

# Temperature Distributions in the Protosatellite Disks of Uranus, Jupiter, Neptune and Saturn and Common Structure in Their Satellite Systems

James C. Lombardi Sr. Professor Emeritus,  
Physics Department, Allegheny College, Meadville, PA, USA; james.lombardi@allegheny.edu

## ABSTRACT

We observe similar structures in the orbital radii of satellite and ring systems of Uranus, Jupiter and Neptune. This stimulates an investigation that the evolution of these systems and Saturn's system is initiated by a common mechanism that involves the interaction of radiation with their subnebulae (protosatellite disks). A model is presented that is characterized by resonance created through stimulated radiative molecular association (SRMA) reactions. In this model thermal energy is extracted from a protoplanetary disk at specific distances from the protoplanet wherever there is a match between the local thermal energy of the disk and the energy of photons impinging on the disk. Radiation is supplied by a portion of the hydrogen molecule's spectrum for the Uranian, Jovian and Neptunian disks and a portion of the hydrogen atom's spectrum for the Saturnian disk. Findings shed light on the early evolution and structure of satellite systems including the complicated ring system of Saturn. They also link satellite orbital radii size distributions to shapes of temperature distributions (TD's) in protosatellite disks. Theoretically determined TD's in the protosatellite disks of Uranus (Mousis 2004) and Saturn (Mousis Gautier and Bockelee-Moran, 2002) are essential to the present investigation.

**KEYWORDS:** planets, protoplanets, protosatellites, protoplanetary disks, subnebula, atoms, molecules, spectra, satellite systems, radiative molecular association

## 1. INTRODUCTION

This investigation deals with the satellites that are created in the mid-planes of the protosatellite disks of Uranus, Jupiter, Neptune and Saturn. These satellites have orbits with small eccentricities and inclinations relative to the planet's equatorial plane and are generally called regular satellites. However for ease of discussion here they are usually simply referred to as satellites. Section 2.1 introduces relationships among the orbital radii of regular satellites and rings of Uranus, Jupiter and Neptune. Subsequent sections present a model that explains these relationships and also the structure of the satellite systems of the four giant gas planets in the solar system. In this model regular satellites are formed where disk temperatures have certain values with these values depending on the energies of photons that exist in the disk. We propose the connection between temperature and photon energy to be a result of resonance involving stimulated radiative molecular association (SRMA).

In this investigation TD stands for the midplane temperature distribution or portion of a temperature distribution in the protosatellite disk of a protoplanet. Results of this investigation rely heavily on the theoretically determined TD's in the protosatellite disks of Uranus (Mousis 2004) and Saturn (Mousis Gautier and Bockelee-Moran, 2002).

This paper is a rewrite of Lombardi (2015a). Lombardi (2015b) was a follow-up for Lombardi (2015a) and now it should be a follow-up for this paper.

## 2. RESULTS AND DISCUSSION

Table 1 contains the names of known regular satellites of Uranus, Jupiter, and Neptune and their orbital radii (lengths of the semi-major axis) in units of  $R_U = 25,559$  km,  $R_J = 71,492$  km and  $R_N = 24,766$  km, the equatorial radii of Uranus, Jupiter and Neptune respectively (NASA 2021). I.e. if  $r$  is the orbital radius of a Uranian satellite, then  $r/R_U$  is its orbital radius in units of  $R_U$ . In the case of Uranus, satellites with orbital radii less than  $r/R_U = 2.3$  are not listed in Table 1 but are considered later. For Jupiter and Neptune all of the known regular satellites are listed. Values for the index  $i$  are listed in the first column of Table 1. It will become apparent why the indices have their particular listed values in later sections. All satellites and orbital radii in the same row are associated with the same value of  $i$ . Note that Cupid in the Uranian system does not have a value of  $i$  assigned to it. The reason for this is in sections 2.1.b.

### 2.1.a. The Linearity of the Uranian vs Jovian and Uranian vs Neptunian Orbital Radii

Fig. 1 is a graph of orbital radii of Uranian satellites versus the orbital radii of the Jovian satellites. Each point is for the two radii with the same  $i$  value. No other set of pairings of Uranian and Jovian satellite orbital radii produces a graph that is fitted by a straight line nearly as well as the one used to make Fig. 1.

