$$y_i \sim N(\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i},\ 1/\tau) \qquad \beta_m \sim N(\mu_m,\ 1/\tau_m)$$

$$L = \prod_{i=1}^{N} N(\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i},\ 1/\tau) \qquad \tau \sim \mathrm{Gamma}(\alpha, \beta)$$

$m = 1, 2$

$$p(\beta_0 \mid \beta_1, \beta_2, \tau, y, x, \tau_m, \alpha, \beta)$$

$$\propto p(\beta_0, \beta_1, \beta_2, \tau, y, x, \tau_m, \alpha, \beta)$$

$$= p(y \mid \beta_0, \beta_1, \beta_2, \tau, x)\, p(\beta_0 \mid \mu_0, \tau_0)\, p(\beta_1 \mid \mu_1, \tau_1)\, p(\beta_2 \mid \mu_2, \tau_2)\, p(\tau)$$

$$\propto p(y \mid \beta_0, \beta_1, \beta_2, \tau, x)\, p(\beta_0 \mid \mu_0, \tau_0) \cdots \bigstar$$

$$N(\beta_0 + \beta_1 x_1 + \beta_2 x_2,\ 1/\tau),\ N(\mu_0,\ 1/\tau_0)$$

$$\bigstar = N(\mu_0, 1/\tau_0) \prod_{i=1}^{N} (\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i},\ 1/\tau)$$

$$-\frac{1}{2}\log\left(2\pi \cdot \frac{1}{\tau_0}\right) - \frac{\tau_0(\beta_0 - \mu_0)^2}{2} + \sum_{i=1}^{N} -\left(\frac{1}{2}\log\left(2\pi \cdot \frac{1}{\tau}\right) + \frac{\tau(y - \beta_0 - \beta_1 x_1 - \beta_2 x_2)^2}{2}\right)$$

$$\propto -\frac{\tau_0(\beta_0 - \mu_0)^2}{2} - \frac{\tau}{2}\sum_{N}(y - \beta_0 - \beta_1 x_1 - \beta_2 x_2)^2$$

β_0に関係ない定数は除いているので比例関係にある

$$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \leftarrow \text{Use Normal!}$$

$$= -\frac{1}{2}\log(2\pi\sigma^2) - \frac{(x-\mu)^2}{2\sigma^2}.$$

1) $\alpha - \frac{\tau_0}{2}\beta_0^2 + \tau_0 \mu_0 \beta_0$

$$-\frac{\tau}{2}\sum_N \left(\beta_0^2 - 2\beta_0(y_i - \beta_1 x_{1i} - \beta_2 x_{2i})\right)$$

$$= \underbrace{-\frac{\tau_0}{2}\beta_0^2 + \tau_0 \mu_0 \beta_0 - \frac{\tau}{2}N\beta_0^2}_{A,\ B} + \tau\sum_N \beta_0(y_i - \beta_1 x_{1i} - \beta_2 x_{2i})$$

$$A = \left(-\frac{\tau_0 + \tau N}{2}\right)\beta_0^2$$

$$B = (\tau_0 + \tau N)\left(\frac{\tau_0}{\tau_0 + \tau N}\right)\mu_0 \beta_0 + (\tau_0 + \tau N)\left(\frac{\tau}{\tau_0 + \tau N}\right)\beta_0 \sum_N (y_i - \beta_1 x_{1i} - \beta_2 x_{2i}) \cdots (*)$$

$\Leftarrow N(u, 1/t)$ なら

$$\alpha - \frac{t}{2}(x-u)^2 \propto -\frac{t}{2}x^2 + tux$$

$(x+y+z+w)^2 = (y + (x+z+w))^2$

この形をつくる！

(Normalの形の変形形)

$$(*) = (\tau_0 + \tau N)\cdot\beta_0\left\{\frac{\tau_0}{\tau_0 + \tau N}\mu_0 + \frac{\tau}{\tau_0 + \tau N}\sum_N(\cdot)\right\}$$

$$= (\tau_0 + \tau N)\cdot\beta_0\left\{\frac{\tau_0 \mu_0 + \tau\sum_N(\cdot)}{\tau_0 + \tau N}\right\}$$

$$\sum k_i m_i = \sum k_i \sum m_i$$

$$\alpha - \frac{\tau_0(\beta_0 - \mu_0)^2}{2} - \frac{\tau}{2}\sum_N (y_i - \beta_0 - \beta_1 x_{1i} - \beta_2 x_{2i})^2 \cdots (1)$$

β_0 まで

① $\alpha - \frac{\tau_0}{2}\beta_0^2 + \tau_0\mu_0\beta_0$

$$-\frac{\tau}{2}\sum_N\left(\beta_0^2 - 2\beta_0(y_i-\beta_1 x_{1i}-\beta_2 x_{2i})\right)$$

$$= \underbrace{-\frac{\tau_0}{2}\beta_0^2 + \tau_0\mu_0\beta_0}_{B} \underbrace{-\frac{\tau}{2}N\beta_0^2}_{A} + \tau\sum_N \beta_0(y_i-\beta_1 x_{1i}-\beta_2 x_{2i})$$

$A = \left(-\frac{\tau_0+\tau N}{2}\right)\beta_0^2$

$B = (\tau_0+\tau N)\left(\frac{\tau_0}{\tau_0+\tau N}\right)\mu_0\beta_0 + (\tau_0+\tau N)\left(\frac{\tau}{\tau_0+\tau N}\right)\beta_0\sum_N(y_i-\beta_1 x_{1i}-\beta_2 x_{2i})$ … (*)

$(*) = (\tau_0+\tau N)\cdot\beta_0\left\{\frac{\tau_0}{\tau_0+\tau N}\mu_0 + \frac{\tau}{\tau_0+\tau N}\sum_N(\cdot)\right\}$

$= (\tau_0+\tau N)\cdot\beta_0\left\{\frac{\tau_0\mu_0 + \tau\sum_N(\cdot)}{\tau_0+\tau N}\right\}$

$\Leftarrow N(u, 1/t)$ から

$-\frac{t}{2}(x-u)^2$

$\alpha - \frac{t}{2}x^2 + t\mu x$

$(x+y+z+w)^2 = (y+(x+z+w))^2$

(β1)

$$-\frac{\tau_1}{2}\beta_1^2 + \tau_1\mu_1\beta_1 - \frac{\tau}{2}\sum_N\left(\beta_1^2 x_{i1}^2 - 2\beta_1 x_{i1}(y_i-\beta_0-\beta_2 x_{2i})\right)$$

$$= \left(-\frac{\tau_1+\tau\sum x_{i1}^2}{2}\right)\beta_1^2 + \tau_1\mu_1\beta_1 + \tau\sum_N\{\beta_1 x_{i1}(y_i-\beta_0-\beta_2 x_{2i})\}$$

$$+\left[\tau_1\mu_1 + \tau\sum_N x_{i1}(y_i-\beta_0-\beta_2 x_{2i})\right]\beta_1$$

$$+\left[(\tau_1+\tau\textstyle\sum x_{i1}^2)\cdot\frac{\tau_1\mu_1+\tau\sum x_{i1}(y_i-\beta_0-\beta_2 x_{2i})}{\tau_1+\tau\sum x_{i1}^2}\right]\beta_1$$

$t = \tau_1 + \tau\sum x_{i1}$

$\mu = \frac{\tau_1\mu_1 + \tau\sum(y_i-\beta_0-\beta_2 x_{2i})}{\tau_1+\tau\sum x_{i1}}$

⇒ $-\frac{t}{2}x^2 + t\mu x$ と同じ形

左上の