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ATA Question 3 - Temperature

1. Introduction.

   A simulation was written to find the temperature gradient (T) of a sun-like object in MATLAB. The goal of this paper is to find a reasonable function that can model the sun's temperature gradient in terms of radius (i.e. a function T(r)). Several assumptions were made and will be outlined.

2. Calculations.

   To start with the simulation, a formula for density must be used. Upon further research, there were two main ways to calculate this,[1] one being:

$$\rho(r) = \rho_0(1 - \frac{r}{R}) \qquad [1]$$

where:

$\rho(r)$ = density function (kg m^{-3})
ρ_0 = density at core (1.662 E 5 kg m^{-3})
r = radius (m)
R = radius of the sun (m)

The initial density was sourced from NASA.[2]

This serves to be a fairly good approximation. The other equation is a polytrope and will not be used.

The remainder of the equations will be taken from lectures.[3] These will often contain integrals which are evaluated using MATLAB.

While analyzing boundary conditions, recall that at the core, mass and radius are both 0, total mass is fixed, and temperature and pressure at the edge of the star are both 0.

For the variable mass, there is a relationship found to be:

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r) \qquad [2]$$

where:

$m(r)$ = mass function (kg)

Multiplying both sides by dr and integrating (with boundary conditions, i.e. mass is 0 at r = 0), you get an expression for m(r). This allows us to use both [1] and [2] to find pressure (derived from Hyrdostatic Equilbrium):

$$\frac{dP}{dr} = \frac{-Gm(r)\rho(r)}{r^2} \qquad [3]$$

where:

P = pressure (N m^2)
G = Gravitational constant (m$^3 kg^{-1} s^{-2}$)

Again, we integrate this, but we recall that in this case, pressure is 0 at the edge (radius of the sun) instead of at the centre. Through this, we can finally find an expression for temperature through the Ideal Gas Law since the gas should largely obey it:

$$P(r) = \frac{k}{\mu m_H}\rho(r)T(r) \qquad [4]$$

where:

k = Boltzmann's Constant (J k^{-1})
m_H = mass of Hydrogen (kg)

However, this is not yet a temperature function of r. So, we simply rearrange:

$$T(r) = \frac{\mu m_H}{k\rho(r)} \qquad [5]$$

This is now the equation we can use to help us plot the temperature gradient.

3. Results and Analysis

   Comparing the plot from MATLAB and NASA:

