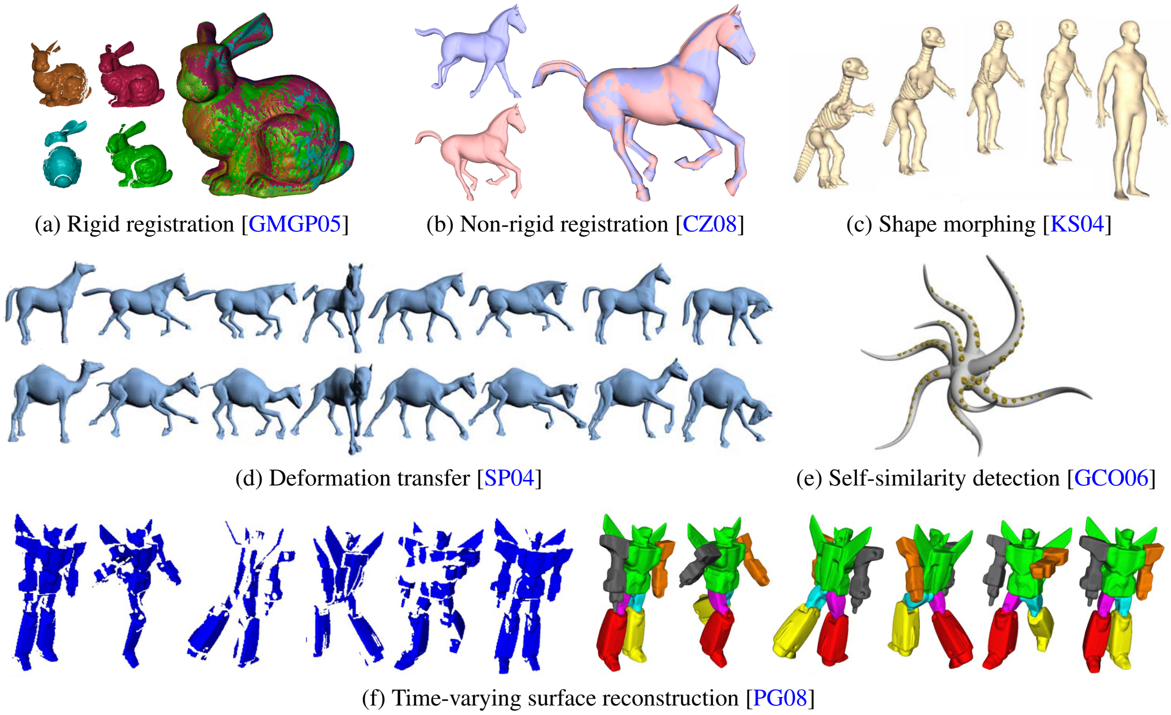


**Figure 15:** *Examples of applications that make use of correspondence methods: (a) a set of scans (to the left) is rigidly aligned to reconstruct the shape of the bunny (to the right), (b) the horse in two different poses (to the left) is non-rigidly aligned (result shown to the right), (c) the dino-skeleton is morphed into the human, (d) the motion defined on the horse (top line) is transferred to the camel (bottom line), (e) an application of partial matching: the suction cups on the tentacles of the octopus are detected as being similar (highlighted in yellow), (f) a set of range scans of an object in motion (shown to the left in blue) provides a single reconstructed model on which the motion is defined (shown to the right in color).*

图15：使用对应方法的应用示例：（a）一组扫描（左侧）严格对齐以重建兔子的形状（向右），（b）两种不同姿势的马 （左边）是非刚性对齐的（结果显示在右边），（c）恐龙骨架变形为人，（d）马上定义的运动（顶线）转移到骆驼（ 底线），（e）部分匹配的应用：章鱼触手上的吸盘被检测为相似（以黄色突出显示），（f）运动物体的一组范围扫描（显示在 左边是蓝色）提供了一个单一的重建模型，运动被定义在该模型上（右边显示颜色）。

