Stock market crashes, Precursors and Replicas

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Abstract

We present an analysis of the time behavior of the $S\&P500$ (Standard and Poors) New York stock exchange index before and after the October 1987 market crash and identify precursory patterns as well as aftershock signatures and characteristic oscillations of relaxation. Combined, they all suggest a picture of a kind of dynamical critical point, with characteristic log-periodic signatures, similar to what has been found recently for earthquakes. These observations are confirmed on other smaller crashes, and strengthen the view of the stockmarket as an example of a self-organizing cooperative system.

Résumé

Nous présentons une analyse du comportement de l'indice boursier americain $S\&P500$ avant et après le crash d'octobre 1987. Nous identifions des motifs précurseurs ainsi que des oscillations de relaxation et des signatures de répliques après le crash. Ces caractéristiques suggèrent toutes ensembles que ce crash peut être vu comme une sorte de point critique dynamique, possédant des signatures spécifiques log-périodiques, comme on l'a découvert précédemment pour les tremblements de terre. Ces observations sont confirmés sur d'autres crashes plus petits et renforcent le concept d'un marché mondial vu comme un exemple de système auto-organisé complexe.

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1 The October 1987 crash

From the opening on October 14, 1987 through the market close on October 19, major indexes of market valuation in the United States declined by 30 percent or more. Furthermore, all major world markets declined substantially in the month, which is itself an exceptional fact that contrasts with the usual modest correlations of returns across countries and the fact that stock markets around the world are amazingly diverse in their organization [1].

In local currency units, the minimum decline was in Austria (-11.4%) and the maximum was in Hong Kong (-45.8%). Out of 23 major industrial countries [2], 19 had a decline greater than 20%. Contrary to a common belief, the US was not the first to decline sharply. Non-Japanese Asian markets began a severe decline on October 19, 1987, their time, and this decline was echoed first on a number of European markets, then in North American, and finally in Japan. However, most of the same markets had experienced significant but less severe declines in the latter part of the previous week. With the exception of the US and Canada, other markets continued downward through the end of October, and some of these declines were as large as the great crash on October 19.

A lot of work has been carried out to unravel the origin(s) of the crash, notably in the properties of trading and the structure of markets; however, no clear cause has been singled out. It is noteworthy that the strong market decline during October 1987 followed what for many countries had been an unprecedented market increase during the first nine months of the year and even before. In the US market for instance, stock prices advanced 31.4% over those nine months. Some commentators have suggested that the real cause of October's decline was that over-inflated prices generated a speculative bubble during the earlier period.

However, the analysis on univariate associations and multiple regressions of these various factors which have been carried out [1] conclude that it is not clear at all what was the origin of the crash. The most precise statement, albeit somewhat self-referencing, is that the most statistically significant explanatory variable in the October crash can be ascribed to the normal response of each country's stock market to a worldwide market movement. A world market index was thus constructed [1] by equally weighting the local currency indexes of 23 major industrial countries [2] and normalized to 100 on september 30. It fell to 73.6 by October 30. The important result is that it was found to be statistically related to monthly returns in

every country during the period from the beginning of 1981 until the month before the crash, albeit with a wildly varying magnitude of the responses across countries [1]. This correlation was found to swamp the influence of the institutional market characteristics. This signals the possible existence of a subtle but nonetheless present world-wide cooperativity.

In addition, we also note the highly nonlinear (threshold like) behavior of traders, following positive and negative feedback patterns [3]. This, in addition to these above facts and the preliminary understanding of market self-organization provided by simple statistical models [3], lead us to ask whether the October 1987 crash could not be the result of a worldwide cooperative phenomenon, with signatures in analogy with critical phase transitions in physics. Here scale invariance and self-similarity are the dominant concepts, which have proven extremely useful in non-equilibrium driven systems, such as earthquakes, avalanches, crack propagation, traffic flow to mention a few.