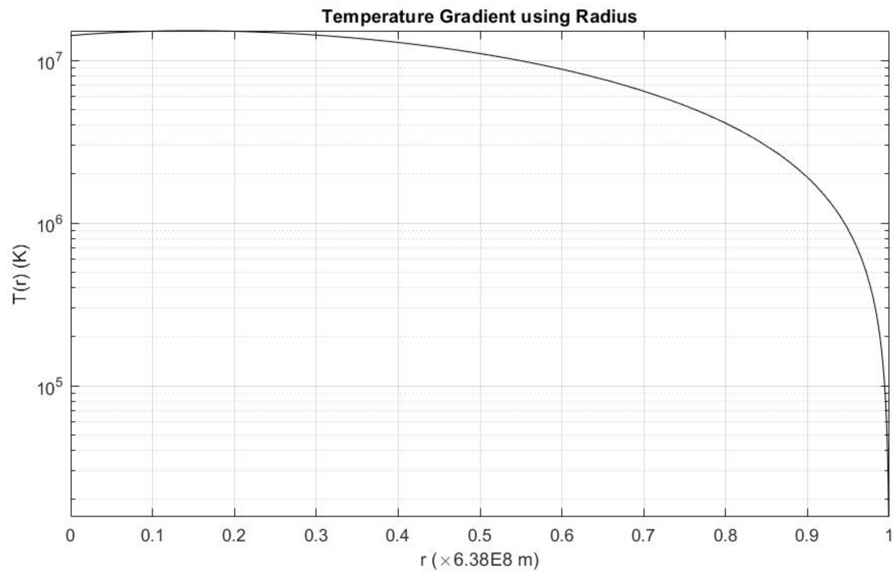
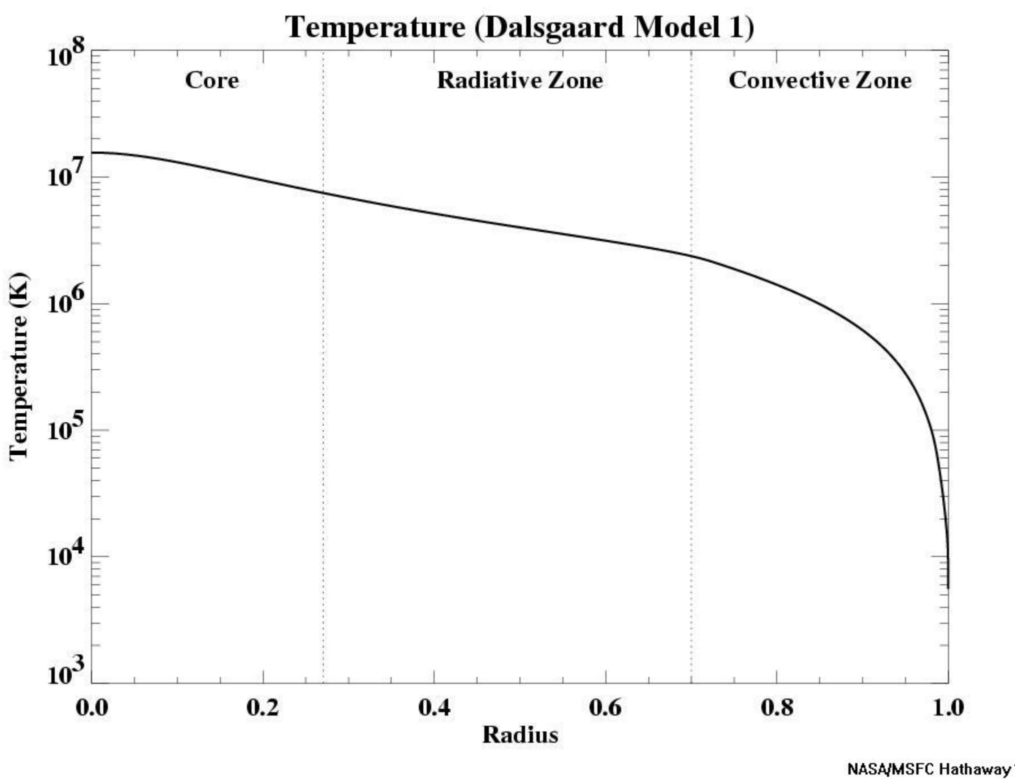


Figure 1: Simulated Temperature Gradient The temperature gradient was generated using [5]. The Y-Axis is on a logarithmic scale. The curve is (incorrectly) very smooth.



[4]

Figure 2: Accepted Gradient from NASA There is a clear discrepancy visually between this and Figure 1. In this, the curve is much less smooth because other factors and equations were considered.

There starts to be a much larger slope in Figure 1. somewhere between 0.6R and 0.8R. This may suggest that one form of energy (temperature) transport has started to surpass the other in its efficiency. Originally, the radiative tends to work much better, but at this point (0.7R), the convective transport starts to become more efficient. Yet, upon graphing the two functions separately, using their respective equations,[3] we find that this switch happens much before this point. This brings us to the conclusion that there may be an equation that may not accurately describe the behaviour.

There are a few reasons for the discrepancies between the values. As previously mentioned, a linear density function was chosen due to simplicity. However, it is likely that the density changes in a much more unpredictable way that can cause fluctuations in the shape of the temperature function.

Another reason is that throughout the calculations, the constants were not symbolically manipulated but instead were used in computations of integrals. Floating point errors may have played a role, especially in cases where the boundary condition suggests that a measurement is 0 at the surface rather than the core.

Finally, as touched up on in section 3, there is an equation that may be dramatically changing the behaviour of the two main energy transport methods. This is likely the Ideal Gas Law, since there may be problems are some densities. A better alternative could perhaps be from the Stefan-Boltzman Law.

References

[1] University of Turku. Lecture 3: Modeling Stars. `http://www.astro.utu.fi/~cflynn/Stars/l3.html`.

[2] NASA. Sun Fact Sheet. `http://www.astro.utu.fi/~cflynn/Stars/l3.html`.

[3] van. Bemmel. AP Physics C 1920 Lectures. 2019.

[4] NASA. NASA/Marshall Solar Physics. `https://solarscience.msfc.nasa.gov/interior.shtml`.