The Doppler Effect and Cosmological Redshift
Read-ahead

Introduction

Most people have heard the Doppler effect, even if they haven't heard *of* it. The Doppler effect is what you are hearing when a siren on a fire truck sounds higher and higher as the truck approaches you, then slowly returns to its original pitch after the truck passes you and drives away. Sound travels as a wave through air, and when the source of the sound is moving with respect to the listener, the shape of the wave is affected (Figure 1).

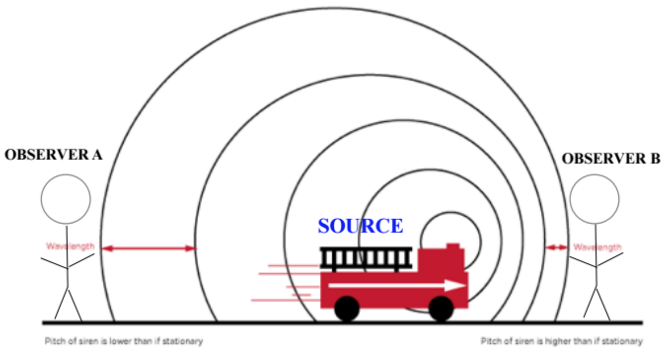

Figure 1: The Doppler Effect

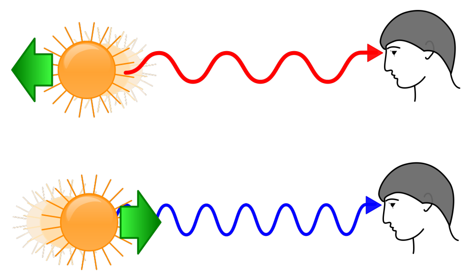

Figure 2: Redshift and Blueshift

The Doppler effect can also be observed with light waves (Figure 2), and is one of the causes of *redshift*. When a star is moving away from us, the wavelength of its light is elongated, shifting it towards the red part of the visible light spectrum.[1] *Cosmological* redshift is caused by the stretching of space itself, and is distinct from the Doppler effect. Cosmological redshift was the key piece of evidence that led physicists to conclude that our universe is expanding.

Instructions

Your first step for this module should be to watch a video on the Doppler effect that is linked from the Canvas site. After watching this video and reading through *The Doppler Effect and Cosmological Redshift* context and questions below, you should complete the reflection assignment in Canvas: Note: *you will have a chance to talk further with your coach before answering the questions below in detail.* The point of this read-ahead and the reflection is to "prime the pump" for further conversations with your coaches.

The Doppler Effect and Cosmological Redshift

For the following application we will look at the mathematics behind the Doppler effect and redshift using trigonometric functions to describe both sound and light waves. These phenomena manifest themselves as transformations of the wave functions. We will denote the velocity of a wave by v, the wavelength by λ, and the frequency by $f = v/\lambda$.

[1] If the stars were moving toward us, we would call the resulting compression of the waves *blueshift*.

Suppose, as in Figure 1 above, the two observers are stationary, and the sound source is moving away from Observer A and towards Observer B. Let v be the velocity of sound in air, w the velocity of the source, λ the natural wavelength of the sound, and $f = v/\lambda$ the natural frequency. If T is the time it takes the wave to move one wavelength λ, i.e., $T = \lambda/v = 1/f$, then the distance d that the source moves in time T is given by $d = wT$. This is the amount by which the wavelength has elongated. The perceived wavelength is thus $\lambda_A = \lambda + d = \lambda + wT = \lambda + w(\lambda/v) = (1+w/v)\lambda = \left(\frac{v+w}{v}\right)\lambda$.

Questions

1. (Graded for completeness only.) Given that $\lambda_A = \left(\frac{v+w}{v}\right)\lambda$, is the sound perceived to be higher by Observer A, or lower? Explain.
2. The speed of sound in dry air is roughly 343 meters per second. What is the perceived frequency of a 440 Hz sound wave whose source moves away from the observer with a velocity of 2 m/s? 4 m/s? 10 m/s? Round your answers to one decimal place.
3. How fast would the source need to be moving away from Observer A for the perceived frequency to be 220 Hz?
4. Consider a general sine function $g(t) = A\sin(B(t+C))$ representing a sound wave. Write the function $h(t)$ that represents the perceived sound if the source of the wave is moving with a velocity w away from the observer. Note that your answer should be written in the form of $h(t) = P\sin(Q(t+R))$, where P, Q and R are in terms of the constants A, B and C, as well as w and the velocity v of sound in air.
5. Using the reasoning given in the second paragraph of the *Doppler Effect and Cosmological Redshift* context above, what is the perceived frequency f_B as heard by Observer B? Your answer should be an expression in terms of f, v, and w. Do not include "$f_B =$" in your answer.
6. Suppose you are standing on the sidewalk of a straight street. A fire truck traveling 30 m/s (about 67 mph) along the street approaches from your left, then passes directly in front of you at a distance of 4 meters, then continues off to the right. An actual siren has varying frequency, but suppose for the purposes of this exercise that the fire truck is emitting sound at 440 Hz. Sketch a graph of the perceived frequency of the sound as a function of time, letting $t = 0$ be the moment the fire truck passes directly in front of you.

Redshift has huge significance for astronomy. Physicists look at stars primarily composed of hydrogen, which radiates a distinct spectrum of light. If the wavelength of the observed spectrum is elongated then it is clear that the light waves have undergone a redshift. Depending on how much the wavelength has changed physicists can determine exactly how far away the object is and how fast it is moving away from the earth. This method is actually extremely accurate and the primary way of determining the size and rate of expansion of our universe.

Cosmological redshift is denoted by $z = \dfrac{\lambda_{\text{OBSV}} - \lambda_{\text{EMIT}}}{\lambda_{\text{EMIT}}}$, where λ_{EMIT} is the emitted wavelength of light, and λ_{OBSV} is the observed wavelength. Light waves are transformed according to the formula

$$1 + z = \frac{a_{\text{NOW}}}{a_{\text{THEN}}},$$

where a_{NOW} is the current rate of expansion of space, and a_{THEN} is the rate of expansion when the light was emitted.

7. (Graded for completeness only.) Because the universe is expanding at an increasing rate, it is always the case that $a_{\text{NOW}} > a_{\text{THEN}}$. Explain why this implies that all galaxies display cosmological redshift when observed from earth.
8. If the ratio between the expansion of space now to when the light was emitted is 2.5, and the light is emitted with wavelength 600 nm, what is the observed wavelength?
9. Suppose the function $g(t) = A\sin(B(t+C))$ represents a light wave emitted at a time when space was expanding at one third the current rate. Write the function $h(t)$ representing the observed light wave. Note that your answer should be written in the form of $h(t) = P\sin(Q(t+R))$, where P, Q and R are in terms of the constants A, B and C.

Instructions, part deux
After reading and reflecting on these questions, complete the application reflection on Canvas. This will give your coach some insight on your thinking in order to best help you before you are required to formally answer these questions.