2 Evidence for cooperative behavior and log-periodic oscillations

2.1 Precursory pattern

Fig.1 shows the evolution as a function of time of the New York stock exchange index $S\&P500$ from July 1985 to the end of 1987. The crosses represent the best fit to a constant rate hypothesis corresponding to an average return of about 30% per year. This first representation does not describe the apparent overall acceleration before the crash, occurring already more than a year in advance. This acceleration (*cusp*-like shape) is represented by the monotonic line corresponding to a fit of the data by an pure power law:

$$F_{pow}\left(t\right) = A_1 + B_1 \left(t_c - t\right)^{m_1},\tag{1}$$

where t_c denotes the time at which the powerlaw fit of the $S\&P500$ presents a diverging slope, announcing an imminent crash. Since the "noise" content of $S\&P500$ is not known, a χ^2-statistic cannot be calculated in order to qualify the fit. Instead, we have used the variance of the fit defined as $\text{var}\left(f\right) = \frac{1}{N-n}\sum_{i=1}^{N}\left(y_i - f\left(t_i\right)\right)^2$, where n is the number of free variables in f. (This assumes that the errors are normally distributed, which is a reasonable null-hypothesis.) The ratio of two variances corresponding to two different hypothesis is now the qualifying statistic. For the constant rate hypothesis to that of the power-law, we find a ratio $\text{var}_{exp}/\text{var}_{pow} \approx 1.1$ indicat-

ing a slightly better performance of the power law in capturing the acceleration, the number of free variables being the same (2).

However, already to the naked eye, the most striking feature in this acceleration is the presence of systematic deviations. Inspired by the analogy with critical phenomena, we have fitted this structure by the following mathematical expression

$$F_{lp}(\tau) = A_2 + B_2 (\tau_c - \tau)^{m_2} [1 + C \cos(\omega \log(\tau_c - \tau))], \qquad (2)$$

where $\tau = t/T$ is the time in units of T and we use natural logarithm. The time scale T comes about because the cosine is expected to have some phase Ψ defined by $\cos(\omega \log(t_c - t) - \psi)$. We can always change variable with $\Psi = \omega \log T$, which allows us to retrieve the notation used in Eq.(2). This shows that the phase Ψ is therefore nothing but a time scale. This equation is the first Fourier component of a general log-periodic correction to a pure power law for an observable exhibiting a cusp singularity at the time t_c of the crash, i.e. which becomes scale-invariant at the critical point [4]. Eq. (2) was first proposed to fit experimental measurements of acoustic emissions prior to rupture of heterogeneous composite systems stressed up to failure [5]. It has also been observed to fit the dependence of the released strain on the time to rupture for various large Californian earthquakes and the seismic activity of the Aleutian-Island seismic zone [4] as well as precursors to the recent Kobe earthquake in Japan [6]. Beside, 'complex exponents' (i.e. log-periodic corrections to power laws) have recently been found in a variety of physical systems which constitute paradigms of self-organization and complexity [7]. On a theoretical ground, they reflect the fact that the system is invariant under a *discrete* (rather than continuous) set of dilatations only. While having been ignored for a long time, it seems that complex exponents and their accompanying log-periodic patterns are actually very common in Nature.

The Log-periodic corrections to scaling imply the existence of a hierarchy of characteristic time intervals $t_c - t_n$, determined from the equation $\omega \log(t_c - t_n) + T = n\pi$, which yields $t_c - t_n = \tau_0 \lambda^n$, with $\lambda = \exp \frac{\pi}{\omega}$, $\tau_0 = \lambda^{-\frac{T}{\pi}}$. For the October 1987 crash, we find $\lambda \simeq 1.5 - 1.7$ (this value is remarkably universal and is found approximately the same for other crashes and earthquakes) and $\tau_0 \simeq 0.85 - 0.95$ years. We expect a cut-off at short time scales (i.e. above $-n \sim$ a few units) and also at large time scales due to the existence of finite size effects. These time scales $t_c - t_n$ are not universal but depend upon the specific market. What is expected to be universal are the ratios $\frac{t_c - t_{n+1}}{t_c - t_n} = \lambda$. These time scales could reflect the characteristic relaxation times

associated with the coupling between traders and the fundamentals of the economy.