时变曲面重建：此任务的目标是重建在移动和变形时随时间扫描的3D形状。挑战在于获取大量生成的数据并将其组织成一个代表变形形状的单一模型。对应方法是时变表面重建的核心，因为必须记录不同时间帧的点集以产生最终模型[MFO 07, WJH 07, SAL 08, PG08, LAGP09, CZ09, TBW 09, ZST 10]。

is to reconstruct a 3D shape that was scanned over time while moving and deforming. The challenge is to take the great amount of generated data and organize it into one single model that represents the deforming shape. Correspondence methods are central to time-varying surface reconstruction, since the point sets of different time frames have to be registered to yield the final model [MFO*07, WJH*07, SAL*08, PG08, LAGP09, CZ09, TBW*09, ZST*10].

形状插值：在插值或变形中，一个形状逐渐转换为另一个形状。 变换必须满足某些美学要求，因此形状的逐渐变化具有视觉上令人愉悦的方面[Ale02]。 在这方面的一个重要特性是参考形状和目标形状之间的对应关系应该是有意义的，即，它应该与在语义上等同的形状中的部分相关。

**Shape interpolation:** in interpolation or morphing, one shape is gradually transformed into another. The transformation has to satisfy certain aesthetic requirements, so that the gradual change of the shape has a visually pleasing aspect [Ale02]. One important property in this aspect is that the correspondence between the reference shape and the target shape should be meaningful, i.e., it should relate parts in the shapes that are semantically equivalent.

**Information transfer:** a task that is becoming common is to transfer information from a source 3D object to a target 3D object, especially to enable the reuse of attributes or motion information associated to the source shape. Examples include transferring a deformation or morph from one mesh to another [SP04], transferring textures while deforming a mesh [DYT05], and transferring the style of one group of shapes to another [XLZ*10]. Such tasks clearly require a correspondence, since the motion, attribute or style of each element in the source shape has to be transferred to its corresponding element in the target shape.

信息传递：变得普遍的任务是将信息从源3D对象传递到目标3D对象，尤其是使得能够重用与源形状相关联的属性或运动信息。 示例包括将变形或变形从一个网格转移到另一个网格[SP04]，在变形网格[DYT05]的同时传输纹理，并将一组形状的样式转移到另一个[XLZ 10]。 这样的任务显然需要对应，因为源形状中的每个元素的运动，属性或样式必须被转移到目标形状中的对应元素。

**Symmetry detection:** the symmetries of a shape can act as an important cue when solving several tasks, such as registration, segmentation, compression, modeling and editing [GPF07]. Detecting the symmetries of a shape (a set of transformations that when applied to the shape do not modify its geometry) can be posed as the problem of finding a correspondence from the shape to itself. Therefore, it is natural that symmetry detection algorithms possess many similarities with correspondence methods, such as the use of transformation or point sampling for extrinsic [MGP06, PSG*06] or intrinsic [XZT*09, KLCF10] symmetry detection, or the use of embeddings [OSG08, LCDF10].

对称性检测：在解决多个任务时，形状的对称性可以作为一个重要的线索，例如注册，分割，压缩，建模和编辑[GPF07]。 检测形状的对称性（当应用于形状时不改变其几何形状的一组变换）可以被提出作为从形状到其自身找到对应关系的问题。 因此，对称性检测算法与对应方法具有许多相似性是很自然的，例如使用外部[MGP06, PSG 06]或内在[XZT 09, KLCF10]对称性检测的变换或点采样，或使用嵌入[ OSG08, LCDF10]。

**Recognition and retrieval:** understanding a scene described by a range image is one of the classic challenges in computer vision [FP03]. Shape correspondence is one of the approaches that can be used for this task. This is realized by computing a correspondence between a query shape and the models in a dataset. The identity of the query shape is inferred from the best match to one of the models (according to a correspondence quality measure). A similar procedure can be utilized for shape retrieval [FS06].