**Fig. 1. All the Jovian satellite orbital radii are linearly related to a certain set of Uranian satellite orbital radii.**

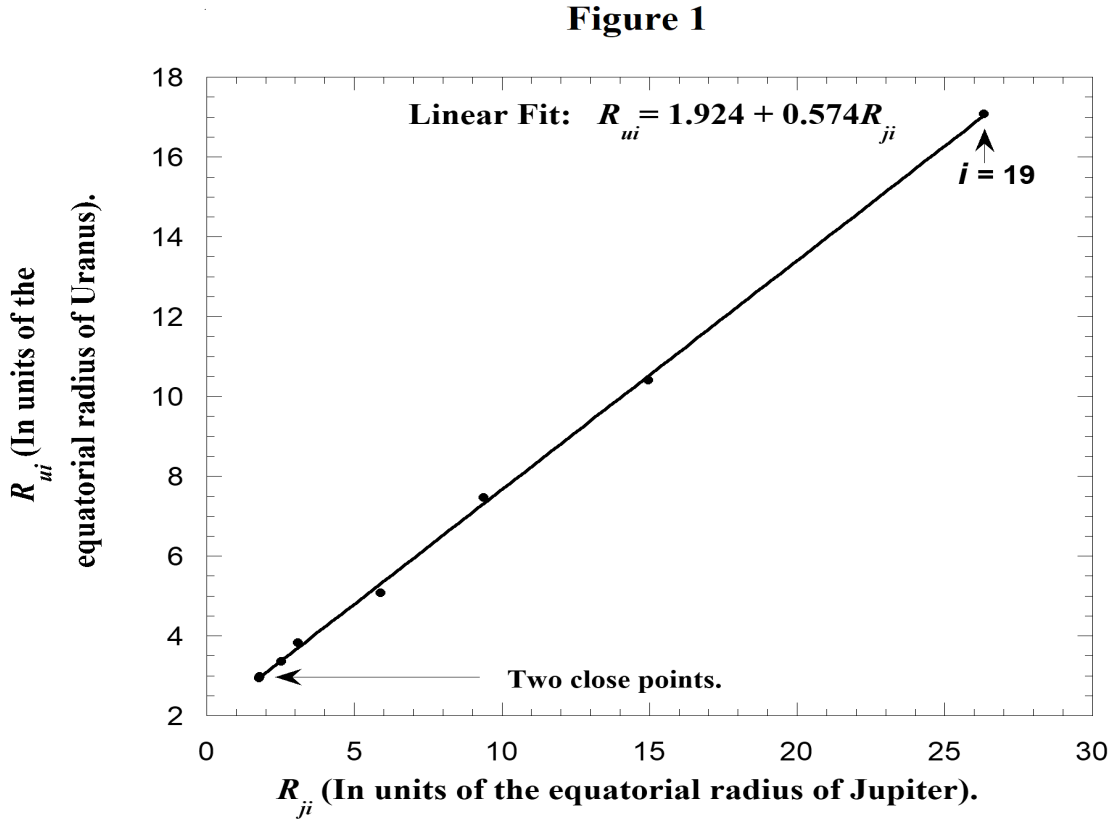
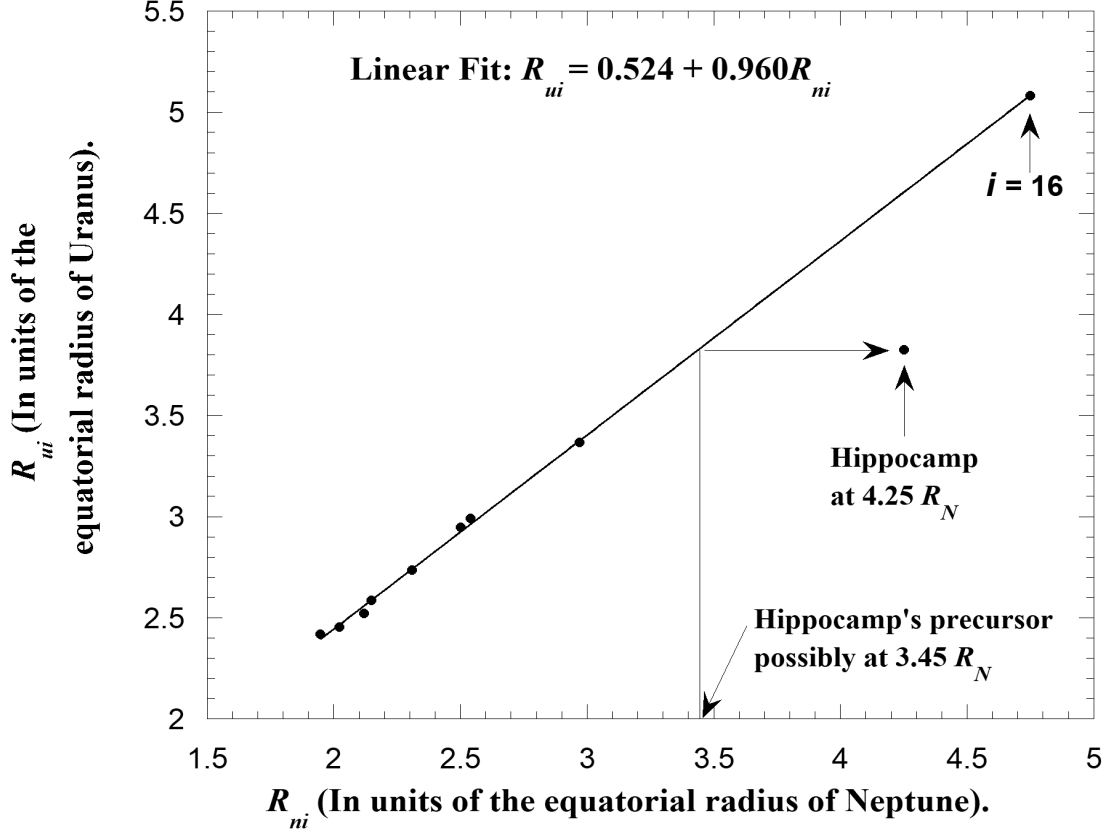