The fit was performed as a minimization of the variance var_{lp}, defined above, of the data. For the three linear variables A_2, B_2, C, the minimization of the variance yields a set of three linear equations which can be solved analytically, thus determining A_2, B_2 and C as functions of the four nonlinear variables m, t_c, ω and T. After this first step and replacing the analytical formulas of the linear variables A_2, B_2 and C in the expression of the variance, we get a 4-parameter fit where the remaining unknown variables are m, t_c, ω and T. We claim this corresponds indeed to a 4-parameter fit (and not to a 7-parameter fit) since we have used an analytical constraint (here the minimization of the variance) to eliminate 3 unknown variables. This is completely similar, say, to the fit of a probability distribution presenting a priori two unknown variables, the normalizing factor C and a characteristic decay rate μ ($Ce^{-\mu x}$ for an exponential distribution), in which the condition of normalization to 1 of the probability distribution imposes $C = \mu$ leading actually to a 1-parameter fit. In addition, we checked that the results are independent of the time unit used (which controls the T variable). This was done by using either time measured in days from the first point in the fit and also performing the fit with decimal years counting from the turn of the century, giving exactly the same value for m, t_c, ω, implying that we face in fact an effective 3-parameter (m, t_c, ω) fit. Moreover, these three parameters m, t_c and ω are the most physically relevant, two of these (m and ω) being expected to exhibit some universality as discussed previously within the renormalization group framework [4, 5, 7].

Due to the "noisy" nature of the data and the fact that we are performing a minimization of the variance with respect to the four remaining non linear parameters m, t_c, ω and T, the 5-dimensional space of the var_{lp} as a function of m, t_c, ω and T has in general several local minima. Hence, a preliminary restricted search (so-called Taboo search [8]) was performed before the full 4-parameter fit was executed, ensuring that the global minimum was found. This search was done on a grid paving the two-dimensional space (t_c, ω): for each given couple (t_c, ω), we minimize the variance with respect to the two other parameters and plot the resulting variance as a function of t_c and ω. Finding the local minima of the variance on this grid, we then launch a simplex algorithm on the four non linear parameters m, t_c, ω and T. The estimation of the position of the critical time t_c is found within a few days from the actual crash time and the critical exponent m is $m_2 = 0.33$. The ratio between var_{lp} and that of the two other hypothesis is more than a factor of 3, which very clearly establishes F_{lp}

as the best performing fit among the three proposed.

We also scanned regions without crashes to ascertain the absence of significant log-periodic fluctuations there.

2.2   Aftershock patterns

If the concept of a crash as a kind of critical point has any value, we should be able to identify post-crash signatures of the underlying cooperativity. In fact, we should expect an at least qualitative symmetry between patterns before and after the crash. In other words, we should be able to document the existence of a critical exponent as well as log-periodic oscillations on relevant quantities after the crash. We have found such a signature in the variance (not to be confused with the variance of the fit) of the $S\&P500$ index, implied from the $S\&P500$ options.

The term "implied variance" has the following meaning. To understand what it means, one must first recall what is an option: this financial instrument is nothing more than an insurance that can be bought or sold on the market to insure oneself against unpleasant price variations [9]. The price of an option on the $S\&P500$ index is therefore a function of the variance (so-called volatility) of the $S\&P500$. The more volatile, the more fluctuating, the more risky is the $S\&P500$, the more expensive is the option. In other words, the price of an option on the market reflects the value of the variance of the stock as estimated by the market with its offer-and-demand rules. In practice, it is very difficult to have a good model for market price volatilities or even to measure it reliably. The standard procedure is then to see what the market forces decide for the option price and then determine the implied volatility by inversion of the Black and Scholes formula for option pricing [10, 9].