识别和检索：理解范围图像描述的场景是计算机视觉中的经典挑战之一[FP03]。 形状对应是可用于此任务的方法之一。 这是通过计算查询形状与数据集中的模型之间的对应关系来实现的。 从与模型之一的最佳匹配推断出查询形状的身份（根据对应质量测量）。 类似的程序可用于形状检索[FS06]。

**Statistical shape modeling:** the analysis of anatomical structures such as organs or bones can be facilitated when a statistical shape model is available. These models are useful for extracting shapes from images, since they are able to describe the valid variations in the appearance and the size of a shape. Such models are typically constructed while computing a group correspondence for a collection of shapes that represent a common anatomical structure [DTT08].

统计形状建模：当统计形状模型可用时，可以促进对诸如器官或骨骼的解剖结构的分析。 这些模型对于从图像中提取形状非常有用，因为它们能够描述形状外观和大小的有效变化。 通常在计算表示共同解剖结构的形状集合的组对应时构建这样的模型[DTT08]。

**Change detection:** another application of correspondence is to track changes in a shape (e.g. displacements, growth) over time. In the medical field, an example application is to track the change in the number and density of moles on a patient's skin (for cancer prediction), which can be posed as a problem of point cloud correspondence [MHL09]. In remote sensing, one example is to track the change over time in the layout of cities and their land usage [LLFM00].

变化检测：对应的另一种应用是跟踪形状（例如位移，增长）随时间的变化。 在医学领域，一个示例应用是跟踪患者皮肤上的痣的数量和密度的变化（用于癌症预测），这可以作为点云对应的问题[MHL09]。 在遥感中，一个例子是跟踪城市布局及其土地使用情况随时间的变化[LLFM00]。

## 8. Previous surveys on shape correspondence

8.以前关于形状对应的调查

Surveys that review a partial list of the topics covered in this report have appeared before. We discuss the most recent and relevant references here.

以前出现过审查本报告所涉及的部分主题列表的调查。 我们在这里讨论最新和相关的参考文献。

Tangelder and Veltkamp [TV08], Iyer et al. [IJL*05], and Bustos et al. [BKS*05] provide comprehensive surveys on content based 3D shape retrieval, mostly discussing global shape signatures. Biasotti et al. [BFF*07] present a tutorial on 3D shape description based on properties of real functions, which can be used as signatures for shape matching.

Tangelder和Veltkamp[TV08]，Iyer等人。 [IJL 05]和Bustos等人。 [BKS 05]提供基于内容的三维形状检索的综合调查，主要讨论全局形状特征。 Biasotti等。 [BFF 07]提供了基于实际函数属性的3D形状描述教程，可以用作形状匹配的签名。

In terms of correspondence surveys, Alexa [Ale02] covers recent advances in mesh morphing, where the computation of dense correspondences from an initial set of sparse correspondences is also part of the discussion. Veltkamp and Hagedoorn [VH01] discuss similarity measures and algorithms for shape matching. This publication is the most similar to this report, and includes a solid introduction to the correspondence problem. However, most of the survey focuses on similarity metrics, while the discussion on the algorithms is brief, and important new developments have appeared since the publication of that survey.

在对应调查方面，Alexa[Ale02]介绍了网格变形的最新进展，其中从初始稀疏对应集合计算密集对应也是讨论的一部分。 Veltkamp和Hagedoorn[VH01]讨论了形状匹配的相似性度量和算法。 本出版物与本报告最为相似，包括对通信问题的可靠介绍。 然而，大多数调查都侧重于相似性指标，而对算法的讨论很简短，自该调查发表以来已经出现了重要的新发展。

The book by Bronstein et al. [BBK08] is a recent and comprehensive discussion on the analysis of non-rigid shapes. It covers problems such as determining the similarity between non-rigid shapes and techniques for correspondence based on MDS.

Bronstein等人的书。 [BBK08]是关于非刚性形状分析的最新综合讨论。 它涵盖了诸如确定非刚性形状之间的相似性和基于MDS的对应技术之类的问题。

Audette et al. [AFP00] present an overview of surface registration techniques for medical imaging. The survey discusses in detail techniques based on applying geometric transformations to the data (rigid-body motion, etc), but does not discuss techniques that work in the correspondence space. Heimann and Meinzer [HM09] survey the topic of statistical shape modeling and briefly review techniques for registration and group correspondence. Statistical shape models and group correspondence are also extensively covered in the recent book by Davies et al. [DTT08].