Fig. 2 is a graph that is similar to Fig. 1 except Fig. 2 is for Uranian and Neptunian satellites. The graph is also well fitted by a straight line except for the Mab-Hippocamp point where  $i = 15$  as seen in Table 1. This discrepancy is discussed in section 2.1.b.

**Figure 2**

**All but one of the Neptunian satellite orbital radii are linearly related to a certain set of Uranian satellite orbital radii.**



The equations of the best-fit lines to the graphs in Figs. 1 and 2 are

$$R_{ui} = 1.924 + 0.574R_{ji} \quad (1)$$

and

$$R_{ui} = 0.524 + 0.960R_{ni}, \quad (2)$$

where  $R_{ui}$  is the orbital radius of the  $i^{\text{th}}$  Uranian satellite in units of  $R_U$ . Similar definitions hold for  $R_{ji}$  and  $R_{ni}$ . Ring Galle in Neptune's system lies inside the innermost satellite. Its width is large and optical depth low (NASA 2021). Ring Galle is not included in the present analysis.

### *2.1.b. Details Concerning Cupid and Hippocamp*

We mentioned above that Cupid in the Uranian system does not have a value of  $i$  assigned to it in Table 1 nor is it included in future analysis. Cupid, Belinda ( $i = 11$ ) and Perdita ( $i = 12$ ) are part of a triplet in

which each member has nearly the same orbital radius. This triplet is associated with doublets in the system of Jovian and Neptunian systems. If we assign a value of  $i$  to each of the three satellites in the triplet, we do not arrive at good linear fits as seen in Figs. 1 and 2. Possibly sometime during the evolution of a doublet in the Uranian system the two members collide resulting in the triplet as seen in Table 1.