Fig.2 presents the time evolution of the implied variance of the $S\&P500$ index after the crash, taken from [11]. As expected, the variance decreases dramatically after the crash, while exhibiting characterizing log-periodic oscillations.

Note the long time scale covering a period of the order of a year involved in the relaxation of the volatility after the crash to a level comparable to before the crash. We also note that the $S\&P500$ index as well as others worldwide have remained around the immediate of the crash level for a long time. For instance, by February 29,1988, the world index stood at 72.7 (reference 100 on September 30, 1987). Thus, the price level established in the October crash seems to have been a virtually unbiased estimate of the average price level over the subsequent months. Note also that the present value of the $S\&P500$ index is much larger than it was even before

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the october 1987 crash, showing again that nothing fundamental happened then. All this is in support of the idea of a critical point, according to which the event is an intrinsic signature of a self-organization of the markets worldwide.

Our analysis with eq.(2), with $t_c - t$ replaced by $t - t_c$ gives again an estimation of the position of the critical time t_c, which is found within a few days. The critical exponent is now $m_2 = -1.2$. The ratio of var_{lp} to var_{pow} and var_{exp}, respectively, is ≈ 2, the power law again performing slightly better than an exponential relaxation hypothesis.

We have found another striking signature of the cooperative behavior of the US market by analyzing the time evolution of the $S\&P500$ index over a time window of a few weeks after the October 19 crash. A fit shown in Fig.3 with a exponentially decaying sinusoidal function suggests that the US market behaved, during a few weeks after the crash, as a *single* dissipative harmonic oscillator. We think that this signature strengthens the view of a market as a cooperative self-organizing system, presenting powerlaw distributions, large events in possible coexistence with synchronized behavior. Such properties have been indeed documented recently in models with threshold dynamics showing the generic coexistence between critical self-organization and a large 'avalanche' regime corresponding to synchronization of threshold oscillators [12]. For the October 19, 1987 crash, we find that the characteristic decay time as well as the period of the oscillations are about a week.

3   Discussion

We have found evidence of log-periodic structures in several others crashes in a variety of markets [13], paralleling previous similar observations on earthquakes [4, 5, 6]. We suggest that this reflects the fundamental cooperative nature of the behavior of stock markets. In general, cooperative behaviors in complex systems cannot be reduced to a simple decomposition on elementary causes, in agreement with the observation [1] that no single source [14] has been identified as a key factor in the October 1987 crash. One must rather look from a more global view point in which the crash can emerge "naturally" as an intrinsic signature of the functioning of the market.

To rationalize these observations, we will report elsewhere [13] on a simple model of stockmarket speculation leading to a crash based on the existence of positive feedback interactions in which traders exchange information according to a hierarchical structure. This structure is intended to model the organization of the market in the

world, where at the highest level of the hierarchy, we find the "currency and trading-blocks" (Yen, US\$, D-mark, ...), at the level immediately below we have countries, at the level below the major banks and institutions within a country, at the level below the various departments of the banks, etc. Hierarchy or, what is the same, discrete scale invariance, be it structurally built-in or dynamically generated, has been recognized as the key ingredient to obtain log-periodic behavior [4, 7]. As expected, the solution of the model indeed shows the existence of a critical point which can be identified as the crash and of well-defined precursory and aftershock log-periodic patterns. Although the model is rather ad-hoc, these results make more plausible our above observation of a qualitative symmetry in the critical behavior of the market before and after the crash. This model analyzes a situation of pure speculation, based on the tendency for traders to imitate each others. When a series of buy orders, say, are issued, an acceleration of demand results, which is self-strengthening. This acceleration cannot be sustained indefinitely and, at some threshold, a crash ends this sequence.