Audette等人。 [AFP00]概述了医学成像的表面配准技术。 该调查详细讨论了基于对数据应用几何变换的技术（刚体运动等），但没有讨论在对应空间中起作用的技术。 Heimann和Meinzer[HM09]调查了统计形状建模的主题，并简要回顾了注册和小组对应的技术。 Davies等人在最近的书中也广泛地介绍了统计形状模型和组对应。[DTT08]。

## 9. Summary and future perspectives

9.总结和未来前景

In Figure 16, we show a progression in the development of solutions to shape correspondence. On one hand, significant progress has been made to compute rigid or pose-invariant alignments (through the correspondence of approximately isometric shapes). Since these problems have well-defined objectives, remaining work is mainly focusing on improving efficiency and accuracy, or on handling more difficult specializations of the problem, such as partial matching. In part, the success in these areas is also due to the fact that the correspondence can be obtained reliably from purely geometric information extracted from the shapes, and shape alignment can be described in terms of unambiguous transformations or objective functions.

在图16中，我们展示了形状对应解决方案开发的进展。 一方面，已经在计算刚性或姿势不变的对齐方面取得了重大进展（通过近似等距形状的对应关系）。 由于这些问题具有明确的目标，因此剩下的工作主要集中在提高效率和准确性，或者处理问题的更困难的专业化，例如部分匹配。 部分地，这些领域的成功也是由于可以从从形状中提取的纯几何信息可靠地获得对应的事实，并且可以根据明确的变换或目标函数来描述形状对准。

On the other hand, finding a meaningful correspondence between shapes belonging to the same class but differing (sometimes significantly) in their geometry, structure, or even topology, remains a challenge. Traditional methods which rely on assumptions of rigidity, isometry, or sufficient geometric similarity between corresponding parts are simply inadequate. Man-made shapes such as the ones shown in Figure 3 are particularly challenging to deal with, since these objects often differ not only by geometric deformations, but also by their part constitutions. Shape correspondence then departs from the low-level sphere of geometry analysis and becomes the higher-level problem of semantic reasoning, where we seek to recognize the parts of shapes and infer their semantics or functionality. A meaningful correspondence can then be established between the recognized parts. The utilization of prior knowledge is seen as a promising solution, where the main difficulty is how to model the knowledge and make use of it effectively.

另一方面，在属于同一类但在几何，结构或甚至拓扑方面不同（有时显着）的形状之间找到有意义的对应关系仍然是一个挑战。依赖于刚性，等距或相应部分之间的足够几何相似性的假设的传统方法是不恰当的。诸如图3中所示的人造形状处理起来特别具有挑战性，因为这些对象通常不仅通过几何变形而且通过它们的部分构造而不同。然后，形状对应脱离了低级几何分析领域，成为语义推理的高级问题，我们寻求识别形状的各个部分并推断它们的语义或功能。然后可以在识别的部分之间建立有意义的对应关系。利用先验知识被视为一种有前景的解决方案，其主要困难是如何对知识进行建模并有效地利用它。

Some recent works have taken the first steps towards knowledge-driven shape correspondence. Knowledge can be incorporated by utilizing a set of examples where a few landmark points have already been matched by an expert user [WH07], by using a set of examples already in full correspondence [PDT07], or by relying on a set of pre-segmented and pre-labeled shapes, so that classifiers can be trained on these examples and applied to label the primitives of an unknown shape [KHS10, vKTS*11]. Another direction is to learn how the terms in the various objective functions should be weighted, depending on the restricted domain of the problem that we are considering [CMC*09]. One more possibility is to consider group information when performing correspondence-related tasks, such as skeletoniza-