The Mab-Hippocamp point (where  $i=15$  for both satellites) in Fig. 2 is far from the linear fit. Mab's orbital radius together with Thebe's orbital radius creates a point in Fig. 1. The fact that the points in Fig. 1 (including Mab's) are well fitted by a straight line suggests strongly that, the Mab-Hippocamp point in Fig. 2 is not positioned well because of the value of Hippocamp's orbital radius, not Mab's. Table 2 indicates that Hippocamp's mass is small compared to the masses of the other nearby satellites. Perhaps a precursor of Hippocamp experiences an impact that shatters it leaving Hippocamp as a fragment. Fig. 2 indicates the orbital radius of  $4.25R_N$  for Hippocamp and the approximate orbital radius of  $3.45R_N$  for the precursor.

### 2.1.c. Eqs. (1) & (2) as Transformation Equations for Orbital Radii

Consider Table 1. The Jovian distribution of orbital radii starts with  $R_{j11}$  and ends with  $R_{j19}$ . Eqs. (1) can be used to scale and shift the radial coordinates that describe the Jovian system so all the  $R_{ji}$  from  $R_{j11}$  to  $R_{j19}$  span the same normalized radial coordinates as does the portion of the Uranian system from  $R_{u11}$  to  $R_{u19}$ . The  $0.574R_{ji}$  term in Eq. (1) scales the radial distance associated with the  $R_{ji}$  and the constant 1.924 shifts the origin of the system. Therefore if we substitute a value of  $R_{ji}$  into Eq. (1) we calculate a transformed value of  $R_{ji}$  (call it  $R_{Tji}$ ) that is close to the corresponding value of  $R_{ui}$ . The same idea holds for the transformation of the  $R_{ni}$  values (call each one  $R_{Tni}$ ) by using Eq. (2). Table 3 contains the scaled (transformed) orbital radii of satellites of Jupiter and Neptune along with the orbital radii of all the regular satellites and rings of Uranus (NASA 2021).

To visualize the usefulness of transformed orbital radii consider Fig. 3 which is constructed using Table 2. The top panel is the distribution of transformed Neptunian orbital radii. The middle panel is the distribution of transformed Jovian orbital radii. The bottom panel is the distribution of all the Uranian satellite orbital radii, not just those that overlap with the Neptunian and Jovian satellites. Notice the Neptunian and Jovian line graphs have points (actually vertical lines or large dots) that line up closely with radii in the Uranian distribution and also with each other. The single exception is Hippocamp's point as expected.

The positions corresponding to the surfaces of Jupiter and Neptune are each determined by substituting the number 1 into the right side of Eq. (1) and Eq. (2) respectively. The transformed surface radial coordinates for the Jovian disk and the Neptunian disk are 2.498 and 1.484 respectively. These are both compared to the Uranian surface coordinate of 1. The fact that they are considerably larger than 1 indicates that the region where the Uranian satellites are created is comparatively larger than the corresponding normalized regions for Jupiter and Neptune. This may help to explain why there are more satellites in the Uranian system than in each of the Jovian and Neptunian systems.

Table 1. Orbital radii of satellites and rings of Uranus, Jupiter and Neptune in units of their equatorial radii. For instance if  $R_U$  is the equatorial radius of Uranus in km and  $r_{ui}$  is the orbital radius of a Uranian satellite in km, then  $R_{ui} = r_{ui}/R_U$  is its orbital radius in units of  $R_U$ .

$i$	Uranian Satellites	$R_{ui}^a$	Jovian Satellites	$R_{ji}^a$	Neptunian Satellites	$R_{ni}^a$
1	Bianca	2.316				
3&4	Cressida	2.418			Naiad	1.947
5&6	Desdemona	2.453			Thalassa	2.022
7	Juliet	2.520			Despina	2.121
9	Portia	2.586			Rings LeV&Las <sup>c</sup>	2.148
10	Rosalind	2.735			Ring Arago	2.310
	Cupid <sup>b</sup>	2.911				
11	Belinda	2.946	Metis	1.790	Galatea & Unnamed ring	2.502
12	Perdita	2.990	Adrastea	1.804	Ring Adams	2.541
13&14	Puck	3.365	Amalthea	2.537	Larissa	2.970
15	Mab	3.824	Thebe	3.104	Hippocamp	4.252
16	Miranda	5.082	Io	5.900	Proteus	4.750
17	Ariel	7.469	Europa	9.387		
18	Umbriel	10.407	Ganymede	14.972		
19	Titania	17.070	Callisto	26.334		
20	Oberon	22.830				

Note: Each satellite is assigned an index ( $i$ ) consisting of one or two integers. Satellites in the same row have the same  $i$ . The indexing system is explained in section 2.2.