To sum up, the acceleration described by a power law is the signature of a critical point. The log-periodic oscillations are the signature of discrete scale invariance in the trading structure given above. There are several mechanisms that can generate this remarkable structure; for instance a built-in hierarchical structure or irreversible non-linear intermittent dynamics are know to generate these patterns [4, 7].

It is intriguing that the log-periodic structures documented here bear some similarity with the 'Elliott waves' of technical analysis [15]. Technical analysis in finance can be broadly defined as the study of financial markets, mainly using graphs of stock prices as a function of time, in the goal of predicting future trends. A lot of efforts has been developed in finance both by academic and trading institutions and more recently by physicists (using some of their statistical tools developed to deal with complex times series) to analyse past data to get informations on the future. The 'Elliott wave' technique is probably the most famous in this field. It has been introduced in the 1930's, based on observations on the human (trader) psychology on one hand and from analogies with the mathematical theory of numbers and more precisely the theory of Fibonacci numbers on the other hand. It describes the time series of a stock price as made of different "waves". These different waves are in relation with each others through the Fibonacci series $F_{n+2} = F_{n+1} + F_n$ (with $F_0 = F_1 = 1$). It is easy to show that $\frac{F_{n+1}}{F_n}$ converges to a constant (the so-called golden mean $g \simeq 1.618$), implying an approximate geometrical series of time scales

$F_{n+1} \simeq gF_n$ in the underlying waves, compatible with our above estimate for the ratio $\lambda \simeq 1.5 - 1.7$. We speculate that the 'Elliott waves', so strongly rooted in the financial analysts' folklore, could be a signature of an underlying critical structure of the stockmarket.

After completion of this work, we learned that James A. Feigenbaum and Peter G.O. Freund (cond-mat/9509033) have obtained, independently, very similar results to ours.

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Figure captions

- Fig.1 : evolution as a function of time of the New York stock exchange index $S\&P500$ from July 1985 to the end of 1987 (557 trading days). The + represent a constant return increase of $\approx 30\%$/year and had $var\,(F_{exp}) \approx 113$. The best fit to a power-law gives $A_1 \approx 327$, $B_1 \approx -79$, $t_c \approx 87.65$, $m_1 \approx 0.7$ and $var_{pow} \approx 107$. The best fit to eq.(2) gives $A_2 \approx 412$, $B_2 \approx -165$, $t_c \approx 87.74$, $C \approx 12$, $\omega \approx 7.4$, $T = 2.0$, $m_2 \approx 0.33$ and $var_{lp} \approx 36$. One can observe four well-defined oscillations fitted by eq.(2), before finite size effects limit the theoretical divergence of the acceleration, at which point the bubble ends in the crash. All the fits are carried over the whole time interval shown, up to 87.6. The fit with eq.(2) turns out to be very robust with respect to this upper bound which can be varied significantly.

- Fig.2 : Time evolution of the implied variance in log-scale of the $S\&P500$ index after the crash, taken from [11]. The + represent an exponential decrease with $var\,(F_{exp}) \approx 15$. The best fit to a power-law, represented by the monotonic line, gives $A_1 \approx 3.9$, $B_1 \approx 0.6$, $t_c = 87.75$, $m_1 \approx 1.5$ and $var_{pow} \approx 12$. The best fit to eq.(2) with $t_c - t$ replaced by $t - t_c$ gives $A_2 \approx 3.4$, $B_2 \approx 0.9$, $t_c \approx 87.77$, $C \approx 0.3$, $\omega \approx 11$, $m_2 \approx -1.2$ and $var_{lp} \approx 7$. One can observe six well-defined oscillations fitted by eq.(2).

- Fig.3 : Time evolution of the $S\&P500$ index over a time window of a few weeks after the October 19 crash. The fit with an exponentially decaying sinusoidal function suggests that a good model for the short-time response of the US market is a *single* dissipative harmonic oscillator.