最近的一些作品迈出了知识驱动的形状对应的第一步。 知识可以通过利用一组示例来合并，其中一些标志点已经由专家用户[WH07]匹配，通过使用已经完全对应[PDT07]的一组示例，或者依赖于一组预先分段的和 预先标记的形状，以便可以在这些示例上训练分类器并应用于标记未知形状的基元[KHS10, vKTS 11]。 另一个方向是了解如何对各种目标函数中的术语进行加权，这取决于我们正在考虑的问题的限制域[CMC 09]。 另一种可能性是在执行与通信相关的任务时考虑组信息，例如骨架化[WH09]或自动分割[GF09, XLZ 10]。 直接寻址非均匀部分缩放可以极大地改善协同分割的结果[XLZ 10]。

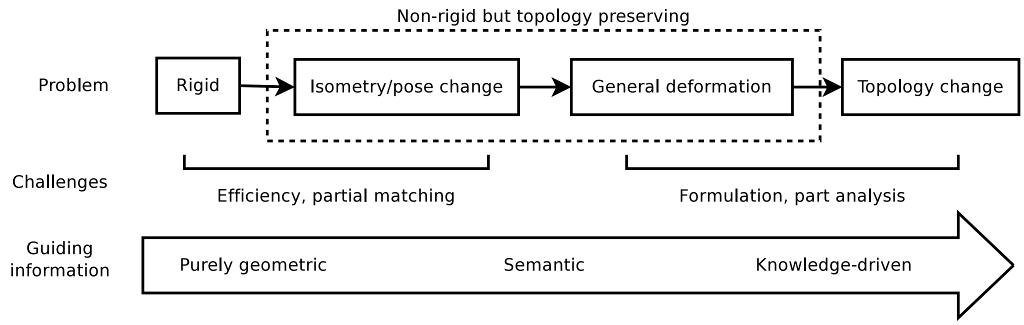


**Figure 16:** *A progression of development in shape correspondence methods.*

tion [WH09] or automatic segmentation [GF09, XLZ*10]. Directly addressing the nonhomogeneous part scaling can greatly improve the results of co-segmentation [XLZ*10].

由于各种对应方法仍在增加，因此需要通过基准数据集进行客观评估。 设想的基准应该包含不同类别的问题（严格的，非刚性的），当然还有足够的人造物体集合来反映它们所代表的挑战。 为了减轻创建这样一个基准所需的工作量，一种可能的策略是采用一组简化的形状，其中已知对应的地面实况，并通过随机（但在统计上或物理上可信）的变换使形状变形，类似于 为解剖学形状做了什么[HJT08]。

As the variety of correspondence methods is still increasing, the need for objective evaluation via benchmark datasets is important. The envisioned benchmark should contain different classes of problems (rigid, non-rigid) and certainly a sufficient collection of man-made objects to reflect the challenge they represent. To alleviate the amount of work required to create such a benchmark, one possible strategy is to take a reduced set of shapes for which the correspondence ground-truth is known and deform the shapes with random (yet statistically or physically plausible) transformations, similar to what is done for anatomical shapes [HJT08].

我们已经看到最近开发的网格分割基准[CGF09]，随后由Kalogerakis等人用语义部分标签标记。[KHS10]。 这些结果自然地反映了人类如何分割形状然后建立这些部分之间的对应关系。 这种对应反映了承认的过程。 因此，很明显三个任务之间存在着深刻的联系：分割，识别和对应。 因此，我们推测形状语义分析的最终方法不会单独处理这些问题，而是在组或先验信息的帮助下同时解决这些问题。

We have seen the recent development of a benchmark for mesh segmentation [CGF09], which is subsequently tagged with semantic part labels by Kalogerakis et al. [KHS10]. These results naturally reflect how humans tend to segment shapes and then establish a correspondence between these segments. Such a correspondence reflects the process of recognition. Hence it seems apparent that there is a deep connection among the three tasks: segmentation, recognition and correspondence. Therefore, we conjecture that the ultimate approach for the semantic analysis of shapes would not treat these problems separately, but solve them all simultaneously and with the aid of group or prior information.


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