<sup>a</sup>NASA(2021)

<sup>b</sup>See section 2.1.b concerning why the orbital radius of Cupid is not an index nor is it used in the analysis.

<sup>c</sup>Rings LeV&Las stands for Rings LeVerrier and Lassell

Table 2. Orbital radii and masses of satellites of Neptune. The list includes all satellites in the group nearest to Neptune most of which are regular satellites. Rings are not included.

$i$		Orbital Radius $R_{ni}^a$	Mass <sup>a</sup> ( $10^{17}$ kg)
3&4	Naiad	1.948	2
5&6	Thalassa	2.022	4
7	Despina	2.121	20
11	Galatea	2.502	40
13&14	Larissa	2.970	50
15	Hippocamp	4.252	0.3
16	Proteus	4.751	500
—	Triton <sup>b</sup>	14.328	214000
—	Nereid <sup>b</sup>	222.67	300

<sup>a</sup>NASA(2021)

<sup>b</sup>Triton and Nereid are irregular satellites. Their orbital plane is not aligned with the orbital plane of the regular satellites.

Table 3. Orbital radii of satellites and rings of Uranus, Jupiter and Neptune in units of equatorial radius of the respective planet. For Jupiter and Neptune,  $R_{Tji}$  and  $R_{Tni}$  are orbital radii transformed using Eqns (1) and (2) respectively.

$i$	Uranian Satellites	$R_{ui}''^a$	$R_{ui}'^a$	$R_{ui}^a$	Jovian Satellites	$R_{Tji}^b$	Neptunian Satellites	$R_{Tji}^c$
11&12	Ring 6	1.637						
13&14	Ring 5	1.652						
15	Ring 4	1.666						
15	Ring alpha		1.750					
13&14	Ring $\beta$		1.787					
11&12	Ring $\eta$		1.846					
10	Ring $\gamma$		1.863					
9	Ring $\delta$		1.900					
7	Cordelia		1.948					
5&6	Ring $\lambda$		1.957					
3&4	Ring $\varepsilon$		2.006					
1	Ophelia		2.105					
1	Bianca			2.316				
3&4	Cressida			2.418			Naiad	2.393
5&6	Desdemona			2.453			Thalassa	2.465
7	Juliet			2.520			Despina	2.560
9	Portia			2.586			Rings LeV&Las <sup>d</sup>	2.586
10	Rosalind			2.735			Ring Arago	2.742
	Cupid <sup>e</sup>			2.911				
11	Belinda			2.946	Metis	2.951	Galatea & Unnamed ring	2.926
12	Perdita			2.990	Adrastea	2.959	Ring Adams	2.963
13&14	Puck			3.365	Amalthea	3.380	Larissa	3.375
15	Mab			3.824	Thebe	3.706	Hippocamp	4.606
16	Miranda			5.082	Io	5.311	Proteus	5.084
17	Ariel			7.469	Europa	7.312		
18	Umbriel			10.407	Ganymede	10.518		
19	Titania			17.070	Callisto	17.040		
20	Oberon			22.830				

Note: Each satellite is assigned an index ( $i$ ) consisting of one or two integers. Satellites in the same row have the same  $i$ . The indexing system is explained in sections 2.1.c, 2.2a and 2.2b.

<sup>a</sup>NASA(2021)

$R_{ui}''$  refers to orbital radii from Ring 6 to Ring 4.

$R_{ui}'$  refers to orbital radii from Ring  $\alpha$  to Ophelia.

$R_{ui}$  refers to orbital radii from Bianca to Oberon.

<sup>b</sup>NASA(2021) transformed with Eqn (1) as described in sections 2.1.c

<sup>c</sup>NASA(2021) transformed with Eqn (2) as described in sections 2.1.c

<sup>d</sup>Rings LeV&Las stands for Rings LeVerrier and Lassell

<sup>e</sup>See text concerning why the orbital radius of Cupid does not have an index nor is it used in the analysis.